Graph Algorithms

CS3104

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Plan for Today

- Introduction: Undirected and Directed Graphs
- Representation of Graphs

Graph Representation

- Adjacency Matrix
- Adjacency List

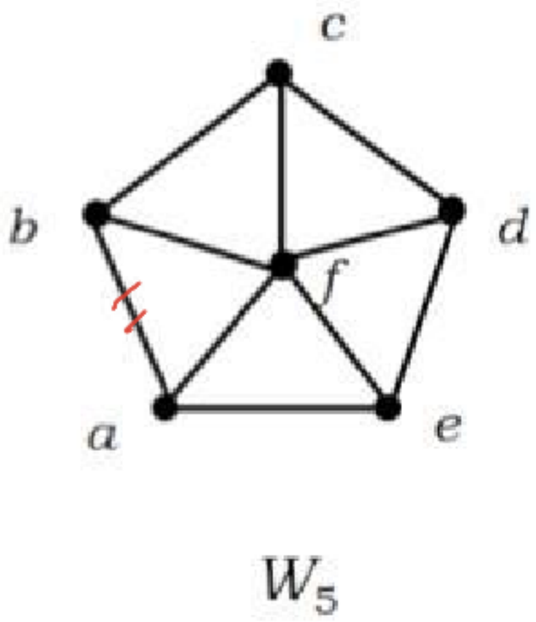
Representation of Graphs

Adjacency Matrix

- A simple graph $G = (V, E)$ with n vertices can be represented by its *adjacency matrix*, A, where the entry a_{ij} in row i and column j is:

$$a_{ij} = \begin{cases} 1 & \text{if } \{v_i, v_j\} \text{ is an edge in } G \\ 0 & \text{otherwise} \end{cases}$$

Adjacency Matrix Example



W_5

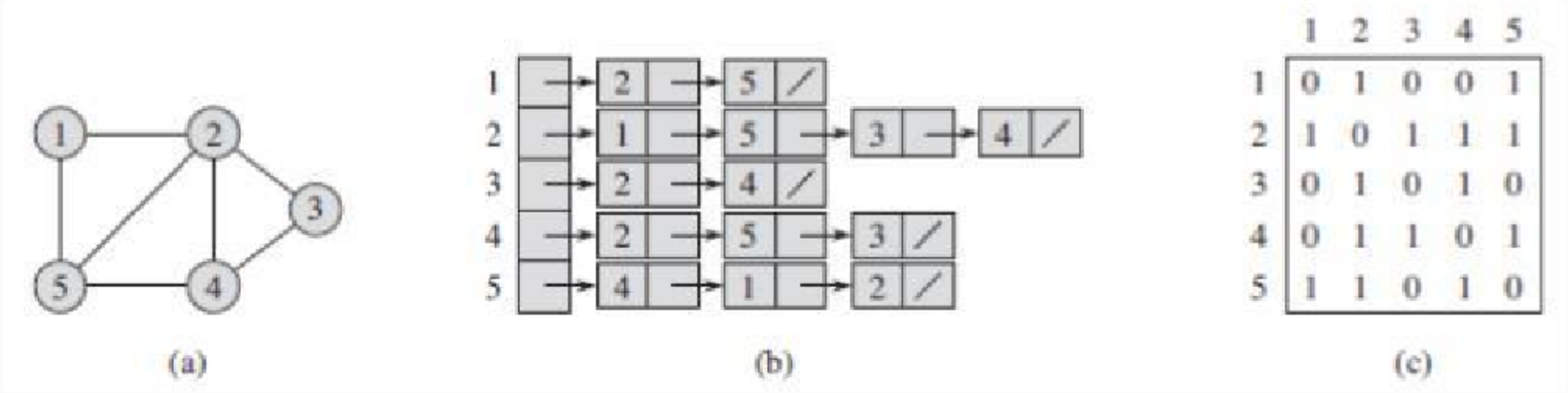
From \ To	a	b	c	d	e	f
a	0	1	0	0	1	1
b	1	0	1	0	0	1
c	0	1	0	1	0	1
d	0	0	1	0	1	1
e	1	0	0	1	0	1
f	1	1	1	1	1	0

Representation of Graphs

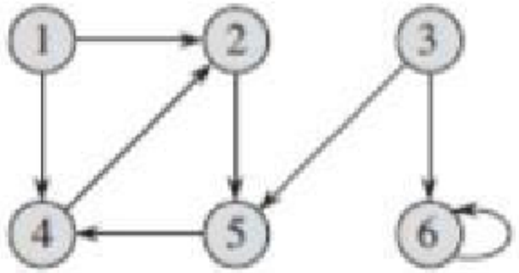
Adjacency List

- The adjacency-list representation of a graph $G = (V, E)$ consists of an array Adj of $|V|$ lists, one for each vertex in V.
  - For each $u \in V$, the adjacency list $Adj[u]$ contains all the vertices such that there is an edge $(u,v) \in E$. That is, $Adj[u]$ consists of all the vertices adjacent to u in G.

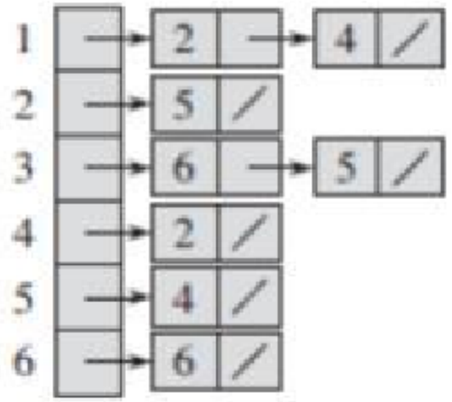
Adjacency List



Adjacency List



(a)



(b)

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

(c)

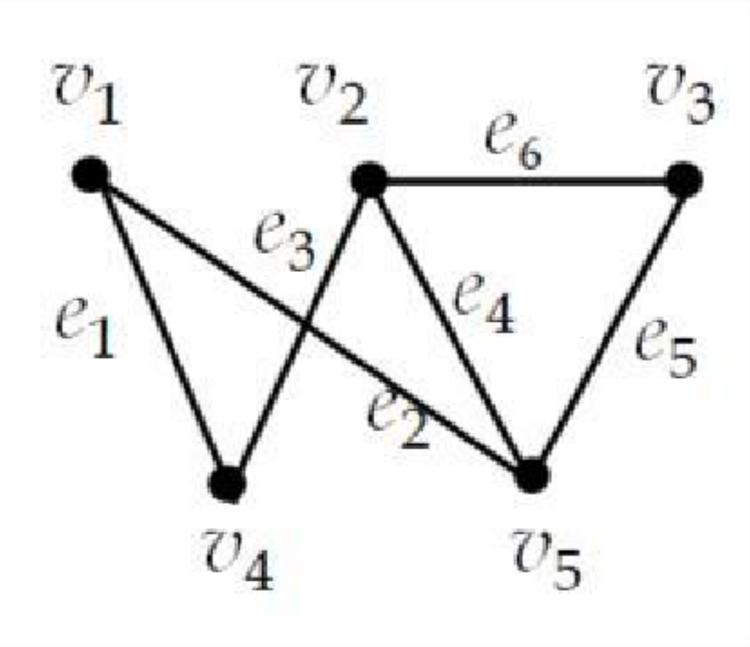
Incidence Matrix

- Let $G = (V,E)$ be an undirected graph. Suppose $v_1, v_2, v_3, \ldots, v_n$ are the vertices and $e_1, e_2, e_3, \ldots, e_m$ are the edges of G. The *incidence matrix w.r.t.* this ordering of V and E is the $n \times m$ matrix $M = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{if edge } e_j \text{ is incident with } v_i \\ 0 & \text{otherwise} \end{cases}$$

Incidence Matrix Example

- Represent the graph shown with an *incidence matrix*.



	e_1	e_2	e_3	e_4	e_5	e_6
v_1	1	1	0	0	0	0
v_2	0	0	1	1	0	1
v_3	0	0	0	0	1	1
v_4	1	0	1	0	0	0
v_5	0	1	0	1	1	0

← edges

↑ vertices

Application of Graph Theory

- Graphs are used to define the **flow of computation**.
- Graphs are used to represent **networks of communication**.
- Graphs are used to represent **data organization**.
- Graph transformation systems work on rule-based in-memory manipulation of graphs. Graph databases ensure **transaction-safe, persistent storing and querying of graph structured data**.
- Graph theory is used to find **shortest path in road** or a network.
- In **Google Maps**, various locations are represented as vertices or nodes and the roads are represented as edges and graph theory is used to find the shortest path between two nodes.

Overview of the Course

- Introduction and review of graphs
- Network Flow
- Planarity of Graphs
- Matching and Covering in graphs
- Graph Coloring
- Large graphs
- Design and analysis of algorithms large graphs

References

1. *Narsing Deo, "Graph Theory with applications to Engineering and Computer Science", Prentice Hall Inc*
2. *Douglas B West, "Introduction To Graph Theory", Pearson Education Inc,*
3. *R. Diestel, "Graph Theory", Springer Verlag*
4. *Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, "Introduction to Algorithms", The MIT Press*