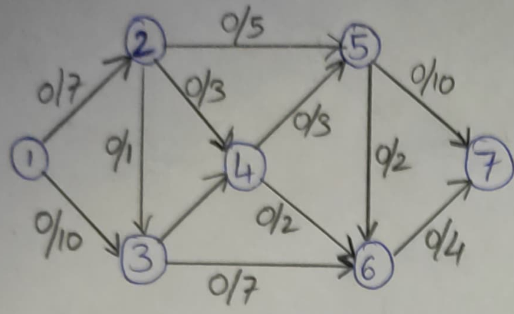
Graph Algorithms:-

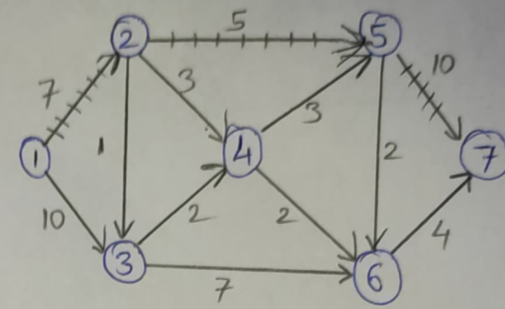
1) Ford-Fulkerson Algorithm:-

Sol:



[Graph G]:-

Residual Network [G_f]:-

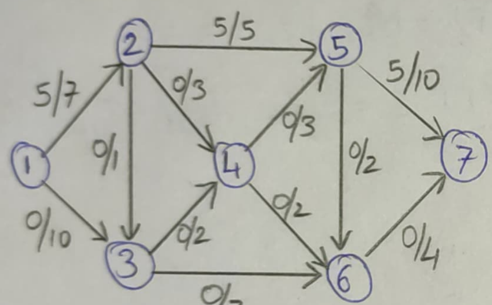


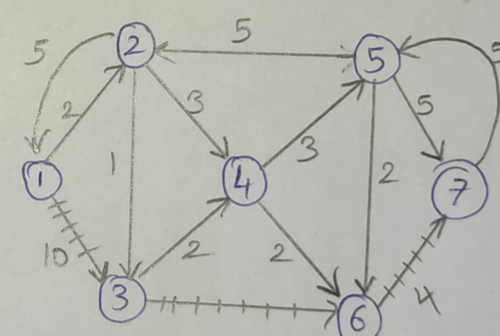
Augumenting path is

$P = (1, 2, 5, 7)$

$\delta(P) = Min(7, 5, 10)$

$\delta(P) = 5 \Rightarrow$ Residual Capacity



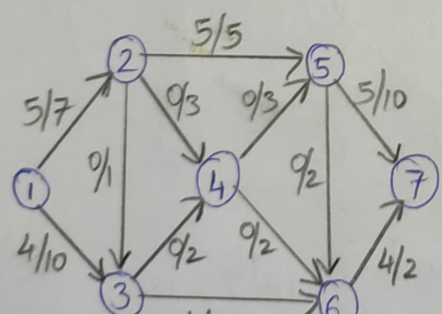


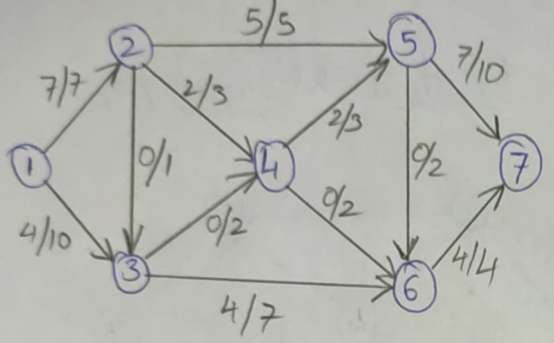
Augmenting path is

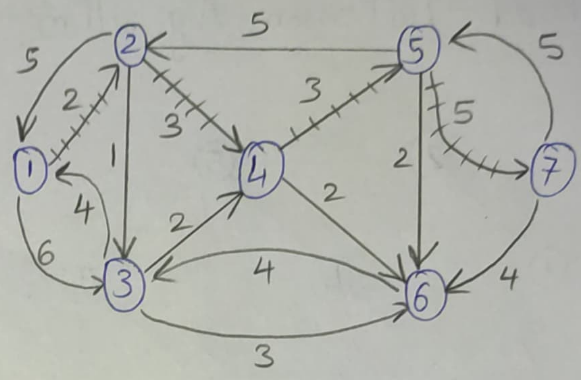
$P = (1, 3, 6, 7)$

$\delta(P) = Min(10, 7, 4)$

$\delta(P) = 4 \longrightarrow$ Residual capacity





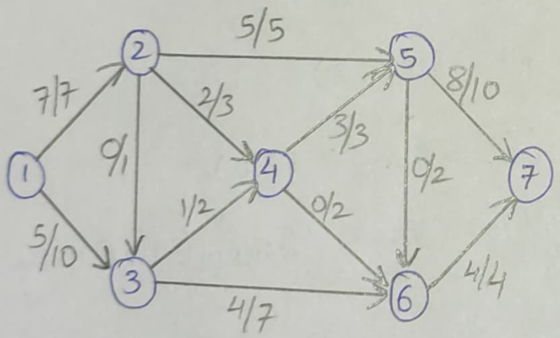


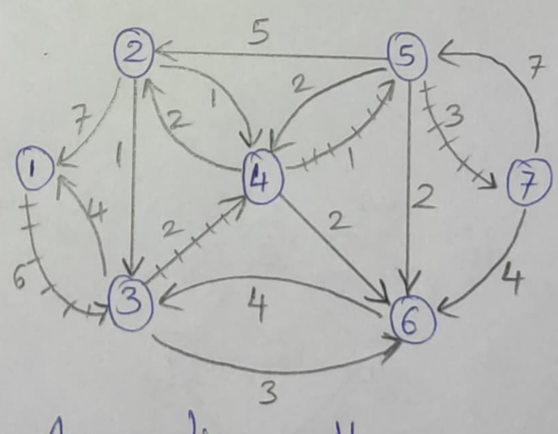
Augumenting path is

P = (1, 2, 4, 5, 7)

$\delta(P)$ = Min {2, 3, 3, 5}

$\delta(P) = 2$ => Residual Capacity.



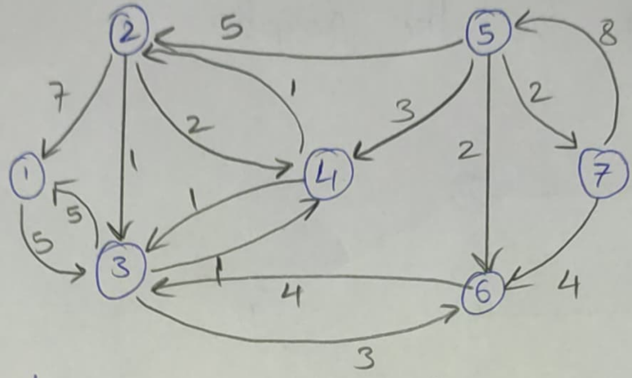


Augumenting path is

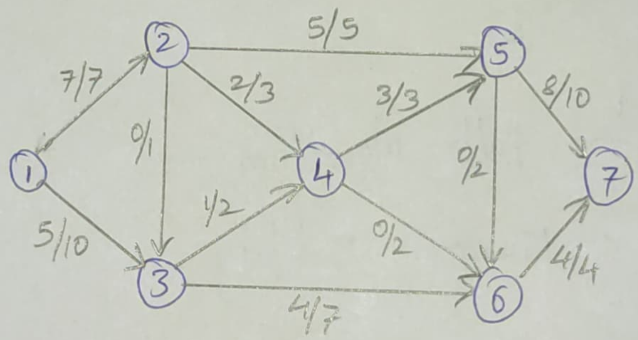
P = (1, 3, 4, 5, 7)

$\delta(P)$ = Min {6, 2, 1, 3}

$\delta(P) = 1$ => Residual Capacity



∴ There is no augmenting path exist.

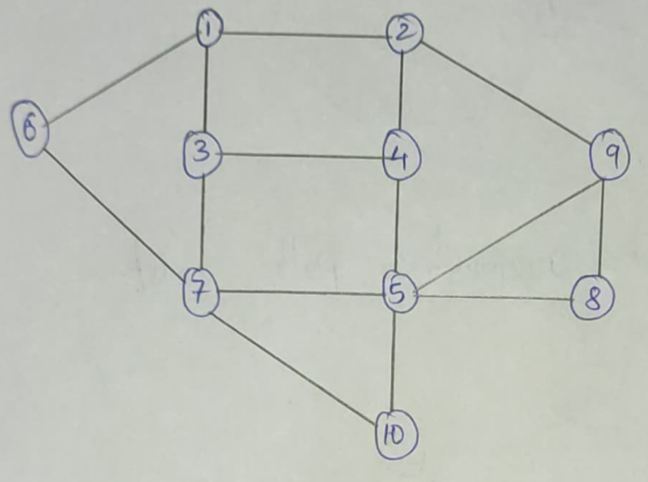


Max flow = f(2,5) + f(2,4) + f(3,4) + f(3,6)

= 5+2+1+4 = 12 units.

$\boxed{\therefore \text{Max flow} = 12 \text{ units.}}$

2) Maximum matching for the graph?



By using augumenting path algorithm:-

INPUT G ; OUTPUT :- Maximum Matching 'M' [set of edges]

<u>Algo:-</u>

1) start with $M = \phi$

2) while (there is augumenting path 'P' w.r.t. M)

{

$$M = \underbrace{M \Delta P}_{\text{Symmetric difference}} = (M-P) \cup (P-M)$$

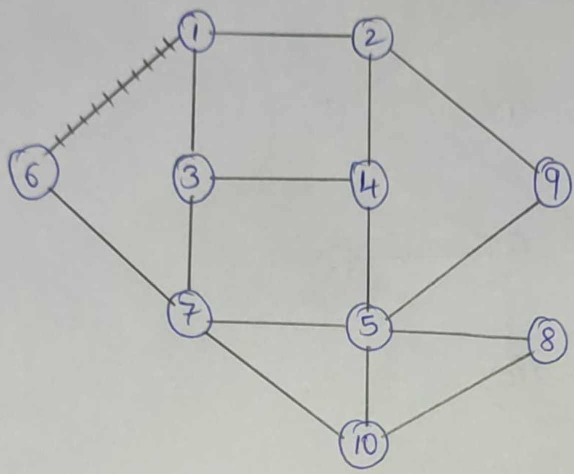
}

(1) Initial $M=\phi$, $P=(6,1)$

$M = M\Delta P = (M-P) \cup (P-M)$

$\Rightarrow M = \{(6,1)\}$

$\boxed{|M| = 1}$



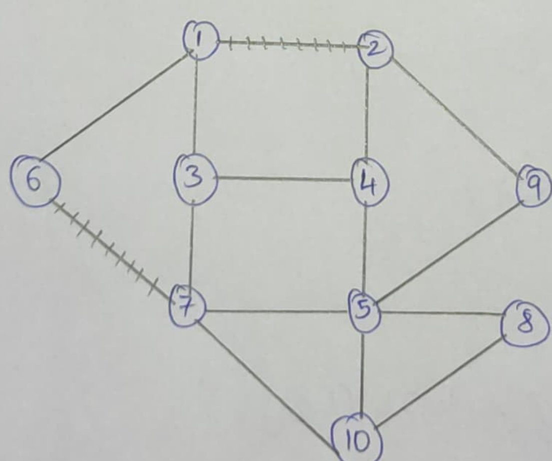
(2) $P = \{(2,1), (1,6), (6,7)\}$

$M \Delta P = (M-P) \cup (P-M)$

$M \Delta P = \{(2,1), (6,7)\}$

$M = \{(2,1), (6,7)\}$

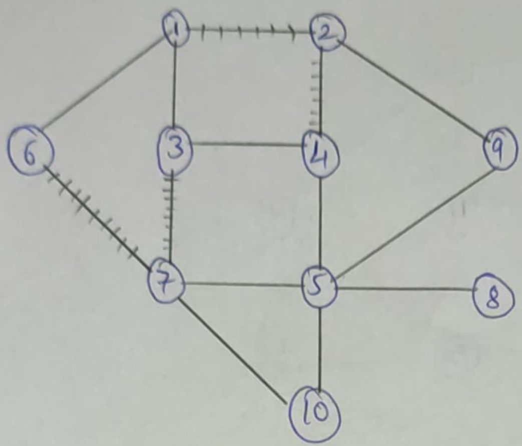
$\boxed{|M| = 2}$



(3) $P = \{(4,2), (2,1), (1,6), (6,7), (7,3)\}$

$M = M \Delta P = \{(4,2), (1,6), (7,3)\}$

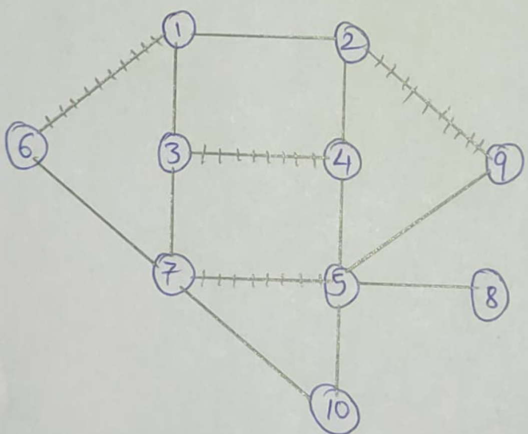
$\boxed{|M| = 3}$



(4) $P = \{(9,2), (2,4), (4,3), (3,7), (7,5)\}$

$M = M \Delta P = \{(9,2), (4,3), (7,5), (1,6)\}$

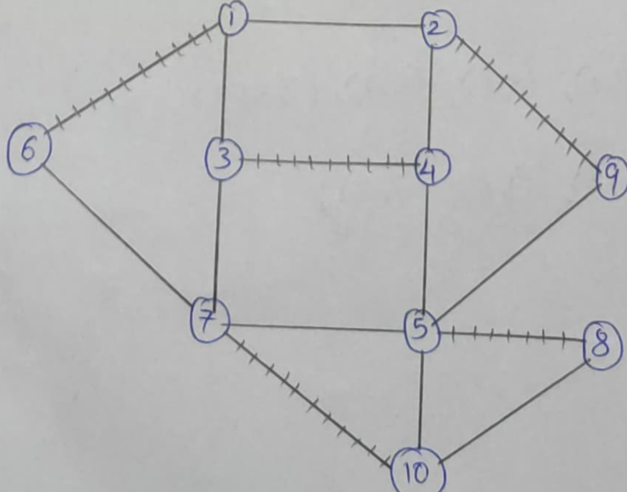
$\boxed{|M| = 4}$



(5) $P = \{(8,5), (5,7), (7,10)\}$

$M = M \Delta P = \{(8,5), (7,10), (9,2), (4,3), (1,6)\}$

$\boxed{|M| = 5}$



(6) Further. No augumenting path is there.

$\therefore$ The matching is maximum and the matching contains all the vertices of γ graph. so the matching is complete (or) perfect.

$$|M| = 5$$

$$M = \{(8,5), (7,10), (9,2), (4,3), (1,6)\}$$

3) closeness centrality $C_c(i)$

$$C_c(i) = \frac{n-1}{\sum_{j=1}^{n} d(i,j)}$$

[distance between i^{th} node and j^{th} node].

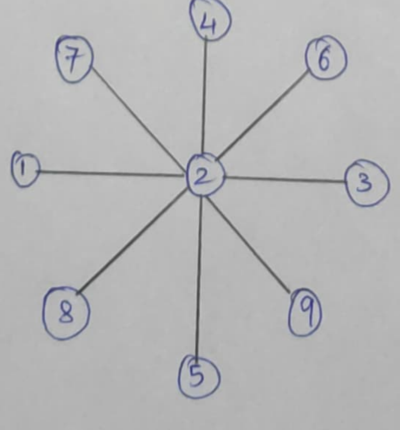
Between centrality $B_c(v)$

$$BC(v) = \sum_{s \neq t \neq v} \frac{\delta_{st}(v)}{\delta_{st}}$$

$\delta_{st} \longrightarrow$ Total no.of shortest paths between s and t.

$\delta_{st}(v) \longrightarrow$ Total no.of shortest paths between vertices s and t that passes through vertex "v".

(a)



here $n=9$

$$C_c(i) = C_c(2) = \frac{9-1}{d(1,2)+d(1,3)+d(1,4)+d(1,4) \\ d(1,5)+d(1,6)+d(1,7)+d(1,8)}$$

$$= \frac{8}{8} = 1 \quad \Rightarrow \quad \boxed{C_c(2)=1}$$

$$C_c(3) = \frac{8}{1+2\times 7} = \frac{8}{14+1} = \frac{8}{15}$$

$$\Rightarrow \boxed{C_c(3) = 0.533}$$

$$BC(v) = \sum_{s \neq t \neq v} \frac{\delta_{st}(v)}{\delta_{st}}$$

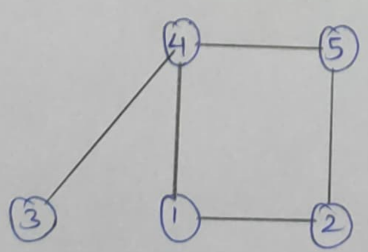
$$BC(2) = \frac{\delta_{st}(2)}{{}^{(n-1)}C_2} = \frac{{}^8C_2}{{}^8C_2} = 1 \quad \Rightarrow \quad \boxed{BC(2)=1}$$

$$BC(v) = \sum_{s \neq t \neq v} \frac{\delta_{st}(v)}{\delta_{st}}$$

$$BC(3) = \frac{\delta_{st}(v)}{{}^{n-1}C_2} = \frac{0}{{}^8C_2} = 0$$

$$\boxed{\therefore BC(2)=1,\ BC(3)=0}$$

(b)



$\boxed{n=5}$

$$Cc(2) = \frac{n-1}{\sum_{j=1}^{n} d(i,j)} = \frac{4}{d(1,2)+d(4,2)+d(5,2)+d(3,2)}$$

$$Cc(2) = \frac{4}{1+1+1+3} = \frac{4}{6} = \frac{2}{3} = 0.66$$

$$Cc(3) = \frac{4}{2+3+0+1+2} = \frac{4}{8} = 0.5$$

$$BC(2) = \sum_{s \neq t \neq 2} \frac{\delta_{st}(2)}{\delta_{st}} = \frac{1}{{}^4C_2} = \frac{1}{6} = 0.167$$

$$BC(3) = \sum_{s \neq t \neq 3} \frac{\delta_{st}(3)}{\delta_{st}} = \frac{0}{{}^4C_2} = 0$$

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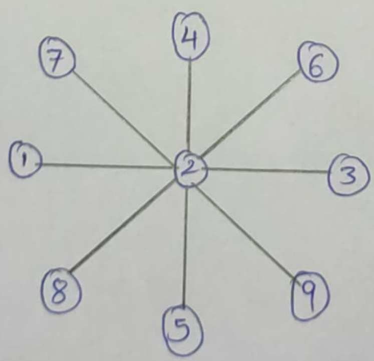
(4) clustering co-efficient $= \dfrac{2nv}{|N(v)|(|N(v)|-1)}$ if $|N(v)| > 1$

(a) else, 0 if $|N(v)| \leq 1$



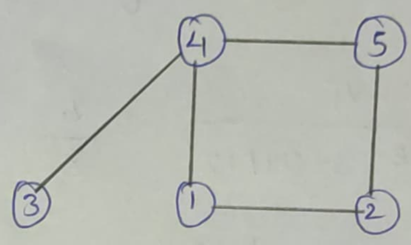
$Cc(1) = 0$ [$\because |N(v)| = 1$]

$$Cc(2) = \frac{0 \times 2}{8 \times (8\cdot1)} = 0$$

[$Cc(3) = Cc(4) = Cc(5) = Cc(6) = Cc(7) = Cc(8) = Cc(9) = 0$]

(b) $$CC = \frac{2nv}{|N(v)|(|N(v)|-1)} \quad \text{if } |N(v)|>1$$

else $CC=0$



$$CC(1) = \frac{2\times 0}{2(2-1)} = 0$$

$$CC(2) = \frac{2\times 0}{2\times(2-1)} = 0$$

$$CC(3) = 2\times 0 = 0 \quad [|N(v)|=1]$$

$$CC(4) = \frac{2\times 0}{2(2-1)} = 0$$

$$CC(5) = \frac{2\times 0}{2\times(2-1)} = 0$$