

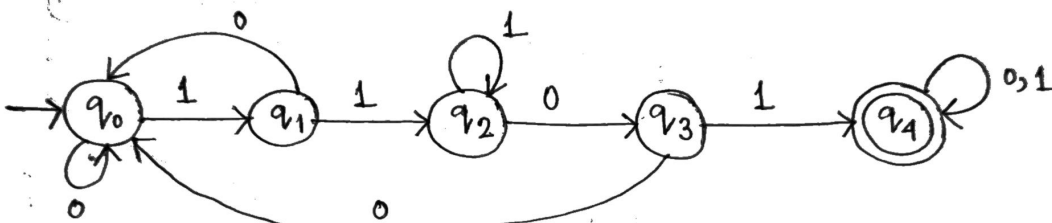
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1. The language

$$L_1 = \{x \in \{0,1\}^* \mid x \text{ contains 1101 as a substring}\}$$

$L_1$  is a regular language since, we can draw DFA  $M_1$  for the language such that  $L(M_1) = L_1$ .

Now, the DFA  $M_1$  is:



$$M_1 = \{Q, \Sigma, \delta, q_0, F\}$$

$$\text{where, } Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{0, 1\}$$

$$F = \{q_4\}$$

$q_0 \rightarrow$  initial state.

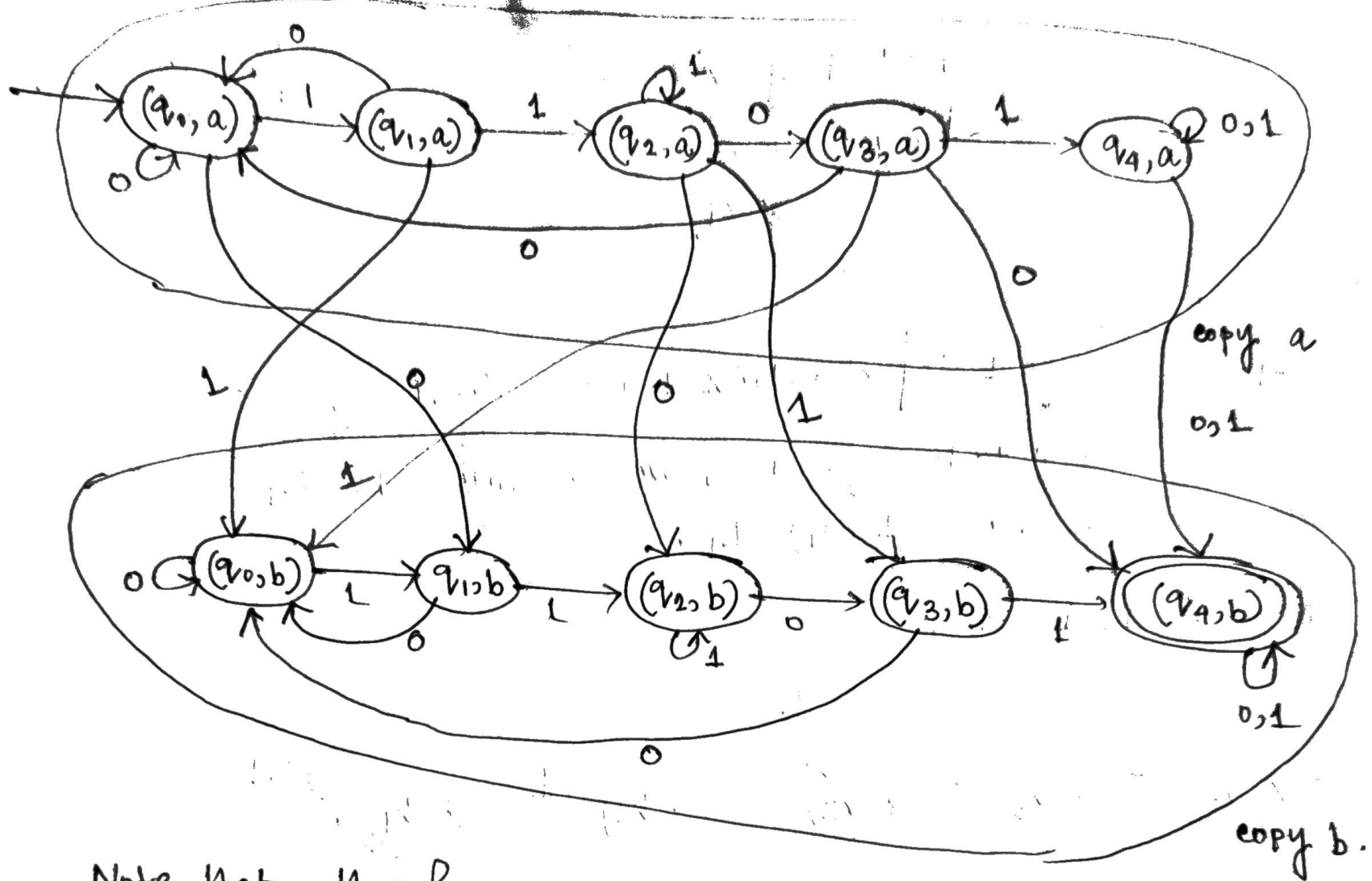
$\delta \rightarrow$  transition function,  $\delta: Q \times \Sigma \rightarrow Q$

$$\begin{aligned} \delta(q_0, 0) &= q_0, & \delta(q_1, 0) &= q_0, & \delta(q_2, 0) &= q_0, & \delta(q_3, 0) &= q_0, & \delta(q_4, 0) &= q_4 \\ \delta(q_0, 1) &= q_1, & \delta(q_1, 1) &= q_2, & \delta(q_2, 1) &= q_2, & \delta(q_3, 1) &= q_4, & \delta(q_4, 1) &= q_4 \end{aligned}$$

Now,  $L_2 = \{y \in \{0,1\}^* \mid \text{there exist a } x \in L_1, \text{ exactly one-bit of which is flipped to obtain } y\}$

we have to design an NFA  $M_2$ , such that  $L(M_2) = L_2$ .

for that, we need two copies of the DFA  $M_1$ .



Note that, though copy a and b are copies of DFA  $M_1$ , copy a doesn't contain the final state and copy b doesn't contain the initial state.

$$M_2 = \{ Q_2, \Sigma, \delta_2, (q_{0,a}), F_2 \}$$

$$\text{where, } Q_2 = \{ (q_i, a), (q_j, b) \mid 0 \leq i, j \leq 4, i, j \in \mathbb{N}^+ \}$$

$$\Sigma = \{ 0, 1 \}$$

$$F_2 = \{ (q_{4,b}) \}$$