Graph Algorithms

CS3104

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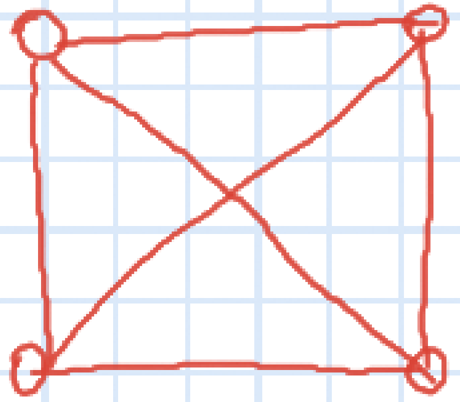
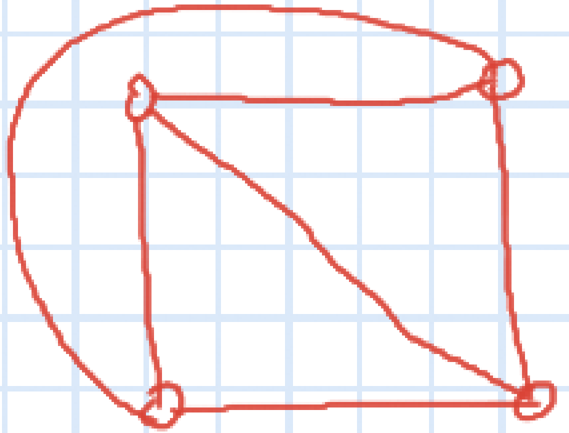
Planar Graph

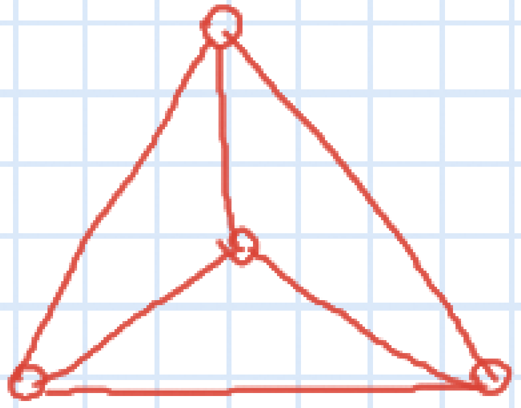
It is a graph which can be drawn in the plane such that no two edges cross except at vertex.

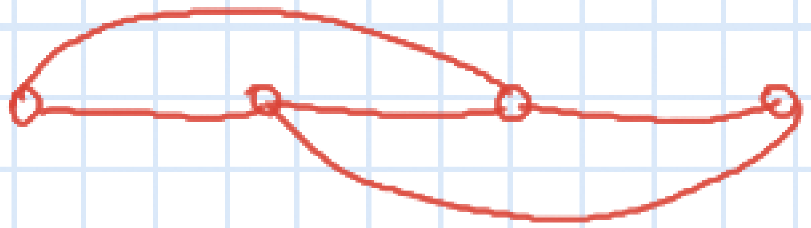


K_4

$\Rightarrow$

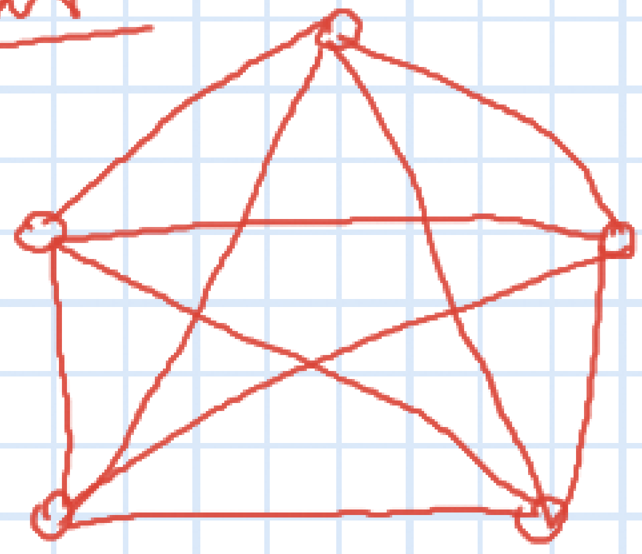




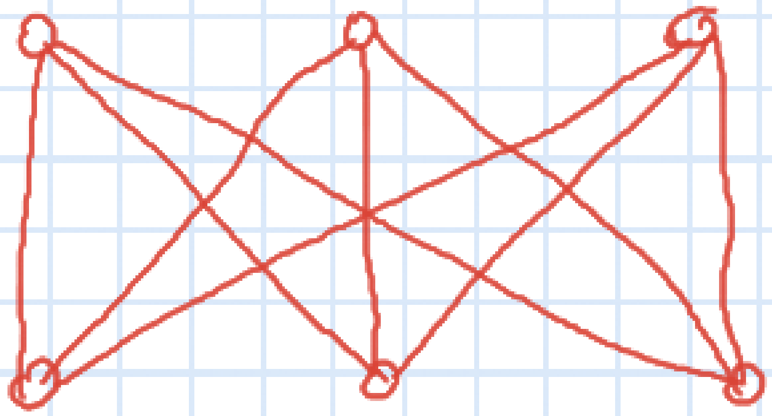


$\boxed{K_1, K_2, K_3, K_4} \Rightarrow$ Planar graph.

non-Planar.

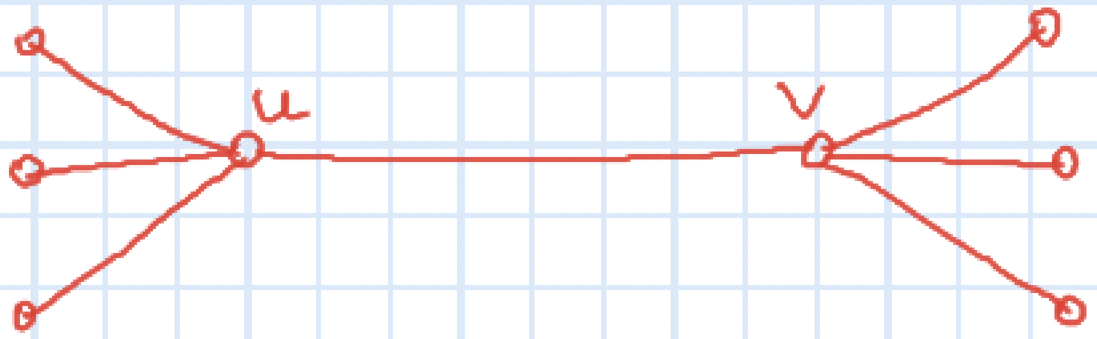


K_5



$K_{3,3}$

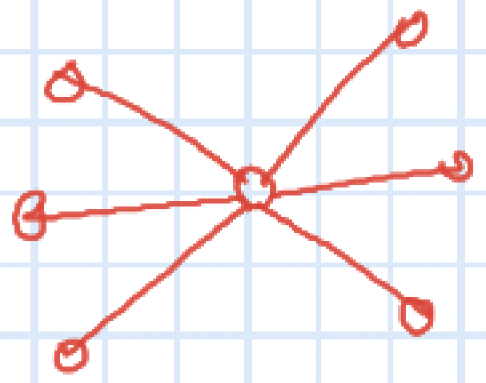
Operations.



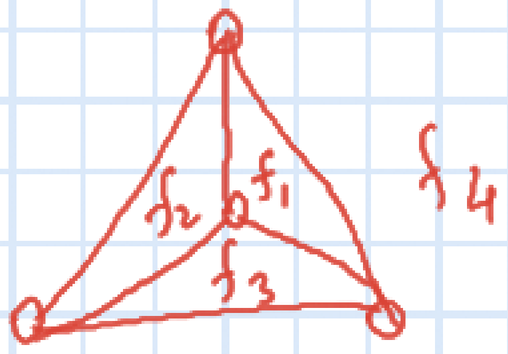
After deletion of (u,v) →

Contraction on an edge (u,v)

remove the edge (u,v) and merge the two vertex (u & v).



Let G be a planar graph and consider the regions bounded by the edges of G. These are known as faces.



K_4

Total number of faces, $f = 4$

Theorem (Euler, 1758)

If a connected planar graph G, has exactly v vertices, e edges and f faces then

$$\boxed{v - e + f = 2}$$

$v = 4$
$e = 6$
$f = 4$

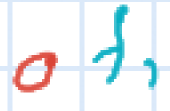
$v - e + f$
$= 4 - 6 + 4$
$= 2$

This is true for every connected planar graph.

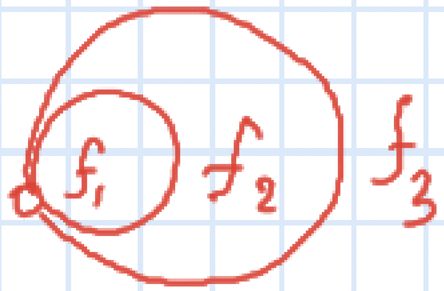
Proof $\qquad\qquad v - e + f = 2$

Induction on the number of vertices

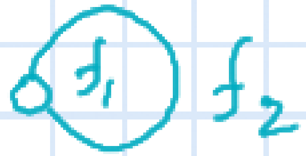
All planar graph with only one vertex, $v = 1$



if $e = 0$ then number of faces $f = 1$
then the formula holds



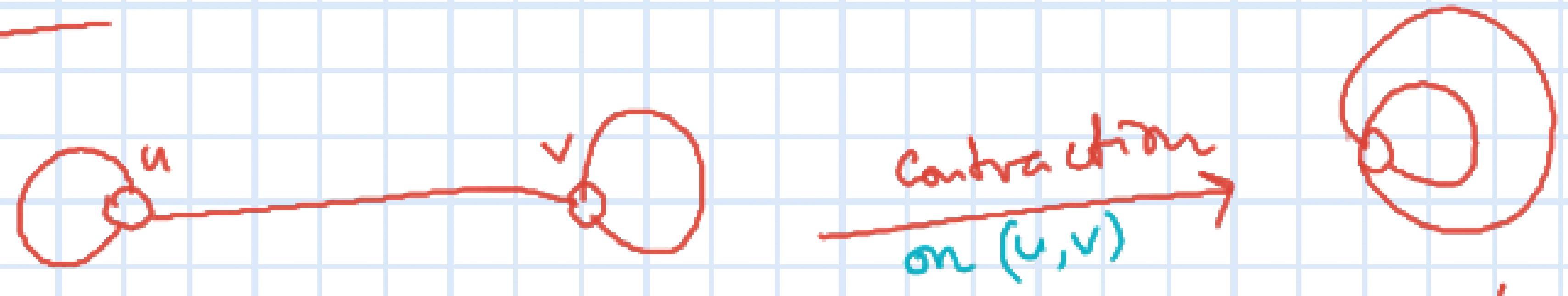
if $e = 1$ then number of faces $f = 2$
then the formula holds



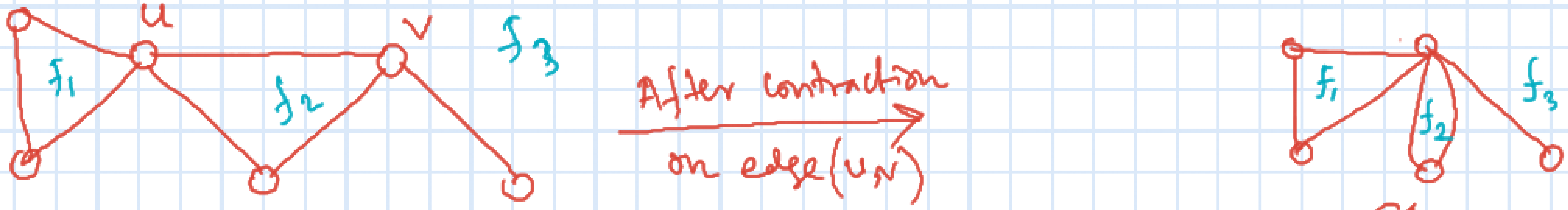
if $e = 2$ then number of faces $f = 3$
then the formula holds.

For $v > 1$

$v - e + f = 2$



G $\qquad$ G'



G $\qquad$ G'

$f = 3$
$v = 6$
$e = 7$

$f' = 3$
$v' = 5$
$e' = 6$

the contraction operation does not change the number of faces, but it reduces the number of vertices and edges by one.

$v' = v - 1$, $e' = e - 1$, $f' = f$

$v' - e' + f' = (v-1) - (e-1) + f$
$= v - e + f = 2$