

Potential difference (p.d.) (V):-

→ The p.d. between two points in an electric field may be defined as the amount of work done in moving a unit positive test charge from one point to other point against electric force.

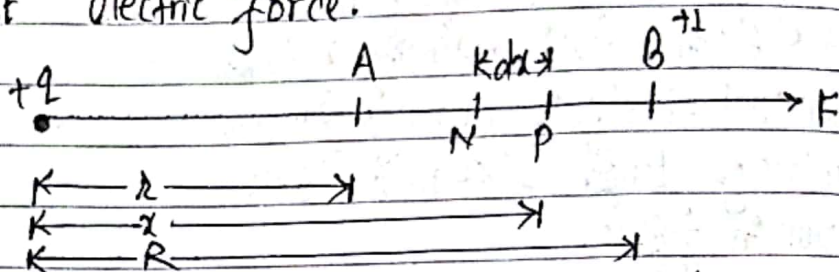


Fig:- potential difference due to point charge.

→ Let us consider a positive charge $+1$ is being taken from point B at distance 'R' from charge 'Q' in the electric field in it.

Suppose at any instant it reaches at point 'P' at distance 'x' from charge 'Q'. Then force acting on the unit charge at point 'P' is given by:

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q \cdot 1}{x^2} \quad \text{--- (i)}$$

Also, suppose the unit charge is displaced by small distance 'dx' toward charge 'Q' at point 'N'. Then small amount of work done is given by:

$$dW = -F \cdot dx$$

$$dW = -\frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} dx \quad \text{--- (ii)}$$

Negative sign shows that unit positive charge moved in opposite direction of electrostatic force.

Now, the total amount of work done in bringing the unit test charge from B to A is given by:

$$W_{BA} = \int_B^A dW$$

$$= \int_R^r -\frac{Q}{4\pi\epsilon_0 x^2} dx$$

$$= \frac{-q}{4\pi\epsilon_0} \int_R^{\infty} x^{-2} dx$$

$$= \frac{-q}{4\pi\epsilon_0} \left[\frac{x^{-2+1}}{-2+1} \right]_R^{\infty}$$

i.e.,

$$W_{BA} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_R^{\infty}$$

$$\Rightarrow W_{BA} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{Y} - \frac{1}{R} \right] \text{ --- (iii)}$$

From definition of p.d.;

$$W_{BA} = V_A - V_B$$

$$\therefore V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{Y} - \frac{1}{R} \right] \text{ --- (iv)}$$

This is required expression for p.d. between two points in electrostatic field.

An electron Volt (ev):-

→ The energy gained by an electron which has been accelerated through a p.d. of 1 volt is called 'an electron volt (ev)'.
From defⁿ of p.d.;

$$V = \frac{\text{Workdone (W)}}{\text{charge (Q)}}$$

$$\text{Or, } \boxed{W = VQ}$$

If an electron of charge $1.6 \times 10^{-19} \text{ C}$ is accelerated through a p.d. of 1 volt, then,

$$W = 1 \text{ V} \times 1.6 \times 10^{-19} \text{ C}$$

$$= 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \boxed{1 \text{ ev} = 1.6 \times 10^{-19} \text{ J}} \quad \#$$

Unit of p.d.:-

→ If W be the amount of work done in moving the test charge $+q_0$ from one point to the other point. then, p.d. between two points is given by:

$$\text{p.d.} = \frac{\text{Work done}}{\text{Charge}} \Rightarrow V = \frac{W}{q_0}$$

Thus, the S.I. unit of p.d. is J/C or Volt.

$$\text{Also, } 1V = \frac{1J}{1C}$$

Hence, the p.d. betⁿ two points is said to be 1 Volt, if 1 joule of work is to be done in bringing 1 Coulomb of charge from one point to other against the electrostatic force.

Electric potential:-

→ The electric potential at a point in an electric field is defined as the amount of work done in moving a unit positive test charge from infinity to that point against the electrostatic forces.

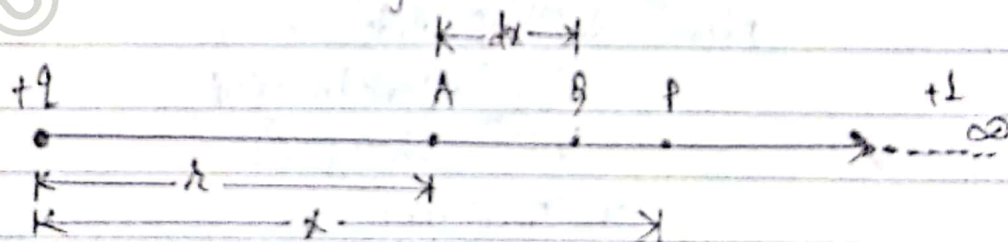


fig:- Electric potential

→ Let us consider a positive $+1$ charge is being taken from infinity towards point A at distance ' r ' from charge $+Q$ in the electric field in it.

Suppose at any instant it reaches at point 'P' at distance ' x ' from charge ' Q '. Then force acting on the unit charge at point 'P' is given by;

$$F = \frac{1}{4\pi\epsilon_0} \frac{q \cdot 1}{x^2} \text{ --- (i)}$$

Suppose the unit charge is displaced by small distance 'dx' towards point 'A' to point 'B'. Then, small amount of work done is given by:

$$dw = -F \cdot dx \\ = -\frac{q}{4\pi\epsilon_0} \cdot \frac{1}{x^2} dx \text{ --- (ii)}$$

Negative sign shows that unit positive charge moved opposite direction of electrostatic force.

Now, the total amount of workdone in bringing the unit test charge from infinity to point 'A' is given by:

$$\begin{aligned} W_{\infty A} &= \int_{\infty}^A dw \\ &= \int_{\infty}^r -\frac{q}{4\pi\epsilon_0} \frac{1}{x^2} dx \\ &= -\frac{q}{4\pi\epsilon_0} \int_{\infty}^r x^{-2} dx \\ &= -\frac{q}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{\infty}^r \\ &= -\frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right] \\ \Rightarrow W_{\infty A} &= \frac{q}{4\pi\epsilon_0 r} \text{ --- (iii)} \end{aligned}$$

From defⁿ of Electric potential;

$$W_{\infty A} = V_A$$

$$\therefore \boxed{V_A = \frac{q}{4\pi\epsilon_0 r}} \text{ --- (iv)}$$

This is required expression for electric potential.

Potential gradient:-

→ The rate of change of potential with respect to distance along the lines of force is called the electric potential gradient (dv/dx).

Relation betⁿ Electric field intensity & potential gradient.

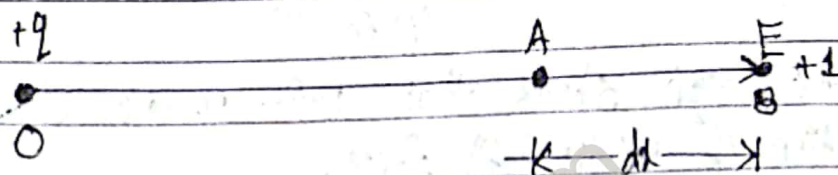


fig: potential gradient betⁿ two points in electric field.

→ Let us consider two points A & B in the electric field of charge 'q' as shown in figure. Let 'dx' be the small distance betⁿ two points A & B. Suppose points A & B are so close to each other that the electric field intensity 'E' between them is constant.

Now, work done in moving unit positive charge from B to A.

$$W = -\text{Force} \times \text{displacement}$$

$$W = -E \times dx \quad \text{--- (i)}$$

If 'dv' is the p.d. between A & B, then work done in moving a unit charge from B to A.

$$W = dv \quad \text{--- (ii)}$$

Comparing eqⁿ (i) & (ii), we get:

$$dv = -E dx$$

$$\therefore \boxed{E = -\frac{dv}{dx}} \quad \text{--- (iii)}$$

This is required relation betⁿ Electric field intensity & potential gradient and this relation shows that Electric field intensity at a point is equal to the negative of potential gradient at that point.

Equipotential Surface:-

↳ An equipotential surface in an electric field is defined as the surface over which the electric potential has the same value.

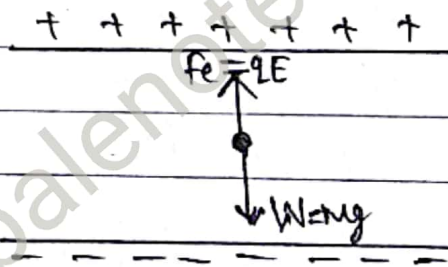
If A & B are two points on an equipotential surface, then,

$$V_A = V_B$$

$$\text{So, } W_{BA} = U_{BA} = V_A - V_B = 0$$

Thus, no work^{is} done to move a unit positive charge on the surface of an equipotential surface.

Note:-



At stationary (rest) or equilibrium condition;

$$F_e = W$$

$$qE = mg$$

$$\text{Also, } E = \frac{V}{d}$$

Where, V = p.d. betⁿ two plates
 d = separation betⁿ two plates.