Graph Algorithms

CS3104

Dr. Samit Biswas, *Assistant Professor,*
Department of Computer Sc. and Technology,
Indian Institute of Engineering Science and Technology, Shibpur

Email: samit@cs.iiests.ac.in

Ford-Fulkersion-Method (G, S, t)

1. Initialize flow, f to Zero.
2. while there exist an augmenting path, p in G_f

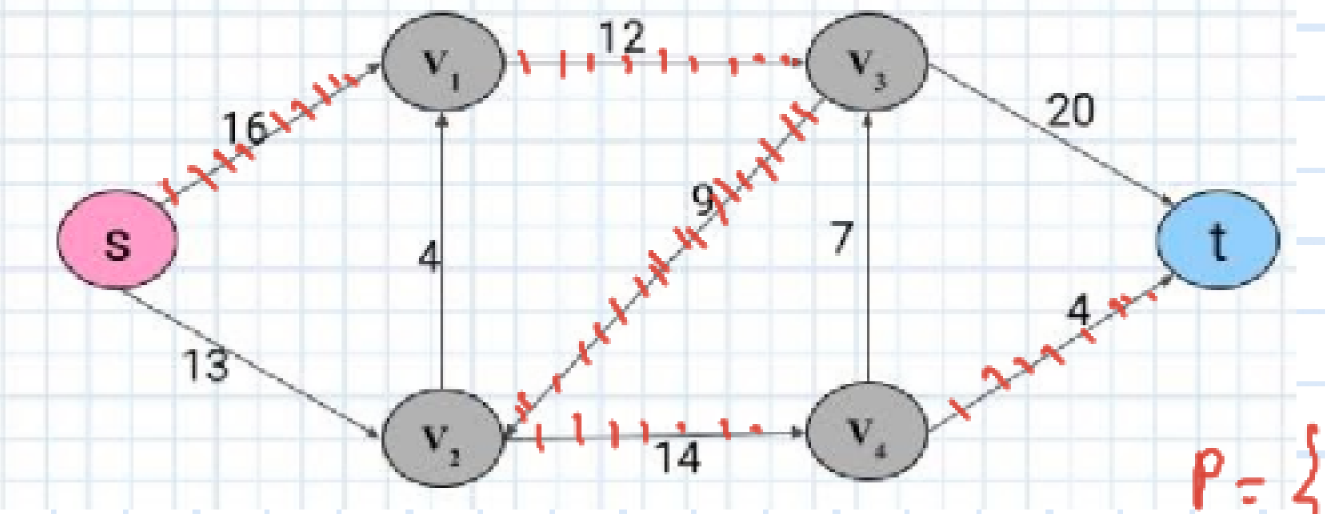
   do augment flow, f along p.

3. return f.

The Ford-Fulkerson Algorithm

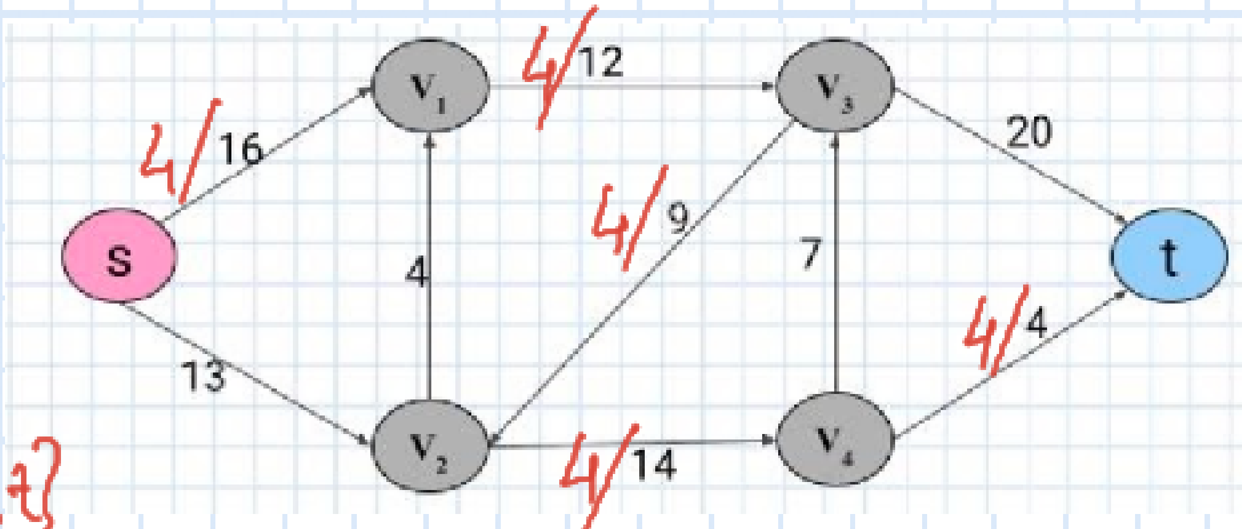
FORD-FULKERSON(G, s, t)

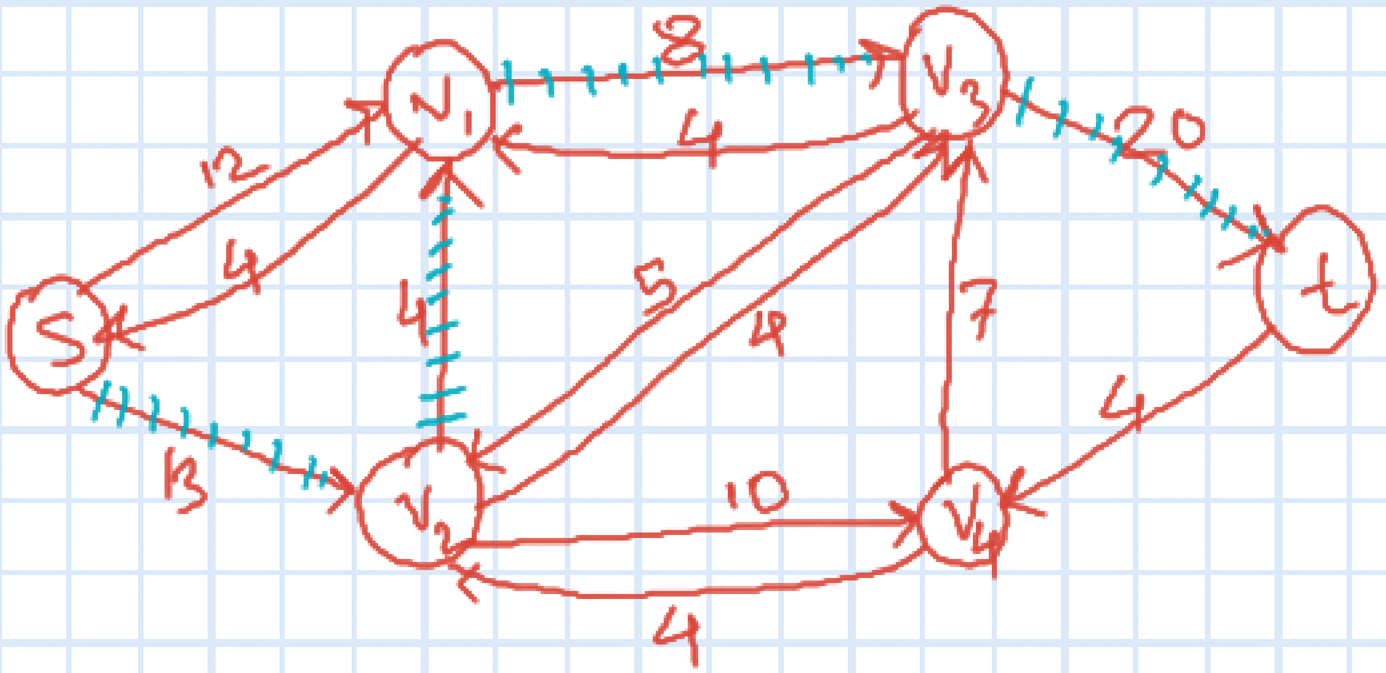
for each edge $(u, v) \in G.E$
    $(u, v).f = 0$
while there exists a path p from s to t in the residual network G_f
    $c_f(p) = \min\{c_f(u, v) : (u, v) \text{ is in } p\}$
    for each edge (u, v) in p
        if $(u, v) \in E$
            $(u, v).f = (u, v).f + c_f(p)$
        else $(v, u).f = (v, u).f - c_f(p)$



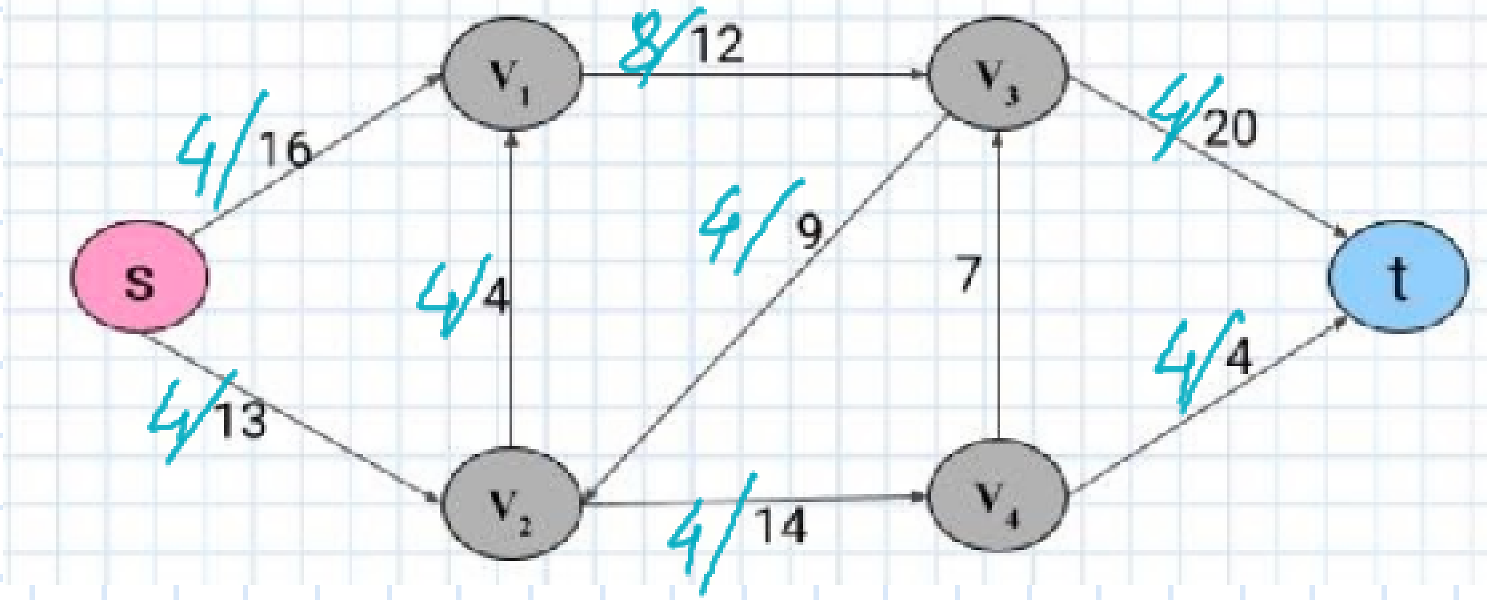
$c_f(p) = 4$

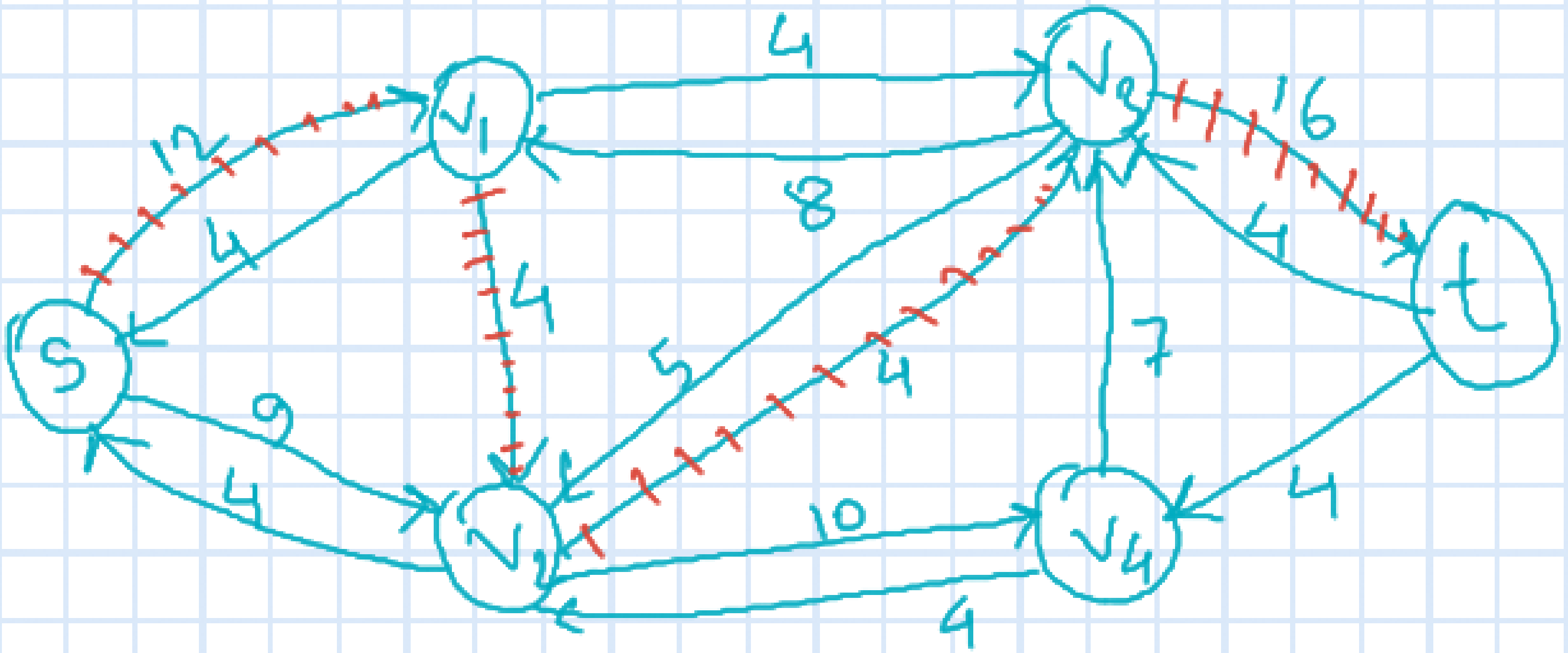
$p = \{s, v_1, v_3, v_2, v_4, t\}$



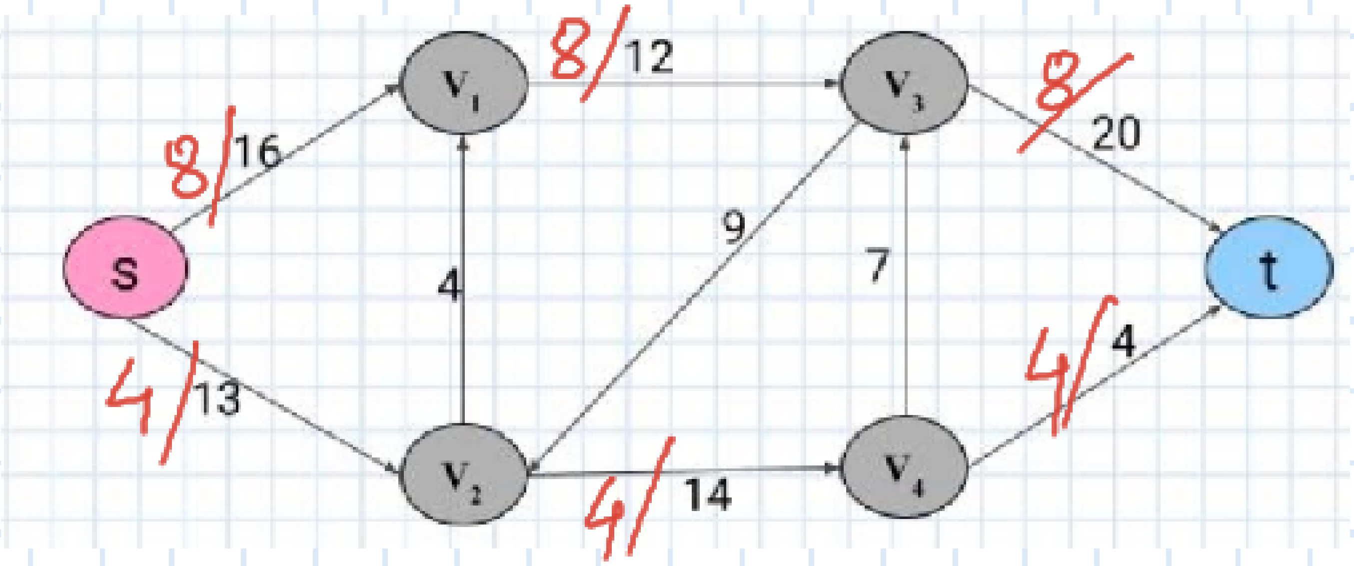


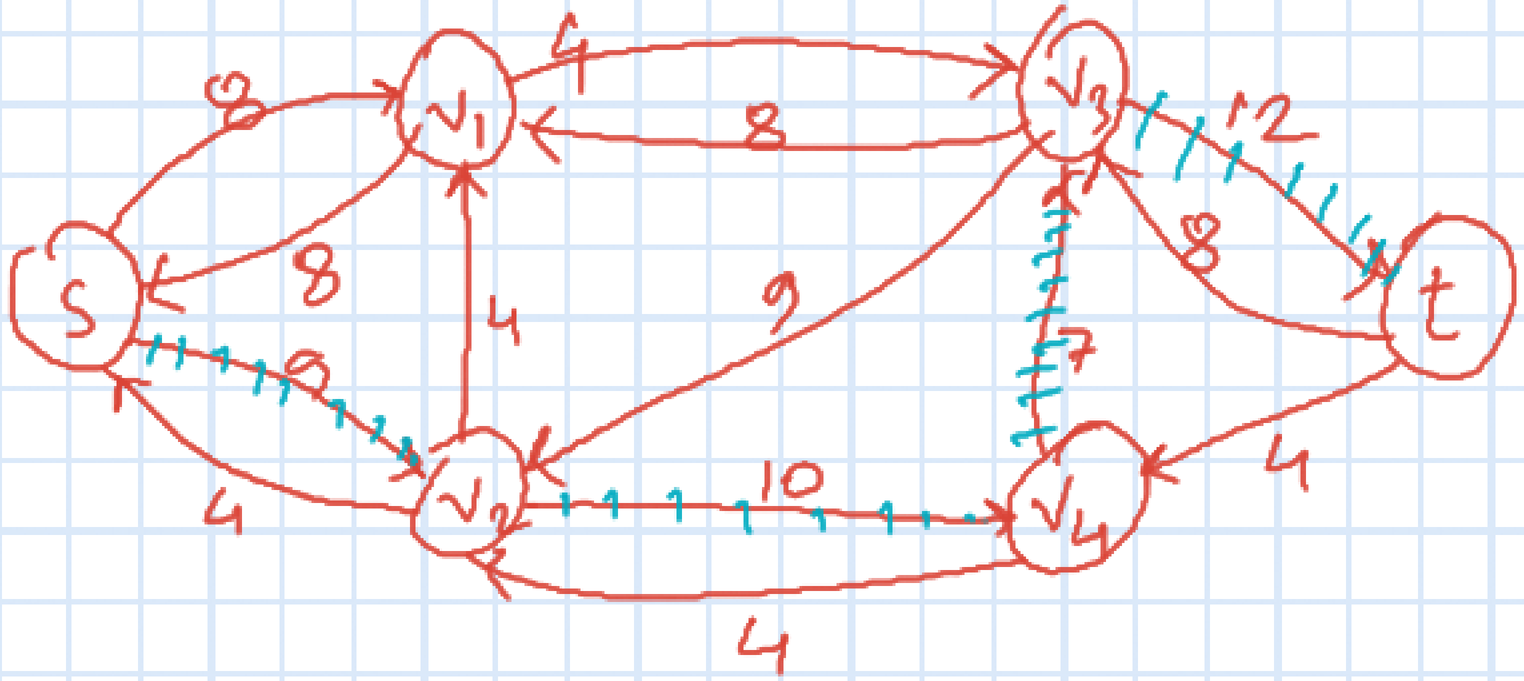
$c_f(p) = 4$



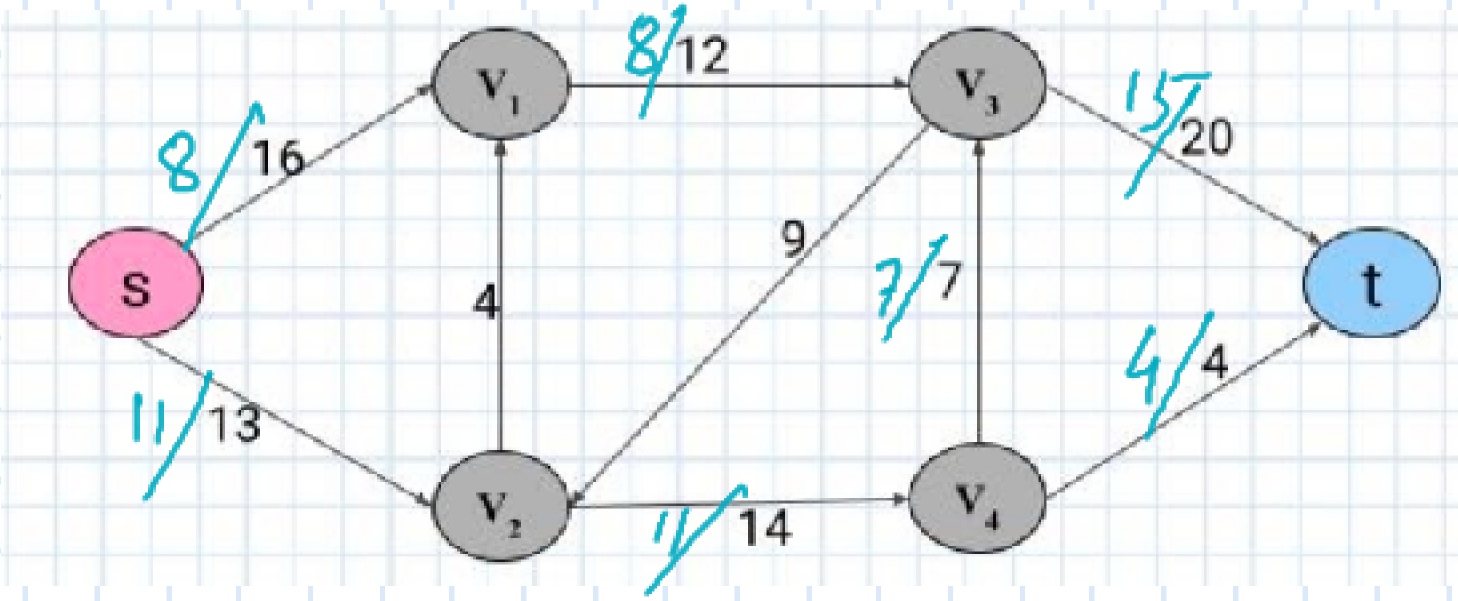


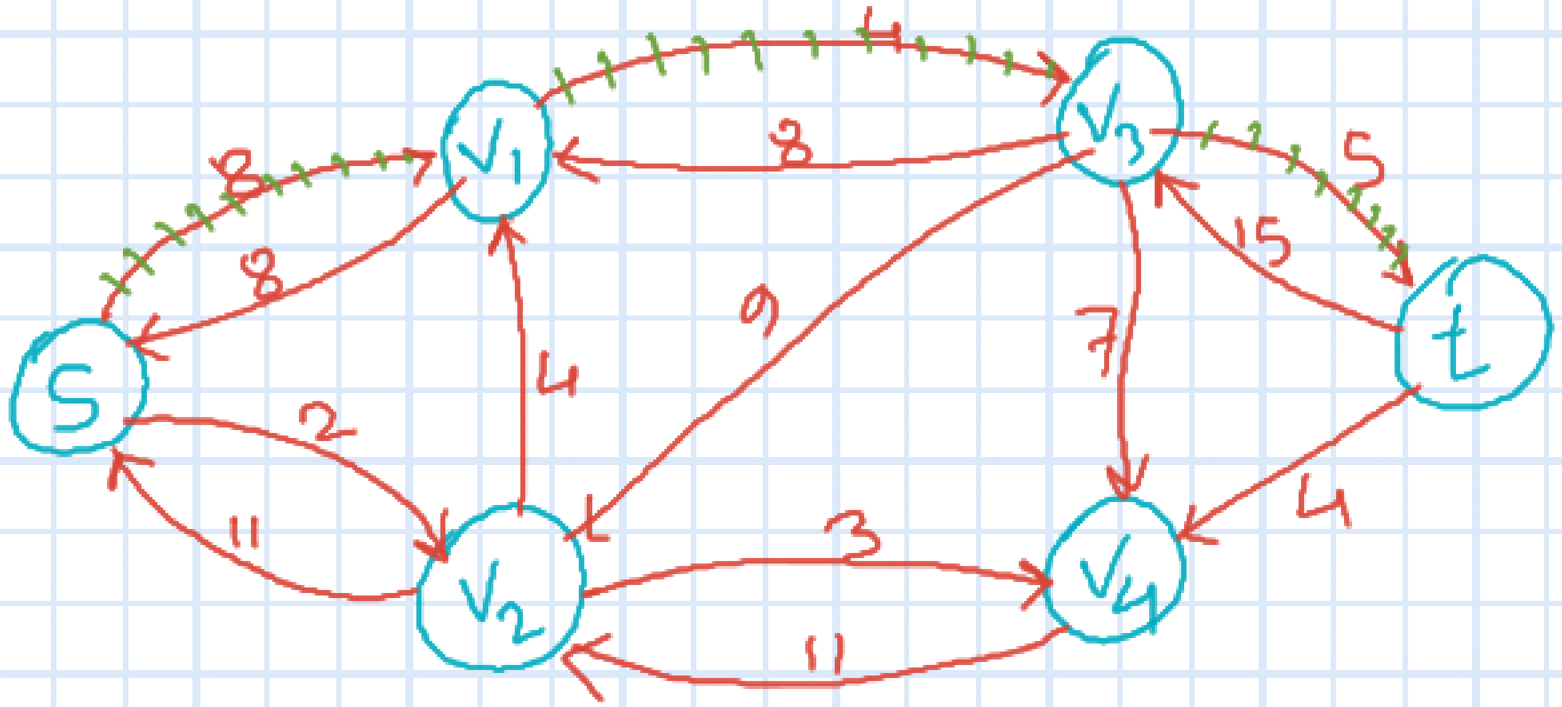
$c_f(p) = 4$



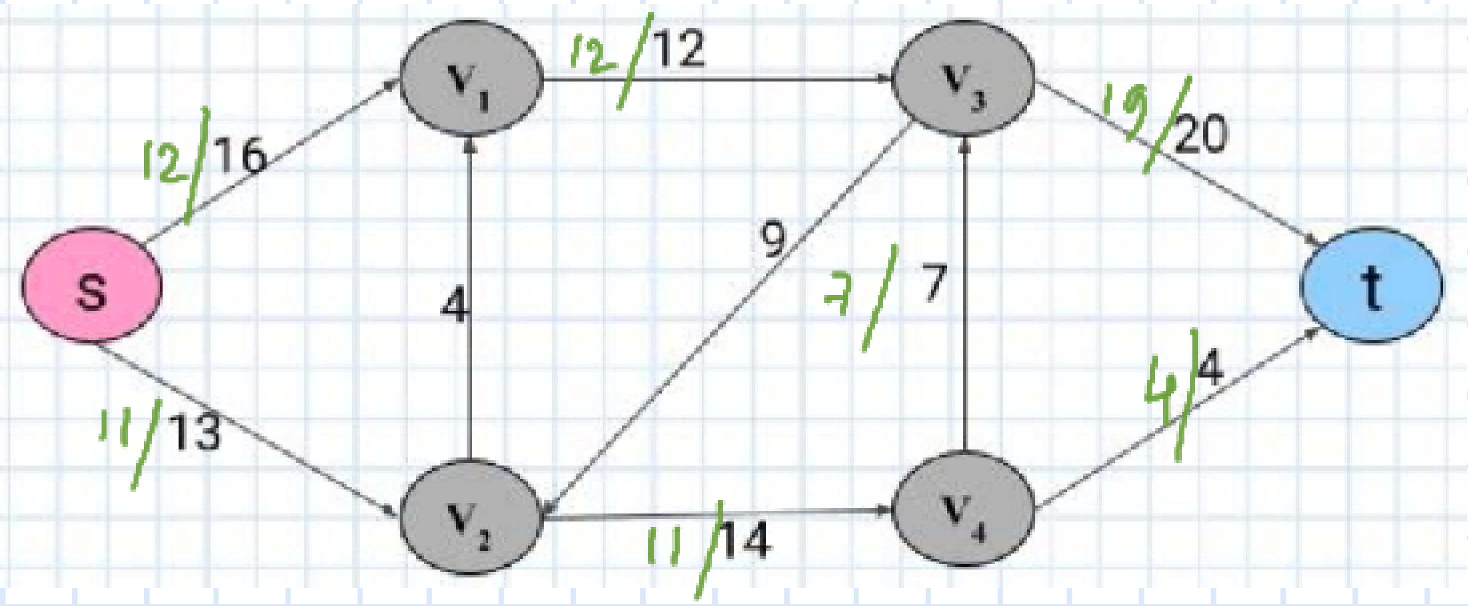


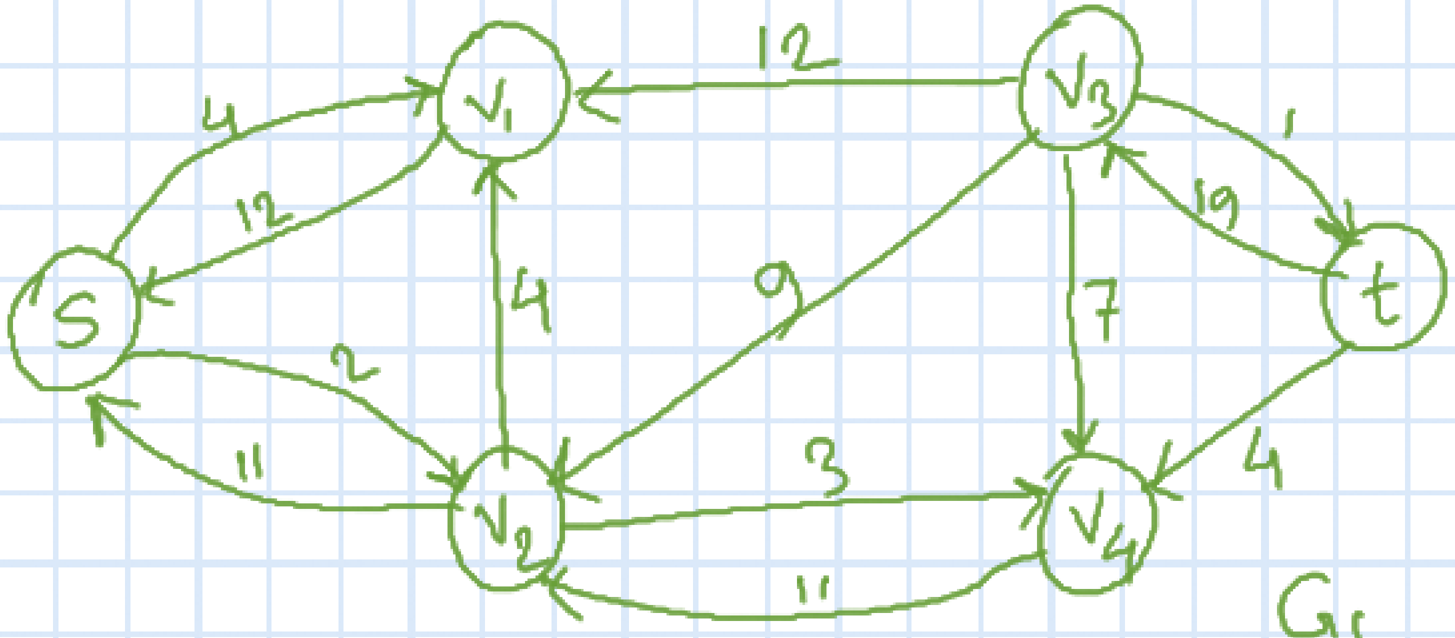
$c_f(p) = 7$





$c_f(p) = 4$





G_f

After this in G_f no augmenting path is possible.

Cuts of the flow Network.

Cut (S, t) of flow network $G = (V, E)$ is a partition of vertices, V into S & $T = V - S$

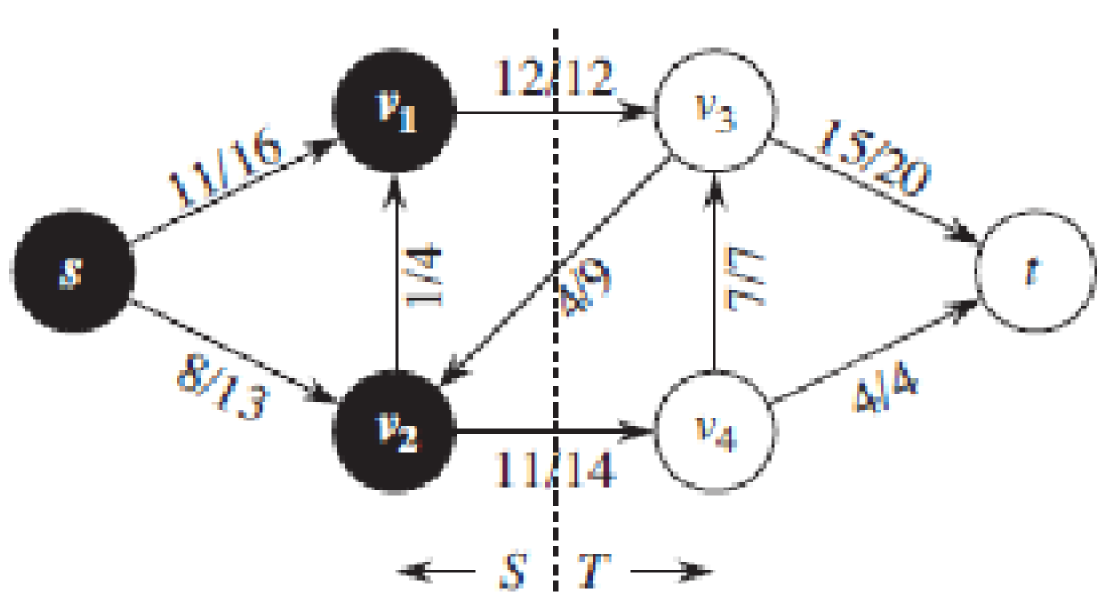
$$s \in S, \; t \in T$$

$$f(S, T) = \sum_{u \in S} \sum_{v \in T} f(u, v) - \sum_{u \in S} \sum_{v \in T} f(v, u)$$

Capacity of the cut (S, T) is

$$c(S, T) = \sum_{u \in S} \sum_{v \in T} c(u, v)$$

minimum Cut $\Rightarrow$ min over all cuts of the network



Cut $(\{s, v_1, v_2\}, \{v_3, v_4, t\})$

Net flow

$$f(v_1, v_3) + f(v_2, v_4) - f(v_3, v_2) = 12 + 11 - 4 = 19$$

Capacity of this cut

$$c(v_1, v_3) + c(v_2, v_4) = 12 + 14 = 26.$$

Flow across any cut is the same and equals the value of the flow

<u>Max-Flow min-cut Theorem.</u>

Let f is a flow in network G. Then the following cond^ns are equivalent

(a) f is a maximum flow in G.

(b) Residual network G_f contains no augmenting path.

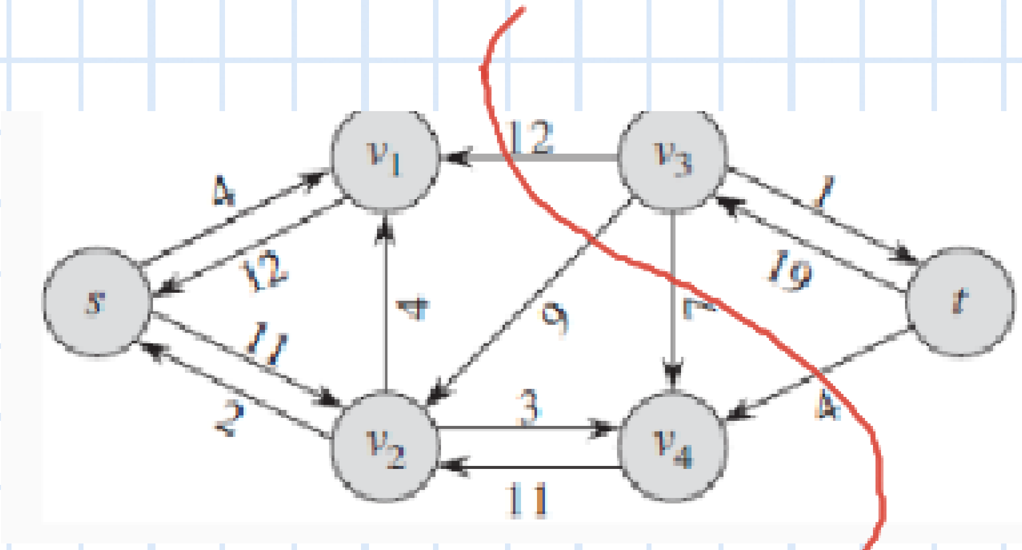
(c) there is a cut (S, T) with $|f| = C(S, T)$

maximum flow = capacity of minimum cut

$S = \{s, v_1, v_2, v_4\}$

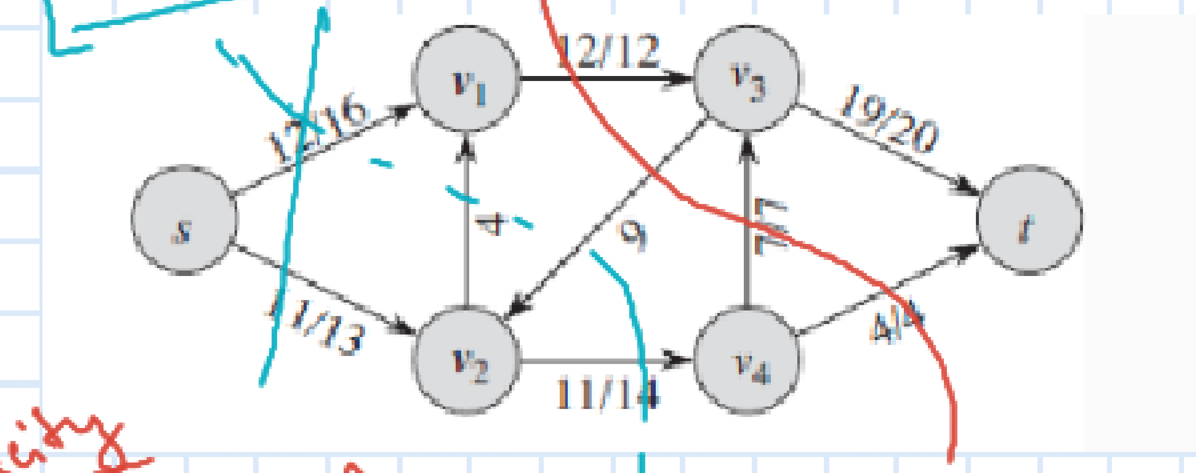
$T = \{v_3, t\}$

Cut (S, T)



G_f

Augmented flow.



Cut capacity $C(S, T) = 23$