

The robotic arm consists of two links of length  $l_1 = 1$  and  $l_2 = 1$ . The configuration is defined by joint angles  $q_1$  and  $q_2$ .

### Task 1: Position Calculations

To visualize the arm, we calculate the coordinates as follows:

- **Base:** Fixed at the origin  $(0, 0)$ .
- Elbow Joint  $(x_1, y_1)$ :

$$x_1 = l_1 \cos(q_1)$$

$$y_1 = l_1 \sin(q_1)$$

- End-Effector  $(x, y)$ :

$$x = l_1 \cos(q_1) + l_2 \cos(q_1 + q_2)$$

$$y = l_1 \sin(q_1) + l_2 \sin(q_1 + q_2)$$

#### Straight Arm ( $q_1=0^\circ, q_2=0^\circ$ ):

- Elbow:  $x_1 = 1\cos(0) = 1, y_1 = 1\sin(0) = 0$
- End-Effector:  $x = 1\cos(0) + 1\cos(0) = 2, y = 1\sin(0) + 1\sin(0) = 0$

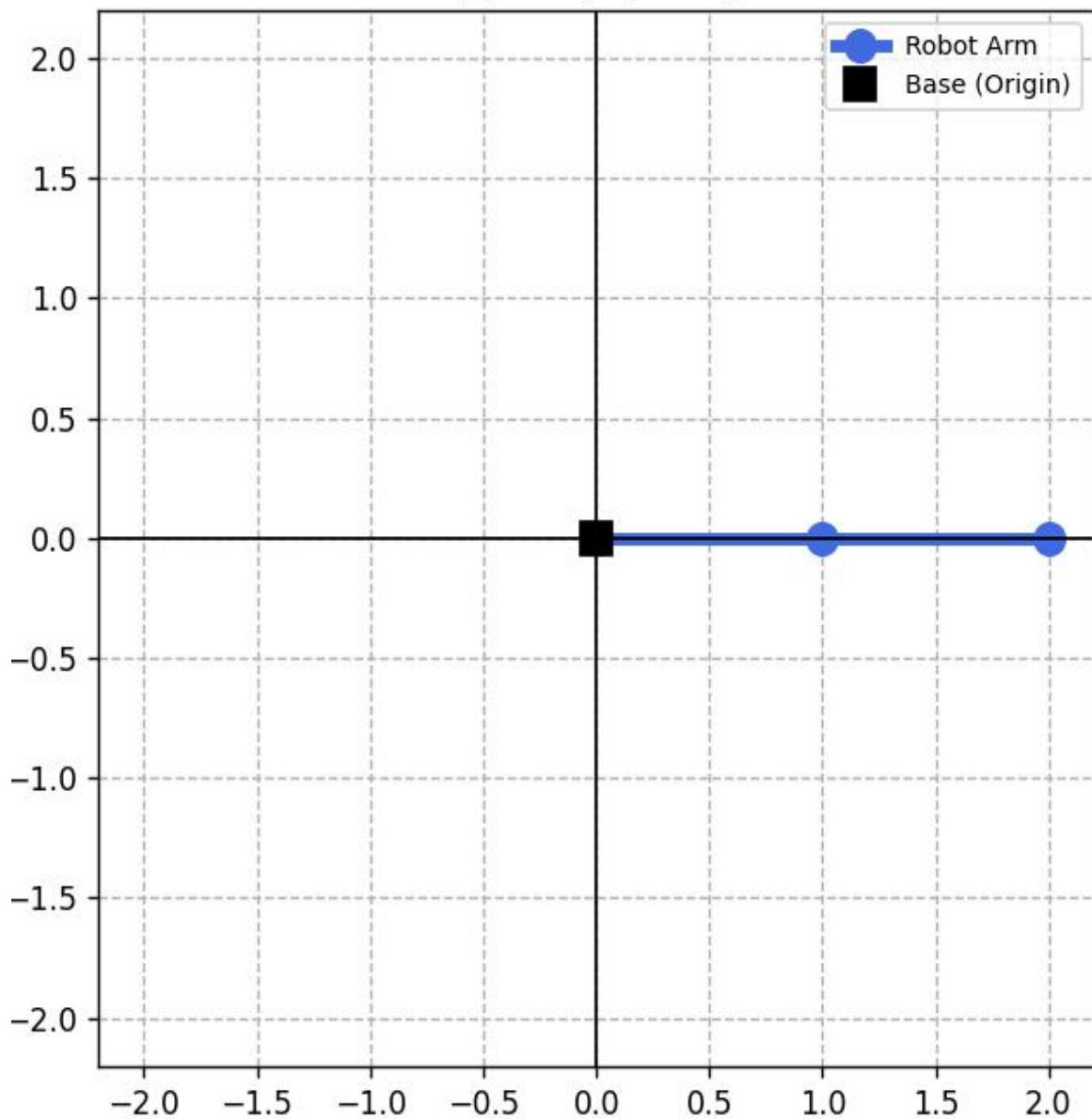
#### Bent Elbow ( $q_1=45^\circ, q_2=90^\circ$ ):

- Elbow:  $x_1 = 0.707, y_1 = 0.707$
- End-Effector:  $x = \cos(45^\circ) + \cos(135^\circ) = 0, y = \sin(45^\circ) + \sin(135^\circ) = 1.414$

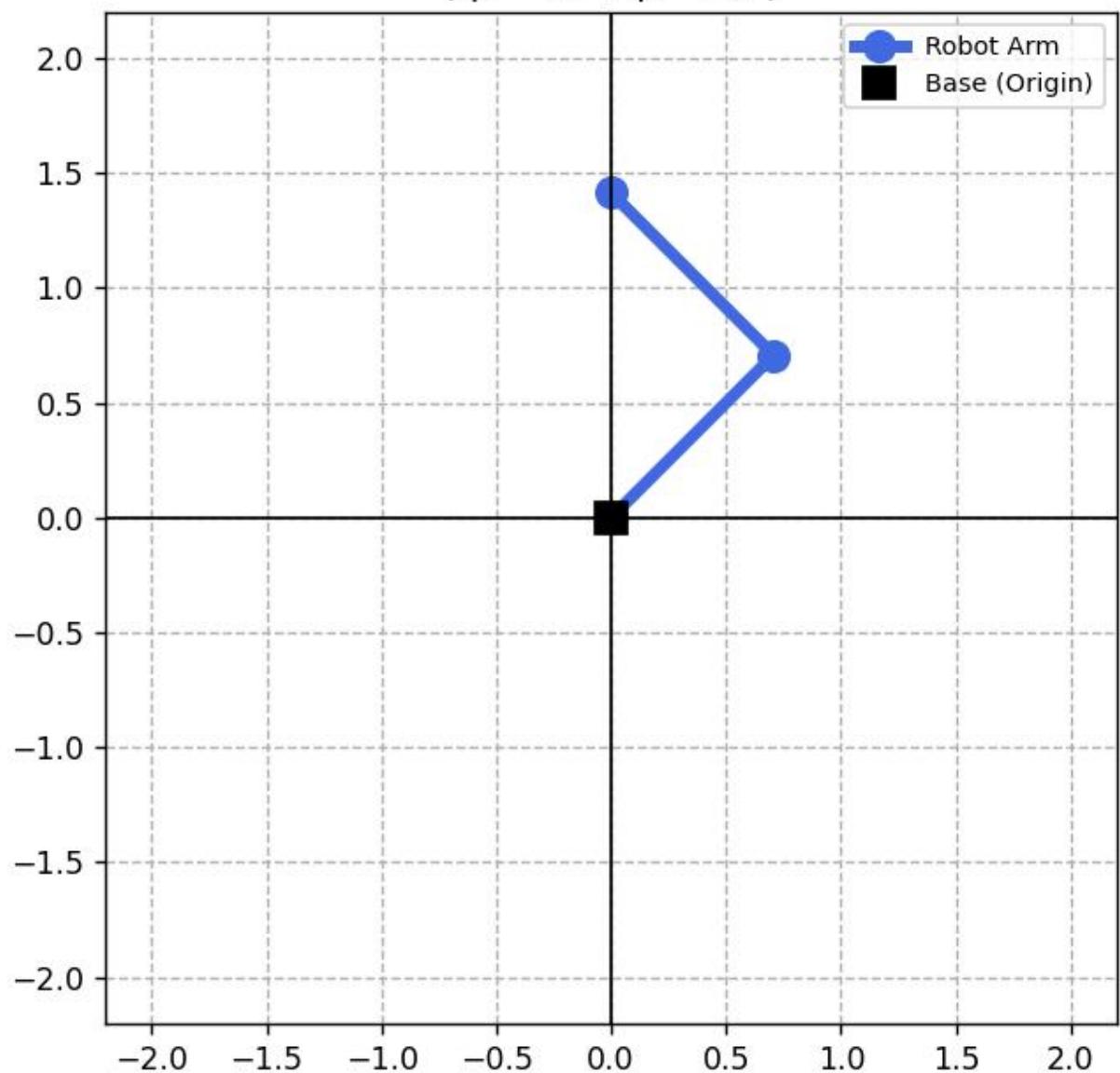
#### Folded Arm ( $q_1=0^\circ, q_2=180^\circ$ ):

- Elbow:  $x_1 = 1, y_1 = 0$
- End-Effector:  $x = \cos(0) + \cos(180^\circ) = 0, y = \sin(0) + \sin(180^\circ) = 0$

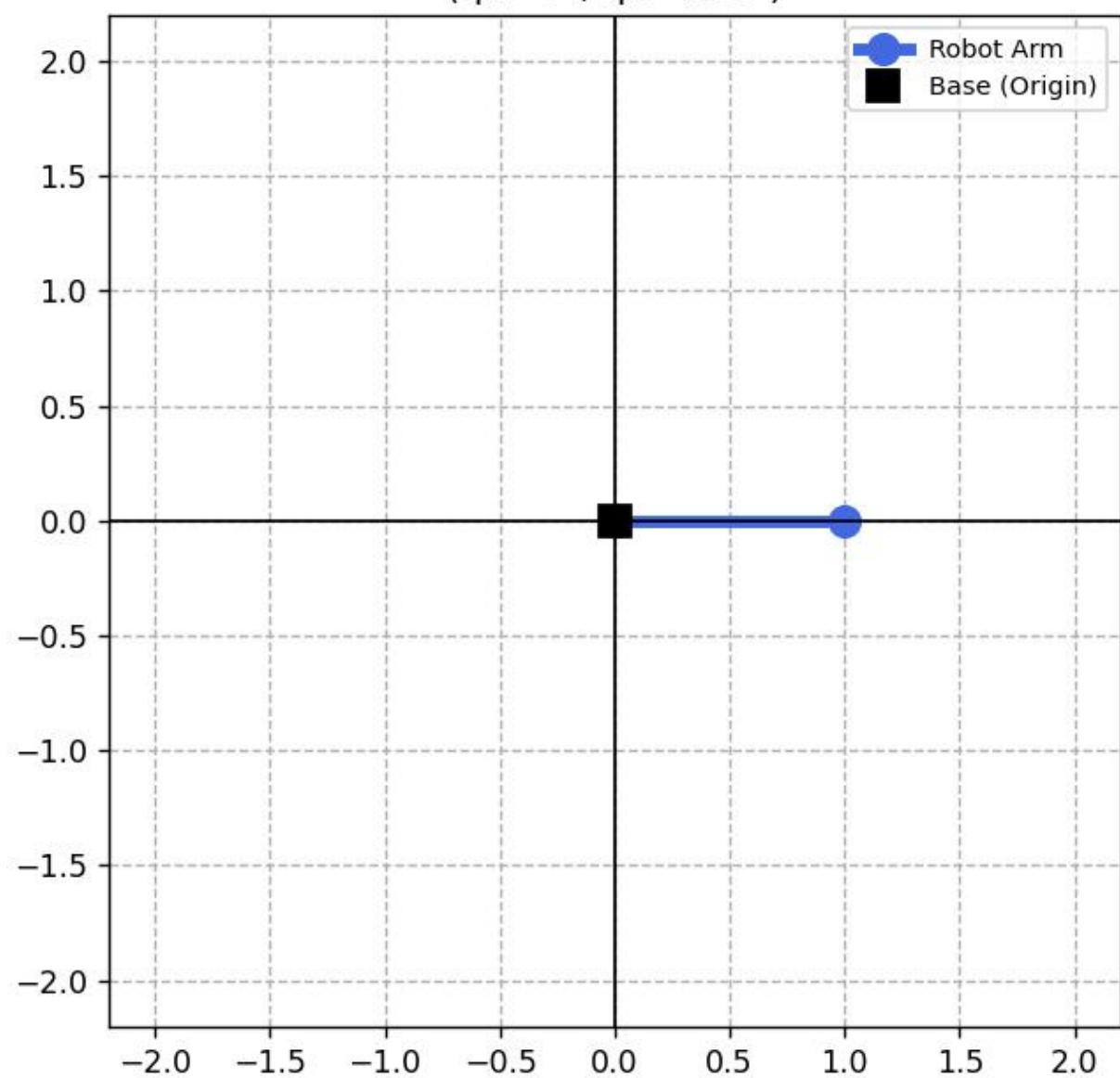
**Straight Arm**  
 $(q_1=0^\circ, q_2=0^\circ)$



Bent Elbow  
( $q_1=45^\circ$ ,  $q_2=90^\circ$ )



Folded Arm  
( $q_1=0^\circ$ ,  $q_2=180^\circ$ )



### Task 3:

Python script-

```
import numpy as np

import matplotlib.pyplot as plt

def plot_robotic_arm(q1_deg, q2_deg, ax, title):

    l1 = 1

    l2 = 1

    q1 = np.radians(q1_deg)

    q2 = np.radians(q2_deg)

    x1 = l1 * np.cos(q1)

    y1 = l1 * np.sin(q1)

    x2 = l1 * np.cos(q1) + l2 * np.cos(q1 + q2)

    y2 = l1 * np.sin(q1) + l2 * np.sin(q1 + q2)

    ax.plot([0, x1, x2], [0, y1, y2], marker='o', markersize=10, linewidth=4, color='royalblue',
            label='Robot Arm')

    ax.plot(0, 0, 'ks', markersize=10, label='Base (Origin)')

    ax.set_title(f"title\nq1={q1_deg}°, q2={q2_deg}°", fontsize=12)

    ax.set_xlim(-2.2, 2.2)

    ax.set_ylim(-2.2, 2.2)

    ax.set_aspect('equal')

    ax.grid(True, linestyle='--')

    ax.axhline(0, color='black', lw=1)

    ax.axvline(0, color='black', lw=1)

    ax.legend(loc='upper right', fontsize='small')

fig, axs = plt.subplots(1, 3, figsize=(18, 6))

plot_robotic_arm(0, 0, axs[0], "Straight Arm")

plot_robotic_arm(45, 90, axs[1], "Bent Elbow")

plot_robotic_arm(0, 180, axs[2], "Folded Arm")

plt.tight_layout()

plt.show()

configs = [(0, 0), (45, 90), (0, 180)]
```

```

print("Coordinate Results for Task 1:")

for q1, q2 in configs:

    r1, r2 = np.radians(q1), np.radians(q2)

    ex, ey = np.cos(r1), np.sin(r1)

    eex = ex + np.cos(r1 + r2)

    eey = ey + np.sin(r1 + r2)

    print(f"Angles({{q1},{q2}}) -> Elbow: {{ex:.2f}, {ey:.2f}}, End-Effector: {{eex:.2f}, {eey:.2f}}")

```

## Analysis of Arm Position and Workspace

The configuration of the 2-link planar robotic arm is governed by the two joint angles,  $q_1$  and  $q_2$ , which determine the position of the elbow and the end-effector in the Cartesian plane.

### 1. Effect of Joint Angles on Position

- **Angle  $q_1$  (Base Joint):** This angle controls the rotation of the entire arm assembly relative to the origin. Changing  $q_1$  moves the arm in a circular arc around the base, determining the direction in which the arm points.
- **Angle  $q_2$  (Elbow Joint):** This angle defines the orientation of the second link relative to the first. It primarily affects the "extension" or "reach" of the arm.
  - When  $q_2 = 0^\circ$ , the arm is **fully extended** (straight), reaching its maximum distance from the origin.
  - When  $q_2 = 180^\circ$ , the arm is **folded back**, bringing the end-effector back toward the base.

### 2. Workspace Analysis

- **Reachable Workspace:** Since the links are of equal length ( $l_1 = l_2 = 1$ ), the total maximum reach is  $l_1 + l_2 = 2$ .
- **Geometry:** The workspace is a **solid disk** centered at the origin with a radius of 2.
- **Accessibility:** Because the links are equal, the arm can reach any point from the boundary ( $r = 2$ ) all the way down to the origin ( $r = 0$ ), providing a full range of motion within that circular area.