2/12/16

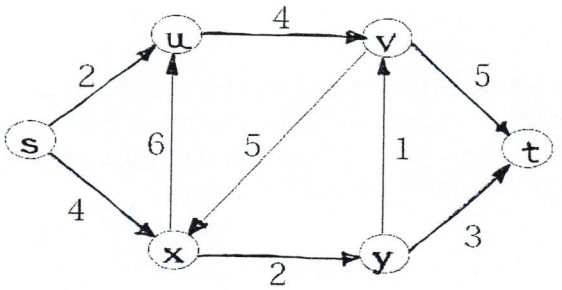
Indian Institute of Engineering Science and Technology, Shibpur
B.E. (CST) 7th Semester Examination 2016
Graph Algorithms (CS 704/5)

Time: 3 hours **Full marks: 70**

Attempt Question 1 and any four from the rest
All parts of the same question are to be answered together

Question 1 [15 + 15]

(a) State the Shortest Path Augmentation (SPA) algorithm to find maximum flow in a directed graph. Consider the following directed graph, where the edge-weights denote the capacities of the edges. Use the SPA algorithm to find a maximum feasible flow from s to t in this graph. Show all steps in solving the problem.



(b) State the Gale Shapley algorithm for stable matching. Use the Gale Shapley algorithm to solve the following instance of the stable matching problem. Show all steps in the solution.
There are four boys (B1, B2, B3, B4) and four girls (G1, G2, G3, G4). Their preferences are as follows, with the first entry in each list specifying the most preferred match:

B1:	G4, G1, G2, G3	G1:	B4, B1, B3, B2
B2:	G2, G3, G1, G4	G2:	B1, B3, B2, B4
B3:	G2, G4, G3, G1	G3:	B1, B2, B3, B4
B4:	G3, G1, G4, G2	G4:	B4, B1, B3, B2

Question 2 [4 + 6]

(a) Draw the non-planar graphs with (i) minimum number of nodes, (ii) minimum number of edges.
(b) Consider the statement: If G is a simple, connected planar graph, G must contain at least one node of degree 5 or less. Is the statement true? If yes, prove the statement; if no, give a counter-example.

Question 3 [4 + 6]

(a) With respect to matching in graphs, define the terms (i) maximal matching, (ii) maximum matching, (iii) augmenting path.
(b) Consider the following statement: If a given matching M is maximal, M is guaranteed to be a maximum matching. Is the statement true? If yes, prove the statement; if no, give a counter-example. Also, state whether the converse of the statement is true, with proper justification.

Question 4 [4 + 6]

(a) Describe an iterative algorithm for computing PageRank of nodes in a directed graph.
(b) Demonstrate the working of the algorithm on the graph given as part of Question 1(a), up to three iterations, thus computing approximate PageRank scores for all nodes in the graph.

Question 5 [4 + 6]

(a) Define a flow over a graph $G = (V, E)$, where V is the set of vertices, and E is the set of edges.
(b) State and prove the Max Flow Min Cut Theorem.

Question 6 [10]

Write a short note on community detection in graphs, outlining any two community detection algorithms, one for detecting disjoint communities, and one for detecting overlapping communities.

Question 7 [10]

Define the terms Independent Set, and Maximal Independent Set (MIS) in a graph. Assume that you are given an algorithm to find MIS in a graph, *findMIS*, which has time complexity $f(n)$ where n is the number of nodes in the graph. How can you use *findMIS* to obtain a proper coloring of a given graph? What will be the time complexity for finding a proper coloring?