## Indian Institute of Engineering Science and Technology, Shibpur Dual Degree (B.Tech. - M.Tech.) 5<sup>th</sup> Semester (CST) Examination (End Semester) November, 2023

Graph Algorithms (CS 3104)

Full Marks: 50

Time: 3 Hours

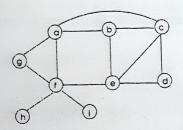
Answer Question-1 and any four from the remaining. Do all parts of a question together. Do not mix up answers to parts of different questions in the answer script.

1. (a) State Max flow min cut theorem and prove it through an example.

- (b) An undirected graph G has 8 edges. Find the number of vertices, if the degree of each vertex in G is 2.
- State and prove the handshaking theorem.

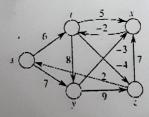
[4+3+3=10]

- (a) Define vertex colouring of a graph G and show that every planar graph is 5-vertex colourable.
  - (b) Construct a possible spanning tree of the graph depicted in following Figure using the Kruskal's algorithm. Show the result of each iteration of the algorithm.



[4+6=10]

- 3. (a) what is an undirected graph? Prove that an undirected graph has even number vertices of odd
  - (b) Run the Bellman-Ford algorithm (for finding Single-source shortest-path) for the following directed graph using the vertex, z as source. In each pass relax the light edges and show the d and  $\pi$  values after each passes.

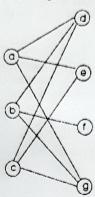


[4+6=10]

- 4. (a) Using Kuratowski's theorem, show that the Petersen's graphs are non-planar.
  - (b) Prove, that every planar graph, which has no loops or multiple edges, and v vertices ( $v \geq 3$ ) and e edges then  $e \leq 3v - 6$ .

[6+4=10]

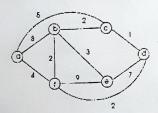
5. (a) Using the augmenting path algorithm, find out the maximum matching in the bipartite graph below. Show the result of each iteration of the algorithm.



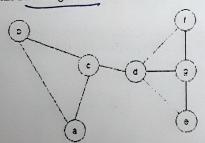
(b) Prove that the chromatic number of a graph will not exceed by more than one the maximum degree of the vertices in a graph.

$$[6+4=10]$$

(a) Compute vertex betweenness centrality of all the vertices of the following graph. According to you, which vertex is the most important based on this centrality value, and why?



(b) Find the clustering coefficient for each vertex in the graph below and work out the average clustering coefficient. Also, find the matching Index of the two nodes c and d.



$$[(4+1)+(4+1)=10]$$