

Indian Institute of Engineering Science and Technology, Shibpur  
B.Tech. - M.Tech. Dual Degree 5<sup>th</sup> Semester (CST) Examination 2022  
Graph Algorithms (CS 3104)

Full Marks: 50

Time: 3 Hours

*Answer Question-1 and any four from the remaining.*

*Do all parts of a question together. Do not mix up answers to parts of different questions in the answer script.*

1. (a) Prove that every component of the symmetric difference of two matchings of a graph is a path or even cycle.  
(b) Let  $G = ({V_1, V_2}, E)$  be a bipartite graph with vertex partitions  $V_1$  and  $V_2$ . Show that

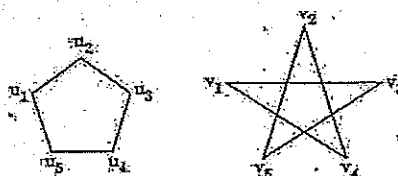
$$\sum_{u \in V_1} \deg(u) = \sum_{v \in V_2} \deg(v)$$

[3 + 3 = 6]

2. (a) Define vertex colouring of a graph  $G$  and show that every planar graph is 5-vertex colorable.  
(b) Given a directed graph  $G = (V, E)$ . Describe a fast algorithm to compute the connected components in a graph  $G$ .

[6 + 5 = 11]

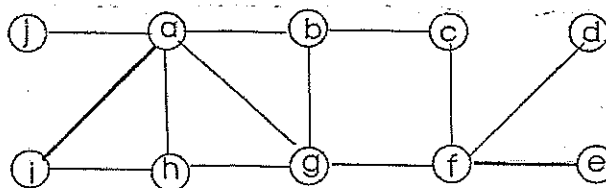
3. (a) Define isomorphism. Determine whether the following pair of graphs are isomorphic. If yes, justify your answer.



- (b) State and prove Handshaking Theorem.

[6 + 5 = 11]

4. (a) Prove that a graph is planar if and only if it contains no subdivision of  $K_5$  or  $K_{3,3}$ .  
(b) Describe an algorithm that finds, as efficiently as possible, a matching of maximum cardinality in any graph. Consider the graph shown below with initial matching shown in bold, find augmenting paths iteratively to obtain a maximum matching for this graph.

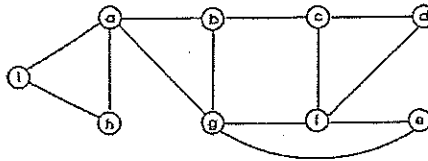


[5 + 6 = 11]

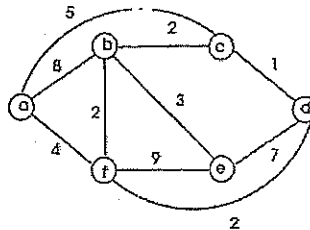
5. (a) Prove that in a connected planar graph with  $n$  vertices and  $e$  edges has  $e - n + 2$  regions.  
(b) Define Chromatic Number of a graph. Prove that a graph with at least one edge is two chromatic if and only if it has no circuits of odd length.

[7 + 4 = 11]

6. (a) Find the clustering coefficient for each vertex in the graph shown below and also work out the average clustering coefficient.



- (b) Compute the vertex betweenness values for each vertex for the graph as shown below. Find two influential nodes in this graph using these measures.



$$[5 + 6 = 11]$$