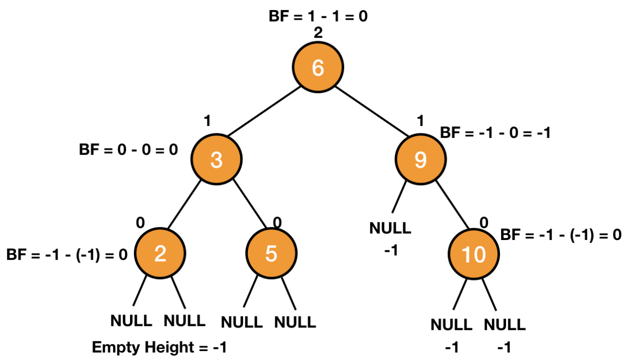
[redacted] (/)

AVL Trees

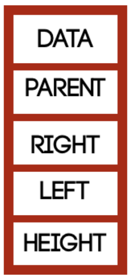
Similar to red-black trees, **AVL (Adelson-Velskii and Landis)** trees are also binary search trees which are pretty good balanced. AVL trees use different rules to achieve that balance.

AVL trees use **balance factor** to get a balanced tree. **Balance factor** of any node is defined as **height(left subtree) - height(right subtree)**.

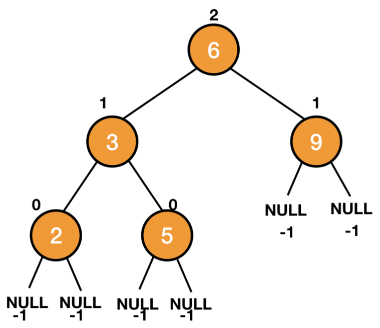


For an AVL tree, the absolute value of balance factor for any node can't be greater than 1 i.e., each node must have a balance factor of either -1, 0 or 1.

Instead of calculating heights of nodes, again and again, we store the current heights in each node. Look at the following picture.



If there is only one child of a node, then the height of the other child (empty height) is assumed to be -1. It is shown in the picture given below.

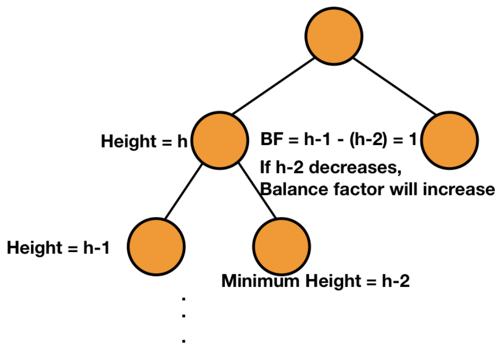


We have claimed that if the balance factors of each node are one of -1, 0 or 1, then the tree is balanced and the height of the tree will be *[Math Processing Error]*. Let's prove this claim.

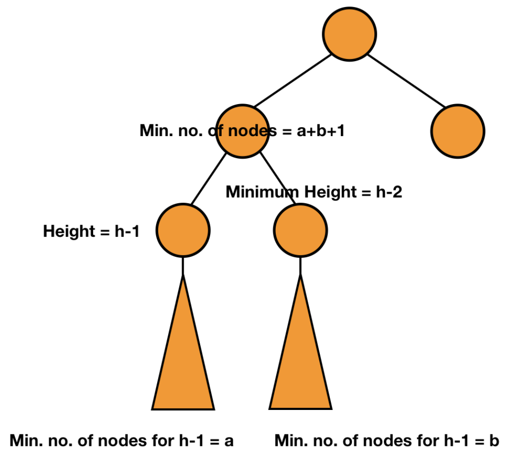
Height of an AVL Tree

Let *[Math Processing Error]* be the minimum number of nodes in an AVL tree of height *[Math Processing Error]*. We can say that *[Math Processing Error]* and *[Math Processing Error]*.

Let there be a node with a height *[Math Processing Error]* and one of its child has a height of *[Math Processing Error]*, then for an AVL tree, the minimum height of the other child will be *[Math Processing Error]*.



It means that the minimum number of nodes at height *[Math Processing Error]* will be the sum of the minimum number of nodes at heights *[Math Processing Error]* and *[Math Processing Error]* + 1 (the node itself).


Min. no. of nodes = a+b+1
Minimum Height = h-2
Height = h-1
Min. no. of nodes for h-1 = a
Min. no. of nodes for h-1 = b

[Math Processing Error]

Replacing *[Math Processing Error]* with *[Math Processing Error]*,

[Math Processing Error]

Replacing the value of *[Math Processing Error]* in the first equation,

[Math Processing Error]

or, *[Math Processing Error]*

We can also say that,

[Math Processing Error]

Similary,

[Math Processing Error]
[Math Processing Error]

We can write,

[Math Processing Error]

Choosing the value of *[Math Processing Error]* such that *[Math Processing Error]*.

or, *[Math Processing Error]*

Putting this value,

[Math Processing Error]

Using this value,

[Math Processing Error]

or, *[Math Processing Error]*

Taking log on both sides,

[Math Processing Error]

or, *[Math Processing Error]*

or, *[Math Processing Error]*

or, *[Math Processing Error]*

or, *[Math Processing Error]*

Since *[Math Processing Error]* is the minimum number of nodes, *[Math Processing Error]*.

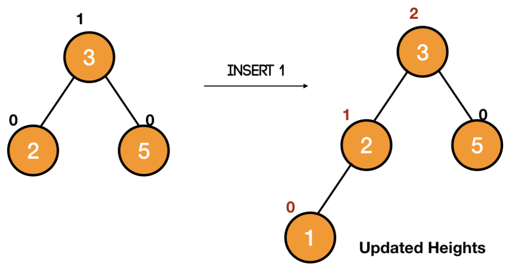
Thus, *[Math Processing Error]*

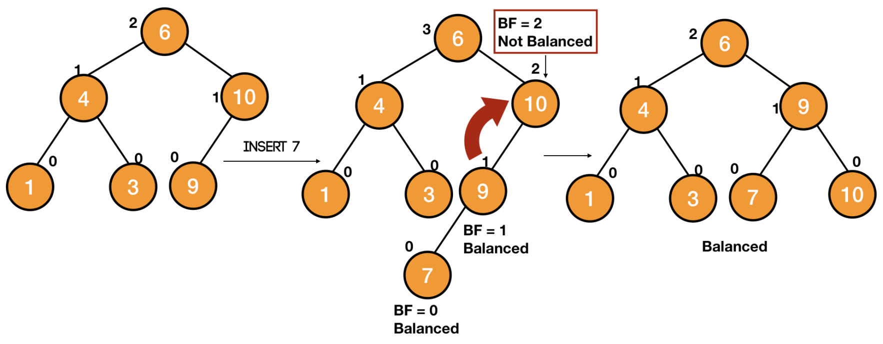
So we have proved that the height of an AVL tree is *[Math Processing Error]*. This means that we can perform basic search tree opertaions in *[Math Processing Error]* time.

Let's learn about the insertion and deletion in an AVL tree.

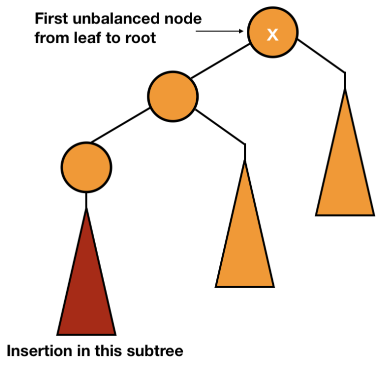
Insertion in AVL Tree

Inserting a new node can cause the balance factor of some node to become 2 or -2. In that case, we fix the balance factors by use of rotations. Also, only the heights of the nodes on the path from the insertion point to the root can be changed. So, after inserting, we go back up to the root and update heights of the nodes and if we find any node with a balance factor of 2 or -2, we fix it by rotation.

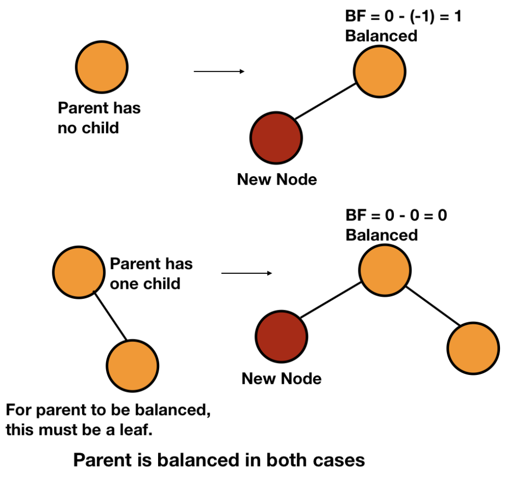




Suppose we have caused unbalance to some node after insertion and the node which needs rebalancing is *x*.



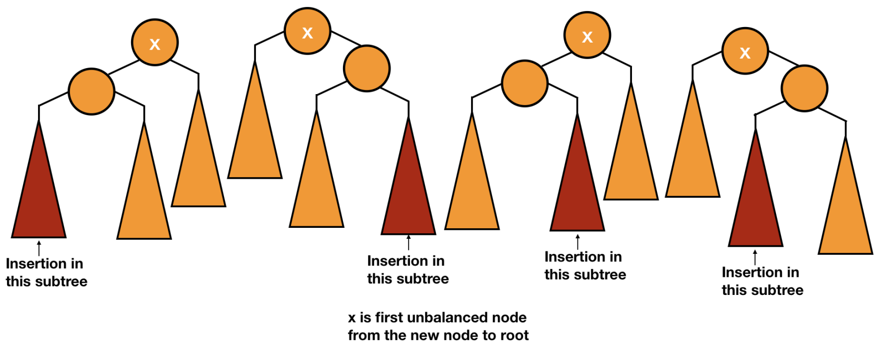
The parent of a newly inserted node will have either one child or none prior to insertion. In both the cases, insertion of the node won't cause unbalance of its parent.



However, the grandparent of the new node might lose its balance.

There can be 4 cases of insertion:

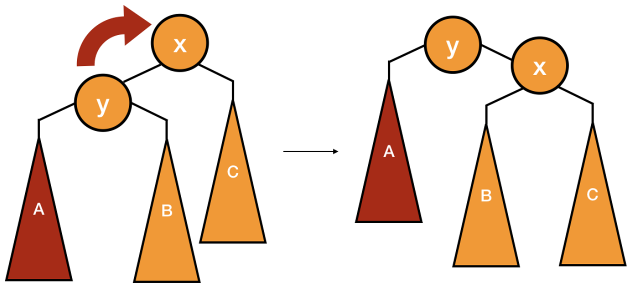
- Outside Cases (require single rotation)
    1. Insertion into left subtree of left child of *x*.
    2. Insertion into right subtree of right child of *x*.
- Inside Cases (require double rotation)
    3. Insertion into right subtree of left child of *x*.
    4. Insertion into left subtree of right child of *x*.


x
x
x
x
Insertion in this subtree
Insertion in this subtree
Insertion in this subtree
Insertion in this subtree
x is first unbalanced node from the new node to root

Let's look at the rotations performed for the 4 cases.

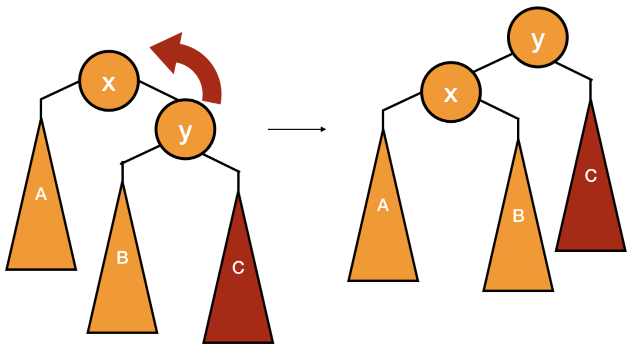
Case 1: Insertion into left subtree of left child of *x*

We do right rotation on the node *x*.



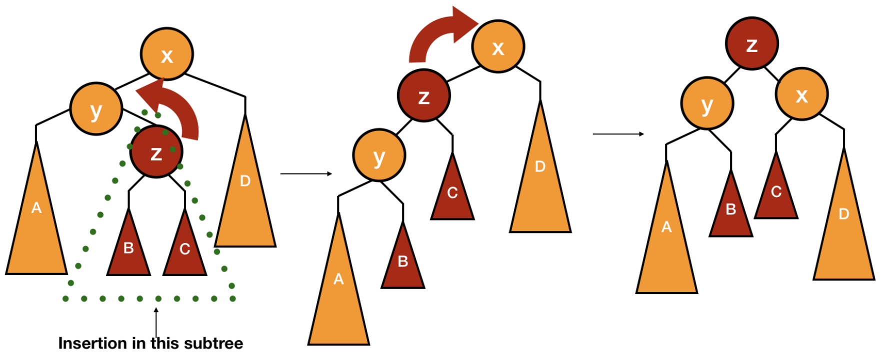
Case 2: Insertion into right subtree of right child of *x*

We do left rotation on the node *x*.



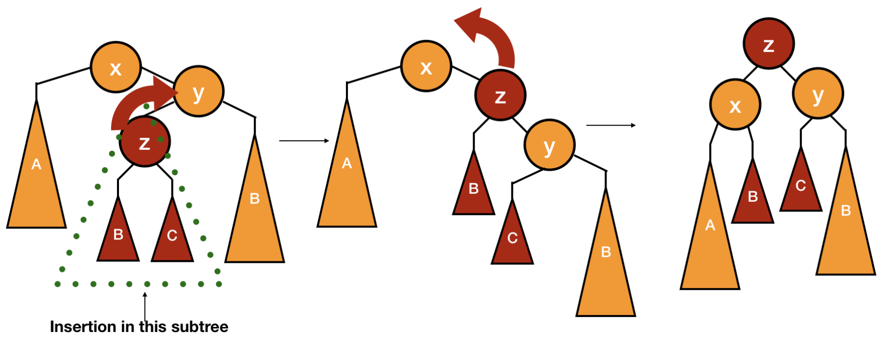
Case 3: Insertion into right subtree of left child of *x*

We first do left rotation on the left child of *x* and then right rotation on *x*.

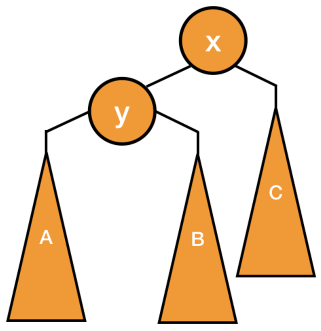


Case 4: Insertion into left subtree of right child of *x*

We first do right rotation on the right child of *x* and then left rotation on *x*.



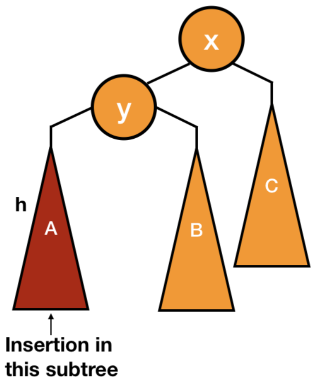
Now, we know what kind of disturbances can occur in the balance of a tree and how to fix them. But we still haven't justified that performing these rotations will fix the balance factor. So, let's look at what happens to the height and the balance factor of the nodes after doing the rotations in inside and outside cases. First, look at the picture of a subtree before insertion.



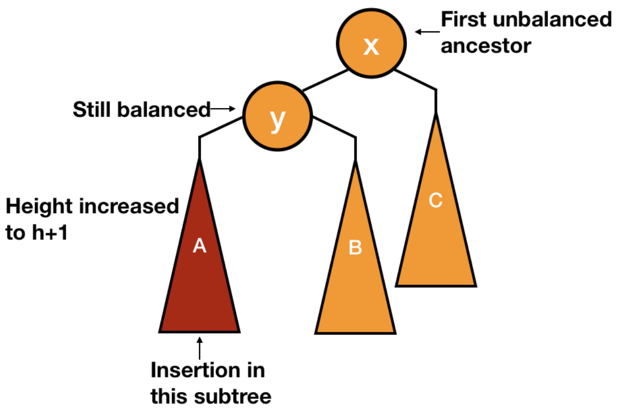
Subtree Before Insertion

Outside Case

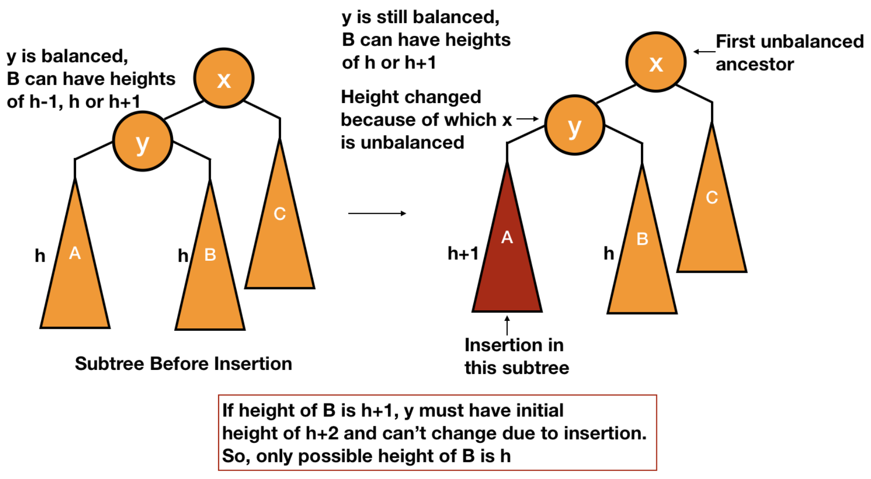
We are going to look at the outside case first and we are assuming that a node is added somewhere in the subtree *A* and because of it, node *x* is the first ancestor node which becomes unbalanced.



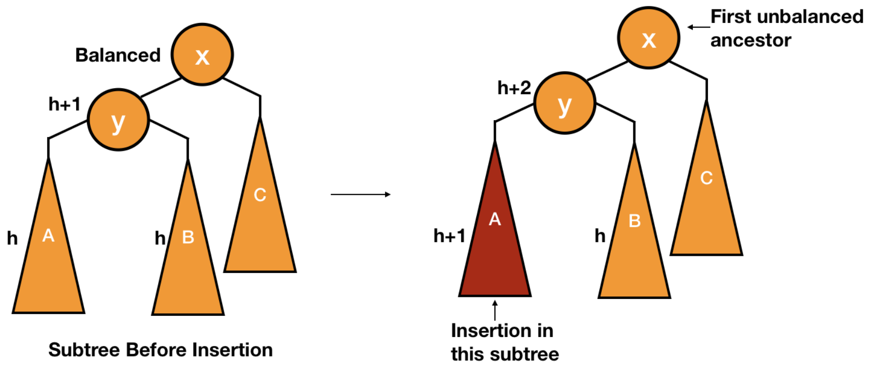
Suppose the height of the root of the node *A* was initially *h* and after the insertion, it is *h+1*. Since root of *x* is the first node which is unbalance, *y* is still balanced and initially it was also balanced but its height has been increased because of which *x* is now unbalanced.



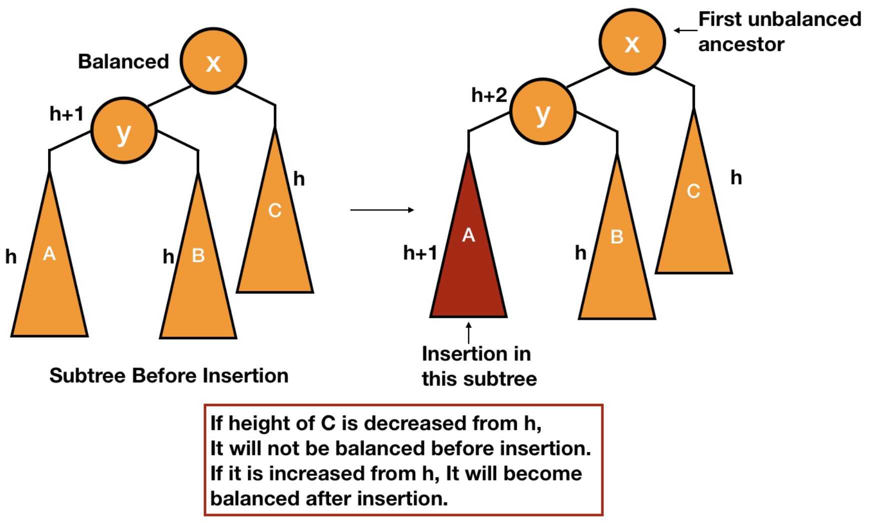
Since *y* is balanced, the root of *B* can have heights of *h*, *h+1* or *h+2*. But *y* was also balanced before insertion, so only possible heights of *B* are *h* or *h+1*. We have also discussed that the height of *y* increased because of the insertion and this wouldn't have been possible if the height of *B* was *h+1*. So, *B* must have a height of *h*.


y is balanced, B can have heights of h-1, h or h+1
x
y
h
A
h
B
C
Subtree Before Insertion
y is still balanced, B can have heights of h or h+1
Height changed because of which x is unbalanced
First unbalanced ancestor
x
y
h+1
A
h
B
C
Insertion in this subtree
If height of B is h+1, y must have initial height of h+2 and can't change due to insertion. So, only possible height of B is h

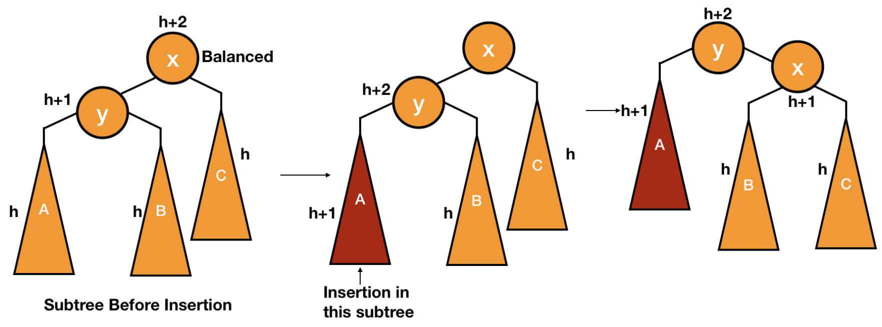
Thus, initially the node *y* had a height of *h+1* and now (after insertion), it is *h+2*. Also, *x* is now unbalanced but before insertion, it was balanced.



The only possible height of *C* which can make *x* unbalanced after the insertion is *h*.


Balanced
x
y
h+1
A
B
C
h
h
h
Subtree Before Insertion
First unbalanced ancestor
h+2
h+1
Insertion in this subtree
If height of C is decreased from h,
It will not be balanced before insertion.
If it is increased from h, It will become
balanced after insertion.

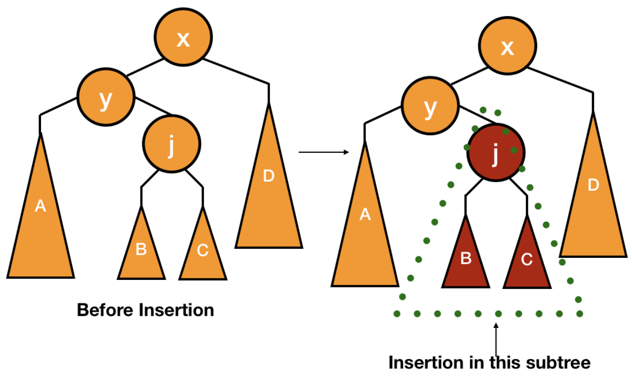
Let's perform the right rotation to fix this unbalanced.



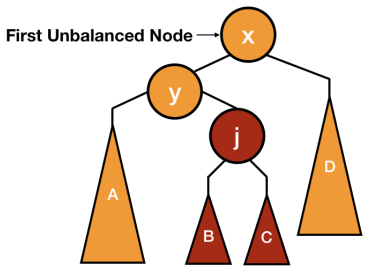
Before insertion, the root of the subtree (*x*) has a height of *h+2* and now the root of the subtree (*y*) has also a height of *h+2*. You can also verify that every node is balanced in the subtree itself. Since the root of the subtree has the same height as before, the unbalance is not going to propagate above it and all the nodes above it will have the same height as before the insertion. So, we have fixed the unbalance.

Let's look at the inside case and see if rotations really work in this case or not.

Inside Case

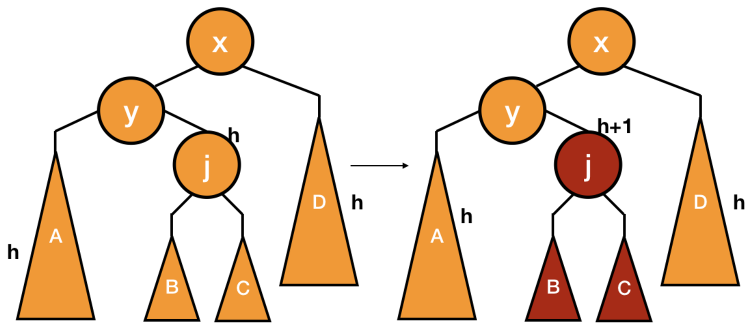


This time, we are assuming that a new node has been added somewhere in the subtree marked in the picture above and because of it, *x* is the first node which becomes unbalanced. This will happen after an increase in the height of the node *y*.



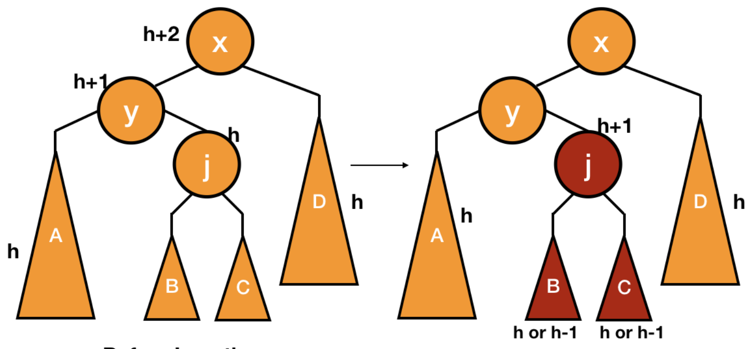
x can be unbalanced because of increase in height of y

By the same arguments as stated in the outside case, we can say that the height of *A* and *D* must be *h*.



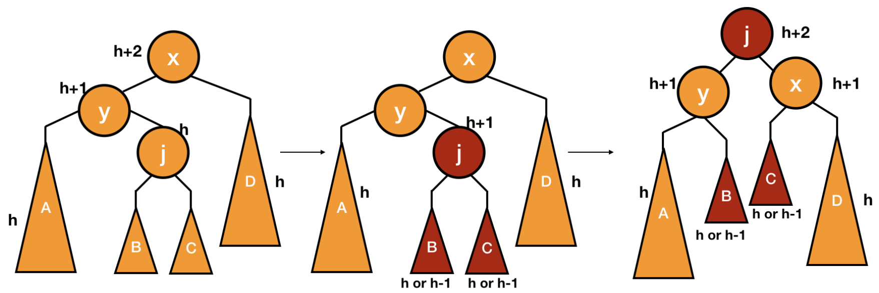
Before Insertion

The node *j* has a height of *h+1* and is balanced, it means that *B* and *C* can have heights of either *h* or *h-1*. However, one of these must have a height of *h* because the height of *j* is *h+1*.



Before Insertion

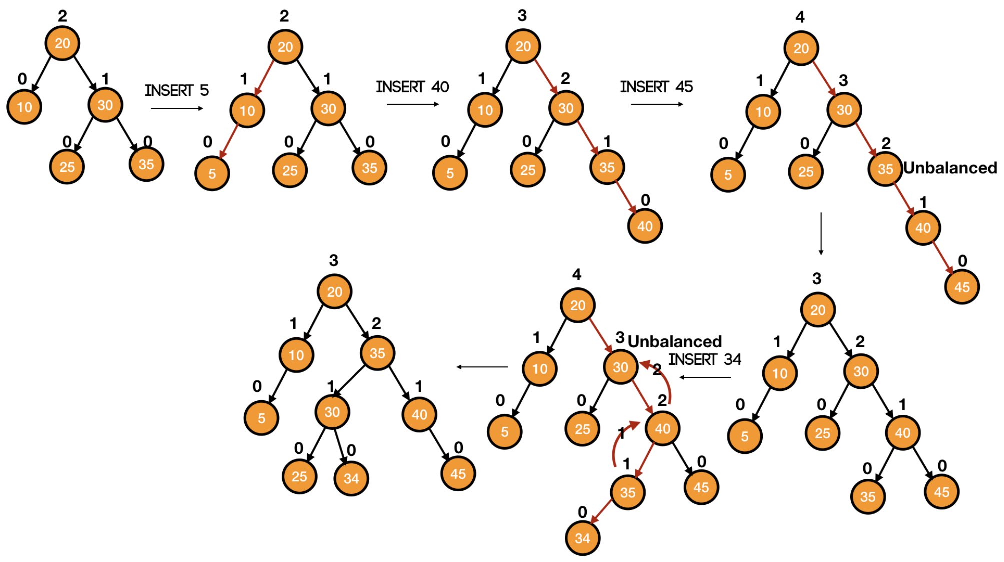
Let's perform the suitable rotations to fix the unbalance.



Before Insertion

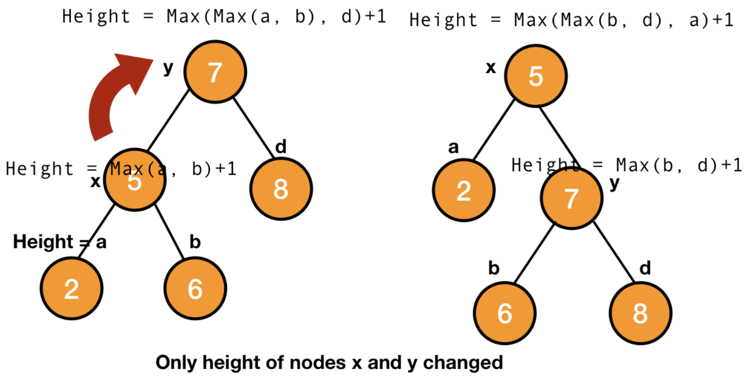
Initially, the root of the subtree has a height of *h+2* and still it has a height of *h+2*. So, all the nodes above it must have the same height. You can also check that every node is balanced in the subtree itself. So, we have fixed the unbalance.

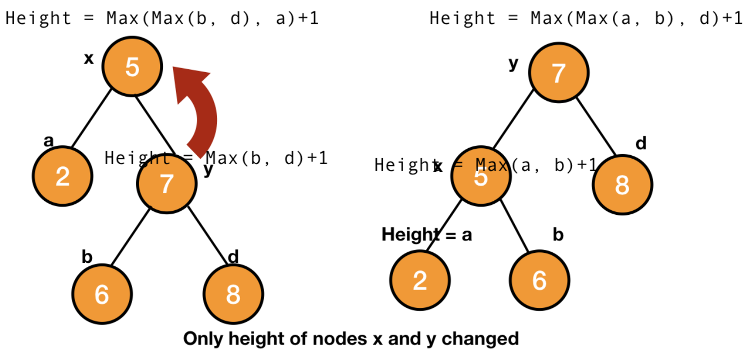
Let's look at some examples of insertion:



Code for Insertion

We have already discussed that we are also going to store the height of every node. And these heights can also change during the rotations. So, let's update our previous rotation functions so that they also update the changed heights after rotations.


Height = Max(Max(a, b), d)+1
Height = Max(Max(b, d), a)+1
Height = Max(a, b)+1
Height = Max(b, d)+1
Height = a
x
y
a
b
d
Only height of nodes x and y changed


Height = Max(Max(b, d), a)+1
Height = Max(Max(a, b), d)+1
Height = Max(b, d)+1
Height = Max(a, b)+1
Height = a
x
y
a
b
d
Only height of nodes x and y changed

As you can see from the above pictures, only the heights of *x* and *y* are going to be changed (heights of their ancestors can also change but we will fix them in the insertion process), so will update the heights of *x* and *y* only.

```
x.height = 1 + MAX(HEIGHT(x.left), HEIGHT(x.right))
y.height = 1 + MAX(HEIGHT(y.left), HEIGHT(y.right))
```

```
LEFT_ROTATE(T, x)
    y = x.right
    x.right = y.left
    if y.left != NULL
        y.left.parent = x
    y.parent = x.parent
    if x.parent == NULL //x is root
        T.root = y
    elseif x == x.parent.left // x is left child
        x.parent.left = y
    else // x is right child
        x.parent.right = y
    y.left = x
    x.parent = y

    x.height = 1 + MAX(HEIGHT(x.left), HEIGHT(x.right))
    y.height = 1 + MAX(HEIGHT(y.left), HEIGHT(y.right))
```

We will simply insert the new node (*z*) as we do in a binary search tree. After that, we will update the heights of its ancestors - `node.height = 1 + MAX(HEIGHT(node.left), HEIGHT(node.right))`.

```
i = z.parent
while i != NULL
  i.height = 1 + MAX(i.left.height, i.right.height)
  i = i.parent
```

After that, we set *x* and *y* as *z*'s grandparent and parent respectively.

```
if x.parent != NULL
  if x.parent.parent != NULL
    x = z.parent.parent
    y = z.parent
```

Then we will check if *x* is balanced or not, and if not, then do suitable rotations among the 4 cases.

```
if BALANCE-FACTOR(x) <= -2 or BALANCE-FACTOR(x) >= 2 //grandparent is unbalanced
  if y == x.left
    if z == x.left.left //case 1
      RIGHT_ROTATE(T, x)
    else if z == x.left.right //case 3
      LEFT_ROTATION(T, y)
      RIGHT_ROTATE(T, x)
  else if y == x.right
    if z == x.right.right //case 2
      LEFT_ROTATE(T, y)
    else if z == x.right.left //case 4
      RIGHT_ROTATE(T, y)
      LEFT_ROTATE(T, x)
```

```
INSERT(T, n)
     temp = T.root
     y = NULL
     while temp != NULL
          y = temp
          if n.data < temp.data
               temp = temp.left
          else
               temp = temp.right
     n.parent = y
     if y==NULL
          T.root = n
     else if n.data < y.data
          y.left = n
     else
          y.right = n

     z = n
     while y != NULL
          y.height = 1 + MAX(HEIGHT(y.left), HEIGHT(y.right))

          x = y.parent
          if BALANCE-FACTOR(x) <= -2 or BALANCE-FACTOR(x) >= 2 //grandparent is unbalanced
               if y == x.left
                    if z == x.left.left //case1
                         RIGHT_ROTATE(T, x)
                    else if z == x.left.right //case 3
                         LEFT_ROTATE(T, y)
                         RIGHT_ROTATE(T, x)
               else if y == x.right
                    if z == x.right.right //case 2
                         LEFT_ROTATET(T, x)
                    else if z == x.right.left //case 4
                         RIGHT_ROTATE(T, y)
                         LEFT_ROTATE(T, x)

               break
          y = y.parent
          z = z.parent
```

</>
We have already updated the heights of ancestors before insertions and also updated heights during rotations. We have also shown that the height of the root of the subtree in which insertion is made is not going to change, so there is no need to re-update the heights of ancestors after rotations.

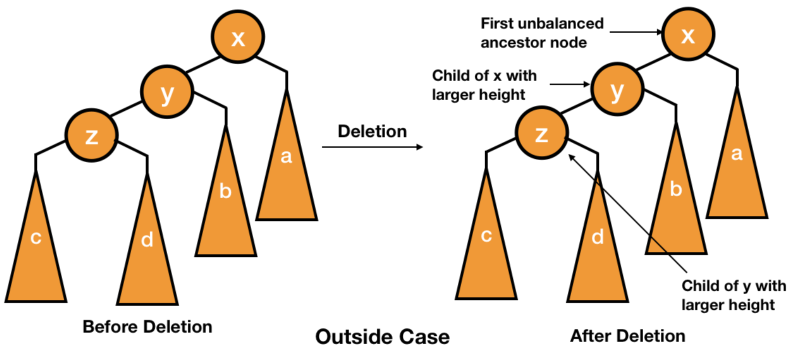
Deletion in AVL Tree

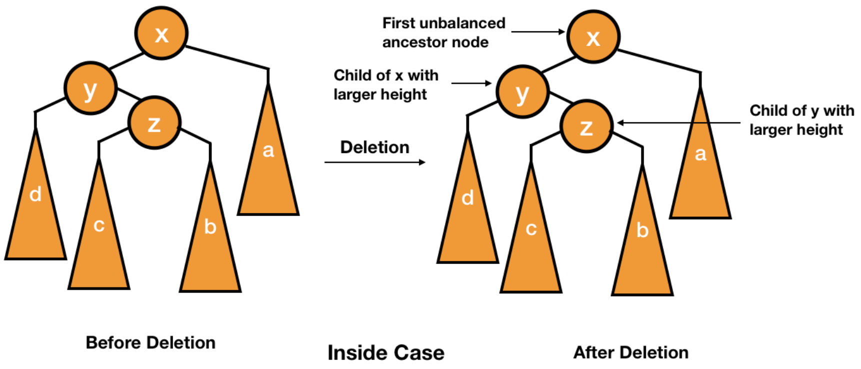
In deletion also, we delete the node to be deleted in the same way as we do with a normal binary search tree. After that, we fix the unbalance of any ancestor node with suitable rotations. The only thing is that unlike insertion, it might be possible that the unbalance propagates above the tree in deletion which makes us rebalance the ancestor nodes. Let's look at some examples of deletion and then the reason for the same will be clear.

As we have already discussed, there can be three cases of deletion - the node to be deleted has no child, the node to be deleted has one child or the node to be deleted has 2 children. In the third case when the node has 2 children, we replace the content of the nodes with its successor and it reduces to the deletion of the node with either one child or none.

After the deletion procedure, the heights of ancestor nodes will decrease and this may cause unbalance in the tree. Let's look at the loss of balances that can happen and how to fix them.

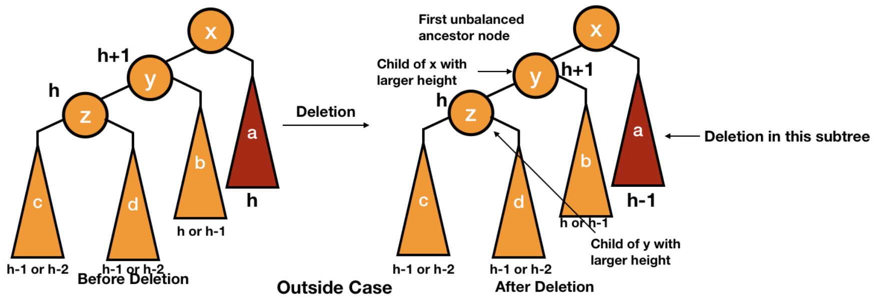
Let's assume that the first ancestor which is unbalanced is *x* and *y* is its child with greater height among the heights of the two children of *x*. And *z* is the child of *y* with greater height among the children of *y*.


x
y
z
a
b
c
d
Before Deletion
Deletion
Outside Case
First unbalanced ancestor node
Child of x with larger height
Child of y with larger height
After Deletion


x
y
z
a
b
c
d
Before Deletion
Deletion
Inside Case
First unbalanced ancestor node
Child of x with larger height
Child of y with larger height
After Deletion

We fix the unbalancing by performing the required rotations involving the nodes *x*, *y* and *z* as we did in the insertion part.

Let's talk about the outside case first in which the node *x* is unbalanced because of the deletion of a node from the subtree *a*.

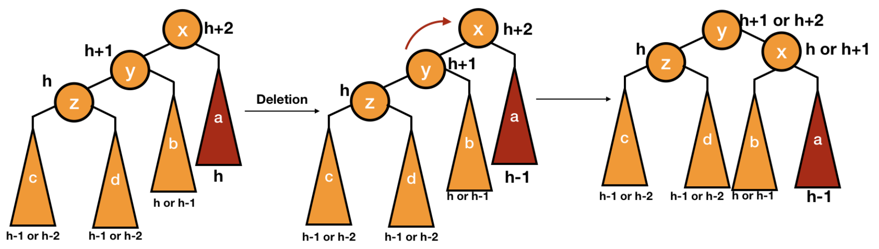


The height of the subtree *a* is *h* before deletion and the node *x* is balanaced. Since *x* is balanced, it means that the height of the node *y* can be *h*, *h-1* or *h+1*. But after deletion, the height of the subtree *a* is changed to *h-1* and the node *x* is unbalanced now. It means that the only possible height of the node *y* can be *h+1*.

Since *z* is the child of *y* with larger height, so its height will be *h*. As *y* is balanced, the height of *b* will be either *h* or *h-1*.

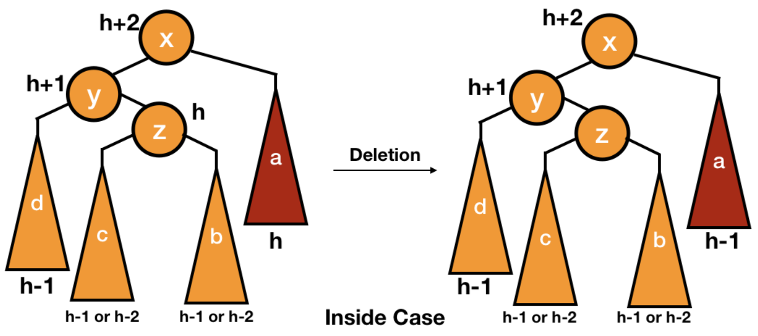
The heights of the nodes *c* and *d* can be either *h-1* or *h-2*. However, one of these must have a height of *h-1* because the height of node *x* is *h*.

After deletion, the height of the node *a* changed to *h-1* and thus caused the unbalance on the node *x*. To fix this, we have performed a right rotation on the node *x* as we did in insertion.

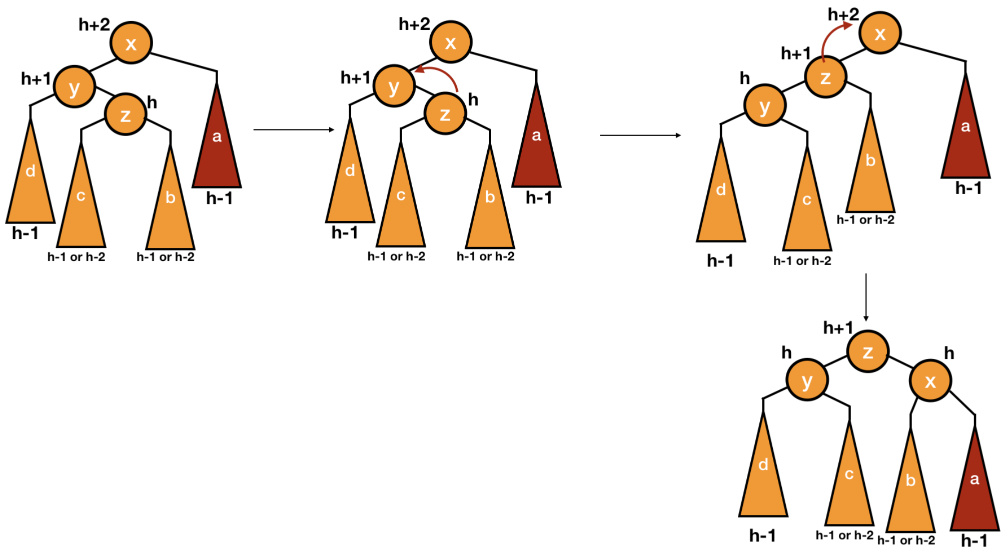


Now, the node *y* is the root of the subtree. Initially, this subtree has a height of *h+2* but now it has a height of *h+1* or *h+2*. If it is changed to *h+1*, it might have caused unbalance somewhere in its ancestor. In this way, the unbalance propagates above the tree in deletion.

Let's take deletion of inside case.

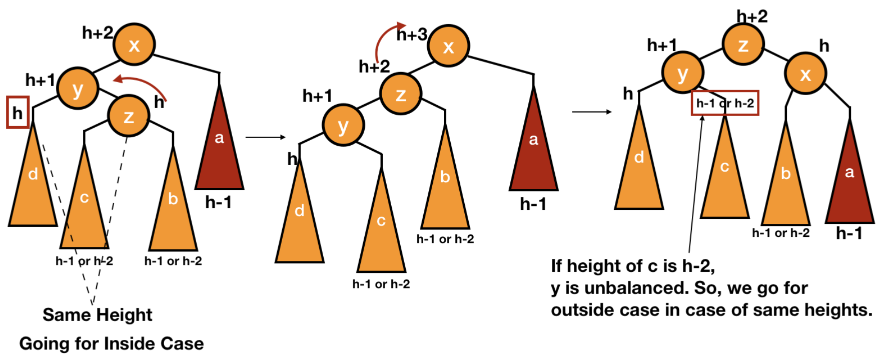


Let's perform the rotations to fix this.



Node *z* is now the root of the subtree with a height of *h+1* which initially was *h+2*, so we need to check further if any ancestor requires any balancing or not.

One thing should be noted that the subtree *d* can have a height of either *h-1* or *h*. But if it is *h* (equal to its sibling), then both children of *y* have same heights and there isn't a taller child. In that case, we will pick the root of the subtree *d* as *z* which will lead to the outside case and thus requiring only one rotation to fix the unbalance. However, the reason for picking the root of the subtree *d* as *z* and going for the outside case is not to fix the unbalance in a single rotation only but if we would have picked the other node and gone for the double rotation (inside case), then it wouldn't fix the unbalance. This is shown in the following picture.



Code for Deletion

The code for the deletion is pretty much similar to that of binary search tree. So, let's focus on the rebalancing part.

Similar to the fixing of insertion, we will start with a loop but we will not stop just after fixing an unbalanced node because we have already discussed that the unbalance might propagate above in case of deletion.

```
AVL_DELETE_FIXUP(T, n)
    p = n

    while p != NULL
        p.height = 1 + MAX(HEIGHT(p.left), HEIGHT(p.right))

        if(BALANCE_FACTOR(p) <= -2 || balance_factor(p) >= 2) //grandparent is unbalanced
            x = p

            //taller child of x will be y
            if x.left.height > x.right.height
                y = x.left
            else
                y = x.right

            //taller child of y will be z
            if y.left.height > y.right.height
                z = y.left
            else if y.left.height < y.right.height
                z = y.right
            else //same height go for single rotation
                if y == x.left
                    z = y.left
                else
                    z = y.right

            //perform rotaions
            if y == x.left
                if z == x.left.left //case1
                    RIGHT_ROTATE(T, x)
                else if z == x.left.right //case 3
                    LEFT_ROTATE(T, y)
                    RIGHT_ROTATE(T, x)
            else if y == x.right
                if z == x.right.right //case 2
                    LEFT_ROTATET(T, x)
                else if z == x.right.left //case 4
                    RIGHT_ROTATE(T, y)
                    LEFT_ROTATE(T, x)
        p = p.parent
```

Firstly, we have set *y* as the taller child of *x* - `if x.left.height > x.right.height → ... else → ....` Similarly, we have set *z* as the taller child of *y* but if the heights of both the children of *y* are same, we would choose *z* such that we would have to go for a single rotation - `if y == x.left → z = y.left else → z = y.right`.

After this, we have just performed the rotation as we did with the insertion part.

```
DELETE(T, z)
    if z.left == NULL
        TRANSPLANT(T, z, z.right)

        if z.right != NULL
            AVL_DELETE_FIXUP(T, z.right)

    elseif z.right == NULL
        TRANSPLANT(T, z, z.left)

        if z.left != NULL
            AVL_DELETE_FIXUP(T, z.left)

    else
        y = MINIMUM(z.right) //minimum element in right subtree
        if y.parent != z //z is not direct child
            TRANSPLANT(T, y, y.right)
            y.right = z.right
            y.right.parent = y
        TRANSPLANT(T, z, y)
        y.left = z.left
        y.left.parent = y

        if y != NULL
            AVL_DELETE_FIXUP(T, y)
```

C **Python** **Java**

(/add_ques

```
#include <stdio.h>
#include <stdlib.h>

typedef struct avl_node {
  int data;
  struct avl_node *left;
  struct avl_node *right;
  struct avl_node *parent;
  int height;
}avl_node;

typedef struct avl_tree {
  avl_node *root;
}avl_tree;

avl_node* new_avl_node(int data) {
  avl_node *n = malloc(sizeof(avl_node));
  n->data = data;
  n->left = NULL;
  n->right = NULL;
  n->parent = NULL;
  n->height = 0;

  return n;
}

avl_tree* new_avl_tree() {
  avl_tree *t = malloc(sizeof(avl_tree));
  t->root = NULL;

  return t;
}

int max(int a, int b) {
  if(a > b)
    return a;
  return b;
}

int height(avl_node *n) {
  if(n == NULL)
    return -1;
  return n->height;
}

avl_node* minimum(avl_tree *t, avl_node *x) {
  while(x->left != NULL)
    x = x->left;
  return x;
}

void left_rotate(avl_tree *t, avl_node *x) {
  avl_node *y = x->right;
  x->right = y->left;
  if(y->left != NULL) {
    y->left->parent = x;
  }
  y->parent = x->parent;
  if(x->parent == NULL) { //x is root
    t->root = y;
  }
  else if(x == x->parent->left) { //x is left child
    x->parent->left = y;
  }
  else { //x is right child
    x->parent->right = y;
  }
  y->left = x;
  x->parent = y;

  x->height = 1 + max(height(x->left), height(x->right));
  y->height = 1 + max(height(y->left), height(y->right));
}

void right_rotate(avl_tree *t, avl_node *x) {
  avl_node *y = x->left;
  x->left = y->right;
```

(/add_ques

```
  if(y->right != NULL) {
    y->right->parent = x;
  }
  y->parent = x->parent;
  if(x->parent == NULL) { //x is root
    t->root = y;
  }
  else if(x == x->parent->right) { //x is left child
    x->parent->right = y;
  }
  else { //x is right child
    x->parent->left = y;
  }
  y->right = x;
  x->parent = y;

  x->height = 1 + max(height(x->left), height(x->right));
  y->height = 1 + max(height(y->left), height(y->right));
}

int balance_factor(avl_node *n) {
  if(n == NULL)
    return 0;
  return(height(n->left) - height(n->right));
}

void insert(avl_tree *t, avl_node *n) {
  avl_node *y = NULL;
  avl_node *temp = t->root;
  while(temp != NULL) {
    y = temp;
    if(n->data < temp->data)
      temp = temp->left;
    else
      temp = temp->right;
  }
  n->parent = y;

  if(y == NULL) //newly added node is root
    t->root = n;
  else if(n->data < y->data)
    y->left = n;
  else
    y->right = n;

  avl_node *z = n;

  while(y != NULL) {
    y->height = 1 + max(height(y->left), height(y->right));


    avl_node *x = y->parent;

    if(balance_factor(x) <= -2 || balance_factor(x) >= 2) {//grandparent is unbalanced
      if(y == x->left) {
        if(z == x->left->left) //case 1
          right_rotate(t, x);

        else if(z == x->left->right) {//case 3
          left_rotate(t, y);
          right_rotate(t, x);
        }
      }
      else if(y == x->right) {
        if(z == x->right->right) //case 2
          left_rotate(t, x);

        else if(z == x->right->left) {//case 4
          right_rotate(t, y);
          left_rotate(t, x);
        }
      }
      break;
    }
    y = y->parent;
    z = z->parent;
  }
}
```

(/add_ques

```
void transplant(avl_tree *t, avl_node *u, avl_node *v) {
  if(u->parent == NULL) //u is root
    t->root = v;
  else if(u == u->parent->left) //u is left child
    u->parent->left = v;
  else //u is right child
    u->parent->right = v;

  if(v != NULL)
    v->parent = u->parent;
}

void avl_delete_fixup(avl_tree *t, avl_node *n) {
  avl_node *p = n;

  while(p != NULL) {
    p->height = 1 + max(height(p->left), height(p->right));

    if(balance_factor(p) <= -2 || balance_factor(p) >= 2) { //grandparent is unbalanced
      avl_node *x, *y, *z;
      x = p;

      //taller child of x will be y
      if(x->left->height > x->right->height)
        y = x->left;
      else
        y = x->right;

      //taller child of y will be z
      if(y->left->height > y->right->height) {
        z = y->left;
      }
      else if(y->left->height < y->right->height) {
        z = y->right;
      }
      else { //same height, go for single rotation
        if(y == x->left)
          z = y->left;
        else
          z = y->right;
      }

      if(y == x->left) {
        if(z == x->left->left) //case 1
          right_rotate(t, x);

        else if(z == x->left->right) {//case 3
          left_rotate(t, y);
          right_rotate(t, x);
        }
      }
      else if(y == x->right) {
        if(z == x->right->right) //case 2
          left_rotate(t, x);

        else if(z == x->right->left) {//case 4
          right_rotate(t, y);
          left_rotate(t, x);
        }
      }
    }
    p = p->parent;
  }
}

void avl_delete(avl_tree *t, avl_node *z) {
  if(z->left == NULL) {
    transplant(t, z, z->right);
    if(z->right != NULL)
      avl_delete_fixup(t, z->right);
    free(z);
  }
  else if(z->right == NULL) {
    transplant(t, z, z->left);
    if(z->left != NULL)
      avl_delete_fixup(t, z->left);
    free(z);
```

(/add_ques

```
  }
  else {
    avl_node *y = minimum(t, z->right); //minimum element in right subtree
    if(y->parent != z) {
      transplant(t, y, y->right);
      y->right = z->right;
      y->right->parent = y;
    }
    transplant(t, z, y);
    y->left = z->left;
    y->left->parent = y;
    if(y != NULL)
      avl_delete_fixup(t, y);
    free(z);
  }
}

void inorder(avl_tree *t, avl_node *n) {
  if(n != NULL) {
    inorder(t, n->left);
    printf("%d\n", n->data);
    inorder(t, n->right);
  }
}

int main() {
  avl_tree *t = new_avl_tree();

  avl_node *a, *b, *c, *d, *e, *f, *g, *h, *i, *j, *k, *l, *m;

  a = new_avl_node(10);
  b = new_avl_node(20);
  c = new_avl_node(30);
  d = new_avl_node(100);
  e = new_avl_node(90);
  f = new_avl_node(40);
  g = new_avl_node(50);
  h = new_avl_node(60);
  i = new_avl_node(70);
  j = new_avl_node(80);
  k = new_avl_node(150);
  l = new_avl_node(110);
  m = new_avl_node(120);

  insert(t, a);
  insert(t, b);
  insert(t, c);
  insert(t, d);
  insert(t, e);
  insert(t, f);
  insert(t, g);
  insert(t, h);
  insert(t, i);
  insert(t, j);
  insert(t, k);
  insert(t, l);
  insert(t, m);

  avl_delete(t, a);
  avl_delete(t, m);

  inorder(t, t->root);

  return 0;
}
```

Analysis of AVL Trees

The insertion of a node is going to take *[Math Processing Error]* time because the tree is balanced. Also, the unbalances in the insertion process get fixed after a fixed number of rotations. So, the entire process is of *[Math Processing Error]* time.

(/add_ques

We can delete any node in constant time i.e., *[Math Processing Error]* and also fix the unbalance with a fixed number of rotations in *[Math Processing Error]* time but since the unbalance might propagate above the tree, the entire deletion process takes *[Math Processing Error]* time.

❝ I learned very early the difference between knowing the name of something and knowing something ❞

- Richard P. Feynman

PREV (/course/data-structures-red-black-trees-deletion/) (/course/data-structures-splay-trees/) NEXT

New Questions

Difference between = and == method In java **- Java**

(/discussion/difference-between-and-method-in-java)

This is a program for displaying multiplication table of any number but when I write program as given it doesn't give proper result but when I declare **- C**

(/discussion/this-is-a-program-for-displaying-multiplication-ta)

What do you mean by Constructor? **- Java**

(/discussion/what-do-you-mean-by-constructor)

setting up an ide for mac.. **- Cpp**

(/discussion/setting-up-an-ide-for-mac)

Please fill the blanks and help me am stuck **- Java**

(/discussion/fill-the-blanks-and-help-me-am-stuck)

Time complexity of the Python Function **- Python**

(/discussion/time-complexity-of-the-python-function)