Indian Institute of Engineering Science and Technology, Shibpur
B.Tech. - M.Tech. Dual Degree 5^{th} Semester (CST) Examination (Mid Semester) 2023
Graph Algorithms (CS 3104)

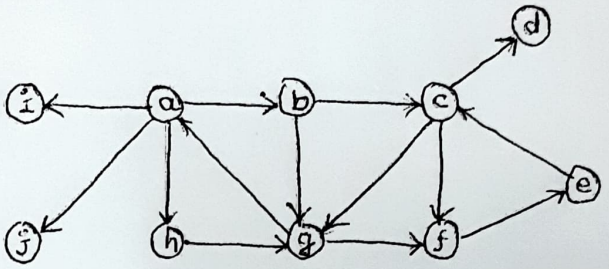
Full Marks: 30 **Time: 2 Hours**

Answer Question-1 and any three from the remaining.
Do all parts of a question together. Do not mix up answers to parts of different questions in the answer script.

1. (a) Let $G = (V, E)$ be a graph where $\forall v \in V$, $deg(v) \geq 2$. Show that G contains a cycle.
   (b) Prove that the number of vertices of odd degree in a graph is always even.

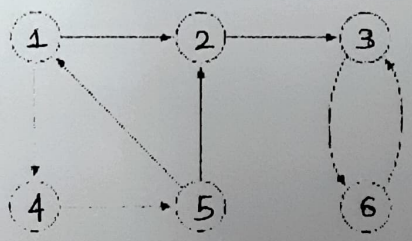
[3 + 3 = 6]

2. Work out a possible DFS tree rooted at vertex, a in the digraph shown in Fig. by showing the discovery and finish times for each vertex. Show also the tree edges, front edges, back edges, and cross edges in the graph.



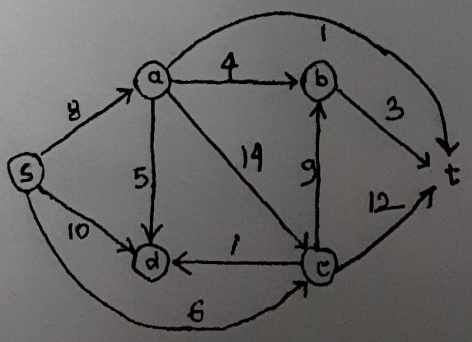
[8]

3. Write an algorithm to find the strong components of a Graph. Consider a digraph $G = (V, E)$ below with the sets $V = \{1, 2, 3, 4, 5, 6\}$ of nodes and $E = \{(1, 2), (1, 4), (2, 3), (3, 6), (4, 5), (5, 1), (5, 2), (6, 3)\}$ of edges and find its strong components (i.e., strongly connected components) by using this algorithm



[3 + 5 = 8]

4. (a) Consider the following directed, weighted graph. The weights on the edges show the capacity of the edges. Compute the maximum flow from the source s to the sink t using the Ford-Fulkerson algorithm. At each step, clearly show the flow.



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[8]

5. (a) There are five cities in a network. The travel time for travelling directly from i to j is the $(i,j)^{th}$ entry in the matrix below. The matrix is not symmetric and an infinity entry indicates that there is no direct route. Determine the least travel time and quickest route from i^{th} city to j^{th} city for each pair (i,j).

$$\begin{bmatrix} 0 & 10 & 20 & \infty & 17 \\ 8 & 0 & 9 & 22 & 33 \\ 14 & 13 & 0 & 15 & 27 \\ 30 & \infty & 17 & 0 & 10 \\ \infty & 15 & 12 & 8 & 0 \end{bmatrix}$$

[8]