Indian Institute of Engineering Science and Technology, Shibpur B.Tech. - M.Tech. Dual Degree 5^{th} Semester Examinations, December, 2021 Graph Algorithms (CS 3104)

Full Marks: 70 Time: 90 minutes

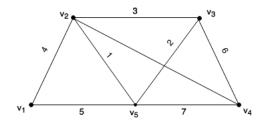
Answer Question No. 1 and any four from the remaining.

Do all parts of a question together. Do not mix up answers to parts of different questions in the answer script.

- 1. Define the following terms.
 - (a) Hamiltonian Path.
 - (b) Graph Density.

$$[3+3=6]$$

2. (a) Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown below.



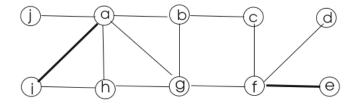
(b) Prove that the chromatic number of a graph will not exceed by more than one the maximum degree of the vertices in a graph.

$$[7+9=16]$$

- 3. (a) Define vertex colouring of a graph G and show that every planar graph is 5-vertex colourable.
 - (b) Given a directed graph G = (V, E). Describe a fast algorithm to compute the connected components in a graph G.

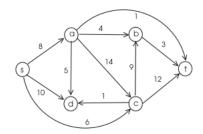
$$[7+9=16]$$

- 4. (a) Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.
 - (b) Consider the graph shown below with initial matching shown in bold, find augmenting paths iteratively to obtain a maximum matching for this graph.



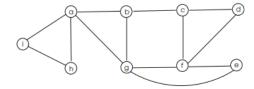
$$[7 + 9 = 16]$$

5. (a) Find the maximum flow in the network shown below using the Ford–Fulkerson algorithm by showing all iterations of the algorithm.

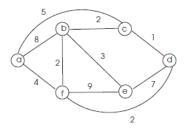


[16]

6. (a) Find the clustering coefficient for each vertex in the graph shown below and also work out the average clustering coefficient.



(b) Compute the vertex betweenness values for each vertex for the graph as shown below. Find two influential nodes in this graph using this measures.



[8 + 8 = 16]