IDEAL GAS

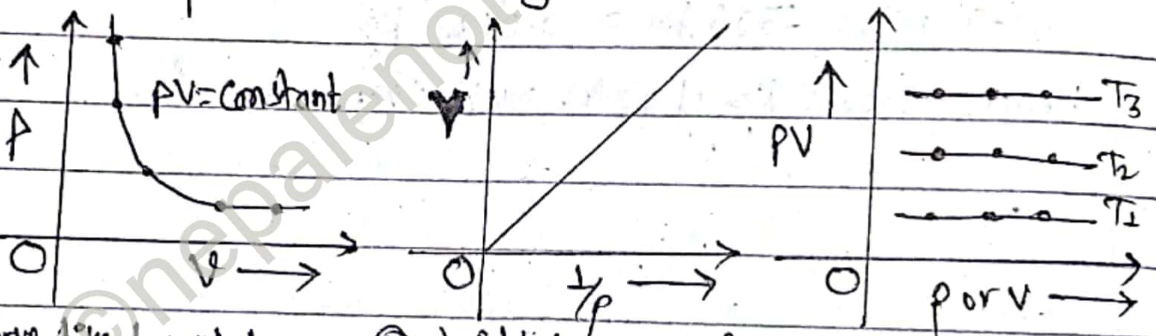
Gas law's:-

[1] Boyle's law:-

↳ It states, "On keeping temperature constant, the volume of given mass of gases is inversely proportional to the pressure."

i.e.,
$$V \propto \frac{1}{P}$$
$$\Rightarrow PV = \text{Constant}$$
$$\Rightarrow \boxed{P_1V_1 = P_2V_2}$$ Which is relation for Boyle's law.

★ Graphical representation of Boyle's law:-



(a) Curve like hyperbola
(b) straight line passing through origin ($y = mx + c$)
(c) straight line parallel to x-axis.

[2] Charle's law:-

↳ It is of two types:-

(i) Charle's law at constant pressure:-

↳ It states, "On keeping pressure constant, the volume of given mass of gases is directly proportional to the temperature.

i.e., $V \propto T$

$$\Rightarrow \frac{V}{T} = \text{constant}$$

$$\Rightarrow \boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$ Which is relation for charle's law at constant pressure.

(ii) Charle's law at Constant Volume:-

↳ It states, "On keeping constant volume, the pressure is directly proportional to the temperature.

i.e., $P \propto T$

$$\Rightarrow \frac{P}{T} = \text{constant}$$

$$\Rightarrow \boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}$$ Which is relation for charle's law at constant Volume.

Absolute zero temperature:-

↳ The hypothetical temperature at which volume of a gas becomes zero is called absolute zero temperature. It is equal to -273°c or 0K.

★ Variation of Charle's law:-

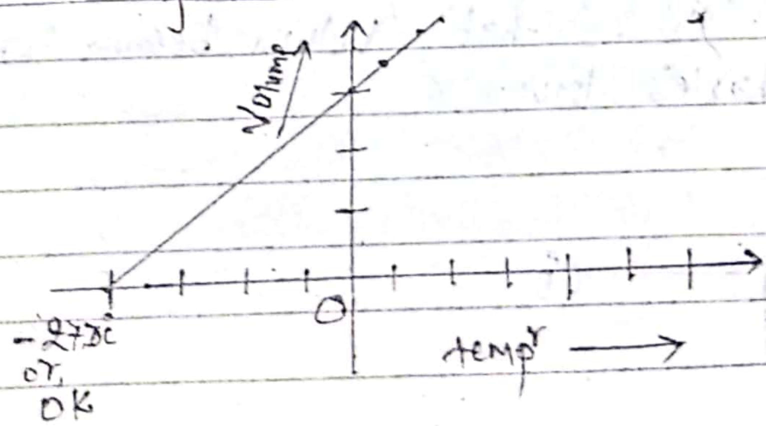


fig: (V) vs (T)

Equation of State for an Ideal gas:-

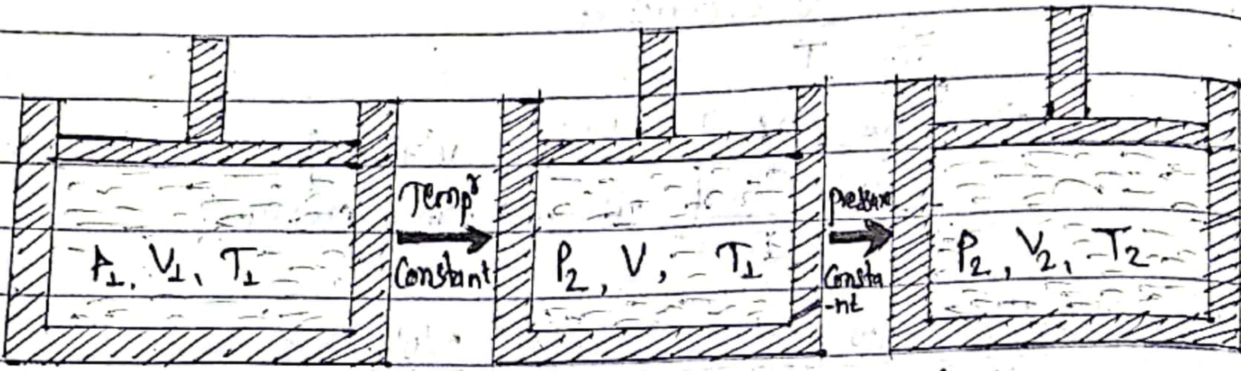


fig: Changing pressure, volume and temperature of gas:-

↳ Let us Consider, One mole of gas is kept in a cylinder provided with a frictionless piston. Let, P_1, V_1 & T_1 and P_2, V_2, T_2 be the initial and final pressure, volume and temp^r respectively. To reach final stage two step are consider as shown in figure above.

In first step, temperature 'T_1' is kept constant & pressure is increase to 'P_2' so that the volume is decreased and become V. then from Boyle's law:-

$$P_1V_1 = P_2V$$

$$\Rightarrow \boxed{V = \frac{P_1V_1}{P_2}} \text{ ——— (i)}$$

In Second step, pressure 'P_2' is kept constant and temperature is increase to 'T_2'. So that volume ~~becom~~ increase to 'V_2'. The from Charle's law;

$$\frac{V}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \boxed{V = \frac{V_2T_1}{T_2}} \text{ ——— (ii)}$$

From eq^n (i) & (ii)

$$\frac{P_1V_1}{P_2} = \frac{V_2T_1}{T_2}$$

or, $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{PV}{T} = \text{constant}$$

$$\Rightarrow \frac{PV}{T} = R$$

Where, R is universal gas constant & it's value is 8.31 $J mol^{-1} K^{-1}$ or, 0.0821 ltr. atm. $mol^{-1} K^{-1}$.

$$\Rightarrow PV = RT \quad \text{—— (iii)}$$

This (eq^n (iii)) is ideal gas eq^n for 1 mole of gas.

For (n) mole, $\boxed{PV = nRT}$

We have, $PV = nRT$

$$\Rightarrow PV = \frac{m}{M} RT \quad [\because m = n.M]$$

$$\Rightarrow PV = m.\frac{R}{M}.T$$

$$\Rightarrow PV = mrT \quad [\text{where } r = R/M, \text{ gas Constant per unit mass}]$$

For unit mass

$$\Rightarrow \boxed{PV = rT} \ \#$$

1SQ.1[A] What are difference between ideal gas and real gas?

→

Ideal gas (perfect gas)	Real gas
* The hypothetical gases that do not exist practically in nature are called ideal gases.	* The gases that exist practically in nature are called real gases.
* Ideal gas obey the Boyle's law, Charle's law and Combined gas eq^n.	* Real gas does not obey Boyle's law, Charle's law & Combined gas eq^n.
* They follows gas law at all temperature and pressure Conditions.	* They follows gas law at low temperature and high pressure.
* Inter atomic force in ideal gas is equal to zero.	* Inter atomic force in real gases is non zero and Significant.

SQ.2[A] Find the dimensional formula of Universal gas Constant 'R'?

Soln:- We have, $PV = nRT$

$$R = \frac{PV}{nT} = \frac{\frac{F}{A}\cdot A\cdot h}{n\cdot T} = \frac{Fh}{nT}$$

Now,

Dimensional formula of R,

$[R] = [MLT^{-2}][L][K^{-1}]$ [$\because$ n is dimensionless]

$\therefore$ Dim. formula of R $= [ML^2T^{-2}K^{-1}]$

KINETIC MOLECULAR THEORY OF GASES:-

The main postulates (or assumptions) of this theory are as:-

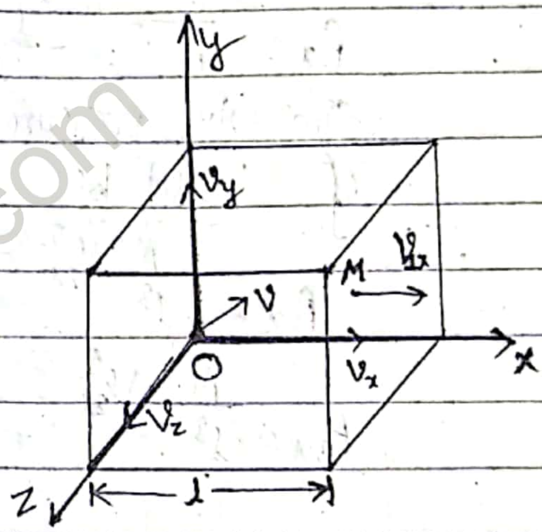
1) Every gas consist of a large no. of small particle called molecules.
2) The gaseous molecules are so small that the volume occupied by a single molecule can be neglected as compared to the total volume of gas.
3) The gaseous molecules are in motion. They collide with each other and also with walls of container.
4) The molecular collision is perfectly elastic i.e. there is no loss of K.E.
5) The pressure exerted by a gas is due to continuous bombardment of gas molecules on the wall of vessel.
6) There is no force of attraction between gas molecules.
7) The average kinetic energy of gas molecules is directly proportional to the absolute temperature. i.e. $K.E. \propto T$
8) There is no effect of gravity on gas molecules.

Root mean Square (rms) speed :-

↳ The root of the mean of the square of the speed of gas molecules is called root mean square speed. It is denoted by $\bar{c}$ and given by:

$$\bar{c} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}}$$

Pressure exerted by gas:-



↳ Let us consider a gas is kept inside a cubical vessel of side 'l' at temperature 'T'. Also, let 'M' be the mass of a gas molecule and 'N' be the total number of gas molecules. So, total mass of gas, $(m = n.M)$. Also let, v_x, v_y and v_z are the component of velocity v along x, y & z-axis respectively. Then resultant velocity is given by;

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad \text{——— (i)}$$

Again, consider a gas molecule moving along x-axis with the velocity v_{1x} and after striking the surface of vessel and rebounced with the same velocity. Then, change in momentum of that molecules be,

$$\Delta P_{1x} = M v_{1x} - [M(-v_{1x})]$$

$$= 2M v_{1x}$$

Let, 't' be the time taken by gas molecules to travell from origin (O) to right face and back to origin. Then,

$$t = \frac{2l}{v_{1x}}$$

Therefore force exerted by that gas molecule on the surface of Cubical vessel,

$$F_{1x} = \frac{\Delta P_{1x}}{t} = \frac{2MV_{1x}}{\frac{2l}{V_{1x}}} = \frac{MV_{1x}^2}{l}$$

Now pressure exerted by that molecules on the wall of vessel,

$$P_{1x} = \frac{F_{1x}}{A} = \frac{\frac{M}{l}V_{1x}^2}{l^2} = \frac{M}{l^3}V_{1x}^2$$

Similarly, pressure exerted by other molecules on the same surface of vessel be,

$$P_{2x} = \frac{M}{l^3}V_{2x}^2$$

$$P_{3x} = \frac{M}{l^3}V_{3x}^2$$

$$\vdots \qquad \vdots$$

$$P_{Nx} = \frac{M}{l^3}V_{Nx}^2$$

Thus, total pressure exerted by gas molecules along x-axis be,

$$P_x = P_{1x} + P_{2x} + \dots + P_{Nx}$$

$$= \frac{M}{l^3}\left[V_{1x}^2 + V_{2x}^2 + \dots + V_{Nx}^2\right] \longrightarrow (ii)$$

Similarly, pressure exerted by gas molecules along y & z-axis.

$$P_y = \frac{M}{l^3}\left[V_{1y}^2 + V_{2y}^2 + \dots + V_{Ny}^2\right] \longrightarrow (iii) \quad \& \quad P_z = \frac{M}{l^3}\left[V_{1z}^2 + V_{2z}^2 + \dots + V_{Nz}^2\right] \longrightarrow (iv)$$

Now,

pressure exerted by gas molecules in a whole vessel is the average of P_x, P_y & P_z.

i.e, $$P = \frac{P_x + P_y + P_z}{3}$$

$$\Rightarrow P = \frac{M}{3l^3}\left[(V_{1x}^2 + V_{2x}^2 + \dots + V_{Nx}^2) + (V_{1y}^2 + V_{2y}^2 + \dots + V_{Ny}^2) + (V_{1z}^2 + V_{2z}^2 + \dots + V_{Nz}^2)\right]$$

$$\Rightarrow P = \frac{M}{3l^3}\left[(V_{1x}^2 + V_{1y}^2 + V_{1z}^2) + (V_{2x}^2 + V_{2y}^2 + V_{2z}^2) + \dots + (V_{Nx}^2 + V_{Ny}^2 + V_{Nz}^2)\right]$$

$$\Rightarrow P = \frac{M}{3l^3}\left[V_1^2 + V_2^2 + \dots + V_N^2\right] \longrightarrow (v)$$

Now, from defination of root mean square speed;

$$\bar{c} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_N^2}{N}}$$

$$\Rightarrow \bar{c}^2 . N = v_1^2 + v_2^2 + \cdots + v_N^2 \quad \text{—— (VI)}$$

Using (VI) in eqn (V)

$$\Rightarrow P = \frac{M}{3l^3} \bar{c}^2 . N = \frac{NM}{3l^3} \bar{c}^2 = \frac{m}{3V} \bar{c}^2 \quad [\because NM = m \ \& \ V = l^3]$$

$$\text{or, } P = \frac{1}{3} . \frac{m}{V} . \bar{c}^2 = \frac{1}{3} \rho . \bar{c}^2 \quad [\because \rho = \frac{m}{V}]$$

$$\Rightarrow \boxed{P = \frac{1}{3} \rho \bar{c}^2}$$ Which is required relation for pressure exerted by gas on the wall of vessel.

K.E. of gas:-

★ Since, pressure exerted by gas;

$$P = \frac{1}{3} \rho \bar{c}^2$$

$$\text{or, } P = \frac{1}{3} \frac{m}{V} \bar{c}^2$$

$$\text{or, } 3PV = m\bar{c}^2$$

$$\text{or, } \frac{1}{2} m\bar{c}^2 = \frac{3}{2} PV$$

$$\therefore \boxed{\text{KE of gas} = \frac{3}{2} PV}$$

Also, $P = \frac{1}{3} \frac{N \cdot M}{V} . \bar{c}^2$

$$\text{or, } \frac{3PV}{N} = M\bar{c}^2$$

$$\text{or, } \frac{1}{3} M\bar{c}^2 = \frac{3}{2} \frac{PV}{N}$$

$$\therefore \boxed{\text{KE of a gas molecule} = \frac{3PV}{2N}}$$

From Ideal gas equation,

$$PV = nRT$$

So, KE of a gas molecule $= \frac{3}{2} \frac{nRT}{N}$

Again,

$$N = n . N_A$$

$$\therefore \text{K.E. of a gas molecule} = \frac{3}{2} \frac{nRT}{n . N_A}$$

$$\text{or, } \frac{3}{2} \frac{R}{N_A} T = \frac{3}{2} K.T$$

Where,

$K = R/N_A$, Called Boltzmann Constant.

Ideal gas equation from Kinetic Molecular Theory (K.M.T):-

We have, pressure exerted by gas molecule,

$$p = \frac{1}{3}\rho\bar{c}^2$$

$$\text{or, } p = \frac{1}{3}\frac{m}{V}\bar{c}^2$$

$$\Rightarrow \boxed{pV = \frac{1}{3}m\bar{c}^2} \quad \text{--- (1)}$$

If 'M' be the mass of molecules and 'N' be the no. of molecules in gas, then, $m = NM$

$$\Rightarrow pV = \frac{1}{3}NM\bar{c}^2 \quad \text{--- (2)}$$

Also,

$$\frac{1}{2}M\bar{c}^2 = \frac{3}{2}KT$$

$$\Rightarrow M\bar{c}^2 = 3KT \quad \text{--- (3)}$$

Using eq^n (3) in eq^n (2)

$$\text{or, } pV = \frac{1}{3}N.3KT$$

$$\Rightarrow \boxed{pV = NKT}$$

Also, $N = n.N_A$

$$\Rightarrow pV = n.N_A\,KT$$

$$\Rightarrow pV = n.\cancel{N_A}.\frac{R}{\cancel{N_A}}T$$

$$\Rightarrow \boxed{pV = nRT}$$

This is ideal gas equation.

Gas law's from K.M.T. of gas:-

[1] Boyle's law:-

↳ We have,

$$P = \frac{1}{3}\rho \bar{c}^2$$

or, $$P = \frac{1}{3}\frac{m}{V}\bar{c}^2$$

$$\Rightarrow \boxed{PV = \frac{1}{3}m\bar{c}^2} \quad \text{--- (1)}$$

Since, $KE\left(\frac{1}{2}m\bar{c}^2\right) \propto T$

& at constant temperature,

$$\frac{1}{2}m\bar{c}^2 = \text{constant}$$

$$\Rightarrow PV = \text{constant}$$

$$\Rightarrow P = \text{Constant} \cdot \frac{1}{V}$$

$$\Rightarrow \boxed{P \propto \frac{1}{V}}$$

Which proves Boyle's law.

[2] Charle's law:-

↳ We have,

$$P = \frac{1}{3}\rho \bar{c}^2$$

or, $$P = \frac{1}{3}\frac{m}{V}\bar{c}^2$$

$$\Rightarrow \boxed{PV = \frac{1}{3}m\bar{c}^2} \quad \text{--- (1)}$$

Since, $K.E.\left(\frac{1}{2}m\bar{c}^2\right) \propto T$

$$\Rightarrow \boxed{PV \propto T}$$

If Volume is Constant, then,

$$\Rightarrow \boxed{P \propto T}$$

And If pressure is Constant, then,

$$\Rightarrow \boxed{V \propto T}$$

Which proves charle's law.