Graph Algorithms

CS3104

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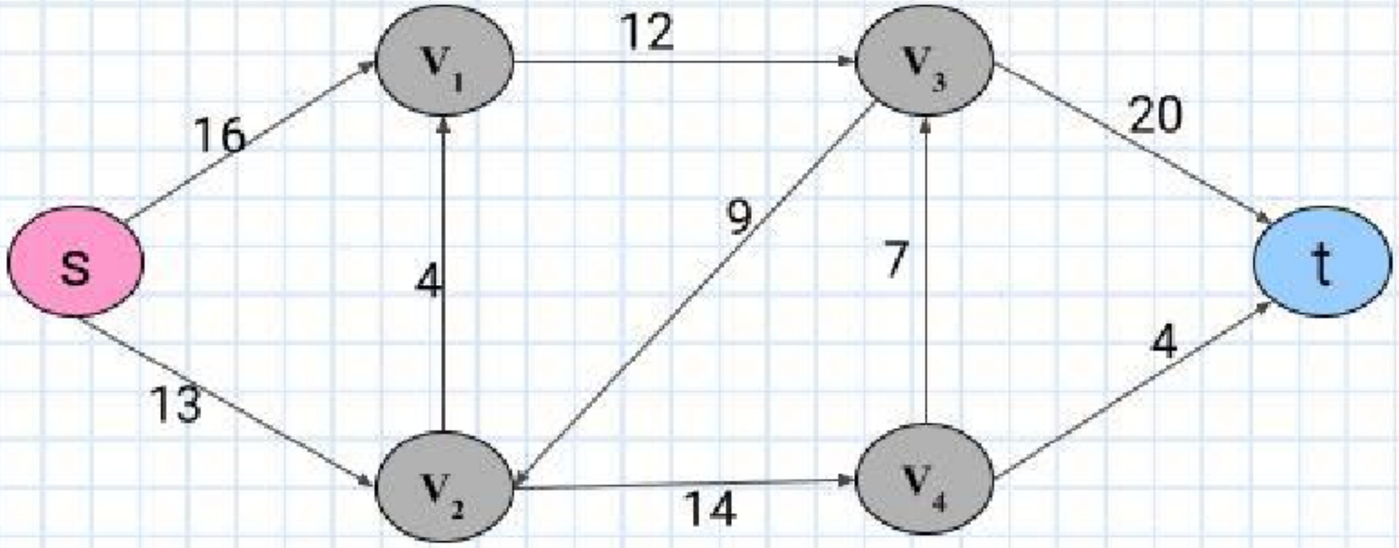
Flow Network

- **What is Flow networks?**
  - A flow network $G=(V, E)$: a directed graph, where each edge $(u,v) \in E$ has a nonnegative capacity $c(u,v)>=0$.
  - If $(u,v) \notin E$, we assume that $c(u,v)=0$.
  - two distinct vertices : **a source, *s* and a sink, *t*.**



$G=(V,E)$, Source s, Sink t, flow in G it will be a real valued function $f: V \times V \to R$

Properties

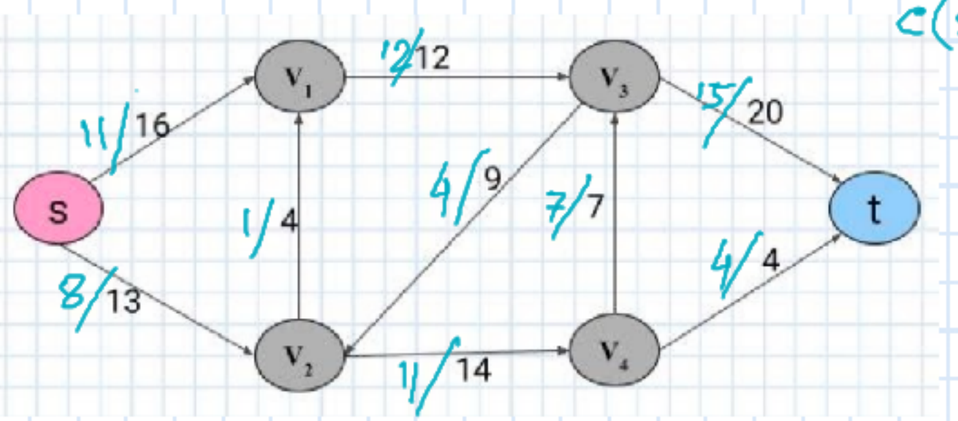
(a) Capacity constraints :-

for all $(u,v) \in V$ we require $0 \le f(u,v) \le c(u,v)$

(b) Skew Symmetry : $f(u,v) = -f(v,u)$

(c) Flow Conservation : for all edge $(u,v) \in V$ we require for every vertex, $v \notin \{s,t\}$

$$\underbrace{\sum_{v \in V} f(v,u)}_{\text{flow into } u} = \underbrace{\sum_{v \in V} f(u,v)}_{\text{flow out of } u}$$



f/c

$f(v_2,v_4) = 11$

$f(v_4,v_2) = -11$

Net flow

The quantity $f(u,v)$ is called the net flow from vertex u to vertex v.

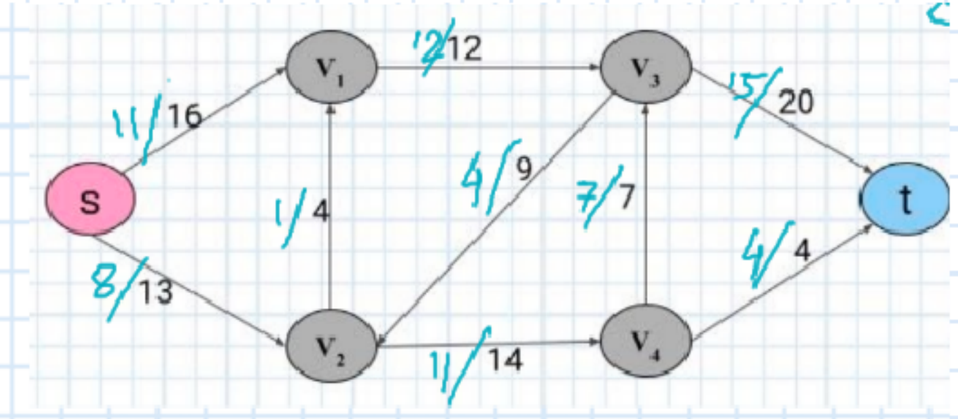
The value of a flow is defined as:-

$$|f| = \sum_{v \in V} f(s,v)$$

The total flow from source to any other vertices is same as the total flow from any vertex to the sink.

$$|f| = \sum_{v \in V} f(s,v) - \sum_{v \in V} f(v,s) = \sum_{v \in V} f(v,t) - \sum_{v \in V} f(t,v)$$

$|f| = 19$



Maximum flow Problem

Given a flow network, G, with s and t

— Find a flow of maximum value from source s to sink t.

— Ford-Fulkerson Algorithm
— The Edmonds-Karp Algorithm

Residual Network, Augmenting Path & Cut

Residual Network

Given a flow network G, flow, f

Residual Network, G_f

Consists edges that can admit more Net flow

if $f(u,v) < c(u,v)$

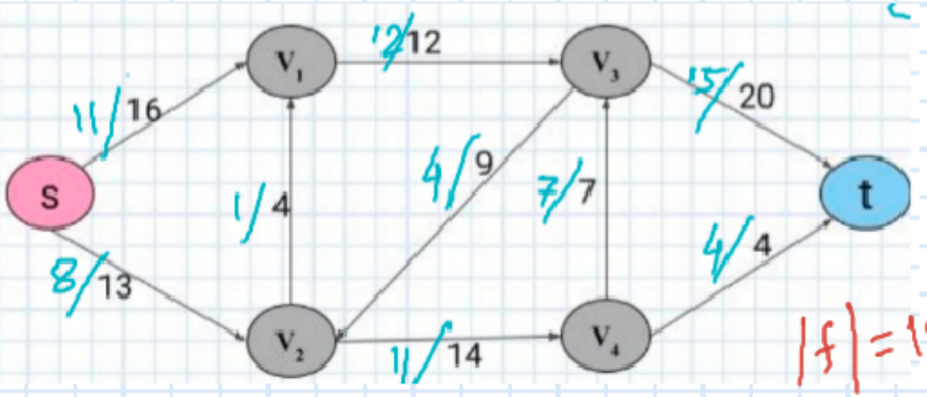
Include a forward edge (u,v) with residual capacity

$$c_f(u,v) = c(u,v) - f(u,v)$$

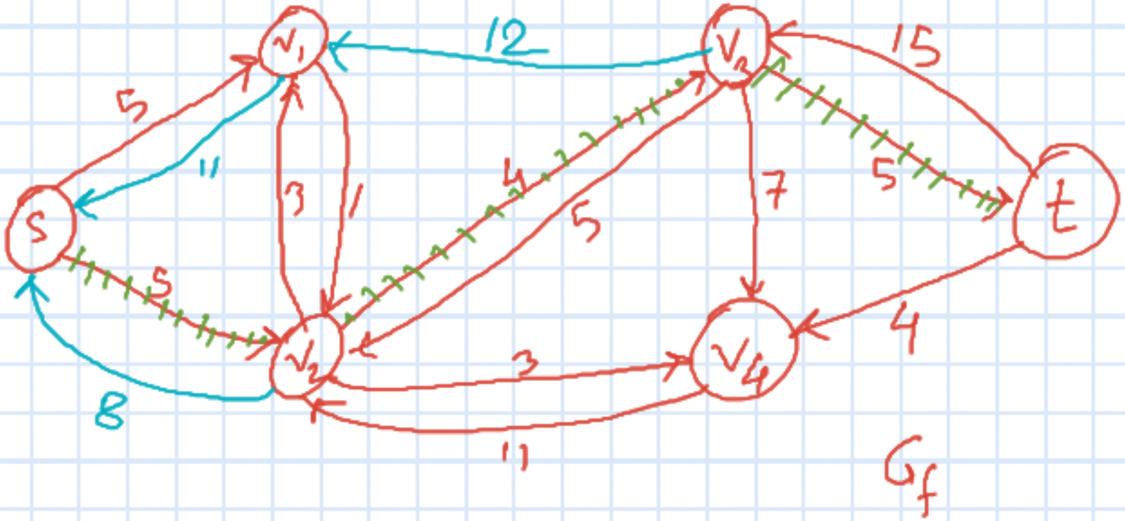
if $f(u,v) > 0$

Include a backward edge (u,v) with capacity

$$c_f(u,v) = f(u,v)$$



$|f| = 19$



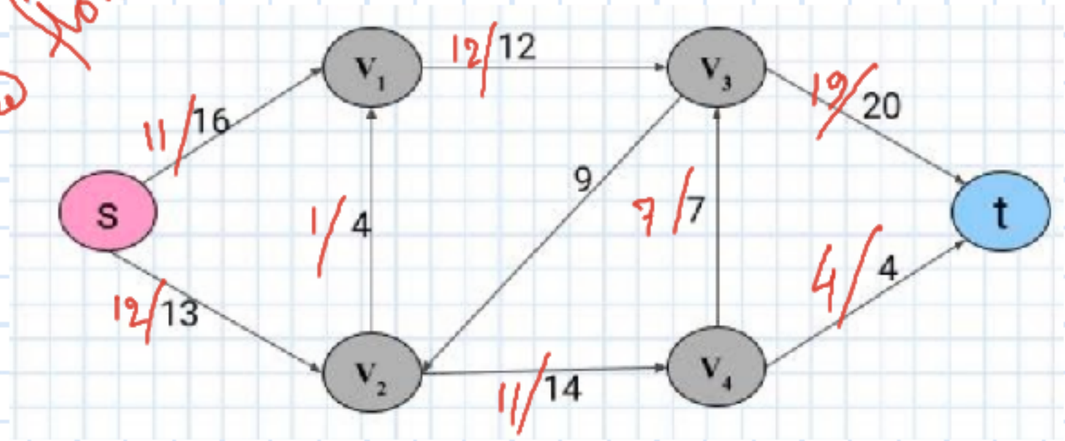
G_f

Augmented Path, $P = (s, v_2, v_3, t)$

$\delta(P) = \min\{c(s, v_2), c(v_2, v_3), c(v_3, t)\}$

$= \{5, 4, 5\} = 4$ → Residual capacity.

Augmented flow



$|f| = 23$