TOC Assignment → 1

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- Suppose, $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA, then
  $L(M) \rightarrow$ set of all strings that are accepted by M.
  $L(M) = \{ w \in \Sigma^* : M \text{ accepts } w \}$

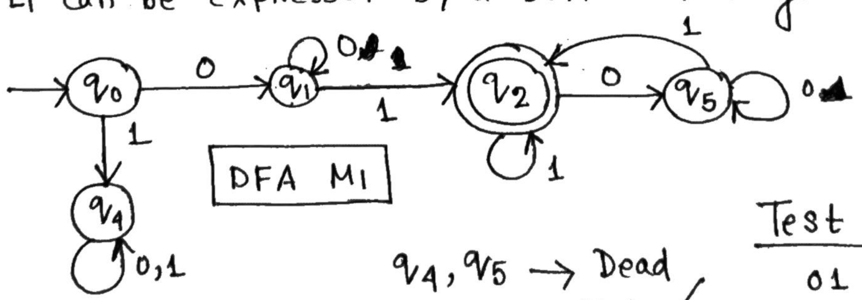
- $\boxed{\text{Regular Language}}$: Let Σ be an alphabet and $A \subseteq \Sigma^*$ be a language over Σ.
  The language A is regular, if there exists a DFA M, such that
  $A = L(M)$.

- $\boxed{\text{Question}}$: a) $L_1 = \{ w \in \{0,1\}^* \mid w$ starts with a 0 and ends with a 1 $\}$

  b) $L_2 = \{ w \in \{0,1\}^* \mid w$ contains 1010 as a substring $\}$

$\Rightarrow$ In both the cases, $\Sigma = \{0,1\}$
$L_1 \subseteq \Sigma^*$, $L_2 \subseteq \Sigma^*$

i) L_1 can be expressed by a DFA state diagram.

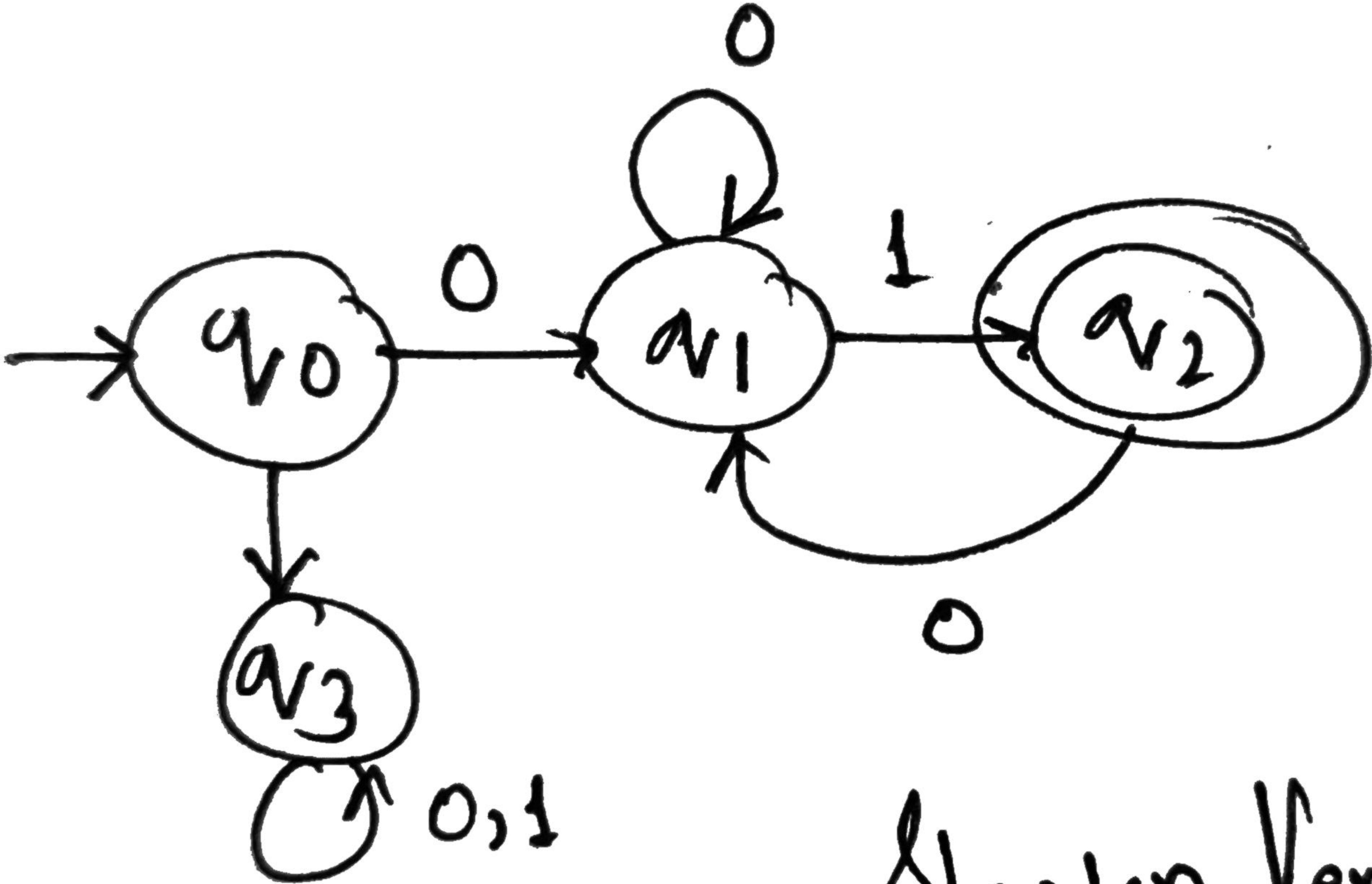


DFA M_1

$q_4, q_5 \rightarrow$ Dead State / Trap State.

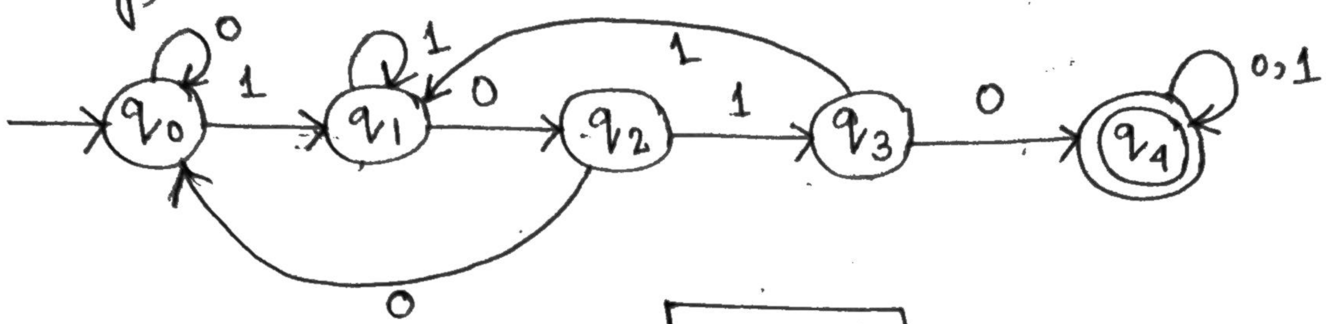
Test case :
01 ✓
0011 ✓
01101 ✓
0110100 ✗

Here, DFA M_1 ~~Satisf~~ takes / recognises L_1.
Hence L_1 is regular language.



Shorter Version

Similarly, DFA for L_2 can also be created.



$\boxed{\text{DFA } M_2}$

Test Case

1010 ✓
001010 ✓
101001 ✓
1001010 ✓
1111010 ✓
0101010 ✓
1001001010 ✓
1001101010 ✓
1011010 ✓

Hence, L_2 is also Regular language, because, $L_2 = L(M_2)$.

- Now, regular languages are closed wrt. regular operations.

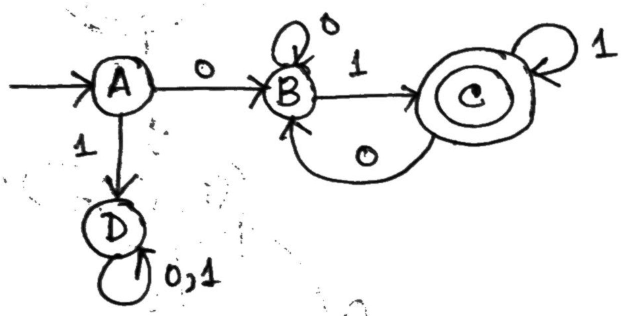
If L_1 and L_2 (both $\subseteq \Sigma^*$) are regular, then $(L_1 \cup L_2)$ will also be a regular language.

Here, both L_1 $(= L(M_1))$ and L_2 $[= L(M_2)]$ are regular languages over the alphabet $\Sigma = \{0, 1\}$

Hence, $(L_1 \cup L_2) = L(M_1) \cup L(M_2)$ is also regular.

Since both M_1 and M_2 states diagram is written with the help of $q_0, q_1, \ldots$ etc, I'm changing the names a bit so that they are distinct while making their union DFA.

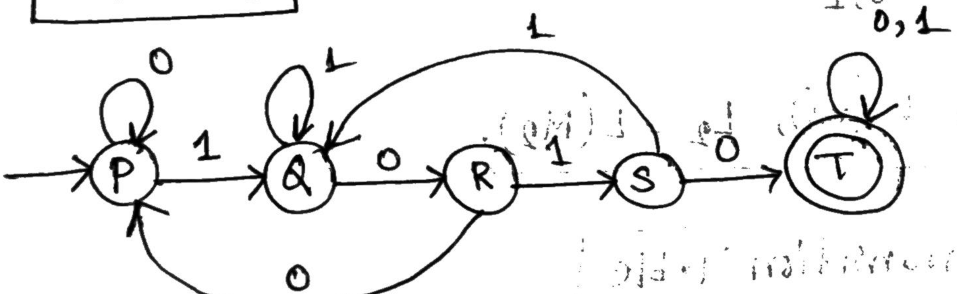
$\boxed{\text{DFA } M_1} = (\{A, B, C, D\}, A, \{C\}, \delta_1, \{0,1\})$



where,

$$\begin{cases} \delta_1(A,0) = B, \ \delta_1(B,0) = B \\ \delta_1(A,1) = D, \ \delta_1(B,1) = C \\ \delta_1(D,0) = D, \ \delta_1(C,0) = B \\ \delta_1(D,1) = D, \ \delta_1(C,1) = C \end{cases}$$

$\boxed{\text{DFA } M_2} =$



$M_2 = (\{P, Q, R, S, T\}, P, \{T\}, \delta_2, \{0,1\})$.

where,

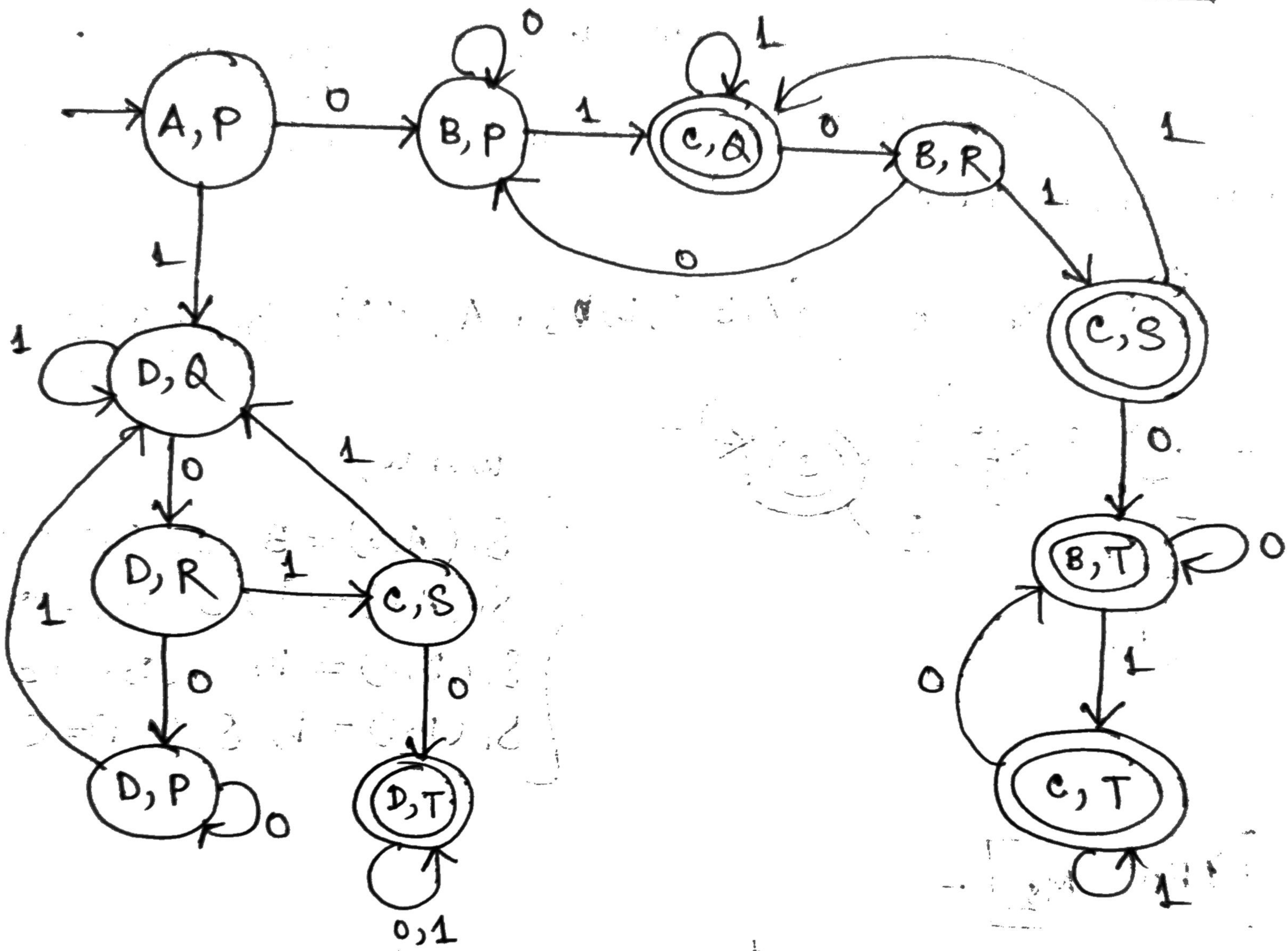
$\delta_2(P,0) = P$	$\delta_2(Q,0) = R$	$\delta_2(R,0) = P$
$\delta_2(P,1) = Q$	$\delta_2(Q,1) = Q$	$\delta_2(R,1) = S$

$\delta_2(S,0) = T, \ \delta_2(S,1) = Q, \ \delta_2(T,0) = T, \ \delta_2(T,1) = T$

STATE Transition Table

State (M_1, M_2) form	0	1
→(A, P) initial	(B, P)	(D, Q)
(B, P)	(B, P)	(C, Q)
O (C, Q) final	(B, R)	~~(C, S)~~ (C, Q)
(B, R)	(B, P)	(C, S)
(C, S)	(B, T)	(C, Q)
O (B, T) final	(B, T)	(C, T)
O (C, T) final	(B, T)	(C, T)
(D, Q)	(D, R)	(D, Q)
(D, R)	(D, P)	(C, S)
O (C, S) final	(D, T)	(D, Q)
(D, P)	(D, P)	(D, Q)
O D, T final.	(D, T)	(D, T)

Now, State Diagram of DFA $L(M_1) \cup L(M_2)$



Here, $L_1 = L(M_1)$, $L_2 = L(M_2)$.