[redacted] (/)

Red-Black Trees | Insertion

Before moving to the procedure of insertion and deletion, let's look at an important operation which is not only used by red-black trees but many other binary search trees also.

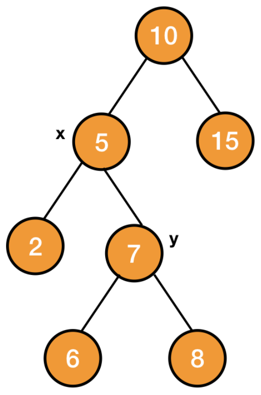
Rotations in Binary Search Tree

There are two types of rotations:

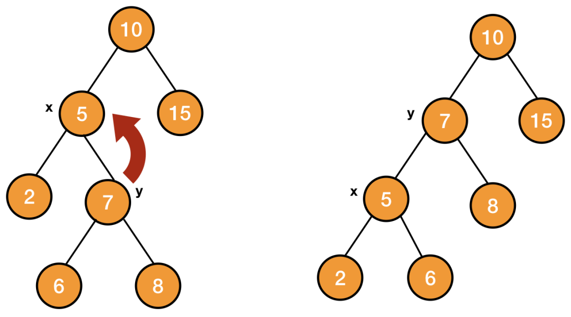
- Left Rotation
- Right Rotation

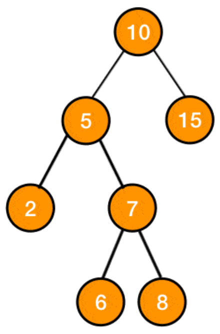
In left rotation, we assume that the right child is not null. Similarly, in the right rotation, we assume that the left child is not null.

Consider the following tree:

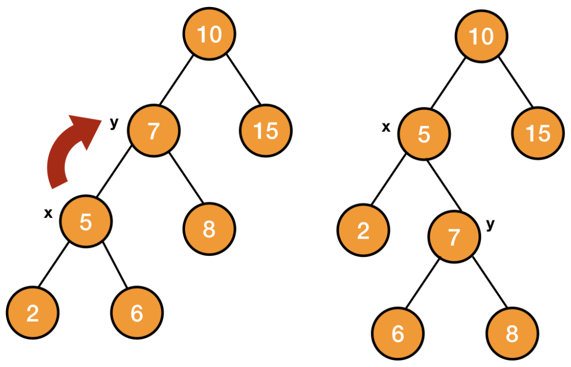


After applying left rotation on the node *x*, the node *y* will become the new root of the subtree and its left child will be *x*. And the previous left child of *y* will now become the right child of *x*.

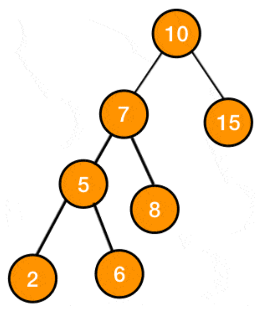




Now applying right rotation on the node *y* of the rotated tree, it will transform back to the original tree.



So right rotation on the node *y* will make *x* the root of the tree, *y* will become *x*'s right child. And the previous right child of *x* will now become the left child of *y*.



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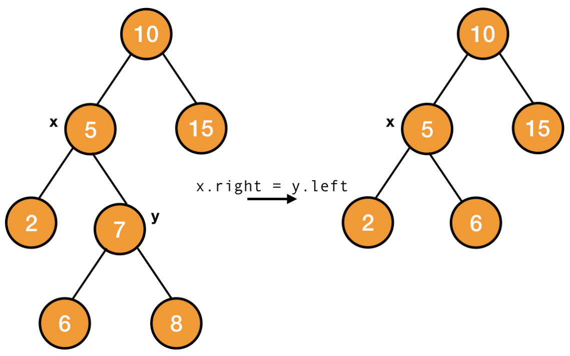
Take a note that rotation doesn't violate the property of binary search trees.

Code of Rotations

We are going to explain the code for left rotation here. The code for the right rotation will be symmetric.

We need the tree *T* and the node *x* on which we are going to apply the rotation - `LEFT_ROTATION(T, x)`.

The left grandchild of *x* (left child of the right child *x*) will become the right child of it after rotation. We will do this but before doing this, let's mark the right child of *x* as *y*.



```
LEFT_ROTATION(T, x)
  y = x.right
  x.right = y.left
```

The left child of *y* is going to be the right child of *x* - `x.right = y.left`. We also need to change the parent of `y.left` to *x*. We will do this if the left child of *y* is not `NULL`.

```
if y.left != NULL
  y.left.parent = x
```

Then we need to put *y* to the position of *x*. We will first change the parent of *y* to the parent of *x* - `y.parent = x.parent`. After this, we will make the node *x* the child of *y*'s parent instead of *y*. We will do so by checking if *y* is the right or left child of its parent. We will also check if *y* is the root of the tree.

```
y.parent = x.parent
if x.parent == NULL //x is root
  T.root = y
elseif x == x.parent.left // x is left child
  x.parent.left = y
else // x is right child
  x.parent.right = y
```

At last, we need to make *x* the left child of *y*.

```
y.left = x
x.parent = y
```

```
LEFT_ROTATE(T, x)
    y = x.right
    x.right = y.left
    if y.left != NULL
        y.left.parent = x
    y.parent = x.parent
    if x.parent == NULL //x is root
        T.root = y
    elseif x == x.parent.left // x is left child
        x.parent.left = y
    else // x is right child
        x.parent.right = y
    y.left = x
    x.parent = y
```

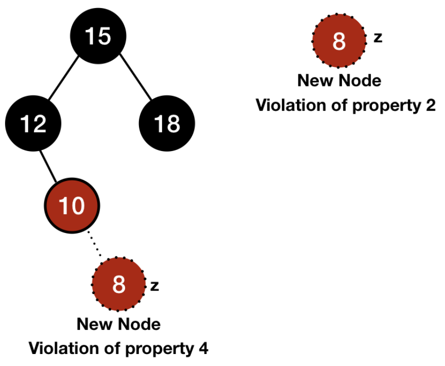
From the above code, you can easily see that rotation is a constant time taking process (*[Math Processing Error]*).

Now that we know how to perform rotation, we will use this to restore red-black properties when they get violated after adding or deleting any node. So, let's learn to insert a new node to a red-black tree.

Insertion

We insert a new node to a red-black tree in a similar way as we do in a normal binary search tree. We just call a function at the last to fix any kind of violations that could have occurred in the process of insertion.

We color any newly inserted node to red. Doing so can violate the property 4 of red-black trees which we will fix after the insertion process as stated above. There can be a violation of property 2 also but it can be easily fixed by coloring the root black. Also, there can't be any other violations.



So, we will fix these properties after inserting the node.

Since there can't be consecutive red nodes, the changes that this violation could happen is half. Also, fixing the violation of property 4 is easy. Think of a case when the newly inserted node is black. This would affect the black height and fixing that would be difficult. This is the reason why we don't color any new node to black.

Code for Insertion

As told earlier, the entire code for insertion is the same and we will just call a function to fix the violation of properties of red-black trees. We will also color the newly added node to red.

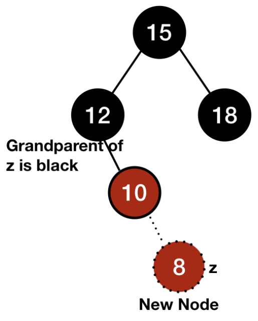
```
INSERT(T, n)
    y = T.NIL
    temp = T.root

    while temp != T.NIL
        y = temp
        if n.data < temp.data
            temp = temp.left
        else
            temp = temp.right
    n.parent = y
    if y==T.NIL
        T.root = n
    else if n.data < y.data
        y.left = n
    else
        y.right = n

    n.left = T.NIL
    n.right = T.NIL
    n.color = RED
    INSERT_FIXUP(T, n)
```

Here, we have used `T.NIL` instead of `NULL` unlike we do with normal binary search tree. Also, those `T.NIL` are the leaves and they all are black, so there won't be a violation of property 3.

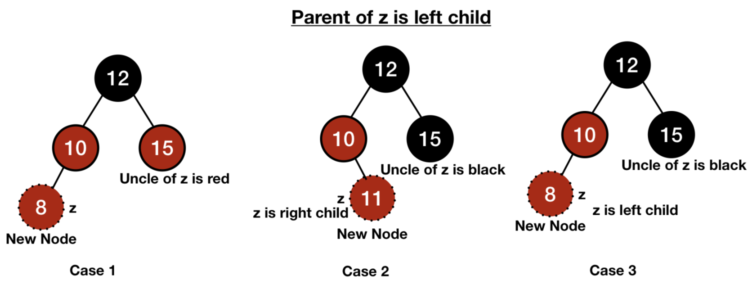
In the last few lines, we are making the *left* and *right* of the new node `T.NIL` and also making it red. At last, we are calling the function to fix the violations of the red-black properties. Rest of the code is similar to a normal binary search tree.

Let's focus on the function to fix the violations.

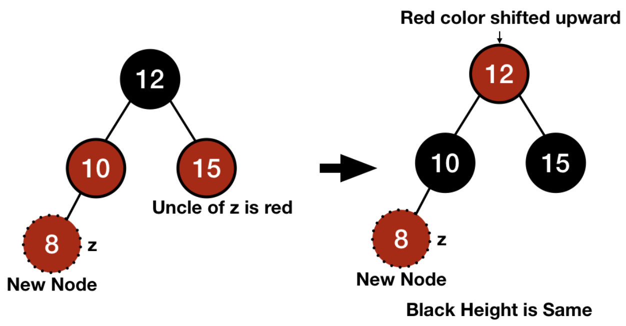
The property 4 will be violated when the parent of the inserted node is red. So, we will fix the violations if the parent of the new node is red. At last, we will color the root black and that will fix the violation of property 2 if it is violated. In the case of violation of property 4 (when the parent of the new node is red), the grandparent will be black.



There can be six cases in the case of violation of the property 4. Let's look at the given pictures first assuming that the parent of the node *z* is a left child of its parent which gives us the first three cases. The other three cases will be symmetric when the node *z* will be the right child of its parent.

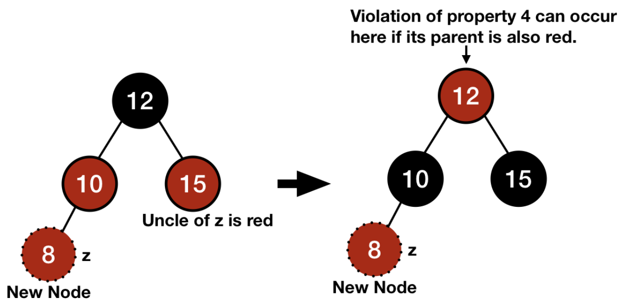


The first case is when the uncle of *z* is also red. In this case, we will shift the red color upward until there is no violation. Otherwise, if it reaches to the root, we can just color it black without any consequences.



So, the process is to make both the parent and the uncle of *z* black and its grandparent red. In this way, the black height of any node won't be affected and we can successfully shift the red color upward.

However, making the grandparent of *z* red might cause violation of the property 4 there. So, we will do the fixup again on that node.

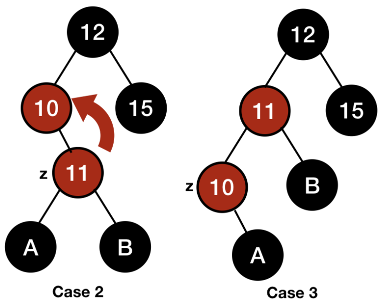


Violation of property 4 can occur here if its parent is also red.
12
10
15
Uncle of z is red
8
z
New Node
12
10
15
8
z
New Node

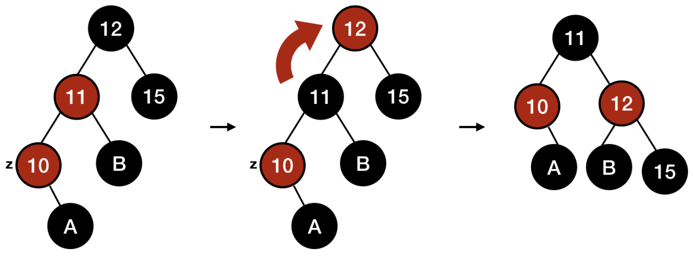
In the second case, the uncle of the node *z* is black and the node *z* is the right child.

In the third case, the uncle of the node *z* is black and the node *z* is the left child.

We can transform the second case into the third one by performing left rotation on the parent of the node *z*. Since both *z* and its parent are red, so rotation won't affect the black height.

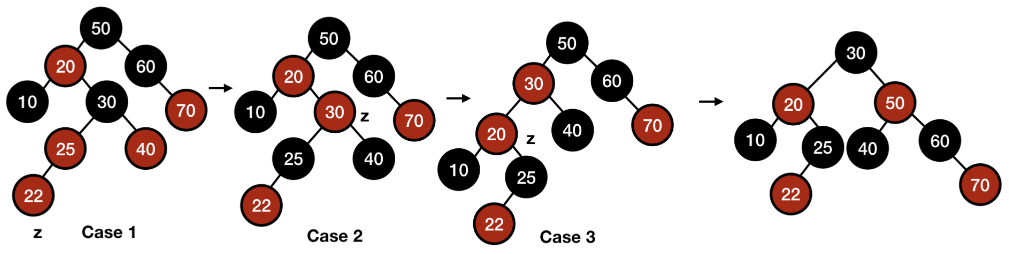


In case 3, we first color the parent of the node *z* black and its grandparent red and then do a right rotation on the grandparent of the node *z*. This fixes the violation of properties completely. This is shown in the picture given below.



Since we are making the parent of *z* black in both case 2 and case 3, the loop will stop in these two cases.

Let's look at the following example first.



Similarly, there will be three cases when the parent of *z* will be the right child but those cases will be symmetric to the above cases only with left and right exchanged.

Let's implement the above concepts into code.

Code for Fixup

Let's look at the code for fixing the violation first.

```
INSERT_FIXUP(T, z)
     while z.parent.color == red

          if z.parent == z.parent.parent.left //z.parent is left child
               y = z.parent.parent.right //uncle of z

               if y.color == red //case 1
                    z.parent.color = black
                    y.color = black
                    z.parent.parent.color = red
                    z = z.parent.parent

               else //case 2 or 3
                    if z == z.parent.right //case 2
                         z = z.parent //marked z.parent as new z
                         LEFT_ROTATE(T, z) //rotated parent of orignal z

                    //case 3
                    z.parent.color = black // made parent black
                    z.parent.parent.color = red // made grandparent red
                    RIGHT_ROTATE(T, z.parent.parent) // right rotation on grandparent

          else // z.parent is right child
               code will be symmetric
     T.root.color = black
```

We need to pass the tree and the node added to the function - `INSERT-FIXUP(T, z)`. As discussed above, in case of the violation of the property 4, the parent of the node will be red, so we need to perform fixup only if the parent of *z* is red. At last, we will color the root black which will fix the violation of the property 2 if it is violated.

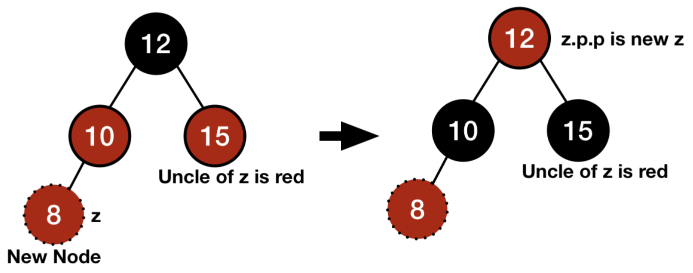
```
INSERT-FIXUP(T, z)
   while z.parent.color == red
      ...
```

We are going to write the code when the parent of *z* is the left child. The code when the parent of *z* is the right child will be symmetric.

```
INSERT-FIXUP(T, z)
   while z.parent.color == red
      if z.parent == z.parent.parent.left //z.parent is left child
      ...
    else // z.parent is right child
      code will be symmetric
```

Next, we have marked the uncle of *z* as *y*. Now, the first case is when the color of the node *y* is red.

In that case, we need to change the color of the parent and the uncle to black and the grandparent to red. After doing this, we shifted *z* two levels up and now we need to check if the grandparent is violating the rules or not. Therefore, we will point *z* to its grandparent i.e., `z = z.parent.parent`. It is shown in the picture given below.



Since `z.parent.parent` is new the *z*, the loop will check it for violations as it is *z* now.

```
INSERT-FIXUP(T, z)
  while z.parent.color == red
    if z.parent == z.parent.parent.left //z.parent is left child
      y = z.parent.parent.right //uncle of z

      if y.color == black // case 1
        z.parent.color = black
        y.color = black
        z.parent.parent.color = red
        z = z.parent.parent // loop will check this new z
```

If *y* is not red, then we will have either case 2 or case 3. If *z* is the right child, then it will be case 2.

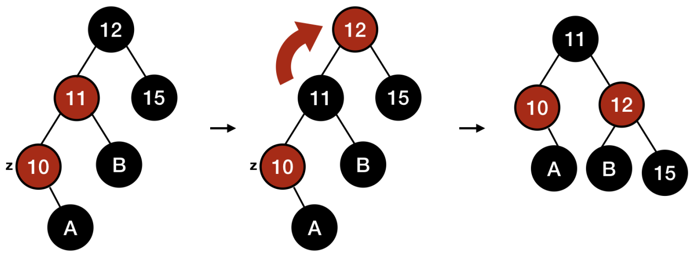
```
INSERT-FIXUP(T, z)
  while z.parent.color == red
    if z.parent == z.parent.parent.left //z.parent is left child
      y = z.parent.parent.right //uncle of z

      if y.color == black // case 1
        ...

      else //case 2 or 3
        if z = z.parent.right // case 2
```

As we have discussed, we are going to transform case 2 to case 3 by doing a left rotation on the parent of *z* and mark it as new *z*.

```
INSERT-FIXUP(T, z)
  while z.parent.color == red
    if z.parent == z.parent.parent.left //z.parent is left child
      y = z.parent.parent.right //uncle of z

      if y.color == black // case 1
        ...

      else //case 2 or 3
```

```
        if z = z.parent.right // case 2
          z = z.parent // marked z.parent as new z
          LEFT_ROTATE(T, z) //rotated parent of orignal z
```

After this, we will handle case 3. We will make the parent of *z* black and its grandparent red. Then, we will do a right rotation on its grandparent.



```
z.parent.color = black
z.parent.parent.color = red
RIGHT_ROTATE(T, z.parent.parent)
```

Either case 2 or case 3 is making the parent of *z* black. In both the cases, the loop is not going to iterate further.

At last, we need to make the root of the tree black because it might be possible that case 1 led the red node up till root.

```
INSERT-FIXUP(T, z)
  while z.parent.color == red
    if z.parent == z.parent.parent.left //z.parent is left child
      ...

      if y.color == black // case 1
        ...

      else
        ...

  T.root.color = black
```

C **Python** **Java**

```c
#include <stdio.h>
#include <stdlib.h>

enum COLOR {Red, Black};

typedef struct tree_node {
  int data;
  struct tree_node *right;
  struct tree_node *left;
  struct tree_node *parent;
  enum COLOR color;
}tree_node;

typedef struct red_black_tree {
  tree_node *root;
  tree_node *NIL;
}red_black_tree;

tree_node* new_tree_node(int data) {
  tree_node* n = malloc(sizeof(tree_node));
  n->left = NULL;
  n->right = NULL;
  n->parent = NULL;
  n->data = data;
  n->color = Red;

  return n;
}

red_black_tree* new_red_black_tree() {
  red_black_tree *t = malloc(sizeof(red_black_tree));
  tree_node *nil_node = malloc(sizeof(tree_node));
  nil_node->left = NULL;
  nil_node->right = NULL;
  nil_node->parent = NULL;
  nil_node->color = Black;
  nil_node->data = 0;
  t->NIL = nil_node;
  t->root = t->NIL;

  return t;
}

void left_rotate(red_black_tree *t, tree_node *x) {
  tree_node *y = x->right;
  x->right = y->left;
  if(y->left != t->NIL) {
    y->left->parent = x;
  }
  y->parent = x->parent;
  if(x->parent == t->NIL) { //x is root
    t->root = y;
  }
  else if(x == x->parent->left) { //x is left child
    x->parent->left = y;
  }
  else { //x is right child
    x->parent->right = y;
  }
  y->left = x;
  x->parent = y;
}

void right_rotate(red_black_tree *t, tree_node *x) {
  tree_node *y = x->left;
  x->left = y->right;
  if(y->right != t->NIL) {
    y->right->parent = x;
  }
  y->parent = x->parent;
  if(x->parent == t->NIL) { //x is root
    t->root = y;
  }
  else if(x == x->parent->right) { //x is left child
    x->parent->right = y;
  }
  else { //x is right child
```

```
    x->parent->left = y;
  }
  y->right = x;
  x->parent = y;
}

void insertion_fixup(red_black_tree *t, tree_node *z) {
  while(z->parent->color == Red) {
    if(z->parent == z->parent->parent->left) { //z.parent is the left child

      tree_node *y = z->parent->parent->right; //uncle of z

      if(y->color == Red) { //case 1
        z->parent->color = Black;
        y->color = Black;
        z->parent->parent->color = Red;
        z = z->parent->parent;
      }
      else { //case2 or case3
        if(z == z->parent->right) { //case2
          z = z->parent; //marked z.parent as new z
          left_rotate(t, z);
        }
        //case3
        z->parent->color = Black; //made parent black
        z->parent->parent->color = Red; //made parent red
        right_rotate(t, z->parent->parent);
      }
    }
    else { //z.parent is the right child
      tree_node *y = z->parent->parent->left; //uncle of z

      if(y->color == Red) {
        z->parent->color = Black;
        y->color = Black;
        z->parent->parent->color = Red;
        z = z->parent->parent;
      }
      else {
        if(z == z->parent->left) {
          z = z->parent; //marked z.parent as new z
          right_rotate(t, z);
        }
        z->parent->color = Black; //made parent black
        z->parent->parent->color = Red; //made parent red
        left_rotate(t, z->parent->parent);
      }
    }
  }
  t->root->color = Black;
}

void insert(red_black_tree *t, tree_node *z) {
  tree_node* y = t->NIL; //variable for the parent of the added node
  tree_node* temp = t->root;

  while(temp != t->NIL) {
    y = temp;
    if(z->data < temp->data)
      temp = temp->left;
    else
      temp = temp->right;
  }
  z->parent = y;

  if(y == t->NIL) { //newly added node is root
    t->root = z;
  }
  else if(z->data < y->data) //data of child is less than its parent, left child
    y->left = z;
  else
    y->right = z;

  z->right = t->NIL;
  z->left = t->NIL;

  insertion_fixup(t, z);
}
```

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```
void inorder(red_black_tree *t, tree_node *n) {
  if(n != t->NIL) {
    inorder(t, n->left);
    printf("%d\n", n->data);
    inorder(t, n->right);
  }
}

int main() {
  red_black_tree *t = new_red_black_tree();

  tree_node *a, *b, *c, *d, *e, *f, *g, *h, *i, *j, *k, *l, *m;
  a = new_tree_node(10);
  b = new_tree_node(20);
  c = new_tree_node(30);
  d = new_tree_node(100);
  e = new_tree_node(90);
  f = new_tree_node(40);
  g = new_tree_node(50);
  h = new_tree_node(60);
  i = new_tree_node(70);
  j = new_tree_node(80);
  k = new_tree_node(150);
  l = new_tree_node(110);
  m = new_tree_node(120);

  insert(t, a);
  insert(t, b);
  insert(t, c);
  insert(t, d);
  insert(t, e);
  insert(t, f);
  insert(t, g);
  insert(t, h);
  insert(t, i);
  insert(t, j);
  insert(t, k);
  insert(t, l);
  insert(t, m);

  inorder(t, t->root);

  return 0;
}
```

Analysis of Insertion

The insertion of a new node will be performed in *[Math Processing Error]* time becasue we have already discussed that the height of a red-black tree is *[Math Processing Error]*. In the code to fix the violation, only case 1 is going to make the loop iterate further but not case 2 or 3 because they are making the parent of *z* black and the loop will only iterate if it is red.

The Case 1 is shifting *z* two levels up. So even if it reaches the root, it will do it in *[Math Processing Error]* time. Therefore, the total that can be taken by the entire process of insertion is still *[Math Processing Error]*.

The last part to get a full-fledged working red-black tree is to delete any node from it and we will learn it in the next chapter.

“ If you wish to make an apple pie from scratch, you must first invent the universe ”

- Carl Sagan

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