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TOC Assignment → 3

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1. a) $\boxed{L_1 = \{ww \mid w \text{ in } \{0,1\}^*\}}$

First, we will assume that L_1 is regular language. Then we'll show by contradiction that L_1 is not regular.

Now, if L_1 is a regular language, there exists an integer $n \geq 1$, for which the property stated in the lemma holds.

⇒ Let's define $\boxed{S = 0^n 1 0^n 1}$ $\left(S = S_1 S_1,\ S_1 \text{ in } \{0,1\}^* \right)$

we see that string $S \in L_1$

$|S| = (2n+2) > n$

Hence, according to the pumping lemma, there exist strings x, y, z $[\in \Sigma^*]$ such that $S = xyz$ and the following conditions are satisfied :

A) $|y| \geq 1$ i.e. $y \neq \varepsilon$

B) $|xy| \leq n$ and

c) $xy^i z \in L_1 \quad \forall i \in \mathbb{N}$

⇒ $\boxed{\text{proof part}}$: we assume that $y = 0^k$

Since, $y \neq \varepsilon$, so, $k \geq 1$.

and, $|xy| \leq n$. Assuming the first two conditions true, we will check the third condition.

$$|xy^iz| = |0^n 1 0^n 1| + |y^{i-1}| = |0^n 1 0^n 1| + |0^{k(i-1)}|$$

$$xy^iz = 0^{n+k(i-1)} 1 0^n 1$$

Now, if $xy^iz \in L_1$ then $\boxed{n + k(i-1) = n}$

take $i = 2$, L.H.S $= n + k \neq n$ (R.H.S)

because $k \geq 1$.

Hence proved, by contradiction, <u>L_1 is not regular language.</u>

b) $\boxed{L_2 = \{0^i 1^j \mid i > j\}}$

First, we will consider L_2 to be a regular language. Then according to the pumping lemma, there exists an integer $n \geq 1$ (pumping length) for which properties stated in the lemma holds.

$\Rightarrow$ let's define a string $\boxed{w = 0^{n+1} 1^n}$

Since, $(n+1) > n$, hence this particular string $w \in L_2$

$|w| = (2n+1) > n$

Hence, according to the pumping lemma, there exist strings x, y, z [$\in \{0,1\}^*$] such that $w = xyz$ and the following conditions are satisfied:

A) $y \neq \varepsilon$ (i.e. $|y| > 0$ or $|y| \geq 1$).

B) $|xy| \leq n$ and,

C) $xy^iz \in L_2 \quad \forall i \in \mathbb{N}$

we, assume that, $y = 0^K$, $K \geq 1$ (since $y \neq \varepsilon$)
also, $|y| \leq |xy| \leq n$, hence, $1 \leq K \leq n$.

now, $|xy^iz| = |xyz| + |y^{i-1}| = |0^{n+1}1^n| + |0^{K(i-1)}|$

$= xy^iz = 0^{n+1+K(i-1)} 1^n$

Since, ~~i is any nat~~ i can also be zero (0), $xy^iz \in L_2$ for $i = 0$ also.

put $i = 0$, $(n+1) - K \leq n$ (since $K \geq 1$)

Here, we see that $xy^0z \notin L_2$.

Hence, by proof of contradiction, we conclude that L_2 is not a regular language.

c) $\boxed{L_3 = \{ w \text{ in } \{0,1\}^* \mid w = w^R \}}$

First, we assume that L_3 is regular language.
If w is a string, then w^R denotes the inverse of the string.
L_3 is regular, so there exist integer $n \geq 1$ for which the pumping lemma properties hold.

$\Rightarrow$ let's take $\boxed{w = 0^n 1 0^n}$, we see that $w = w^R$ and hence $w \in L_3$.

$|w| = (2n+1) > n$

Hence, according to pumping lemma, there exist strings x, y, z $[\in \{0,1\}^*$ such that $w = xyz$ and the following properties hold:

A) $y \neq \varepsilon$ (i.e. $|y| \geq 1$)

B) $|xy| \leq n$

C) $xy^iz \in L_3 \quad \forall i \in \mathbb{N}$

$\boxed{\text{proof part}}$: We take $y = 0^K$, since $y \neq \varepsilon$, hence $K \geq 1$.

Now, since $|xy| \leq n$, it's safe to take $y = 0^K$. $(1 \leq K \leq n)$.

$$xy^iz = 0^{n+(i-1)K} 1 0^n$$

if $xy^iz \in L_3$ i.e. palindrome string, then

$n + (i-1)K$ must be equal to n for all $i \in N$.

choose $i = 2$, $n + (i-1) \cdot K = (n+K) \neq n$ (since $K \geq 1$, $K \neq 0$).

Hence, xy^2z is NOT a palindrome string, i.e.

$xy^2z \notin L_3$. This contradicts our third (C) condition.

Hence, L_3 is not regular.

(d) $\boxed{L_4 = \{(10)^p 1^q \mid p, q \in \mathbb{N}, p \geq q\}}$

First, we assume that L_4 is a regular language. Then we'll prove by contradiction that L_4 is not regular language.

According to our assumption, L_4 is regular and hence, according to the pumping lemma, there exists an integer $n \geq 1$ for which the property of the lemma holds.

Let's define $\boxed{w = (10)^{n+1} 1^n}$

we see that $w \in L_3$.

Now, $|w| = 2(n+1) + n = (3n+2) \geq n$

So, according to the pumping lemma, there exists strings $x, y, z \in \{0,1\}^*$ such that $w = xyz$ and

A) $y \neq \varepsilon$ (i.e. $|y| \geq 1$)

B) $|xy| \leq n$

C) $xy^iz \in L_4$ $\forall i \in N$ (i.e. $i \geq 0$)

$\boxed{\text{proof part}}$: we assume that $y = (10)^K$

Since, $y \neq \varepsilon$, $\therefore$ $K \geq 1$

also, $|xy| \leq n$ and hence, $1 \leq K \leq n$.

Now, we have to satisfy our third condition.

$$xy^iz = (10)^{n+1+(i-1)K} \, 1^n$$

Since, $xy^iz \in L_4$ for all $i \geq 0$, the condition should hold true for $i = 0$ also.

put $i = 0$, $\quad xy^0z = (10)^{n+1-K} \cdot 1^n$.

now, since $K \geq 1$, $(n+1-K) \leq n$

if that so, $xy^0z \notin L_4$, since in L_4, if $w = (10)^p 1^q$, then $p \geq q$.

Hence, we are getting contradiction.

This proves our initial assumption was not true.

$\therefore$ L_4 isn't regular.