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**1 Combinatorics****1.1 nCr [22 lines]**

```
const int MOD=1e9+7;
const int N=4e5+2;
int fact[N], ifact[N], reciprocal[N];

struct combinit {
    combinit() {
        reciprocal[1] = 1;
        for (int i = 2; i < N; ++i) {
            reciprocal[i] = (MOD - MOD / i) * (long
                long)reciprocal[MOD % i] % MOD;
        }
        fact[0] = ifact[0] = 1;
        for (int i = 1; i < N; ++i) {
            fact[i] = (long long)fact[i - 1] * i %
                MOD;
            ifact[i] = (long long)ifact[i - 1] *
                reciprocal[i] % MOD;
        }
    }
} combinitX;

long long comb(long long n, long long m) {
    if (n < m || m < 0) return 0;
    return (long long)fact[n] * ifact[m] % MOD
        * ifact[n - m] % MOD;
}

2 Data Structure
2.1 2D Fenwick Tree [19 lines]
struct FenwickTree2D
{
    vector<vector<int>> tree;
    int n;
    void init(int size){n = size, tree.assign(n
        + 1, vector<int>(n + 1, 0));}
    void update(int x, int y, int delta)
    {
        for (int i = x; i <= n; i += (i & -i))
            for (int j = y; j <= n; j += (j &
                -j))tree[i][j] += delta;
    }
    int query(int x, int y)
    {
        int sum = 0;
        for (int i = x; i > 0; i -= (i & -i))
            for (int j = y; j > 0; j -= (j &
                -j))sum += tree[i][j];
        return sum;
    }
    int query(int x1, int y1, int x2, int
        y2){return query(x2, y2) - query(x1 -
        1, y2) - query(x2, y1 - 1) + query(x1
        - 1, y1 - 1);}
};
```

**2.2 2D Prefix Sum [3 lines]**

```
void build(){for (int i = 1; i <= n; ++i)
    for (int j = 1; j <= m; ++j)prefix[i][j]
        = arr[i][j] + prefix[i - 1][j] +
        prefix[i][j - 1] - prefix[i - 1][j -
        1];}
```

```
int query(int x1, int y1, int x2, int
    y2){return prefix[x2][y2] - prefix[x1 -
    1][y2] - prefix[x2][y1 - 1] + prefix[x1 -
    1][y1 - 1];}
```

**2.3 Centroid Decomposition [133 lines]**

```
#include <bits/stdc++.h>
using namespace std;

struct CentroidDecomposition
{
    int n;
    int INF = INT_MAX;
    vector<vector<int>> adj;
    vector<bool> centroidMarked;
    vector<int> subSize;
    vector<int> parentCentroid;
    vector<vector<pair<int, int>>>
        centroidPath;
    vector<multiset<int>> bestSet;
    CentroidDecomposition(int _n) : n(_n)
    {
        adj.assign(n + 1, {});
        centroidMarked.assign(n + 1, false);
        subSize.assign(n + 1, 0);
        parentCentroid.assign(n + 1, 0);
        centroidPath.assign(n + 1, {});
        bestSet.assign(n + 1, {});
    }
    void addEdge(int u, int v)
    {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void computeSubSizeDFS(int u, int p)
    {
        subSize[u] = 1;
        for (int v : adj[u])
        {
            if (v == p || centroidMarked[v])
                continue;
            computeSubSizeDFS(v, u);
            subSize[u] += subSize[v];
        }
    }
    int findCentroidDFS(int u, int p, int
        totSize)
    {
        for (int v : adj[u])
        {
            if (v == p || centroidMarked[v])
                continue;
            if (subSize[v] > totSize / 2)
            {
                return findCentroidDFS(v, u,
                    totSize);
            }
        }
        return u;
    }
    void addCentroidDistances(int c)
    {
        queue<pair<int, int>> q;
        vector<char> visited(n + 1, 0);
        q.push({c, 0});
        visited[c] = 1;
```

```

centroidPath[c].push_back({c, 0});
while (!q.empty())
{
    auto [u, d] = q.front();
    q.pop();
    for (int v : adj[u])
    {
        if (centroidMarked[v] ||
            visited[v])
            continue;
        visited[v] = 1;
        centroidPath[v].push_back({c,
            d + 1});
        q.push({v, d + 1});
    }
}

void decompose(int entry, int pCent)
{
    computeSubSizeDFS(entry, 0);
    int totalSize = subSize[entry];

    int c = findCentroidDFS(entry, 0,
        totalSize);

    centroidMarked[c] = true;
    parentCentroid[c] = (pCent == 0 ? c :
        pCent);

    addCentroidDistances(c);

    for (int v : adj[c])
    {
        if (!centroidMarked[v])
        {
            decompose(v, c);
        }
    }
}

void build()
{
    decompose(1, 0);
}

void update(int v)
{
    for (auto &pr : centroidPath[v])
    {
        int c = pr.first;
        int d = pr.second;
        bestSet[c].insert(d);
    }
}

void remove(int v)
{
    for (auto &pr : centroidPath[v])
    {
        int c = pr.first;
        int d = pr.second;
        auto it = bestSet[c].find(d);
        if (it != bestSet[c].end())
        {
            bestSet[c].erase(it);
        }
    }
}

int query(int v)

```

```

{
    int res = INF;
    for (auto &pr : centroidPath[v])
    {
        int c = pr.first;
        int d = pr.second;
        if (!bestSet[c].empty())
        {
            res = min(res, d +
                *bestSet[c].begin());
        }
    }
    return res;
}
};

```

## 2.4 DSU with Rollback [33 lines]

```

struct DSU
{
    vector<int> parent, size;
    vector<pair<int, int>> history;
    int componentCount;
    DSU(int n) : parent(n + 1), size(n + 1,
        1), componentCount(n){iota(parent.
        begin(), parent.end(), 0);}
    int find(int v)
    {
        if (parent[v] != v) return
            find(parent[v]);
        return parent[v];
    }
    bool merge(int u, int v)
    {
        u = find(u), v = find(v);
        if (u == v){return false;}
        if (size[u] < size[v]) swap(u, v);
        history.push_back({v, size[v]});
        history.push_back({u, size[u]});
        parent[v] = u, size[u] +=
            size[v], --componentCount;
        return true;
    }
    void rollback()
    {
        auto [u, oldSizeU] = history.back();
        history.pop_back();
        auto [v, oldSizeV] = history.back();
        history.pop_back();
        parent[v] = v, size[u] =
            oldSizeU, size[v] =
            oldSizeV, componentCount++;
    }
    int getComponentCount(){return
        componentCount;}
    bool same(int u, int v){return find(u) ==
        find(v);}
    int getSize(int v){return size[find(v)];}
};

```

## 2.5 Fenwick Tree [46 lines]

```

struct FenwickTree
{
    vector<int> tree;
    int n;
    FenwickTree(int n){this->n =
        n, tree.resize(n + 1, 0);}
}

```

```

void update(int idx, int val)
{
    while (idx <= n) tree[idx] += val, idx
        += idx & (-idx);
}

int query(int idx)
{
    int sum = 0;
    while (idx > 0) sum += tree[idx], idx
        -= idx & (-idx);
    return sum;
}

int rangeQuery(int l, int r){ return
    query(r) - query(l - 1);}

int inversion_count(vector<int> &arr)
{
    int icount = 0;
    for (int i = n; i > 0; i--)
    {
        int smallerCount = query(arr[i] -
            1);
        icount += smallerCount;
        update(arr[i], 1);
    }
    return icount;
}

int kth(int k)
{
    if (k <= 0 || query(n) < k) return -1;
    int idx = 0, mask = 1 << (31 -
        __builtin_clz(n));
    int acc = 0;
    while (mask)
    {
        int next = idx + mask;
        if (next <= n && acc + tree[next]
            < k)
        {
            idx = next;
            acc += tree[next];
        }
        mask >>= 1;
    }
    return idx + 1;
}
};

```

## 2.6 HeavyLight Decomposition [72 lines]

```

struct HLD{
    vector<vector<int>> adj;
    vector<int> par, dep, heavy, head, pos,
        sub;
    SegTree segTree;
    int curPos;
    HLD(int n) : adj(n + 1), par(n + 1,
        -1), dep(n + 1, 0), heavy(n + 1,
        -1), head(n + 1), pos(n + 1), sub(n +
        1, 0), segTree(n, curPos(1) {}
    int dfs(int u)
    {
        sub[u] = 1;
        int mxsub = 0;
        for (int v : adj[u])
        {
            if (v == par[u])

```

```

                continue;
            par[v] = u;
            dep[v] = dep[u] + 1;
            int subtree = dfs(v);
            sub[u] += subtree;
            if (subtree > mxsub)
            {
                mxsub = subtree;
                heavy[u] = v;
            }
            return sub[u];
        }
    }
    void decompose(int u, int h)
    {
        head[u] = h;
        pos[u] = curPos++;
        if (heavy[u] != -1)
            decompose(heavy[u], h);
        for (int v : adj[u])
        {
            if (v != par[u] && v != heavy[u])
                decompose(v, v);
        }
    }
    void init(int root = 1)
    {
        dfs(root);
        decompose(root, root);
    }
    void updatePath(int u, int v, ll value,
        bool isEdge = false)
    {
        while (head[u] != head[v])
        {
            if (dep[head[u]] < dep[head[v]])
                swap(u, v);
            segTree.update(pos[head[u]],
                pos[u], value);
            u = par[head[u]];
        }
        if (dep[u] > dep[v])
            swap(u, v);
        segTree.update(pos[u] + isEdge,
            pos[v], value);
    }
    ll queryPath(int u, int v, bool isEdge =
        false)
    {
        ll result = 0;
        while (head[u] != head[v])
        {
            if (dep[head[u]] < dep[head[v]])
                swap(u, v);
            result = max(result,
                segTree.query(pos[head[u]],
                    pos[u]));
            u = par[head[u]];
        }
        if (dep[u] > dep[v])
            swap(u, v);
        result = max(result,
            segTree.query(pos[u] + isEdge,
                pos[v]));
    }
}

```

```

    }
    return result;
};

2.7 LCA [121 lines]
#include <bits/stdc++.h>
using namespace std;

class LCA_BinaryLifting
{
public:
    int n, max_log;
    vector<vector<int>> adj;
    vector<vector<int>> ancestor;
    vector<int> depth;

    LCA_BinaryLifting(int n) : n(n)
    {
        adj.resize(n + 1);
        max_log = log2(n) + 1;
        ancestor.assign(n + 1,
            vector<int>(max_log, -1));
        depth.resize(n + 1);
    }
    void add_edge(int u, int v)
    {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void dfs(int node, int parent)
    {
        ancestor[node][0] = parent;
        for (int i = 1; i < max_log; i++)
        {
            if (ancestor[node][i - 1] != -1)
            {
                ancestor[node][i] =
                    ancestor[ancestor[node][i - 1]][i - 1];
            }
        }
        for (int neighbor : adj[node])
        {
            if (neighbor != parent)
            {
                depth[neighbor] = depth[node]
                    + 1;
                dfs(neighbor, node);
            }
        }
    }
    void preprocess(int root)
    {
        depth[root] = 0;
        dfs(root, -1);
    }
    int get_lca(int u, int v)
    {
        if (depth[u] < depth[v])
        {
            swap(u, v);
        }
        for (int i = max_log - 1; i >= 0; i--)
        {
            if (ancestor[u][i] != -1 &&
                depth[ancestor[u][i]] >=
                    depth[v])
        }
    }

```

```

        {
            u = ancestor[u][i];
        }
    }
    if (u == v)
    {
        return u;
    }
    for (int i = max_log - 1; i >= 0; i--)
    {
        if (ancestor[u][i] !=
            ancestor[v][i])
        {
            u = ancestor[u][i];
            v = ancestor[v][i];
        }
    }
    return ancestor[u][0];
}
int get_kth_ancestor(int u, int k)
{
    for (int i = 0; i < max_log; i++)
    {
        if (k & (1 << i))
        {
            u = ancestor[u][i];
            if (u == -1)
                break;
        }
    }
    return u;
}
int query_distance(int u, int v)
{
    int lca = get_lca(u, v);
    return depth[u] + depth[v] - 2 *
        depth[lca];
}
int get_kth_node_in_path(int u, int v, int k)
{
    int lca = get_lca(u, v);
    int dist_u_lca = depth[u] -
        depth[lca];
    int dist_lca_v = depth[v] -
        depth[lca];
    int total_distance = dist_u_lca +
        dist_lca_v;
    if (k > total_distance)
    {
        return -1;
    }
    if (k <= dist_u_lca)
    {
        return get_kth_ancestor(u, k);
    }
    else
    {
        return get_kth_ancestor(v,
            dist_u_lca + dist_lca_v - k);
    }
}
int get_ancestor_depth(int u, int target_depth)
{
    if (depth[u] < target_depth)
    {

```

```

        {
            return -1;
        }
        int diff = depth[u] - target_depth;
        return get_kth_ancestor(u, diff);
    }
};

2.8 Lazy Segment Tree with Hashing [113 lines]
struct Node{
    int a;
    Node():a(0){}
    Node(int _a): a(_a){}
    friend Node merge(const Node &A, const
        Node &B)
    {
        return Node(((1ll)A.a+B.a)%M);
    }
};
struct SegmentTree{
    //1 based string
    int n, base, M;
    int size;
    vector<Node> t;
    vector<ll> lazy;
    vector<bool> isLazy;
    oset<int> in;
    vector<int> pw;
    vector<int> inv;
    SegmentTree(int _n, int _base, int
        _M){init(_n, _base, _M);}
    void init(int _n, int _base, int _M)
    {
        n=_n, base=_base, M=_M;
        for(int i=1;i<=n;i++) in.insert(i);
        size=1;
        while(size<max(1ll,n)) size<<=1;
        t.assign(2*size,Node());
        lazy.assign(2*size,0);
        isLazy.assign(2*size,0);
        pw.resize(n+1,1);
        inv.resize(n+1,1);
        int finv=power(b1,M-2);
        for(int i=1;i<=n;i++)
            pw[i]=((1ll)pw[i-1]*base)%M,
            inv[i]=((1ll)inv[i-1]*finv)%M;
    }
    inline void pull(int nd){
        t[nd]=merge(t[nd<<1],t[nd<<1|1]);
    }
    inline void apply_node(int nd, int st, int
        en, ll val)
    {
        t[nd].a=((1ll)t[nd].a*inv[val])%M;
        lazy[nd]=(lazy[nd]+val);
        isLazy[nd]=1;
    }
    inline void push(int nd, int st, int en)
    {
        if(!isLazy[nd] || nd==size) return;
        int mid=(st+en)>>1;
        apply_node(nd<<1,st,mid,lazy[nd]);
        apply_node(nd<<1|1,mid+1,en,
            lazy[nd]);
        isLazy[nd]=0;
    }

```

```

        lazy[nd]=0;
    }
    void update(int l, int r, int val)
    {update(1,1,size,l,r,val);}
    void update(int nd, int st, int en, int
        l, int r, ll val)
    {
        if(l>en || r<st) return;
        if(l<=st && en<=r){
            apply_node(nd,st,en,val);
            return;
        }
        push(nd,st,en);
        int mid=(st+en)>>1;
        update(nd<<1,st,mid,l,r,val);
        update(nd<<1|1,mid+1,en,l,r,val);
        pull(nd);
    }
    Node query(int l, int r){return
        query(1,1,size,l,r);}
    Node query(int nd, int st, int en, int l,
        int r){
        if(l>en || r<st) return Node();
        if(l<=st && en<=r) return t[nd];
        push(nd,st,en);
        int mid=(st+en)>>1;
        Node L=query(nd<<1,st,mid,l,r);
        Node R=query(nd<<1|1,mid+1,en,l,
            r);
        return merge(L,R);
    }
    void updatePoint(int p, ll
        v){updatePoint(1,1,size,p,v);}
    void updatePoint(int nd, int st, int en,
        int p, ll v)
    {
        if(st==en){
            t[nd]=Node(v);
            lazy[nd]=0;
            isLazy[nd]=0;
            return;
        }
        push(nd,st,en);
        int mid=(st+en)>>1;
        if(p<=mid)
            updatePoint(nd<<1,st,mid,p,v);
        else updatePoint(nd<<1|1,mid+1,en,p,
            v);
        pull(nd);
    }
    void updateIdx(int pos, char ch)
    {
        assert(pos>0 && pos<=in.size());
        int fache=in.find_by_order(pos-1);
        updatePoint(fache,((1ll)(ch+1007)*
            pw[in.size()-pos])%M);
    }
    void deleteIdx(int pos)
    {
        assert(pos>0 && pos<=in.size());
        int rpos=in.find_by_order(pos-1);
        updatePoint(rpos,0);
        update(1,rpos,1);
    }

```

```

        in.erase(rpos);
    }
    int range_hash(int l, int r)
    {
        assert(l>0 && r<=in.size());
        int ff=*in.find_by_order(l-1),
            fs=*in.find_by_order(r-1);
        ll hash=query(ff,fs).a;
        return (hash*inv[n-r])%M;
    }
};

```

## 2.9 Mo's Algorithm [29 lines]

```

const int N = 2e5 + 5;
const int Q = 2e5 + 5;
const int SZ = sqrt(N) + 1;
struct qry {
    int l, r, id, blk;
    bool operator<(const qry& p) const {
        if(blk == p.blk) return blk%2?r>p.r:r<p.r;
        return blk<p.blk;
    }
};
qry query[Q];
ll ans[Q];
void add(int id) {}
void remove(int id) {}
ll get() {}
int n, q;
void MO() {
    sort(query, query + q);
    int cur_l = 0, cur_r = -1;
    for (int i = 0; i < q; i++) {
        qry q = query[i];
        while (cur_l > q.l) add(--cur_l);
        while (cur_r < q.r) add(++cur_r);
        while (cur_l < q.l) remove(cur_l++);
        while (cur_r > q.r) remove(cur_r--);
        ans[q.id] = get();
    }
}
/* 0 indexed. */

```

## 2.10 Persistent Segment Tree [142 lines]

```

#include <bits/stdc++.h>
using namespace std;
#define ll long long

struct Persistent
{
    struct Node
    {
        long long sum, min_val, max_val;
        Node *left, *right;

        Node() : sum(0), min_val(LLONG_MAX),
                max_val(LLONG_MIN),
                left(nullptr), right(nullptr) {}
    };

    Node *root;
    int n;

    Persistent(int n)
    {
        this->n = n;
        root = build(1, n);
    }
};

```

```

}

Node *build(int st, int ed)
{
    Node *node = new Node();
    if (st == ed)
    {
        return node;
    }
    int mid = (st + ed) / 2;
    node->left = build(st, mid);
    node->right = build(mid + 1, ed);
    return node;
}

Node merge(const Node &left, const Node
&right)
{
    Node result;
    result.sum = left.sum + right.sum;
    result.min_val =
        std::min(left.min_val,
                right.min_val);
    result.max_val =
        std::max(left.max_val,
                right.max_val);
    return result;
}

Node *update(Node *node, int st, int ed,
int idx, long long val)
{
    Node *nd = new Node(*node);
    if (st == ed)
    {
        nd->sum = val;
        nd->min_val = val;
        nd->max_val = val;
        return nd;
    }
    int mid = (st + ed) / 2;
    if (idx <= mid)
    {
        nd->left = update(node->left, st,
                        mid, idx, val);
    }
    else
    {
        nd->right = update(node->right,
                        mid + 1, ed, idx, val);
    }
    Node left_result = nd->left ?
        *(nd->left) : Node();
    Node right_result = nd->right ?
        *(nd->right) : Node();
    Node merged_result =
        merge(left_result, right_result);
    nd->sum = merged_result.sum;
    nd->min_val = merged_result.min_val;
    nd->max_val = merged_result.max_val;
    return nd;
}

Node query(Node *node, int st, int ed, int
l, int r)
{

```

```

    if (!node || st > r || ed < l)
    {
        return Node();
    }
    if (st >= l && ed <= r)
    {
        return *node;
    }
    int mid = (st + ed) / 2;
    Node left_result = query(node->left,
        st, mid, l, r);
    Node right_result =
        query(node->right, mid + 1, ed,
            l, r);
    return merge(left_result,
        right_result);
}

Node *updateIndex(Node *root, int idx,
long long val)
{
    return update(root, 1, n, idx, val);
}

Node queryRange(Node *root, int l, int r)
{
    return query(root, 1, n, l, r);
}

};

void solve()
{
    int n, q;
    cin >> n >> q;
    vector<ll> v(n + 1);
    for (int i = 1; i <= n; i++)
    {
        cin >> v[i];
    }
    Persistent obj(n);
    Persistent::Node *root = obj.root;
    for (int i = 1; i <= n; i++)
    {
        root = obj.updateIndex(root, i, v[i]);
    }
    vector<Persistent::Node *> version;
    version.push_back(root);
    while (q--)
    {
        int op;
        cin >> op;
        if (op == 1)
        {
            ll k, a, x;
            cin >> k >> a >> x;
            Persistent::Node *newRoot =
                obj.updateIndex(version[k -
                    1], a, x);
            version[k - 1] = newRoot;
        }
        else if (op == 2)
        {
            ll k, a, b;
            cin >> k >> a >> b;
            cout << obj.queryRange(version[k
                - 1], a, b).sum << endl;
        }
    }
}

```

```

    }
    else
    {
        int k;
        cin >> k;
        version.push_back(version[k - 1]);
    }
}
}

```

## 2.11 Sparse Table [47 lines]

```

struct sparse_table
{
    // for converting 1 based indexing
    // initialize n=size+1,s[0]=dummy;
    ll mxn;
    ll k;
    vector<vector<ll>> table1, table2;
    vector<ll> logs;
    sparse_table(ll n)
    {
        mxn = n;
        logs.resize(n + 1, 0);
        for (int i = 2; i <= mxn; i++)
        {
            logs[i] = logs[i >> 1] + 1;
        }
        k = logs[mxn] + 1;
        table1.resize(mxn + 1, vector<ll>(k));
        table2.resize(mxn + 1, vector<ll>(k));
    }
    void create_table(vector<ll> &v)
    {
        for (int i = 0; i < mxn; i++)
        {
            table1[i][0] = table2[i][0] =
                v[i];
        }
        for (ll i = 1; i < k; i++)
        {
            for (ll j = 0; j + (1 << i) - 1 <
                mxn; j++)
            {
                table1[j][i] = min(table1[j][i
                    - 1], table1[j + (1 << (i
                        - 1))][i - 1]);
                table2[j][i] = max(table2[j][i
                    - 1], table2[j + (1 << (i
                        - 1))][i - 1]);
            }
        }
    }
    ll query1(ll x, ll y)
    {
        ll gap = y - x + 1;
        ll lg = logs[gap];
        return min(table1[x][lg], table1[y -
            (1 << lg) + 1][lg]);
    }
    ll query2(ll x, ll y)
    {
        ll gap = y - x + 1;
        ll lg = logs[gap];
    }
}

```

```

        return max(table2[x][lg], table2[y -
            (1 << lg) + 1][lg]);
    }
};

```

## 2.12 TrieString [93 lines]

```

struct TNode
{
    TNode *child[26];
    bool isTerm;
    int cnt;
    TNode()
    {
        isTerm = false;
        cnt = 0;
        for (int i = 0; i < 26; i++)
        {
            child[i] = nullptr;
        }
    }
};

struct Trie
{
    TNode *root;
    Trie()
    {
        root = new TNode();
    }
    void insert(const string &s)
    {
        TNode *rt = root;
        for (char ch : s)
        {
            int idx = ch - 'a';
            if (rt->child[idx] == nullptr)
            {
                rt->child[idx] = new TNode();
            }
            rt = rt->child[idx];
            rt->cnt++;
        }
        rt->isTerm = true;
    }
    bool search(const string &s)
    {
        TNode *rt = root;
        for (int i = 0; i < s.size(); i++)
        {
            int idx = s[i] - 'a';
            if (rt->child[idx] == nullptr)
            {
                return false;
            }
            rt = rt->child[idx];
        }
        return rt->isTerm;
    }
    bool deleteHelper(TNode *rt, const string
        &s, int pos)
    {
        if (pos == s.length())
        {
            if (!rt->isTerm) return false;
            rt->isTerm = false;

```

```

            return (rt->cnt == 0);
        }
        int idx = s[pos] - 'a';
        if (rt->child[idx] == nullptr) return
            false;
        bool delch =
            deleteHelper(rt->child[idx], s,
                pos + 1);
        rt->child[idx]->cnt--;
        if (delch)
        {
            delete rt->child[idx];
            rt->child[idx] = nullptr;
        }
        return (rt->child[idx] == nullptr) &&
            !rt->isTerm;
    }
    void remove(const string &s)
    {
        deleteHelper(root, s, 0);
    }
    void clear(TNode *rt)
    {
        if (!rt)
        {
            return;
        }
        for (int i = 0; i < 26; i++)
        {
            clear(rt->child[i]);
        }
        delete rt;
    }
    ~Trie()
    {
        clear(root);
    }
};

```

## 2.13 TrieXOR [90 lines]

```

#include <bits/stdc++.h>
using namespace std;
#define ll long long

class Trie
{
public:
    static const int B = 32;
    struct Node
    {
        Node *nxt[2];
        int sz;
        Node() : sz(0)
        {
            nxt[0] = nxt[1] = nullptr;
        }
    };
    Node *root;
    Trie()
    {
        root = new Node();
    }
    // Insert a number into the Trie
    void insert(int val)
    {
        Node *cur = root;

```

```

        cur->sz++;
        for (int i = B - 1; i >= 0; --i)
        {
            int bit = (val >> i) & 1;
            if (!cur->nxt[bit]) cur->nxt[bit]
                = new Node();
            cur = cur->nxt[bit];
            cur->sz++;
        }
    }
    // Query: count numbers 'val' in the Trie
    such that (val XOR x) < k.
    int query(int x, int k)
    {
        Node *cur = root;
        int count = 0;
        for (int i = B - 1; i >= 0; --i)
        {
            if (!cur) break;
            int xBit = (x >> i) & 1, kBit = (k
                >> i) & 1;
            if (kBit == 1)
            {
                if (cur->nxt[xBit]) count +=
                    cur->nxt[xBit]->sz;
                cur = cur->nxt[1 - xBit];
            }
            else cur = cur->nxt[xBit];
        }
        return count;
    }
    // Given x, return the maximum XOR
    achievable with any inserted number.
    int getMaxXor(int x)
    {
        Node *cur = root;
        int result = 0;
        for (int i = B - 1; i >= 0; --i)
        {
            int xBit = (x >> i) & 1;
            int desired = 1 - xBit;
            if (cur->nxt[desired])
            {
                result |= (1 << i);
                cur = cur->nxt[desired];
            }
            else cur = cur->nxt[xBit];
        }
        return result;
    }
    // Given x, return the minimum XOR
    achievable with any inserted number.
    int getMinXor(int x)
    {
        Node *cur = root;
        int result = 0;
        for (int i = B - 1; i >= 0; --i)
        {
            int xBit = (x >> i) & 1;
            if (cur->nxt[xBit]) cur =
                cur->nxt[xBit];
            else
            {
                result |= (1 << i);
                cur = cur->nxt[1 - xBit];

```

```

        }
    }
    return result;
};

```

## 2.14 Wavelet Tree [101 lines]

```

using ll = long long;
struct WaveletTree
{
    int lo, hi;
    WaveletTree *L = nullptr, *R = nullptr;
    // b[i] = number of elements from
    prefix[0..i-1] that went to left
    child.
    // b.size() == n+1 where b[0] = 0.
    vector<int> b;
    vector<ll> pref;
    WaveletTree() = default;
    WaveletTree(const vector<int> &arr, int
        x, int y : lo(x), hi(y), L(nullptr),
        R(nullptr))
    {
        int n = (int)arr.size();
        b.reserve(n + 1);
        pref.reserve(n + 1);
        b.push_back(0);
        pref.push_back(0);
        if (n == 0) return;
        if (lo == hi)
        {
            for (int v : arr)
            {
                b.push_back(b.back() + 1);
                pref.push_back(pref.back() +
                    v);
            }
            return;
        }
        int mid = (lo + hi) >> 1;
        vector<int> leftArr;
        leftArr.reserve(n);
        vector<int> rightArr;
        rightArr.reserve(n);
        for (int v : arr)
        {
            if (v <= mid)
            {
                leftArr.push_back(v);
                b.push_back(b.back() + 1);
            }
            else
            {
                rightArr.push_back(v);
                b.push_back(b.back());
            }
            pref.push_back(pref.back() + v);
        }
        if (!leftArr.empty())
            L = new WaveletTree(leftArr, lo,
                mid);
        if (!rightArr.empty())
            R = new WaveletTree(rightArr, mid
                + 1, hi);
    }
};

```



```
WaveletTree()
{
    delete L;
    delete R;
}
// kth smallest in [l, r], 1-based. returns
// -1 if invalid k.
int kth(int l, int r, int k) const
{
    if (l > r || k <= 0 || k > r - l + 1)
        return -1;
    if (lo == hi) return lo;
    int inLeft = b[r] - b[l - 1];
    if (k <= inLeft) return L->kth(b[l - 1] + 1, b[r], k);
    else return R->kth(l - b[l - 1], r - b[r], k - inLeft);
}
// count of numbers <= k in [l, r]
int countLTE(int l, int r, int k) const
{
    if (l > r || k < lo) return 0;
    if (hi <= k) return r - l + 1;
    int lb = b[l - 1], rb = b[r];
    return (L ? L->countLTE(lb + 1, rb, k) : 0) + (R ? R->countLTE(l - lb, r - rb, k) : 0);
}
// count equal to value in [l, r]
int countEqual(int l, int r, int value) const
{
    if (l > r || value < lo || value > hi)
        return 0;
    if (lo == hi) return r - l + 1;
    int mid = (lo + hi) >> 1;
    int lb = b[l - 1], rb = b[r];
    if (value <= mid) return L ? L->countEqual(lb + 1, rb, value) : 0;
    else return R ? R->countEqual(l - lb, r - rb, value) : 0;
}
// sum of values < k in [l, r]
// Requires pref to be present (we store original prefix sums at construction)
ll sumLess(int l, int r, int k) const
{
    if (l > r || k <= lo) return 0;
    if (hi < k) return pref[r] - pref[l - 1];
    int lb = b[l - 1], rb = b[r];
    ll leftSum = L ? L->sumLess(lb + 1, rb, k) : 0;
    ll rightSum = R ? R->sumLess(l - lb, r - rb, k) : 0;
    return leftSum + rightSum;
}
// utility: returns total number of
// elements stored at this node
int size() const
{
    return (int)pref.size() - 1;
}
};
```

### 3 Dynamic Programming

#### 3.1 Cartesian [42 lines]

```
// Build a max-heap Cartesian tree over
// h[0..n-1].
// Returns the index of the root.
// Outputs left-child in L[], right-child in
// R[].
int buildCartesianTree(const vector<int> &h,
    vector<int> &L, vector<int> &R)
{
    int n = h.size();
    L.assign(n, -1);
    R.assign(n, -1);
    vector<int> parent(n, -1);
    stack<int> st;

    for (int i = 0; i < n; i++)
    {
        int last = -1;
        while (!st.empty() && h[st.top()] < h[i])
        {
            last = st.top();
            st.pop();
        }
        if (!st.empty())
        {
            parent[i] = st.top();
            R[st.top()] = i;
        }
        if (last != -1)
        {
            parent[last] = i;
            L[i] = last;
        }
        st.push(i);
    }
    int root = -1;
    for (int i = 0; i < n; i++)
    {
        if (parent[i] == -1)
        {
            root = i;
            break;
        }
    }
    return root;
}
```

#### 3.2 Convex Hull Trick [99 lines]

```
*****Offline*****
*****
// dpcur[i] depends on dppre[1--n]
const ll M = 1e16 + 7;
const ll N = 1e5 + 3;
ll dis[N];
ll xcross(ll m1, ll c1, ll m2, ll c2, ll n)
{
    ld ans = ((ld)c1 - c2) / (m2 - m1);
    if (ans > n) ans = n + 1;
    return ceil(ans);
}
// main
int main()
{
    fastio int n, m, k;
```

```
cin >> n >> m >> k;
for (int i = 1; i <= n; i++) dis[i] = M;
for (ll i = 1; i <= k; i++)
{
    vector<ll> str;
    str.push_back(1);
    str.push_back(2);
    for (ll i = 3; i <= n; i++)
    {
        while (str.size() > 1)
        {
            ll lst = str.back();
            ll slst = str[str.size() - 2];
            ll cur = xcross(-2 * lst,
                dis[lst] + lst * lst, -2
                * i, dis[i] + i * i, n);
            ll pre = xcross(-2 * lst,
                dis[lst] + lst * lst, -2
                * slst, dis[slst] + slst
                * slst, n);
            if (cur <= pre)
                str.pop_back();
            else break;
        }
        str.push_back(i);
    }
    ll pre = 1;
    ll diss[n + 1];
    for (ll i = 1; i < str.size(); i++)
    {
        ll lst = str[i];
        ll cur = xcross(-2 * lst, dis[lst]
            + lst * lst, -2 * i, dis[i] +
            i * i, n);
    }
    for (ll i = 1; i < str.size(); i++)
    {
        ll lst = str[i - 1];
        ll slst = str[i];
        ll cur = xcross(-2 * lst, dis[lst]
            + lst * lst, -2 * slst,
            dis[slst] + slst * slst, n);
        for (ll i = pre; i < cur; i++)
            diss[i] = (i - lst) * (i -
                lst) + dis[lst];
        pre = cur;
    }
    ll lst = str.back();
    for (ll i = pre; i <= n; i++) diss[i]
        = (i - lst) * (i - lst) +
            dis[lst];
    for (int i = 1; i <= n; i++) dis[i] =
        diss[i];
}
for (int i = 1; i <= n; i++) cout <<
    dis[i] << " ";
cout << "\n";
}
```

```
// for(i>j) dp[i]=min(dp[j]+(i-j)^2+c);
// [1]=C0;
ll xcross(ll m1, ll c1, ll m2, ll c2, ll n)
{
    ld ans = ((ld)c1 - c2) / (m2 - m1);
    if (ans > n) ans = n + 1;
    return ceil(ans);
}
```

```
}
ll dp[N];
void CHT(ll n, ll C1, ll C)
{
    dp[1] = C1;
    vector<ll> str;
    str.push_back(1);
    int cur = 0;
    for (ll i = 2; i <= n; i++)
    {
        dp[i] = C + (i - str[cur]) * (i -
            str[cur]) + dp[str[cur]];
        while (cur < (int)str.size() - 1)
        {
            cur++;
            ll tem = C + (i - str[cur]) * (i -
                str[cur]) + dp[str[cur]];
            if (tem > dp[i])
            {
                cur--;
                break;
            }
            dp[i] = tem;
        }
        while (str.size() > 1)
        {
            ll lst = str.back();
            ll slst = str[str.size() - 2];
            ll cur = xcross(-2 * lst, dp[lst]
                + lst * lst, -2 * i, dp[i] +
                * i, n);
            ll pre = xcross(-2 * lst, dp[lst]
                + lst * lst, -2 * slst,
                dp[slst] + slst * slst, n);
            if (cur <= pre) str.pop_back();
            else break;
        }
        if (cur >= str.size()) cur =
            str.size() - 1;
        str.push_back(i);
    }
}
```

#### 3.3 Divide And Conquer Trick [41 lines]

```
int dcp(int st, int ed, int opt1, int opt2,
    int n)
{
    if (st > ed) return opt1;
    int mid = (st + ed) / 2;
    int opt = opt1;
    ll anss = inf;
    ll cost = ans(1, 1, n, min(mid, opt2) +
        1, mid, mid);
    for (int i = min(opt2, mid); i >= opt1;
        i--)
    {
        if (nxt[i] <= mid) cost = cost +
            nxt[i] - i;
        if (cost + tem[i - 1] < anss)
        {
            anss = cost + tem[i - 1];
            opt = i;
        }
    }
}
```

```

    }
    tk[mid] = anss;
    if (st == ed) return opt;
    opt1 = dcp(st, mid - 1, opt1, opt, n);
    opt2 = dcp(mid + 1, ed, opt, opt2, n);
    return opt2;
}
//main
int main()
{
    fastio
    int n,k;
    cin>>n>>k;
    for(int i=1; i<=n; i++)cin>>a[i];
    gen(n);
    make(1,1,n);
    tk[0]=0;
    for(int i=1; i<=n; i++)tk[i]=0;
    for(int kk=1; kk<=k; kk++)
    {
        for(int i=0; i<=n; i++)tem[i]=tk[i];
        int opt=dcp(1,n,1,n,n);
    }
    cout<<tk[n]<<"\n";
}

```

### 3.4 SOS DP [88 lines]

```

const ll MLOG = 20;
const ll MAXN = (1<<MLOG);
ll dp[MAXN]; ll freq[MAXN];
void forward1(){ // adding element to all its
    super set
    for(ll bit = 0; bit < MLOG; bit++){
        for(ll i = 0; i < MAXN; i++){
            if(i&(1<<bit)){
                dp[i]+=dp[i^(1<<bit)]; //add
                a[i] to a[j] if j&i = i
            }
        }
    }
}
void backward1(){
    for(ll bit = 0; bit < MLOG; bit++){
        for(ll i = MAXN-1; i >= 0; i--){
            if(i&(1<<bit)){
                dp[i]-=dp[i^(1<<bit)];
            }
        }
    }
}
void forward2(){ // add elements to its
    subsets
    for(ll bit = 0; bit < MLOG; bit++){
        for(ll i = MAXN-1; i >= 0; i--){
            if(i&(1<<bit)){
                dp[i^(1<<bit)]+=dp[i];
            }
        }
    }
}
void backward2(){
    for(ll bit = 0; bit < MLOG; bit++){
        for(ll i = 0; i < MAXN; i++){
            if(i&(1<<bit)){

```

```

        dp[i^(1<<bit)]=-dp[i];
    }
}
//F[i] = A[i]2 +(SUM(G[j]))2 where (i&j) = j
//and j < i
//G[i] = (SUM(F[j]2)) where (i&j) = j and j <=
//i
int main()
{
    fastio
    int n;
    cin>>n;
    ll a[n];
    for(int i=0;i<n;i++)cin>>a[i];
    int lg=log2(n)+2;
    ll f[n],g[n],gs[n][lg],f2s[n][lg];
    for(int i=0;i<n;i++)
    {
        f[i]=0;
        for(int j=0;j+1<lg;j++)
        {
            if(i&(1<<j))
            {
                f[i]=f[i]+gs[i^(1<<j)][j];
            }
        }
        f[i]%=M;
        f[i]=(f[i]*f[i]+a[i]*a[i])%M;
        f2s[i][0]=f[i]*f[i]%M;
        for(int j=0;j+1<lg;j++)
        {
            if(i&(1<<j))
            {
                f2s[i][j+1]=(f2s[i][j]+
                f2s[i^(1<<j)][j])%M;
            }else f2s[i][j+1]=f2s[i][j];
        }
        g[i]=f2s[i][lg-1];
        gs[i][0]=g[i];
        for(int j=0;j+1<lg;j++)
        {
            if(i&(1<<j))
            {
                gs[i][j+1]=(gs[i][j]+gs[i^(1<<
                j)][j])%M;
            }else gs[i][j+1]=gs[i][j];
        }
    }
    ll sum=0;
    for(int i=0;i<n;i++)
    {
        sum=(sum+(f[i]*g[i]%M)*i)%M;
    }
    cout<<sum<<"\n";
}

```

## 4 Flow

### 4.1 Dinic [76 lines]

```

// O(V^2*E)
struct Dinic
{
    vector<vector<int>> adj, capacity;

```

```

    vector<int> level, ptr;
    int n;

    Dinic(int n) : n(n)
    {
        adj.resize(n + 1);
        capacity.assign(n + 1, vector<int>(n
            + 1, 0));
        level.resize(n + 1);
        ptr.resize(n + 1);
    }
    void add_edge(int u, int v, int cap)
    {
        capacity[u][v] += cap;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    bool bfs(int s, int t)
    {
        fill(level.begin(), level.end(), -1);
        level[s] = 0;
        queue<int> q;
        q.push(s);

        while (!q.empty())
        {
            int u = q.front();
            q.pop();
            for (int v : adj[u])
            {
                if (level[v] == -1 &&
                    capacity[u][v] > 0)
                {
                    level[v] = level[u] + 1;
                    q.push(v);
                }
            }
        }
        return level[t] != -1;
    }
    int dfs(int u, int t, int flow)
    {
        if (u == t || flow == 0)
            return flow;
        for (int &i = ptr[u]; i <
            adj[u].size(); i++)
        {
            int v = adj[u][i];
            if (level[v] == level[u] + 1 &&
                capacity[u][v] > 0)
            {
                int bottleneck = dfs(v, t,
                    min(flow,
                        capacity[u][v]));
                if (bottleneck > 0)
                {
                    capacity[u][v] -=
                        bottleneck;
                    capacity[v][u] +=
                        bottleneck;
                    return bottleneck;
                }
            }
        }
        return 0;
    }
}

```

```

    int max_flow(int s, int t)
    {
        int flow = 0;
        while (bfs(s, t))
        {
            fill(ptr.begin(), ptr.end(), 0);
            while (int new_flow = dfs(s, t,
                INF))
            {
                flow += new_flow;
            }
        }
        return flow;
    }
};

```

### 4.2 Edmonds Karp [60 lines]

```

// O(V*E^2)
struct EdmondsKarp
{
    vector<vector<int>> capacity, adj;
    int n;

    EdmondsKarp(int n) : n(n)
    {
        capacity.assign(n + 1, vector<int>(n
            + 1, 0));
        adj.resize(n + 1);
    }
    void add_edge(int u, int v, int cap)
    {
        capacity[u][v] += cap;
        adj[u].push_back(v);
        adj[v].push_back(u); // Reverse edge
    }
    int bfs(int s, int t, vector<int> &parent)
    {
        fill(parent.begin(), parent.end(),
            -1);
        parent[s] = -2;
        queue<pair<int, int>> q;
        q.push({s, INF});

        while (!q.empty())
        {
            auto [u, flow] = q.front();
            q.pop();
            for (int v : adj[u])
            {
                if (parent[v] == -1 &&
                    capacity[u][v] > 0)
                {
                    parent[v] = u;
                    int new_flow = min(flow,
                        capacity[u][v]);
                    if (v == t) return
                        new_flow;
                    q.push({v, new_flow});
                }
            }
        }
        return 0;
    }
    int max_flow(int s, int t)
    {

```

```

int flow = 0, new_flow;
vector<int> parent(n + 1);
while ((new_flow = bfs(s, t, parent))
    > 0)
{
    flow += new_flow;
    int cur = t;
    while (cur != s)
    {
        int prev = parent[cur];
        capacity[prev][cur] -=
            new_flow;
        capacity[cur][prev] +=
            new_flow;
        cur = prev;
    }
    return flow;
}
};

```

#### 4.3 Ford Fulkerson [50 lines]

//  $O(E \cdot F)$ , where  $F$  is the maximum flow value.

```

struct FordFulkerson
{
    vector<vector<int>> capacity, adj;
    vector<bool> visited;
    int n;

    FordFulkerson(int n) : n(n)
    {
        capacity.assign(n + 1, vector<int>(n
            + 1, 0));
        adj.resize(n + 1);
    }

    void add_edge(int u, int v, int cap)
    {
        capacity[u][v] += cap;
        adj[u].push_back(v);
        adj[v].push_back(u); // Reverse edge
                               for residual graph
    }

    int dfs(int u, int t, int flow)
    {
        if (u == t)
            return flow;
        visited[u] = true;
        for (int v : adj[u])
        {
            if (!visited[v] && capacity[u][v]
                > 0)
            {
                int bottleneck = dfs(v, t,
                    min(flow,
                        capacity[u][v]));
                if (bottleneck > 0)
                {
                    capacity[u][v] -=
                        bottleneck;
                    capacity[v][u] +=
                        bottleneck;
                    return bottleneck;
                }
            }
        }
    }

    return 0;
}

```

```

}
int max_flow(int s, int t)
{
    int flow = 0, new_flow;
    do
    {
        visited.assign(n + 1, false);
        new_flow = dfs(s, t, INF);
        flow += new_flow;
    } while (new_flow > 0);
    return flow;
}
};

```

#### 4.4 Hopcroft Karp [78 lines]

```

//  $O(E \cdot \sqrt{V})$ 
class HopcroftKarp
{
public:
    int n, m;
    vector<vector<int>> adj;
    vector<int> pairU, pairV, dist;
    HopcroftKarp(int n, int m) : n(n), m(m)
    {
        adj.resize(n + 1);
        pairU.assign(n + 1, 0);
        pairV.assign(m + 1, 0);
        dist.assign(n + 1, 0);
    }

    void addEdge(int u, int v)
    {
        adj[u].push_back(v);
    }

    bool bfs()
    {
        queue<int> q;
        for (int u = 1; u <= n; ++u)
        {
            if (pairU[u] == 0)
            {
                dist[u] = 0;
                q.push(u);
            }
            else dist[u] = INT_MAX;
        }
        dist[0] = INT_MAX;

        while (!q.empty())
        {
            int u = q.front();
            q.pop();
            if (dist[u] < dist[0])
            {
                for (int v : adj[u])
                {
                    if (dist[pairV[v]] ==
                        INT_MAX)
                    {
                        dist[pairV[v]] =
                            dist[u] + 1;
                        q.push(pairV[v]);
                    }
                }
            }
        }
    }

    return dist[0] != INT_MAX;
}

```

```

}
bool dfs(int u)
{
    if (u == 0) return true;
    for (int v : adj[u])
    {
        if (dist[pairV[v]] == dist[u] + 1
            && dfs(pairV[v]))
        {
            pairV[v] = u;
            pairU[u] = v;
            return true;
        }
    }
    dist[u] = INT_MAX;
    return false;
}

int maxMatching()
{
    int matching = 0;
    while (bfs())
    {
        for (int u = 1; u <= n; ++u)
            if (pairU[u] == 0 && dfs(u))
                matching++;
    }
    return matching;
}
};

```

#### 4.5 Max Flow Path [99 lines]

```

struct MaxFlowpath
{
    struct Edge
    {
        int v, rev, cap, flow;
    };
    int n;
    vector<vector<Edge>> adj;

    MaxFlowpath(int n) : n(n)
    {
        adj.resize(n + 1);
    }

    void add_edge(int u, int v)
    {
        adj[u].push_back({v,
            (int)adj[v].size(), 1, 0});
        adj[v].push_back({u,
            (int)adj[u].size() - 1, 0, 0});
    }

    bool bfs(int s, int t, vector<int>
        &parent, vector<int> &edge_index)
    {
        vector<bool> visited(n + 1, false);
        queue<int> q;
        q.push(s);
        visited[s] = true;
        while (!q.empty())
        {
            int u = q.front();
            q.pop();
            for (int i = 0; i <
                adj[u].size(); i++)
            {
                Edge &e = adj[u][i];
                if (!visited[e.v] && e.cap >
                    0)
                {
                    visited[e.v] = true;
                    parent[e.v] = u;
                    edge_index[e.v] = i;
                    q.push(e.v);
                }
            }
            return false;
        }
    }

    int max_flow(int s, int t)
    {
        int total_flow = 0;
        while (true)
        {
            vector<int> parent(n + 1, -1),
                edge_index(n + 1, -1);
            if (!bfs(s, t, parent,
                edge_index))
                break;
            int push_flow = INT_MAX, cur = t;
            while (cur != s)
            {
                int prev = parent[cur];
                Edge &e =
                    adj[prev][edge_index[cur]];

                push_flow = min(push_flow,
                    e.cap);
                cur = prev;
            }
            cur = t;
            while (cur != s)
            {
                int prev = parent[cur];
                Edge &e =
                    adj[prev][edge_index[cur]];

                e.cap -= push_flow;
                e.flow += push_flow;
                adj[cur][e.rev].cap +=
                    push_flow;
                adj[cur][e.rev].flow -=
                    push_flow;
                cur = prev;
            }
            total_flow += push_flow;
        }
        return total_flow;
    }

    void dfs(int u, vector<int> &p)
    {
        p.push_back(u);
        for (auto &e : adj[u])
        {
            if (e.flow > 0)
            {
                e.flow--;
                dfs(e.v, p);
            }
        }
    }
}

```



```

        break;
    }
}
vector<vector<int>>
find_disjoint_paths(int s, int f)
{
    vector<vector<int>> paths;
    while (f--)
    {
        vector<int> path;
        dfs(s, path);
        paths.push_back(path);
    }
    return paths;
}
};

```

#### 4.6 Max Flow [93 lines]

```

struct MaxFlow
{
    struct Edge
    {
        int v, rev, cap;
    };
    int n;
    vector<vector<Edge>> adj;
    MaxFlow(int n) : n(n)
    {
        adj.resize(n + 1);
    }
    void add_edge(int u, int v)
    {
        adj[u].push_back({v,
            (int)adj[v].size(), 1});
        adj[v].push_back({u,
            (int)adj[u].size() - 1, 0});
    }
    bool bfs(int s, int t, vector<int>
        &parent, vector<int> &edge_index)
    {
        vector<bool> visited(n + 1, false);
        queue<int> q;
        q.push(s);
        visited[s] = true;
        while (!q.empty())
        {
            int u = q.front();
            q.pop();
            for (int i = 0; i <
                adj[u].size(); i++)
            {
                Edge &e = adj[u][i];
                if (!visited[e.v] && e.cap >
                    0)
                {
                    visited[e.v] = true;
                    parent[e.v] = u;
                    edge_index[e.v] = i;
                    if (e.v == t) return true;
                    q.push(e.v);
                }
            }
        }
        return false;
    }
};

```

```

int max_flow(int s, int t)
{
    int total_flow = 0;
    while (true)
    {
        vector<int> parent(n + 1, -1),
            edge_index(n + 1, -1);
        if (!bfs(s, t, parent,
            edge_index))
        {
            break;
        }
        int push_flow = INT_MAX, cur = t;
        while (cur != s)
        {
            int prev = parent[cur];
            Edge &e =
                adj[prev][edge_index[cur]];

            push_flow = min(push_flow,
                e.cap);
            cur = prev;
        }
        cur = t;
        while (cur != s)
        {
            int prev = parent[cur];
            Edge &e =
                adj[prev][edge_index[cur]];

            e.cap -= push_flow;
            adj[cur][e.rev].cap +=
                push_flow;
            cur = prev;
        }
        total_flow += push_flow;
    }
    return total_flow;
}
void dfs(int u, vector<bool> &visited)
{
    visited[u] = true;
    for (const auto &e : adj[u])
    {
        if (e.cap > 0 && !visited[e.v])
            dfs(e.v, visited);
    }
}
vector<pair<int, int>> find_min_cut(int s)
{
    vector<bool> visited(n + 1, false);
    dfs(s, visited);
    vector<pair<int, int>> cut_edges;
    for (int u = 1; u <= n; u++)
    {
        if (!visited[u]) continue;
        for (const auto &e : adj[u])
        {
            if (!visited[e.v] && e.cap ==
                0)
            {
                cut_edges.emplace_back(u,
                    e.v);
            }
        }
        return cut_edges;
    }
}
};

```

#### 4.7 Min Cost Max Flow [87 lines]

```

struct MinCostMaxFlow

```

```

{
    struct Edge
    {
        int v, rev; // Destination and
            index of the reverse edge
        int cap, cost; // Capacity and cost of
            the edge
    };
    vector<vector<Edge>> adj;
    int n;
    MinCostMaxFlow(int n) : n(n)
    {
        adj.resize(n + 1); // 1-based indexing
    }
    void add_edge(int u, int v, int cap, int
        cost)
    {
        adj[u].push_back({v,
            (int)adj[v].size(), cap, cost});
        adj[v].push_back({u,
            (int)adj[u].size() - 1, 0,
            -cost});
    }
    bool spfa(int s, int t, vector<int>
        &parent, vector<int> &edge_index,
        vector<int> &dist)
    {
        dist.assign(n + 1, INF);
        parent.assign(n + 1, -1);
        edge_index.assign(n + 1, -1);
        vector<bool> in_queue(n + 1, false);
        queue<int> q;

        dist[s] = 0;
        q.push(s);
        in_queue[s] = true;

        while (!q.empty())
        {
            int u = q.front();
            q.pop();
            in_queue[u] = false;
            for (int i = 0; i <
                adj[u].size(); i++)
            {
                Edge &e = adj[u][i];
                if (e.cap > 0 && dist[u] +
                    e.cost < dist[e.v])
                {
                    dist[e.v] = dist[u] +
                        e.cost;
                    parent[e.v] = u;
                    edge_index[e.v] = i;
                    if (!in_queue[e.v])
                    {
                        q.push(e.v);
                        in_queue[e.v] = true;
                    }
                }
            }
        }
        return dist[t] != INF;
    }
    pair<int, int> max_flow(int s, int t)
    {

```

```

        int total_flow = 0, total_cost = 0;
        vector<int> parent, edge_index, dist;

        while (spfa(s, t, parent, edge_index,
            dist))
        {
            // Find the bottleneck capacity
                along the path
            int push_flow = INF, cur = t;
            while (cur != s)
            {
                int prev = parent[cur];
                Edge &e =
                    adj[prev][edge_index[cur]];

                push_flow = min(push_flow,
                    e.cap);
                cur = prev;
            }
            // Update the residual graph along
                the path
            cur = t;
            while (cur != s)
            {
                int prev = parent[cur];
                Edge &e =
                    adj[prev][edge_index[cur]];

                e.cap -= push_flow;
                adj[e.v][e.rev].cap +=
                    push_flow;
                cur = prev;
            }
            total_flow += push_flow;
            total_cost += push_flow * dist[t];
        }

        return {total_flow, total_cost};
    }
};

```

## 5 Game Theory

### 5.1 Points to be noted [14 lines]

>[First Write a Brute Force solution]  
 >Nim = all xor  
 >Misere Nim = Nim + corner case: if all piles  
 are 1, reverse(nim)  
 >Bogus Nim = Nim  
 >Staircase Nim = Odd indexed pile Nim (Even  
 indexed pile doesn't matter, as one player  
 can give bogus moves to drop all even  
 piles to ground)  
 >Sprague Grundy: [Every impartial game under  
 the normal play convention is equivalent  
 to a one-heap game of nim]  
 Every tree = one nim pile = tree root value;  
 tree leaf value = 0; tree node value = mex  
 of all child nodes.  
 [Careful: one tree node can become multiple  
 new tree roots(multiple elements in one  
 node), then the value of that node = xor  
 of all those root values]

>Hackenbush(Given a rooted tree; cut an edge in one move; subtree under that edge gets removed; last player to cut wins):

Colon:  
 $G(u) = (G(v_1)+1) \oplus (G(v_2)+1) \oplus \dots [v_1, v_2, \dots \text{are childs of } u]$

For multiple trees ans is their xor

>Hackenbush on graph (instead of tree given an rooted graph):

fusion: All edges in a cycle can be fused to get a tree structure; build a super node, connect some single nodes with that super node, number of single nodes is the number of edges in the cycle.

Sol: [Bridge component tree] mark all bridges, a group of edges that are not bridges, becomes one component and contributes number of edges to the hackenbush. (even number of edges contributes 0, odd number of edges contributes 1)

## 6 Geometry

### 6.1 Convex Hull [60 lines]

```
struct pt {
double x, y;
pr(){
pt(double x,double y)
{
this->x=x;
this->y=y;
}
bool operator == (pt const& t) const {
return x == t.x && y == t.y;
};
int orientation(pt a, pt b, pt c) {
double v =
a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y);
if (v < 0) return -1; // clockwise
if (v > 0) return +1; // counter-clockwise
return 0;
}
bool cw(pt a, pt b, pt c, bool
include_collinear) {
int o = orientation(a, b, c);
return o < 0 || (include_collinear && o == 0);
}
bool collinear(pt a, pt b, pt c) { return
orientation(a, b, c) == 0; }
void convex_hull(vector<pt>& a, bool
include_collinear
= false) {
pt p0 = *min_element(a.begin(), a.end(), [](pt
a, pt
b) {
return make_pair(a.y, a.x) < make_pair(b.y,
b.x);
});
sort(a.begin(), a.end(), [&p0](const pt& a,
const
pt& b) {
int o = orientation(p0, a, b);
if (o == 0)
return (p0.x-a.x)*(p0.x-a.x) +
```

```
(p0.y-a.y)*(p0.y-a.y)
< (p0.x-b.x)*(p0.x-b.x) +
(p0.y-b.y)*(p0.y-b.y);
return o < 0;
});
if (include_collinear) {
int i = (int)a.size()-1;
while (i >= 0 && collinear(p0, a[i],
a.back()))
i--;
reverse(a.begin()+i+1, a.end());
}
vector<pt> st;
for (int i = 0; i < (int)a.size(); i++) {
while (st.size() > 1 && !cw(st[st.size()-2],
st.back(), a[i], include_collinear))
st.pop_back();
st.push_back(a[i]);
}
if (include_collinear == false && st.size()
== 2 &&
st[0] == st[1])
st.pop_back();
a = st;
}
```

### 6.2 Geometry [891 lines]

```
int sign(T x) { return (x > eps) - (x <
-eps); }
struct PT {
T x, y;
PT() { x = 0, y = 0; }
PT(T x, T y) : x(x), y(y) {}
PT(const PT &p) : x(p.x), y(p.y) {}
PT operator + (const PT &a) const { return
PT(x + a.x, y + a.y); }
PT operator - (const PT &a) const { return
PT(x - a.x, y - a.y); }
PT operator * (const T a) const { return
PT(x * a, y * a); }
friend PT operator * (const T &a, const PT
&b) { return PT(a * b.x, a * b.y); }
PT operator / (const T a) const { return
PT(x / a, y / a); }
bool operator == (PT a) const { return
sign(a.x - x) == 0 && sign(a.y - y) ==
0; }
bool operator != (PT a) const { return
!(*this == a); }
bool operator < (PT a) const { return
sign(a.x - x) == 0 ? y < a.y : x < a.x; }
bool operator > (PT a) const { return
sign(a.x - x) == 0 ? y > a.y : x > a.x; }
T norm() { return sqrt(x * x + y * y); }
T norm2() { return x * x + y * y; }
PT perp() { return PT(-y, x); }
T arg() { return atan2(y, x); }
PT truncate(T r) { // returns a vector with
norm r and having same direction
T k = norm();
if (!sign(k)) return *this;
r /= k;
return PT(x * r, y * r);
}
```

```
};
istream &operator >> (istream &in, PT &p) {
return in >> p.x >> p.y; }
ostream &operator << (ostream &out, PT &p) {
return out << "(" << p.x << ", " << p.y <<
")"; }
inline T dot(PT a, PT b) { return a.x * b.x +
a.y * b.y; }
inline T dist2(PT a, PT b) { return dot(a -
b, a - b); }
inline T dist(PT a, PT b) { return sqrt(dot(a
- b, a - b)); }
inline T cross(PT a, PT b) { return a.x * b.y
- a.y * b.x; }
inline T cross2(PT a, PT b, PT c) { return
cross(b - a, c - a); }
inline int orientation(PT a, PT b, PT c) {
return sign(cross(b - a, c - a)); }
PT perp(PT a) { return PT(-a.y, a.x); }
PT rotateccw90(PT a) { return PT(-a.y, a.x); }
PT rotatecw90(PT a) { return PT(a.y, -a.x); }
PT rotateccw(PT a, T t) { return PT(a.x *
cos(t) - a.y * sin(t), a.x * sin(t) + a.y
* cos(t)); }
PT rotatecw(PT a, T t) { return PT(a.x *
cos(t) + a.y * sin(t), -a.x * sin(t) + a.y
* cos(t)); }
T rad_to_deg(T r) { return (r * 180.0 / PI); }
T deg_to_rad(T d) { return (d * PI / 180.0); }
T get_angle(PT a, PT b) {
T costheta = dot(a, b) / a.norm() /
b.norm();
return acos(max((T)-1.0, min((T)1.0,
costheta))); }
}
bool is_point_in_angle(PT b, PT a, PT c, PT p)
{ // does point p lie in angle <bac
assert(orientation(a, b, c) != 0);
if (orientation(a, c, b) < 0) swap(b, c);
return orientation(a, c, p) >= 0 &&
orientation(a, b, p) <= 0;
}
bool half(PT p) {
return p.y > 0.0 || (p.y == 0.0 && p.x <
0.0);
}
void polar_sort(vector<PT> &v) { // sort
points in counterclockwise
sort(v.begin(), v.end(), [](PT a,PT b) {
return make_tuple(half(a), 0.0, a.norm2())
< make_tuple(half(b), cross(a, b),
b.norm2());
});
}
void polar_sort(vector<PT> &v, PT o) { // sort
points in counterclockwise with respect to
point o
sort(v.begin(), v.end(), [&](PT a,PT b) {
return make_tuple(half(a - o), 0.0, (a -
o).norm2()) < make_tuple(half(b - o),
cross(a - o, b - o), (b -
o).norm2());
});
}
struct line {
```

```
PT a, b; // goes through points a and b
PT v; T c; //line form: direction vec
[<cross>] (x, y) = c
line() {}
//direction vector v and offset c
line(PT v, T c) : v(v), c(c) {
auto p = get_points();
a = p.first; b = p.second;
}
// equation ax + by + c = 0
line(T _a, T _b, T _c) : v({_b, -_a}),
c(-_c) {
auto p = get_points();
a = p.first; b = p.second;
}
// goes through points p and q
line(PT p, PT q) : v(q - p), c(cross(v,
p)), a(p), b(q) {}
pair<PT, PT> get_points() { //extract any
two points from this line
PT p, q; T a = -v.y, b = v.x; // ax + by = c
if (sign(a) == 0) {
p = PT(0, c / b);
q = PT(1, c / b);
}
else if (sign(b) == 0) {
p = PT(c / a, 0);
q = PT(c / a, 1);
}
else {
p = PT(0, c / b);
q = PT(1, (c - a) / b);
}
return {p, q};
}
// ax + by + c = 0
array<T, 3> get_abc() {
T a = -v.y, b = v.x;
return {a, b, -c};
}
// 1 if on the left, -1 if on the right, 0 if
on the line
int side(PT p) { return sign(cross(v, p) -
c); }
// line that is perpendicular to this and
goes through point p
line perpendicular_through(PT p) { return
{p, p + perp(v)}; }
// translate the line by vector t i.e.
shifting it by vector t
line translate(PT t) { return {v, c +
cross(v, t)}; }
// compare two points by their orthogonal
projection on this line
// a projection point comes before another if
it comes first according to vector v
bool cmp_by_projection(PT p, PT q) { return
dot(v, p) < dot(v, q); }
line shift_left(T d) {
PT z = v.perp().truncate(d);
return line(a + z, b + z);
}
};
// find a point from a through b with distance
d
```

```

PT point_along_line(PT a, PT b, T d) {
    assert(a != b);
    return a + (((b - a) / (b - a).norm()) * d);
}
// projection point c onto line through a and b
// assuming a != b
PT project_from_point_to_line(PT a, PT b, PT c) {
    return a + (b - a) * dot(c - a, b - a) / (b - a).norm2();
}
// reflection point c onto line through a and b
// assuming a != b
PT reflection_from_point_to_line(PT a, PT b, PT c) {
    PT p = project_from_point_to_line(a, b, c);
    return p + p - c;
}
// minimum distance from point c to line
// through a and b
T dist_from_point_to_line(PT a, PT b, PT c) {
    return fabs(cross(b - a, c - a) / (b - a).norm());
}
// returns true if point p is on line segment
// ab
bool is_point_on_seg(PT a, PT b, PT p) {
    if (fabs(cross(p - b, a - b)) < eps) {
        if (p.x < min(a.x, b.x) - eps || p.x > max(a.x, b.x) + eps) return false;
        if (p.y < min(a.y, b.y) - eps || p.y > max(a.y, b.y) + eps) return false;
        return true;
    }
    return false;
}
// minimum distance point from point c to
// segment ab that lies on segment ab
PT project_from_point_to_seg(PT a, PT b, PT c) {
    T r = dist2(a, b);
    if (sign(r) == 0) return a;
    r = dot(c - a, b - a) / r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b - a) * r;
}
// minimum distance from point c to segment ab
T dist_from_point_to_seg(PT a, PT b, PT c) {
    return dist(c, project_from_point_to_seg(a, b, c));
}
// 0 if not parallel, 1 if parallel, 2 if
// collinear
int is_parallel(PT a, PT b, PT c, PT d) {
    T k = fabs(cross(b - a, d - c));
    if (k < eps) {
        if (fabs(cross(a - b, a - c)) < eps &&
            fabs(cross(c - d, c - a)) < eps)
            return 2;
        else return 1;
    }
    else return 0;
}
// check if two lines are same
bool are_lines_same(PT a, PT b, PT c, PT d) {
    if (fabs(cross(a - c, c - d)) < eps &&
        fabs(cross(b - c, c - d)) < eps) return
        true;
    return false;
}
// bisector vector of <abc
PT angle_bisector(PT &a, PT &b, PT &c) {
    PT p = a - b, q = c - b;
    return p + q * sqrt(dot(p, p) / dot(q, q));
}
// 1 if point is ccw to the line, 2 if point is
// cw to the line, 3 if point is on the line
int point_line_relation(PT a, PT b, PT p) {
    int c = sign(cross(p - a, b - a));
    if (c < 0) return 1;
    if (c > 0) return 2;
    return 3;
}
// intersection point between ab and cd
// assuming unique intersection exists
bool line_line_intersection(PT a, PT b, PT c, PT d, PT &ans) {
    T a1 = a.y - b.y, b1 = b.x - a.x, c1 = cross(a, b);
    T a2 = c.y - d.y, b2 = d.x - c.x, c2 = cross(c, d);
    T det = a1 * b2 - a2 * b1;
    if (det == 0) return 0;
    ans = PT((b1 * c2 - b2 * c1) / det, (c1 * a2 - a1 * c2) / det);
    return 1;
}
// intersection point between segment ab and
// segment cd assuming unique intersection
// exists
bool seg_seg_intersection(PT a, PT b, PT c, PT d, PT &ans) {
    T oa = cross2(c, d, a), ob = cross2(c, d, b);
    T oc = cross2(a, b, c), od = cross2(a, b, d);
    if (oa * ob < 0 && oc * od < 0) {
        ans = (a * ob - b * oa) / (ob - oa);
        return 1;
    }
    else return 0;
}
// intersection point between segment ab and
// segment cd assuming unique intersection may
// not exists
// se.size()==0 means no intersection
// se.size()==1 means one intersection
// se.size()==2 means range intersection
set<PT> seg_seg_intersection_inside(PT a, PT b, PT c, PT d) {
    PT ans;
    if (seg_seg_intersection(a, b, c, d, ans))
        return {ans};
    set<PT> se;
    if (is_point_on_seg(c, d, a)) se.insert(a);
    if (is_point_on_seg(c, d, b)) se.insert(b);
    if (is_point_on_seg(a, b, c)) se.insert(c);
    if (is_point_on_seg(a, b, d)) se.insert(d);
    return se;
}
// intersection between segment ab and line
// cd
// 0 if do not intersect, 1 if proper
// intersect, 2 if segment intersect
int seg_line_relation(PT a, PT b, PT c, PT d) {
    T p = cross2(c, d, a);
    T q = cross2(c, d, b);
    if (sign(p) == 0 && sign(q) == 0) return 2;
    else if (p * q < 0) return 1;
    else return 0;
}
// intersection between segament ab and line cd
// assuming unique intersection exists
bool seg_line_intersection(PT a, PT b, PT c, PT d, PT &ans) {
    bool k = seg_line_relation(a, b, c, d);
    assert(k != 2);
    if (k) line_line_intersection(a, b, c, d, ans);
    return k;
}
// minimum distance from segment ab to segment
// cd
T dist_from_seg_to_seg(PT a, PT b, PT c, PT d) {
    PT dummy;
    if (seg_seg_intersection(a, b, c, d, dummy))
        return 0.0;
    else return min({dist_from_point_to_seg(a, b, c), dist_from_point_to_seg(a, b, d), dist_from_point_to_seg(c, d, a), dist_from_point_to_seg(c, d, b)});
}
// minimum distance from point c to ray
// (starting point a and direction vector b)
T dist_from_point_to_ray(PT a, PT b, PT c) {
    b = a + b;
    T r = dot(c - a, b - a);
    if (r < 0.0) return dist(c, a);
    return dist_from_point_to_line(a, b, c);
}
// starting point as and direction vector ad
bool ray_ray_intersection(PT as, PT ad, PT bs, PT bd) {
    T dx = bs.x - as.x, dy = bs.y - as.y;
    T det = bd.x * ad.y - bd.y * ad.x;
    if (fabs(det) < eps) return 0;
    T u = (dy * bd.x - dx * bd.y) / det;
    T v = (dy * ad.x - dx * ad.y) / det;
    if (sign(u) >= 0 && sign(v) >= 0) return 1;
    else return 0;
}
T ray_ray_distance(PT as, PT ad, PT bs, PT bd) {
    if (ray_ray_intersection(as, ad, bs, bd))
        return 0.0;
    T ans = dist_from_point_to_ray(as, ad, bs);
    ans = min(ans, dist_from_point_to_ray(bs, bd, as));
    return ans;
}
struct circle {
    PT p; T r;
    circle() {}
    circle(PT _p, T _r): p(_p), r(_r) {};
}
// center (x, y) and radius r
circle(T x, T y, T _r): p(PT(x, y)), r(_r) {}
// circumcircle of a triangle
// the three points must be unique
circle(PT a, PT b, PT c) {
    b = (a + b) * 0.5;
    c = (a + c) * 0.5;
    line_line_intersection(b, b + rotatecw90(a - b), c, c + rotatecw90(a - c), p);
    r = dist(a, p);
}
// inscribed circle of a triangle
// pass a bool just to differentiate from
// circumcircle
circle(PT a, PT b, PT c, bool t) {
    line u, v;
    T m = atan2(b.y - a.y, b.x - a.x), n = atan2(c.y - a.y, c.x - a.x);
    u.a = a;
    u.b = u.a + (PT(cos((n + m)/2.0), sin((n + m)/2.0)));
    v.a = b;
    m = atan2(a.y - b.y, a.x - b.x), n = atan2(c.y - b.y, c.x - b.x);
    v.b = v.a + (PT(cos((n + m)/2.0), sin((n + m)/2.0)));
    line_line_intersection(u.a, u.b, v.a, v.b, p);
    r = dist_from_point_to_seg(a, b, p);
}
bool operator == (circle v) { return p == v.p && sign(r - v.r) == 0; }
T area() { return PI * r * r; }
T circumference() { return 2.0 * PI * r; }
};
//0 if outside, 1 if on circumference, 2 if
// inside circle
int circle_point_relation(PT p, T r, PT b) {
    T d = dist(p, b);
    if (sign(d - r) < 0) return 2;
    if (sign(d - r) == 0) return 1;
    return 0;
}
// 0 if outside, 1 if on circumference, 2 if
// inside circle
int circle_line_relation(PT p, T r, PT a, PT b) {
    T d = dist_from_point_to_line(a, b, p);
    if (sign(d - r) < 0) return 2;
    if (sign(d - r) == 0) return 1;
    return 0;
}
//compute intersection of line through points a
// and b with
//circle centered at c with radius r > 0
vector<PT> circle_line_intersection(PT c, T r, PT a, PT b) {
    vector<PT> ret;
    b = b - a; a = a - c;
    T A = dot(b, b), B = dot(a, b);
    T C = dot(a, a) - r * r, D = B * B - A * C;
    if (D < -eps) return ret;
}

```

```

ret.push_back(c + a + b * (-B + sqrt(D +
    eps)) / A);
if (D > eps) ret.push_back(c + a + b * (-B
    - sqrt(D)) / A);
return ret;
}
//5 - outside and do not intersect
//4 - intersect outside in one point
//3 - intersect in 2 points
//2 - intersect inside in one point
//1 - inside and do not intersect
int circle_circle_relation(PT a, T r, PT b, T
    R) {
    T d = dist(a, b);
    if (sign(d - r - R) > 0) return 5;
    if (sign(d - r - R) == 0) return 4;
    T l = fabs(r - R);
    if (sign(d - r - R) < 0 && sign(d - l) > 0)
        return 3;
    if (sign(d - l) == 0) return 2;
    if (sign(d - l) < 0) return 1;
    assert(0); return -1;
}
vector<PT> circle_circle_intersection(PT a, T
    r, PT b, T R) {
    if (a == b && sign(r - R) == 0) return
        {PT(1e18, 1e18)};
    vector<PT> ret;
    T d = sqrt(dist2(a, b));
    if (d > r + R || d + min(r, R) < max(r,
        R)) return ret;
    T x = (d * d - R * R + r * r) / (2 * d);
    T y = sqrt(r * r - x * x);
    PT v = (b - a) / d;
    ret.push_back(a + v * x + rotateccw90(v)
        * y);
    if (y > 0) ret.push_back(a + v * x -
        rotateccw90(v) * y);
    return ret;
}
// returns two circle c1, c2 through points a,
// b and of radius r
// 0 if there is no such circle, 1 if one
// circle, 2 if two circle
int get_circle(PT a, PT b, T r, circle &c1,
    circle &c2) {
    vector<PT> v =
        circle_circle_intersection(a, r, b, r);
    int t = v.size();
    if (!t) return 0;
    c1.p = v[0], c1.r = r;
    if (t == 2) c2.p = v[1], c2.r = r;
    return t;
}
// returns two circle c1, c2 which is tangent
// to line u, goes through
// point q and has radius r1; 0 for no circle,
// 1 if c1 = c2, 2 if c1 != c2
int get_circle(line u, PT q, T r1, circle
    &c1, circle &c2) {
    T d = dist_from_point_to_line(u.a, u.b, q);
    if (sign(d - r1 * 2.0) > 0) return 0;
    if (sign(d) == 0) {
        cout << u.v.x << ' ' << u.v.y << '\n';
        c1.p = q + rotateccw90(u.v).truncate(r1);
        c2.p = q + rotatecw90(u.v).truncate(r1);
        c1.r = c2.r = r1;
        return 2;
    }
    line u1 = line(u.a +
        rotateccw90(u.v).truncate(r1), u.b +
        rotateccw90(u.v).truncate(r1));
    line u2 = line(u.a +
        rotatecw90(u.v).truncate(r1), u.b +
        rotatecw90(u.v).truncate(r1));
    circle cc = circle(q, r1);
    PT p1, p2; vector<PT> v;
    v = circle_line_intersection(q, r1, u1.a,
        u1.b);
    if (!v.size()) v =
        circle_line_intersection(q, r1, u2.a,
        u2.b);
    v.push_back(v[0]);
    p1 = v[0], p2 = v[1];
    c1 = circle(p1, r1);
    if (p1 == p2) {
        c2 = c1;
        return 1;
    }
    c2 = circle(p2, r1);
    return 2;
}
// returns area of intersection between two
// circles
T circle_circle_area(PT a, T r1, PT b, T r2) {
    T d = (a - b).norm();
    if (r1 + r2 < d + eps) return 0;
    if (r1 + d < r2 + eps) return PI * r1 * r1;
    if (r2 + d < r1 + eps) return PI * r2 * r2;
    T theta_1 = acos((r1 * r1 + d * d - r2 * r2)
        / (2 * r1 * d)),
        theta_2 = acos((r2 * r2 + d * d - r1 *
        r1) / (2 * r2 * d));
    return r1 * r1 * (theta_1 - sin(2 *
        theta_1) / 2.) + r2 * r2 * (theta_2 -
        sin(2 * theta_2) / 2.);
}
// tangent lines from point q to the circle
int tangent_lines_from_point(PT p, T r, PT q,
    line &u, line &v) {
    int x = sign(dist2(p, q) - r * r);
    if (x < 0) return 0; // point in cricle
    if (x == 0) { // point on circle
        u = line(q, q + rotateccw90(q - p));
        v = u;
        return 1;
    }
    T d = dist(p, q);
    T l = r * r / d;
    T h = sqrt(r * r - l * l);
    u = line(q, p + ((q - p).truncate(l) +
        (rotateccw90(q - p).truncate(h))));
    v = line(q, p + ((q - p).truncate(l) +
        (rotatecw90(q - p).truncate(h))));
    return 2;
}
// returns outer tangents line of two circles
// if inner == 1 it returns inner tangent
// lines
int tangents_lines_from_circle(PT c1, T r1, PT
    c2, T r2, bool inner, line &u, line &v) {
    if (inner) r2 = -r2;
    PT d = c2 - c1;
    T dr = r1 - r2, d2 = d.norm2(), h2 = d2 - dr
        * dr;
    if (d2 == 0 || h2 < 0) {
        assert(h2 != 0);
        return 0;
    }
    vector<pair<PT, PT>>out;
    for (int tmp: {-1, 1}) {
        PT v = (d * dr + rotateccw90(d) * sqrt(h2)
            * tmp) / d2;
        out.push_back({c1 + v * r1, c2 + v * r2});
    }
    u = line(out[0].first, out[0].second);
    if (out.size() == 2) v = line(out[1].first,
        out[1].second);
    return 1 + (h2 > 0);
}
// -1 if strictly inside, 0 if on the polygon,
// 1 if strictly outside
int is_point_in_triangle(PT a, PT b, PT c, PT
    p) {
    if (sign(cross(b - a, c - a)) < 0) swap(b,
        c);
    int c1 = sign(cross(b - a, p - a));
    int c2 = sign(cross(c - b, p - b));
    int c3 = sign(cross(a - c, p - c));
    if (c1 < 0 || c2 < 0 || c3 < 0) return 1;
    if (c1 + c2 + c3 != 3) return 0;
    return -1;
}
T perimeter(vector<PT> &p) {
    T ans=0; int n = p.size();
    for (int i = 0; i < n; i++) ans +=
        dist(p[i], p[(i + 1) % n]);
    return ans;
}
T area(vector<PT> &p) {
    T ans = 0; int n = p.size();
    for (int i = 0; i < n; i++) ans +=
        cross(p[i], p[(i + 1) % n]);
    return fabs(ans) * 0.5;
}
// centroid of a (possibly non-convex) polygon,
// assuming that the coordinates are listed in
// a clockwise or
// counterclockwise fashion. Note that the
// centroid is often known as
// the "center of gravity" or "center of
// mass".
PT centroid(vector<PT> &p) {
    int n = p.size(); PT c(0, 0);
    T sum = 0;
    for (int i = 0; i < n; i++) sum +=
        cross(p[i], p[(i + 1) % n]);
    T scale = 3.0 * sum;
    for (int i = 0; i < n; i++) {
        int j = (i + 1) % n;
        c = c + (p[i] + p[j]) * cross(p[i], p[j]);
    }
    return c / scale;
}
// 0 if cw, 1 if ccw
bool get_direction(vector<PT> &p) {
    T ans = 0; int n = p.size();
    for (int i = 0; i < n; i++) ans +=
        cross(p[i], p[(i + 1) % n]);
    if (sign(ans) > 0) return 1;
    return 0;
}
// it returns a point such that the sum of
// distances
// from that point to all points in p is
// minimum
// O(n log^2 MX)
PT geometric_median(vector<PT> p) {
    auto tot_dist = [&](PT z) {
        T res = 0;
        for (int i = 0; i < p.size(); i++) res +=
            dist(p[i], z);
        return res;
    };
    auto findY = [&](T x) {
        T yl = -1e5, yr = 1e5;
        for (int i = 0; i < 60; i++) {
            T ym1 = yl + (yr - yl) / 3;
            T ym2 = yr - (yr - yl) / 3;
            T d1 = tot_dist(PT(x, ym1));
            T d2 = tot_dist(PT(x, ym2));
            if (d1 < d2) yl = ym2;
            else yl = ym1;
        }
        return pair<T, T> (yl, tot_dist(PT(x,
            yl)));
    };
    T xl = -1e5, xr = 1e5;
    for (int i = 0; i < 60; i++) {
        T xm1 = xl + (xr - xl) / 3;
        T xm2 = xr - (xr - xl) / 3;
        T y1, d1, y2, d2;
        auto z = findY(xm1); y1 = z.first; d1 =
            z.second;
        z = findY(xm2); y2 = z.first; d2 =
            z.second;
        if (d1 < d2) xr = xm2;
        else xl = xm1;
    }
    return {xl, findY(xl).first };
}
vector<PT> convex_hull(vector<PT> &p) {
    if (p.size() <= 1) return p;
    vector<PT> v = p;
    sort(v.begin(), v.end());
    vector<PT> up, dn;
    for (auto& p : v) {
        while (up.size() > 1 &&
            orientation(up[up.size() - 2],
            up.back(), p) >= 0) {
            up.pop_back();
        }
        while (dn.size() > 1 &&
            orientation(dn[dn.size() - 2],
            dn.back(), p) <= 0) {
            dn.pop_back();
        }
        up.push_back(p);
        dn.push_back(p);
    }
}

```



```

v = dn;
if (v.size() > 1) v.pop_back();
reverse(up.begin(), up.end());
up.pop_back();
for (auto& p : up) {
    v.push_back(p);
}
if (v.size() == 2 && v[0] == v[1])
    v.pop_back();
return v;
}

//checks if convex or not
bool is_convex(vector<PT> &p) {
    bool s[3]; s[0] = s[1] = s[2] = 0;
    int n = p.size();
    for (int i = 0; i < n; i++) {
        int j = (i + 1) % n;
        int k = (j + 1) % n;
        s[sign(cross(p[j] - p[i], p[k] - p[i])) + 1] = 1;
        if (s[0] && s[2]) return 0;
    }
    return 1;
}

// -1 if strictly inside, 0 if on the polygon,
// 1 if strictly outside
// it must be strictly convex, otherwise make
// it strictly convex first
int is_point_in_convex(vector<PT> &p, const
    PT& x) { // O(log n)
    int n = p.size(); assert(n >= 3);
    int a = orientation(p[0], p[1], x), b =
        orientation(p[0], p[n - 1], x);
    if (a < 0 || b > 0) return 1;
    int l = 1, r = n - 1;
    while (l + 1 < r) {
        int mid = l + r >> 1;
        if (orientation(p[0], p[mid], x) >= 0) l = mid;
        else r = mid;
    }
    int k = orientation(p[l], p[r], x);
    if (k <= 0) return -k;
    if (l == 1 && a == 0) return 0;
    if (r == n - 1 && b == 0) return 0;
    return -1;
}

bool is_point_on_polygon(vector<PT> &p, const
    PT& z) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        if (is_point_on_seg(p[i], p[(i + 1) % n],
            z)) return 1;
    }
    return 0;
}

// returns 1e9 if the point is on the polygon
int winding_number(vector<PT> &p, const PT& z)
    { // O(n)
    if (is_point_on_polygon(p, z)) return 1e9;
    int n = p.size(), ans = 0;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) % n;
        bool below = p[i].y < z.y;
        if (below != (p[j].y < z.y)) {
            auto orient = orientation(z, p[j],
                p[i]);
            if (orient == 0) return 0;
            if (below == (orient > 0)) ans += below
                ? 1 : -1;
        }
    }
    return ans;
}

// -1 if strictly inside, 0 if on the polygon,
// 1 if strictly outside
int is_point_in_polygon(vector<PT> &p, const
    PT& z) { // O(n)
    int k = winding_number(p, z);
    return k == 1e9 ? 0 : k == 0 ? 1 : -1;
}

// id of the vertex having maximum dot product
// with z
// polygon must need to be convex
// top - upper right vertex
// for minimum dot product negate z and return
// -dot(z, p[id])
int extreme_vertex(vector<PT> &p, const PT
    &z, const int top) { // O(log n)
    int n = p.size();
    if (n == 1) return 0;
    T ans = dot(p[0], z); int id = 0;
    if (dot(p[top], z) > ans) ans = dot(p[top],
        z), id = top;
    int l = 1, r = top - 1;
    while (l < r) {
        int mid = l + r >> 1;
        if (dot(p[mid + 1], z) >= dot(p[mid], z))
            l = mid + 1;
        else r = mid;
    }
    if (dot(p[l], z) > ans) ans = dot(p[l], z),
        id = l;
    l = top + 1, r = n - 1;
    while (l < r) {
        int mid = l + r >> 1;
        if (dot(p[(mid + 1) % n], z) >=
            dot(p[mid], z)) l = mid + 1;
        else r = mid;
    }
    l %= n;
    if (dot(p[l], z) > ans) ans = dot(p[l], z),
        id = l;
    return id;
}

// maximum distance from any point on the
// perimeter to another point on the
// perimeter
T diameter(vector<PT> &p) {
    int n = (int)p.size();
    if (n == 1) return 0;
    if (n == 2) return dist(p[0], p[1]);
    T ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (cross(p[(i + 1) % n] - p[i], p[(j
            + 1) % n] - p[j]) >= 0) {
            ans = max(ans, dist2(p[i], p[j]));
            j = (j + 1) % n;
        }
        ans = max(ans, dist2(p[i], p[j]));
    }
}

i++;
}
return sqrt(ans);
}

// given n points, find the minimum enclosing
// circle of the points
// call convex_hull() before this for faster
// solution
// expected O(n)
circle minimum_enclosing_circle(vector<PT> &p)
    {
    random_shuffle(p.begin(), p.end());
    int n = p.size();
    circle c(p[0], 0);
    for (int i = 1; i < n; i++) {
        if (sign(dist(c.p, p[i]) - c.r) > 0) {
            c = circle(p[i], 0);
            for (int j = 0; j < i; j++) {
                if (sign(dist(c.p, p[j]) - c.r) > 0) {
                    c = circle((p[i] + p[j]) / 2,
                        dist(p[i], p[j]) / 2);
                    for (int k = 0; k < j; k++) {
                        if (sign(dist(c.p, p[k]) - c.r) >
                            0) {
                            c = circle(p[i], p[j], p[k]);
                        }
                    }
                }
            }
        }
    }
    return c;
}

// not necessarily convex, boundary is included
// in the intersection
// returns total intersected length
// it returns the sum of the lengths of the
// portions of the line that are inside the
// polygon
T polygon_line_intersection(vector<PT> p, PT
    a, PT b) {
    int n = p.size();
    p.push_back(p[0]);
    line l = line(a, b);
    T ans = 0.0;
    vector< pair<T, int> > vec;
    for (int i = 0; i < n; i++) {
        int s1 = orientation(a, b, p[i]);
        int s2 = orientation(a, b, p[i + 1]);
        if (s1 == s2) continue;
        line t = line(p[i], p[i + 1]);
        PT inter = (t.v * l.c - l.v * t.c) /
            cross(l.v, t.v);
        T tmp = dot(inter, l.v);
        int f;
        if (s1 > s2) f = s1 && s2 ? 2 : 1;
        else f = s1 && s2 ? -2 : -1;
        vec.push_back(make_pair((f > 0 ? tmp - eps
            : tmp + eps), f)); // keep eps very
            small like 1e-12
    }
    sort(vec.begin(), vec.end());
    for (int i = 0, j = 0; i + 1 <
        (int)vec.size(); i++) {
        j += vec[i].second;
    }
}

if (j) ans += vec[i + 1].first -
    vec[i].first; // if this portion is
    inside the polygon
// else ans = 0; // if we want the maximum
    intersected length which is totally
    inside the polygon, uncomment this and
    take the maximum of ans
}

ans = ans / sqrt(dot(l.v, l.v));
p.pop_back();
return ans;
}

// given a convex polygon p, and a line ab and
// the top vertex of the polygon
// returns the intersection of the line with
// the polygon
// it returns the indices of the edges of the
// polygon that are intersected by the line
// so if it returns i, then the line intersects
// the edge (p[i], p[(i + 1) % n])
array<int, 2>
    convex_line_intersection(vector<PT> &p, PT
        a, PT b, int top) {
    int end_a = extreme_vertex(p, (a -
        b).perp(), top);
    int end_b = extreme_vertex(p, (b -
        a).perp(), top);
    auto cmp_l = [&](int i) { return
        orientation(a, p[i], b); };
    if (cmp_l(end_a) < 0 || cmp_l(end_b) > 0)
        return {-1, -1}; // no intersection
    array<int, 2> res;
    for (int i = 0; i < 2; i++) {
        int lo = end_b, hi = end_a, n = p.size();
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) /
                2) % n;
            (cmp_l(m) == cmp_l(end_b) ? lo : hi) =
                m;
        }
        res[i] = (lo + !cmp_l(hi)) % n;
        swap(end_a, end_b);
    }
    if (res[0] == res[1]) return {res[0], -1};
    // touches the vertex res[0]
    if (!cmp_l(res[0]) && !cmp_l(res[1]))
        switch ((res[0] - res[1] + (int)p.size()
            + 1) % p.size()) {
            case 0: return {res[0], res[0]}; //
                touches the edge (res[0], res[0] +
                1)
            case 2: return {res[1], res[1]}; //
                touches the edge (res[1], res[1] +
                1)
        }
    return res; // intersects the edges (res[0],
        res[0] + 1) and (res[1], res[1] + 1)
}

pair<PT, int> point_poly_tangent(vector<PT>
    &p, PT Q, int dir, int l, int r) {
    while (r - l > 1) {
        int mid = (l + r) >> 1;
        bool pvs = orientation(Q, p[mid], p[mid -
            1]) != -dir;
    }
}

```



```

bool nxt = orientation(Q, p[mid], p[mid + 1]) != -dir;
if (pvs && nxt) return {p[mid], mid};
if (!(pvs || nxt)) {
    auto p1 = point_poly_tangent(p, Q, dir, mid + 1, r);
    auto p2 = point_poly_tangent(p, Q, dir, 1, mid - 1);
    return orientation(Q, p1.first, p2.first) == dir ? p1 : p2;
}
if (!pvs) {
    if (orientation(Q, p[mid], p[l]) == dir) r = mid - 1;
    else if (orientation(Q, p[l], p[r]) == dir) r = mid - 1;
    else l = mid + 1;
}
if (!nxt) {
    if (orientation(Q, p[mid], p[l]) == dir) l = mid + 1;
    else if (orientation(Q, p[l], p[r]) == dir) r = mid - 1;
    else l = mid + 1;
}
}
pair<PT, int> ret = {p[l], l};
for (int i = l + 1; i <= r; i++) ret = orientation(Q, ret.first, p[i]) != dir ? make_pair(p[i], i) : ret;
return ret;
}
// (ccw, cw) tangents from a point that is outside this convex polygon
// returns indexes of the points
// ccw means the tangent from Q to that point is in the same direction as the polygon ccw direction
pair<int, int>
tangents_from_point_to_polygon(vector<PT> &p, PT Q){
    int ccw = point_poly_tangent(p, Q, 1, 0, (int)p.size() - 1).second;
    int cw = point_poly_tangent(p, Q, -1, 0, (int)p.size() - 1).second;
    return make_pair(ccw, cw);
}

// minimum distance from a point to a convex polygon
// it assumes point lie strictly outside the polygon
T dist_from_point_to_polygon(vector<PT> &p, PT z) {
    T ans = inf;
    int n = p.size();
    if (n <= 3) {
        for(int i = 0; i < n; i++) ans = min(ans, dist_from_point_to_seg(p[i], p[(i + 1) % n], z));
    }
    return ans;
}
auto [r, l] = tangents_from_point_to_polygon(p, z);
if(l > r) r += n;

```

```

while (l < r) {
    int mid = (l + r) >> 1;
    T left = dist2(p[mid % n], z), right = dist2(p[(mid + 1) % n], z);
    ans = min({ans, left, right});
    if(left < right) r = mid;
    else l = mid + 1;
}
ans = sqrt(ans);
ans = min(ans, dist_from_point_to_seg(p[l % n], p[(l + 1) % n], z));
ans = min(ans, dist_from_point_to_seg(p[l % n], p[(l - 1 + n) % n], z));
return ans;
}
// minimum distance from convex polygon p to line ab
// returns 0 is it intersects with the polygon
// top - upper right vertex
T dist_from_polygon_to_line(vector<PT> &p, PT a, PT b, int top) { //O(log n)
    PT orth = (b - a).perp();
    if (orientation(a, b, p[0]) > 0) orth = (a - b).perp();
    int id = extreme_vertex(p, orth, top);
    if (dot(p[id] - a, orth) > 0) return 0.0;
    //if orth and a are in the same half of the line, then poly and line intersects
    return dist_from_point_to_line(a, b, p[id]); //does not intersect
}
// minimum distance from a convex polygon to another convex polygon
// the polygon doesnot overlap or touch
T dist_from_polygon_to_polygon(vector<PT> &p1, vector<PT> &p2) { // O(n log n)
    T ans = inf;
    for (int i = 0; i < p1.size(); i++) {
        ans = min(ans, dist_from_point_to_polygon(p2, p1[i]));
    }
    for (int i = 0; i < p2.size(); i++) {
        ans = min(ans, dist_from_point_to_polygon(p1, p2[i]));
    }
    return ans;
}
// calculates the area of the union of n polygons (not necessarily convex).
// the points within each polygon must be given in CCW order.
// complexity: O(N^2), where N is the total number of points
T rat(PT a, PT b, PT p) {
    return !sign(a.x - b.x) ? (p.y - a.y) / (b.y - a.y) : (p.x - a.x) / (b.x - a.x);
};
T polygon_union(vector<vector<PT>> &p) {
    int n = p.size();
    T ans=0;
    for(int i = 0; i < n; ++i) {
        for (int v = 0; v < (int)p[i].size(); ++v) {

```

```

PT a = p[i][v], b = p[i][(v + 1) % p[i].size()];
vector<pair<T, int>> segs;
segs.emplace_back(0, 0);
segs.emplace_back(1, 0);
for(int j = 0; j < n; ++j) {
    if(i != j) {
        for(size_t u = 0; u < p[j].size(); ++u) {
            PT c = p[j][u], d = p[j][(u + 1) % p[j].size()];
            int sc = sign(cross(b - a, c - a)), sd = sign(cross(b - a, d - a));
            if(!sc && !sd) {
                if(sign(dot(b - a, d - c)) > 0 && i > j) {
                    segs.emplace_back(rat(a, b, c), 1);
                    segs.emplace_back(rat(a, b, d), -1);
                }
            }
            else {
                T sa = cross(d - c, a - c), sb = cross(d - c, b - c);
                if(sc >= 0 && sd < 0)
                    segs.emplace_back(sa / (sa - sb), 1);
                else if(sc < 0 && sd >= 0)
                    segs.emplace_back(sa / (sa - sb), -1);
            }
        }
    }
}
sort(segs.begin(), segs.end());
T pre = min(max(segs[0].first, 0.0), 1.0), now, sum = 0;
int cnt = segs[0].second;
for(int j = 1; j < segs.size(); ++j) {
    now = min(max(segs[j].first, 0.0), 1.0);
    if (!cnt) sum += now - pre;
    cnt += segs[j].second;
    pre = now;
    ans += cross(a, b) * sum;
}
return ans * 0.5;
}
// returns the area of the intersection of the circle with center c and radius r
// and the triangle formed by the points c, a, b
T _triangle_circle_intersection(PT c, T r, PT a, PT b) {
    T sd1 = dist2(c, a), sd2 = dist2(c, b);
    if(sd1 > sd2) swap(a, b), swap(sd1, sd2);
    T sd = dist2(a, b);
    T d1 = sqrt1(sd1), d2 = sqrt1(sd2), d = sqrt(sd);
    T x = abs(sd2 - sd - sd1) / (2 * d);
    T h = sqrt1(sd1 - x * x);

```

```

if(r >= d2) return h * d / 2;
T area = 0;
if(sd + sd1 < sd2) {
    if(r < d1) area = r * r * (acos(h / d2) - acos(h / d1)) / 2;
    else {
        area = r * r * (acos(h / d2) - acos(h / r)) / 2;
        T y = sqrt1(r * r - h * h);
        area += h * (y - x) / 2;
    }
}
else {
    if(r < h) area = r * r * (acos(h / d2) + acos(h / d1)) / 2;
    else {
        area += r * r * (acos(h / d2) - acos(h / r)) / 2;
        T y = sqrt1(r * r - h * h);
        area += h * y / 2;
        if(r < d1) {
            area += r * r * (acos(h / d1) - acos(h / r)) / 2;
            area += h * y / 2;
        }
        else area += h * x / 2;
    }
}
return area;
}
// Closest-Pair of Points (O(n log n))
// Returns minimal distance among all pairs in v
T closest(vector<PT>& v){
    sort(v.begin(), v.end(), [](PT a, PT b){
        return a.x < b.x; });
    function<T(int,int)> rec = [&](int l,int r){
        if(r-l < 2) return numeric_limits<T>::infinity();
        int m = (l+r)/2;
        T d = min(rec(l,m), rec(m,r));
        inplace_merge(v.begin()+l, v.begin()+m, v.begin()+r, [](PT a, PT b){ return a.y < b.y; });
        vector<PT> buf;
        for(int i=l; i<r; i++){
            if(fabs(v[i].x - v[m].x) < d){
                for(int j=(int)buf.size()-1; j>=0 && v[i].y - buf[j].y < d; --j)
                    d = min(d, dist(v[i], buf[j]));
                buf.push_back(v[i]);
            }
        }
        return d;
    };
    return rec(0, v.size());
}

```

### 6.3 Minkowski Sum [44 lines]

```

struct pt{
    long long x, y;
    pt operator + (const pt & p) const {
        return pt{x + p.x, y + p.y};
    }
    pt operator - (const pt & p) const {

```

## 8 Math

### 8.1 Berlekamp massey [180 lines]

```
#include <bits/stdc++.h>
using namespace std;

using int64 = long long;
const int64 MOD = 1000000007;

int64 modpow(int64 a, int64 e){
    int64 r=1%MOD;
    a%=MOD;
    while(e){
        if(e&1) r = (__int128)r*a % MOD;
        a = (__int128)a*a % MOD;
        e >>= 1;
    }
    return r;
}

int64 modinv(int64 x){ return
    modpow((x%MOD+MOD)%MOD, MOD-2); }

// --- Fast doubling for Fibonacci (returns
// pair (F_n, F_{n+1})) ---
pair<int64,int64> fib_pair(long long n){
    if(n==0) return {0,1};
    auto p = fib_pair(n>>1);
    int64 a = p.first, b = p.second;
    int64 c = ( (__int128)a * ( (2*b - a +
        MOD) % MOD ) ) % MOD; // F(2k)
    int64 d = ( (__int128)a*a + (__int128)b*b
        ) % MOD; // F(2k+1)
    if(n&1) return {d, (c+d)%MOD};
    else return {c,d};
}

int64 fib(long long n){ return
    fib_pair(n).first; }

// --- Matrix utilities for small matrices
// (square) ---
using Mat = vector<vector<int64>>;

Mat matMul(const Mat &A, const Mat &B){
    int n = A.size();
    int m = B[0].size();
    int p = B.size();
    Mat C(n, vector<int64>(m,0));
    for(int i=0;i<n;i++){
        for(int k=0;k<p;k++){
            if(A[i][k]==0) continue;
            int64 av = A[i][k];
            for(int j=0;j<m;j++){
                C[i][j] = (C[i][j] + av *
                    B[k][j]) % MOD;
            }
        }
    }
    return C;
}

Mat matPow(Mat base, long long e){
    int n = base.size();
    Mat R(n, vector<int64>(n,0));
    for(int i=0;i<n;i++) R[i][i]=1;
```

```
rdfs(it,tem);
}

}
//main
int main()
{
    int n,m;
    cin>>n>>m;
    g.assign(n+1,vector<int>(0));
    rg.assign(n+1,vector<int>(0));
    vis.assign(n+1,0);
    timer=0;
    for(int i=0;i<m;i++){
        int x,y;
        cin>>x>>y;
        g[x].push_back(y);
        rg[y].push_back(x);
    }
    vector<pair<int,int>>str;
    for(int i=1;i<=n;i++){
        dfs(i);
        str.push_back({etime[i],i});
    }
    sort(all(str));
    reverse(all(str));
    vis.assign(n+1,0);
    for(auto [ww,i]:str)
    {
        if(vis[i])continue;
        vector<int>tem;
        rdfs(i,tem);
        for(auto it:tem)cout<<it<<" ";
        cout<<"\n";
    }
}

7.5 TopoSort [26 lines]
vector<vector<int>>>g;
vector<int>toposort(int n)
{
    queue<int>str;
    vector<int>in(n+1,0);
    for(int i=1;i<=n;i++){
        for(auto it:g[i])in[it]++;
    }
    for(int i=1;i<=n;i++)if(in[i]==0)str.push(i);
    vector<int>ans;
    while(!str.empty())
    {
        auto x=str.front();
        str.pop();
        ans.push_back(x);
        for(auto it:g[x])
        {
            in[it]--;
            if(in[it]==0)str.push(it);
        }
    }
    return ans;
}
//g.resize(n+1,vector<int>(0));
//if(ans.size()!=n)cout<<"-1\n";
```

```
}
else if (v != par) lo[src] =
    min(lo[src], dis[v]);
}
if (par == -1 && child > 1) isAP[src] =
    true;
}

7.2 Bellman Ford [21 lines]
void bellman()
{
    for(int i=0;i<n;i++){
        for(int node=1;node<=n;node++){
            for(auto adj_node:adj_list[node])
            {
                int u=node;
                int v=adj_node.first;
                int w=adj_node.second;
                if(d[v]>d[u]+w)
                {
                    d[v]=d[u]+w;
                    parent[v]=u;
                    if(i==n-1)
                        negative_cycle=true,
                        selected_node=v;
                }
            }
        }
    }
}

7.3 Floyd Warshall [6 lines]
void floyd()
{
    for(int k=1;k<=n;k++){
        for(int u=1;u<=n;u++){
            for(int v=1;v<=n;v++){
                d[u][v]=min(d[u][k]+d[k][v],
                    d[u][v]);
            }
        }
    }
}

7.4 SCC [59 lines]
vector<vector<int>>>g,rg;
vector<bool>vis;
int timer;
int stime[N];
int etime[N];
void dfs(int node)
{
    if(vis[node])return;
    vis[node]=1;
    stime[node]=++timer;
    for(auto it:g[node])
    {
        dfs(it);
    }
    etime[node]=++timer;
}
void rdfs(int node,vector<int>&tem)
{
    if(vis[node])return;
    vis[node]=1;
    tem.push_back(node);
    for(auto it:rg[node])
    {
```

```
        return pt{x - p.x, y - p.y};
    }
    long long cross(const pt & p) const {
        return x * p.y - y * p.x;
    }
};

void reorder_polygon(vector<pt> & P){
    size_t pos = 0;
    for(size_t i = 1; i < P.size(); i++){
        if(P[i].y < P[pos].y || (P[i].y ==
            P[pos].y && P[i].x < P[pos].x))
            pos = i;
    }
    rotate(P.begin(), P.begin() + pos,
        P.end());
}

vector<pt> minkowski(vector<pt> P, vector<pt>
    Q){
    // the first vertex must be the lowest
    reorder_polygon(P);
    reorder_polygon(Q);
    // we must ensure cyclic indexing
    P.push_back(P[0]);
    P.push_back(P[1]);
    Q.push_back(Q[0]);
    Q.push_back(Q[1]);
    // main part
    vector<pt> result;
    size_t i = 0, j = 0;
    while(i < P.size() - 2 || j < Q.size() -
        2){
        result.push_back(P[i] + Q[j]);
        auto cross = (P[i + 1] -
            P[i]).cross(Q[j + 1] - Q[j]);
        if(cross >= 0 && i < P.size() - 2)
            ++i;
        if(cross <= 0 && j < Q.size() - 2)
            ++j;
    }
    return result;
}
```

## 7 Graph

### 7.1 Articulation point and bridge [22 lines]

```
const int N=1e5+5;
vector<int> g[N];
int vis[N],dis[N],lo[N],isAP[N];
int timer;
vector<pair<int,int>> ans;
void dfs(int src, int par)
{
    int child = 0;
    vis[src] = true;
    dis[src] = lo[src] = ++timer;
    for (auto v : g[src]) {
        if (!vis[v]) {
            child++;
            dfs(v,src);
            lo[src] = min(lo[src], lo[v]);
            if (par != -1 && lo[v] >=
                dis[src]) isAP[src] = true;
            if(lo[v]>dis[src])
                ans.push_back({min(v,src),
                    max(v,src)});
        }
```

```

while(e){
    if(e&1) R = matMul(R, base);
    base = matMul(base, base);
    e >>= 1;
}
return R;
}

// --- Berlekamp-Massey: returns coefficients c
// (length L) so that
// a_n = c[0]*a_{n-1} + c[1]*a_{n-2} + ...
// + c[L-1]*a_{n-L}
// (i.e., recurrence of order L). Works
// modulo MOD. ---
vector<int64> berlekamp_massey(const
vector<int64>& s){
    vector<int64>& s){
        int n = s.size();
        vector<int64> C(1,1), B(1,1);
        int L = 0, m = 1;
        int64 b = 1;
        for(int i=0;i<n;i++){
            // compute discrepancy
            int64 d = 0;
            for(int j=0;j<=L;j++){
                d = (d + C[j] * s[i-j]) % MOD;
            }
            if(d==0){
                m++;
            } else {
                vector<int64> T = C;
                int64 coef = d * modinv(b) % MOD;
                // C = C - coef * x^m * B
                int need = max((int)C.size(),
                    (int)B.size() + m);
                C.resize(need);
                for(size_t j=0;j<B.size();j++){
                    int idx = j + m;
                    C[idx] = (C[idx] - coef *
                        B[j]) % MOD;
                    if(C[idx]<0) C[idx]+=MOD;
                }
                if(2*L <= i){
                    L = i+1 - L;
                    B = T;
                    b = d;
                    m = 1;
                } else {
                    m++;
                }
            }
        }
        // currently C represents polynomial with
        // leading 1 and C[0]=1
        // We want recurrence a_n = sum_{i=1..L}
        // coeff[i-1] * a_{n-i}
        // For that, coeff[i-1] = (MOD - C[i]) %
        // MOD for i=1..L
        vector<int64> res;
        for(int i=1;i<(int)C.size();i++){
            res.push_back( (MOD - C[i]) % MOD );
        }
        return res;
    }
}

// --- Helper: compute sum_{i=0..N} a_i given
// recurrence and initial values ---

```

```

// recurrence length L, coeffs c[0..L-1],
// initial a[0..L-1]
int64 sum_linear_recurrence(const
vector<int64>& c, const vector<int64>&
init, long long N){
    int L = c.size();
    if(N < (int)init.size()){
        int64 s=0;
        for(int i=0;i<=N;i++) s = (s +
            init[i]) % MOD;
        return s;
    }
    // Build augmented matrix M of size
    // (L+1)x(L+1)
    int sz = L+1;
    Mat M(sz, vector<int64>(sz,0));
    // top row -> next value a_{n+1} = sum
    // c[j]*a_{n-j}
    for(int j=0;j<L;j++) M[0][j] = c[j] % MOD;
    // shift rows
    for(int i=1;i<L;i++){
        M[i][i-1] = 1;
    }
    // last row: S_{n+1} = S_n + a_{n+1} = S_n
    // + dot(c, top L entries)
    for(int j=0;j<L;j++) M[L][j] = c[j] % MOD;
    M[L][L] = 1;
    // initial state vector at index n = L-1:
    // u_{L-1} = [a_{L-1}, a_{L-2}, ..., a_0,
    // S_{L-1}]^T
    Mat u(sz, vector<int64>(1,0));
    int64 S_init = 0;
    for(int i=0;i<L;i++) S_init = (S_init +
        init[i]) % MOD;
    for(int i=0;i<L;i++){
        u[i][0] = init[L-1 - i]; // top
        // element a_{L-1}
    }
    u[L][0] = S_init;
    long long exp = N - (L - 1);
    Mat P = matPow(M, exp);
    Mat res = matMul(P, u);
    int64 ans = res[L][0] % MOD; if(ans<0)
        ans+=MOD;
    return ans;
}

int main(){
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
    int T;
    if(!(cin>>T)) return 0;
    for(int tc=1; tc<=T; ++tc){
        long long N; long long C; int K;
        cin >> N >> C >> K;
        // We need sequence a_n = (F_{n*C})^K
        // for n=0..N
        // Prepare initial terms: need at least
        // 2*(K+1) terms to run BM safely.
        int need = 2*(K+1) + 5; // a bit extra
        // margin
        vector<int64> seq;
        seq.reserve(need);
        for(int i=0;i<need;i++){
            int64 f = fib((long long)i * C);
            // F_{iC} mod MOD

```

```

        int64 val = modpow(f, K);
        seq.push_back(val);
    }
    // Run Berlekamp-Massey on seq to get
    // recurrence
    vector<int64> rec =
        berlekamp_massey(seq);
    int L = rec.size();
    if(L==0){
        // sequence is all zero -> sum=0
        cout << "Case " << tc << ": 0\n";
        continue;
    }
    // initial a[0..L-1]
    vector<int64> init(L);
    for(int i=0;i<L;i++) init[i] = seq[i]
        % MOD;
    // Compute sum from 0..N
    int64 ans =
        sum_linear_recurrence(rec, init,
            N);
    cout << "Case " << tc << ": " << ans %
        MOD << "\n";
    }
    return 0;
}

8.2 Convolution [192 lines]
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
using i128 = __int128_t;

```

```

ll modpow(ll a, ll e, ll mod)
{
    ll r = 1;
    a %= mod;
    while (e)
    {
        if (e & 1) r = (i128)r * a % mod;
        a = (i128)a * a % mod;
        e >>= 1;
    }
    return r;
}
ll modinv(ll a, ll mod) { return modpow((a %
    mod + mod) % mod, mod - 2, mod); }

```

```

struct XOR
{
    static inline void fwd(ll &u, ll &v)
    {
        ll x = u + v, y = u - v;
        u = x;
        v = y;
    }
    static inline void inv(ll &u, ll &v)
    {
        ll x = u + v, y = u - v;
        u = x;
        v = y;
    }
};
struct OR
{
    static inline void fwd(ll &u, ll &v) { v
        += u; }

```

```

    static inline void inv(ll &u, ll &v) { v
        -= u; }
};
struct AND
{
    static inline void fwd(ll &u, ll &v) { u
        += v; }
    static inline void inv(ll &u, ll &v) { u
        -= v; }
};

template <class Op>
void fwht(vector<ll> &a, bool invflag = false)
{
    int n = (int)a.size();
    for (int len = 1; len < n; len <= 1)
    {
        for (int i = 0; i < n; i += len << 1)
        {
            for (int j = 0; j < len; ++j)
            {
                ll &x = a[i + j], &y = a[i + j
                    + len];
                if (!invflag) Op::fwd(x, y);
                else Op::inv(x, y);
            }
        }
    }
    if (invflag && is_same<Op, XOR>::value)
        for (int i = 0; i < n; ++i) a[i] /= n;
}

```

```

template <class Op>
void fwht_mod(vector<ll> &a, ll mod, bool
    invflag = false)
{
    int n = (int)a.size();
    for (int len = 1; len < n; len <= 1)
    {
        for (int i = 0; i < n; i += len << 1)
        {
            for (int j = 0; j < len; ++j)
            {
                ll x = a[i + j], y = a[i + j
                    + len];
                if (!invflag)
                {
                    if constexpr (is_same<Op,
                        XOR>::value)
                    {
                        ll nx = (x + y) %
                            mod, ny = (x - y)
                                % mod;
                        if (ny < 0) ny += mod;
                        a[i + j] = nx;
                        a[i + j + len] = ny;
                    }
                    else if constexpr
                        (is_same<Op,
                            OR>::value)
                        a[i + j + len] = (y +
                            x) % mod;

```

```

        else a[i + j] = (x + y) %
            mod;
    }
    else
    {
        if constexpr (is_same<Op,
            XOR>::value)
        {
            ll nx = (x + y) %
                mod, ny = (x - y)
                    % mod;
            if (ny < 0) ny += mod;
            a[i + j] = nx;
            a[i + j + len] = ny;
        }
        else if constexpr
            (is_same<Op,
                OR>::value)
        {
            ll ny = (y - x) % mod;
            if (ny < 0) ny += mod;
            a[i + j + len] = ny;
        }
        else
        {
            ll nx = (x - y) % mod;
            if (nx < 0) nx += mod;
            a[i + j] = nx;
        }
    }
}

}

if (invflag && is_same<Op, XOR>::value)
{
    ll invn = modinv(n, mod);
    for (int i = 0; i < n; ++i)
        a[i] = (a[i] * (i128)invn) % mod;
}
}

template <class Op>
vector<ll> conv(vector<ll> a, vector<ll> b)
{
    int n = 1;
    while (n < (int)max(a.size(), b.size())) n
        <<= 1;
    a.resize(n);
    b.resize(n);
    fwht<Op>(a, false);
    fwht<Op>(b, false);
    for (int i = 0; i < n; ++i) a[i] = a[i] *
        b[i];
    fwht<Op>(a, true);
    return a;
}

```

```

template <class Op>
vector<ll> conv_mod(vector<ll> a, vector<ll>
    b, ll mod)
{
    int n = 1;
    while (n < (int)max(a.size(), b.size())) n
        <<= 1;
    a.resize(n);

```

```

    b.resize(n);
    fwht_mod<Op>(a, mod, false);
    fwht_mod<Op>(b, mod, false);
    vector<ll> c(n);
    for (int i = 0; i < n; ++i)
        c[i] = (i128)a[i] * b[i] % mod;
    fwht_mod<Op>(c, mod, true);
    return c;
}

// quick usage demo
int main()
{
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    // XOR integer conv
    {
        vector<ll> A = {1, 2, 3, 4}, B = {5,
            6, 7, 8};
        auto C = conv<XOR>(A, B);
        for (auto x : C)
            cout << x << ' ';
        cout << "\n";
    }

    // OR integer conv (size must be pow2)
    {
        vector<ll> A(8), B(8);
        A[0] = 1;
        A[1] = 2;
        B[0] = 3;
        B[4] = 1;
        auto C = conv<OR>(A, B);
        for (auto x : C)
            cout << x << ' ';
        cout << "\n";
    }

    // XOR modular conv
    {
        ll MOD = 1000000007;
        vector<ll> A = {1, 2, 3, 4}, B = {5,
            6, 7, 8};
        auto C = conv_mod<XOR>(A, B, MOD);
        for (auto x : C)
            cout << x << ' ';
        cout << "\n";
    }
    return 0;
}

```

### 8.3 FFT [73 lines]

```

#include <bits/stdc++.h>
using namespace std;
const int N = 3e5 + 9;
const double PI = acos(-1);
struct base {
    double a, b;
    base(double a = 0, double b = 0) : a(a),
        b(b) {}
    const base operator + (const base &c) const
    { return base(a + c.a, b + c.b); }
    const base operator - (const base &c) const
    { return base(a - c.a, b - c.b); }
    const base operator * (const base &c) const

```

```

    { return base(a * c.a - b * c.b, a * c.b
        + b * c.a); }
};
void fft(vector<base> &p, bool inv = 0) {
    int n = p.size(), i = 0;
    for(int j = 1; j < n - 1; ++j) {
        for(int k = n >> 1; k > (i ^ k); k >=
            1);
        if(j < i) swap(p[i], p[j]);
    }
    for(int l = 1, m; (m = l << 1) <= n; l <<=
        1) {
        double ang = 2 * PI / m;
        base wn = base(cos(ang), (inv ? 1. : -1.)
            * sin(ang)), w;
        for(int i = 0, j, k; i < n; i += m) {
            for(w = base(1, 0), j = i, k = i + l; j
                < k; ++j, w = w * wn) {
                base t = w * p[j + l];
                p[j + l] = p[j] - t;
                p[j] = p[j] + t;
            }
        }
    }
    if(inv) for(int i = 0; i < n; ++i) p[i].a
        /= n, p[i].b /= n;
}
vector<long long> multiply(vector<int> &a,
    vector<int> &b) {
    int n = a.size(), m = b.size(), t = n + m -
        1, sz = 1;
    while(sz < t) sz <<= 1;
    vector<base> x(sz), y(sz), z(sz);
    for(int i = 0; i < sz; ++i) {
        x[i] = i < (int)a.size() ? base(a[i], 0) :
            base(0, 0);
        y[i] = i < (int)b.size() ? base(b[i], 0) :
            base(0, 0);
    }
    fft(x), fft(y);
    for(int i = 0; i < sz; ++i) z[i] = x[i] *
        y[i];
    fft(z, 1);
    vector<long long> ret(sz);
    for(int i = 0; i < sz; ++i) ret[i] = (long
        long) round(z[i].a);
    while((int)ret.size() > 1 && ret.back() ==
        0) ret.pop_back();
    return ret;
}
long long ans[N];
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n, x; cin >> n >> x;
    vector<int> a(n + 1, 0), b(n + 1, 0), c(n +
        1, 0);
    int nw = 0;
    a[0]++; b[n]++;
    long long z = 0;
    for (int i = 1; i <= n; i++) {
        int k; cin >> k;
        nw += k < x;
        a[nw]++; b[-nw + n]++;
        z += c[nw] + !nw; c[nw]++;
    }
}

```

```

auto res = multiply(a, b);
for (int i = n + 1; i < res.size(); i++) {
    ans[i - n] += res[i];
}
ans[0] = z;
for (int i = 0; i <= n; i++) cout << ans[i]
    << ' ';
cout << '\n';
return 0;
}

```

### 8.4 NTT [65 lines]

```

#include <bits/stdc++.h>
using namespace std;

const int N = 1 << 20;
const int mod = 998244353;
const int root = 3;
int lim, rev[N], w[N], wn[N], inv_lim;
void reduce(int &x) { x = (x + mod) % mod; }
int POW(int x, int y, int ans = 1) {
    for (; y; y >= 1, x = (long long) x * x %
        mod) if (y & 1) ans = (long long) ans *
            x % mod;
    return ans;
}
void precompute(int len) {
    lim = wn[0] = 1; int s = -1;
    while (lim < len) lim <<= 1, ++s;
    for (int i = 0; i < lim; ++i) rev[i] = rev[i
        >> 1] >> 1 | (i & 1) << s;
    const int g = POW(root, (mod - 1) / lim);
    inv_lim = POW(lim, mod - 2);
    for (int i = 1; i < lim; ++i) wn[i] = (long
        long) wn[i - 1] * g % mod;
}
void ntt(vector<int> &a, int typ) {
    for (int i = 0; i < lim; ++i) if (i <
        rev[i]) swap(a[i], a[rev[i]]);
    for (int i = 1; i < lim; i <= 1) {
        for (int j = 0, t = lim / i / 2; j < i;
            ++j) w[j] = wn[j * t];
        for (int j = 0; j < lim; j += i << 1) {
            for (int k = 0; k < i; ++k) {
                const int x = a[k + j], y = (long
                    long) a[k + j + i] * w[k] % mod;
                reduce(a[k + j] += y - mod),
                    reduce(a[k + j + i] = x - y);
            }
        }
    }
    if (!typ) {
        reverse(a.begin() + 1, a.begin() + lim);
        for (int i = 0; i < lim; ++i) a[i] = (long
            long) a[i] * inv_lim % mod;
    }
}
vector<int> multiply(vector<int> &f,
    vector<int> &g) {
    if (f.empty() or g.empty()) return {};
    int n = (int)f.size() + (int)g.size() - 1;
    if (n == 1) return {(int)((long long) f[0]
        * g[0] % mod)};
    precompute(n);
}

```



```
vector<int> a = f, b = g;
a.resize(lim); b.resize(lim);
ntt(a, 1), ntt(b, 1);
for (int i = 0; i < lim; ++i) a[i] = (long
    long) a[i] * b[i] % mod;
ntt(a, 0);
a.resize(n + 1);
return a;
}

int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n, m; cin >> n >> m;
    vector<int> a(n), b(m);
    for (int i = 0; i < n; i++) {
        cin >> a[i];
    }
    for (int i = 0; i < m; i++) {
        cin >> b[i];
    }
    auto ans = multiply(a, b);
    ans.resize(n + m - 1);
    for (auto x: ans) cout << x << ' '; cout <<
        '\n';
    return 0;
}
```

## 8.5 NTT with Any prime MOD [125 lines]

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
const int N = 3e5 + 9, mod = 998244353;
```

```
struct base {
    double x, y;
    base() { x = y = 0; }
    base(double x, double y): x(x), y(y) { }
};

inline base operator + (base a, base b) {
    return base(a.x + b.x, a.y + b.y); }
inline base operator - (base a, base b) {
    return base(a.x - b.x, a.y - b.y); }
inline base operator * (base a, base b) {
    return base(a.x * b.x - a.y * b.y, a.x *
        b.y + a.y * b.x); }
inline base conj(base a) { return base(a.x,
    -a.y); }

int lim = 1;
vector<base> roots = {{0, 0}, {1, 0}};
vector<int> rev = {0, 1};
const double PI = acos(- 1.0);
void ensure_base(int p) {
    if(p <= lim) return;
    rev.resize(1 << p);
    for(int i = 0; i < (1 << p); i++) rev[i] =
        (rev[i >> 1] >> 1) + ((i & 1) << (p -
            1));
    roots.resize(1 << p);
    while(lim < p) {
        double angle = 2 * PI / (1 << (lim + 1));
        for(int i = 1 << (lim - 1); i < (1 <<
            lim); i++) {
            roots[i << 1] = roots[i];
            double angle_i = angle * (2 * i + 1 - (1
                << lim));
            roots[(i << 1) + 1] =
                base(cos(angle_i), sin(angle_i));
        }
    }
}
```

```
    }
    lim++;
}
}

void fft(vector<base> &a, int n = -1) {
    if(n == -1) n = a.size();
    assert((n & (n - 1)) == 0);
    int zeros = __builtin_ctz(n);
    ensure_base(zeros);
    int shift = lim - zeros;
    for(int i = 0; i < n; i++) if(i < (rev[i]
        >> shift)) swap(a[i], a[rev[i] >>
            shift]);
    for(int k = 1; k < n; k <= 1) {
        for(int i = 0; i < n; i += 2 * k) {
            for(int j = 0; j < k; j++) {
                base z = a[i + j + k] * roots[j + k];
                a[i + j + k] = a[i + j] - z;
                a[i + j] = a[i + j] + z;
            }
        }
    }
}

//eq = 0: 4 FFTs in total
//eq = 1: 3 FFTs in total
vector<int> multiply(vector<int> &a,
    vector<int> &b, int eq = 0) {
    int need = a.size() + b.size() - 1;
    int p = 0;
    while((1 << p) < need) p++;
    ensure_base(p);
    int sz = 1 << p;
    vector<base> A, B;
    if(sz > (int)A.size()) A.resize(sz);
    for(int i = 0; i < (int)a.size(); i++) {
        int x = (a[i] % mod + mod) % mod;
        A[i] = base(x & ((1 << 15) - 1), x >> 15);
    }
    fill(A.begin() + a.size(), A.begin() + sz,
        base{0, 0});
    fft(A, sz);
    if(sz > (int)B.size()) B.resize(sz);
    if(eq) copy(A.begin(), A.begin() + sz,
        B.begin());
    else {
        for(int i = 0; i < (int)b.size(); i++) {
            int x = (b[i] % mod + mod) % mod;
            B[i] = base(x & ((1 << 15) - 1), x >>
                15);
        }
        fill(B.begin() + b.size(), B.begin() +
            sz, base{0, 0});
        fft(B, sz);
    }
    double ratio = 0.25 / sz;
    base r2(0, - 1), r3(ratio, 0), r4(0, -
        ratio), r5(0, 1);
    for(int i = 0; i <= (sz >> 1); i++) {
        int j = (sz - i) & (sz - 1);
        base a1 = (A[i] + conj(A[j])) * r2;
        base b1 = (B[i] + conj(B[j])) * r3, b2 =
            (B[i] - conj(B[j])) * r4;
        if(i != j) {
            base c1 = (A[j] + conj(A[i])) * r2 =
                (A[j] - conj(A[i])) * r2;
        }
    }
}
```

```
    base d1 = (B[j] + conj(B[i])) * r3, d2
        = (B[j] - conj(B[i])) * r4;
    A[i] = c1 * d1 + c2 * d2 * r5;
    B[i] = c1 * d2 + c2 * d1;
}
A[j] = a1 * b1 + a2 * b2 * r5;
B[j] = a1 * b2 + a2 * b1;
}

fft(A, sz); fft(B, sz);
vector<int> res(need);
for(int i = 0; i < need; i++) {
    long long aa = A[i].x + 0.5;
    long long bb = B[i].x + 0.5;
    long long cc = A[i].y + 0.5;
    res[i] = (aa + ((bb % mod) << 15) + ((cc %
        mod) << 30))%mod;
}

return res;
}
```

```
vector<int> pow(vector<int>& a, int p) {
    vector<int> res;
    res.emplace_back(1);
    while(p) {
        if(p & 1) res = multiply(res, a);
        a = multiply(a, a, 1);
        p >>= 1;
    }
    return res;
}

int main() {
    int n, k; cin >> n >> k;
    vector<int> a(10, 0);
    while(k--) {
        int m; cin >> m;
        a[m] = 1;
    }
    vector<int> ans = pow(a, n / 2);
    int res = 0;
    for(auto x: ans) res = (res + 1LL * x * x %
        mod) % mod;
    cout << res << '\n';
    return 0;
}
```

## 9 Matrix

### 9.1 Inverse Matrix [66 lines]

```
vector<vector<int>>
    find_cofactor(vector<vector<int>> a, int
        r, int c)
{
    vector<vector<int>> mat;
    int n = a.size();
    for (int i = 0; i < n; i++)
    {
        vector<int> row;
        if (r == i) continue;
        for (int j = 0; j < n; j++)
        {
            if (j == c) continue;
            row.push_back(a[i][j]);
        }
        mat.push_back(row);
    }
    return mat;
}
```

```
int determinant(vector<vector<int>> a)
{
    if (a.size() == 1) return a[0][0];
    int n = a.size();
    int sign = +1;
    int det = 0;
    for (int i = 0; i < n; i++)
    {
        vector<vector<int>> cf_mat =
            find_cofactor(a, 0, i);
        int cofactor = determinant(cf_mat);
        det += cofactor * sign * a[0][i];
        sign = -sign;
    }
    return det;
}

vector<vector<int>>
    transpose(vector<vector<int>> a)
{
    int n = a.size();
    vector<vector<int>> res(n,
        vector<int>(n));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            res[i][j] = a[j][i];
    return res;
}

vector<vector<double>>
    find_inverse(vector<vector<int>> &a)
{
    int n = a.size();
    int det = determinant(a);
    if (det == 0)
    {
        cout << "Inverse Impossible\n";
        return {{}};
    }
    vector<vector<int>>
        cofactor_matrix(n, vector<int>(n));
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
        {
            int sign = (i + j) % 2 ? -1 : +1;
            cofactor_matrix[i][j] = sign *
                determinant(find_cofactor(a,
                    i, j));
        }
    }
    auto adj_matrix =
        transpose(cofactor_matrix);
    auto inverse_mat = vector<vector<double>>(
        n, vector<double>(n));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            inverse_mat[i][j] = 1.0 *
                adj_matrix[i][j] /det;
    return inverse_mat;
}
```

### 9.2 Matrix Multiplication [43 lines]

```
const ll M=1e9+7;
const int N=103;
```



```

int m;
ll mat[N][N];
ll ans[N][N];
void pow(int po)
{
    for(int i=0; i<m; i++)
        for(int j=0; j<m; j++) ans[i][j] = (i==j);
    while(po)
    {
        if(po%2)
        {
            ll tem[m][m];
            for(int i=0; i<m; i++)
            {
                for(int j=0; j<m; j++)
                {
                    tem[i][j]=0;
                    for(int k=0; k<m; k++)
                        tem[i][j] = (tmp[i][j] +
                            mat[i][k] *
                            ans[k][j]) % M;
                }
            }
            for(int i=0; i<m; i++)
                for(int j=0; j<m; j++)
                    ans[i][j] = tem[i][j];
        }
        po=po/2;
        ll tem[m][m];
        for(int i=0; i<m; i++)
        {
            for(int j=0; j<m; j++)
            {
                tem[i][j]=0;
                for(int k=0; k<m; k++)
                    tem[i][j] = (tmp[i][j] +
                        mat[i][k] *
                        mat[k][j]) % M;
            }
        }
        for(int i=0; i<m; i++)
            for(int j=0; j<m; j++)
                mat[i][j] = tem[i][j];
    }
}

```

### 9.3 Matrix [126 lines]

```

#include <bits/stdc++.h>
using namespace std;
//must 0 based
template<typename T = double>
struct Matrix {
    int n, m;
    vector<vector<T>>> a;
    Matrix(): n(0), m(0) {}
    Matrix(int n_, int m_, T init = T()):
        n(n_), m(m_), a(n_, vector<T>(m_,
            init)) {}
    vector<T>& operator[](int i){ return
        a[i]; }
    const vector<T>& operator[](int i) const {
        return a[i]; }
    static Matrix Identity(int k) {
        Matrix I(k,k,T());
        for(int i=0; i<k; i++) I.a[i][i] = T(1);
    }
}

```

```

        return I;
    }
    Matrix operator+(Matrix& o){
        assert(n==o.n && m==o.m);
        Matrix r(n,m);
        for(int i=0; i<n; i++) for(int
            j=0; j<m; j++) r.a[i][j] = a[i][j]
            + o.a[i][j];
        return r;
    }
    Matrix operator-(Matrix& o){
        assert(n==o.n && m==o.m);
        Matrix r(n,m);
        for(int i=0; i<n; i++) for(int
            j=0; j<m; j++) r.a[i][j] = a[i][j]
            - o.a[i][j];
        return r;
    }
    Matrix operator*(Matrix& o){
        assert(m == o.n);
        Matrix r(n, o.m, T());
        for(int i=0; i<n; i++){
            for(int k=0; k<m; k++){
                T aik = a[i][k];
                for(int j=0; j<o.m; j++){
                    r.a[i][j] += aik *
                        o.a[k][j];
                }
            }
        }
        return r;
    }
    Matrix pow(long long e){
        assert(n==m);
        if (e == 0) return Identity(n);
        if (e < 0) return inverse().pow(-e);
        Matrix base = *this, res =
            Identity(n);
        while(e){
            if (e & 1) res = res * base;
            base = base * base;
            e >>= 1;
        }
        return res;
    }
    long double determinant(double eps =
        1e-12) const {
        assert(n==m);
        int N = n;
        vector<vector<long double>>> b(N,
            vector<long double>(N));
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j)
                b[i][j] = a[i][j];
        long double det = 1.0L;
        for (int col = 0; col < N; ++col) {
            int pivot = col;
            for (int i = col + 1; i < N; ++i)
                if (fabs(b[i][col]) >
                    fabs(b[pivot][col])) pivot =
                    i;
            if (fabs(b[pivot][col]) < eps)
                return 0; // singular
            if (pivot != col) {
                swap(b[pivot], b[col]);
                det = -det;
            }
        }
    }
}

```

```

    }
    det *= b[col][col];
    long double inv_pivot = 1.0L /
        b[col][col];
    for (int i = col + 1; i < N; ++i)
    {
        long double factor = b[i][col]
            * inv_pivot;
        if (fabs(factor) < 1e-18L)
            continue;
        for (int j = col; j < N;
            ++j) b[i][j] -= factor *
            b[col][j];
    }
    return det;
}
Matrix inverse(double eps = 1e-12){
    if (n != m) throw
        runtime_error("inverse requires
            square matrix");
    int N = n;
    vector<vector<double>>> aug(N,
        vector<double>(2*N));
    for(int i=0; i<N; i++){
        for(int j=0; j<N; j++) aug[i][j] =
            double(a[i][j]);
        for(int j=0; j<N; j++) aug[i][N+j]
            = (i==j) ? 1.0 : 0.0;
    }
    for(int col=0; col<N; ++col){
        int pivot = col;
        for(int i=col+1; i<N; i++) if
            (fabs(aug[i][col]) >
                fabs(aug[pivot][col])) pivot
                = i;
        if (fabs(aug[pivot][col]) < eps)
            throw runtime_error("singular
                matrix (or near-singular)");
        swap(aug[col], aug[pivot]);
        double div = aug[col][col];
        for(int j=0; j<2*N; j++) aug[col][j]
            /= div;
        for(int i=0; i<N; i++){
            if (i==col) continue;
            double factor = aug[i][col];
            if (fabs(factor) < 1e-18)
                continue;
            for(int j=col; j<2*N; j++)
                aug[i][j] -= factor *
                aug[col][j];
        }
    }
    Matrix res(N,N);
    for(int i=0; i<N; i++) for(int
        j=0; j<N; j++) res.a[i][j] =
        T(aug[i][N+j]);
    return res;
}
void print(int precision = 10) {
    ios::fmtflags f = cout.flags();
    cout.setf(ios::fixed);
    cout<<setprecision(precision);
    for(int i=0; i<n; i++){
        for(int j=0; j<m; j++){

```

```

            if (j) cout<<' ';
            cout<<a[i][j];
        }
        cout<<"\n";
    }
    cout.flags(f);
}
int main()
{
    int n,m;
    cin >> n >> m;
    Matrix mat(n,m);
    for(int i=0; i<n; i++){
        for(int j=0; j<m; j++) cin >> mat[i][j];
    }
    mat.print();
}

```

## 10 Misc

### 10.1 Bit hacks [25 lines]

# x & -x is the least bit in x.  
 # iterate over all the subsets of the mask  
 for (int s=m; ; s=(s-1)&m) {  
 ... you can use s ...  
 if (s==0) break;  
 }  
 # c = x&-x, r = x+c; (((r^x) >> 2)/c) | r is  
 the  
 next number after x with the same number of  
 bits set.  
 # \_\_builtin\_popcount(x) //number of ones in  
 binary  
 \_\_builtin\_popcountll(x) // for long long  
 # \_\_builtin\_clz(x) // number of leading zeros  
 \_\_builtin\_ctz(x) // number of trailing  
 zeros, they also have long long version  
 # Some properties of bitwise operations:  
 a&b = a xor b + a&b  
 a xor (a&b) = (a/b) xor b  
 b xor (a&b) = (a/b) xor a  
 (a&b) xor (a/b) = a xor b  
 # Addition:  
 a+b = a/b + a&b  
 a+b = a xor b + 2(a&b)  
 # Subtraction:  
 a-b = (a xor (a&b)) - ((a/b) xor a)  
 a-b = ((a/b) xor b) - ((a/b) xor a)  
 a-b = (a xor (a&b)) - (b xor (a&b))  
 a-b = ((a/b) xor b) - (b xor (a&b))

### 10.2 Bitset C++ [18 lines]

```

bitset<17>BS;
BS[1] = BS[7] = 1;
cout<<BS._Find_first()<<endl; // prints 1
bs._Find_next(idx). This function returns
    first set bit after index idx. for example:
bitset<17>BS;
BS[1] = BS[7] = 1;
cout<<BS._Find_next(1)<<'<<BS._Find_next(3)
    <<endl; // prints 7,7
So this code will print all of the set bits of
BS:

```

```
for(int i=BS._Find_first();i< BS.size();i =
    BS._Find_next(i))
    cout<<i<<endl;
//Note that there isn't any set bit after idx,
BS._Find_next(idx) will return BS.size();
same as calling BS._Find_first() when
bitset is clear;
b[i], b.test(i)
b.set(), b.set(i), b.set(i, val)
b.reset(), b.reset(i)
b.flip(), b.flip(i)
b.count(), b.size(), b.any(), b.none(),
    b.all()
b.to_ulong(), b.to_ullong(), b.to_string()
```

### 10.3 Template [33 lines]

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <typename T>using orderedSet =
    tree<T, null_type, less_equal<T>,
        rb_tree_tag,
        tree_order_statistics_node_update>;
//order_of_key(k) - number of element strictly
    less than k
//find_by_order(k) - k'th element in set.(0
    indexed)(iterator)
mt19937_64 rnd(chrono::steady_clock::now().
    time_since_epoch().count());
long long get_rand(long long l, long long r) {
    // random number from l to r
    assert(l <= r);
    return l + rnd() % (r - l + 1);
}
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().
                time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
//pair (a, b) er jonne a * MOD + b
gp_hash_table<int, int, custom_hash> mp;
```

```
int main(int argc, char* argv[]) {
    ios_base::sync_with_stdio(false);//DON'T
        C++
    cin.tie(NULL);//DON'T use for interactive
    int seed = atoi(argv[1]);
    //cout << dist(mt) << '\n';
}
```

### 10.4 XOR Gaussian Elimination [24 lines]

```
const int N=1505;
vector<int> g[N];
signed main()
{
    fast
```

```
int n=1500;
bitset<N> bit[n+1];
for(int i=1;i<=n;i++){
    cin >> bit[i];
}
vector<int> pivot(n+1,-1);
int basis[N];
for(int i=1501;i>=0;i--){
    int in=-1;
    for(int j=1;j<=n;j++){
        if(pivot[j]==-1) continue;
        if(in!=-1 && bit[j][in])
            bit[j]^=bit[in],
            g[j].push_back(in);
        else if(bit[j][in])
            in=j,pivot[j]=i, basis[i]=j;
    }
}
return 0;
}
```

## 11 Number Theory

### 11.1 Linear Sieve [16 lines]

```
vector<int> pri;
vector<int> lp; // lowest prime factor
void sieve(int n) {
    lp.assign(n + 1, 0);
    pri.clear();
    for (int i = 2; i <= n; i++) {
        if (lp[i] == 0) {
            lp[i] = i;
            pri.push_back(i);
        }
        for (int p : pri) {
            if (p > lp[i] || 1LL * p * i > n)
                break;
            lp[p * i] = p;
        }
    }
}
```

### 11.2 Number Theory all concepts [108 lines]

```
#include <bits/stdc++.h>
using namespace std;

using ll = long long;

struct numth
{
    // Struct to hold results of the Extended
        Euclidean Algorithm
    struct exgcd
    {
        ll gcd, x, y; // gcd of a and b,
            coefficients x and y for the
            equation ax + by = gcd(a, b)
    };

    // Extended Euclidean Algorithm to solve ax
        + by = gcd(a, b)
    exgcd exEuclid(ll a, ll b)
    {
        if (b == 0)
```

```
{
    exgcd nd = {a, 1, 0};
    return nd;
}
exgcd sml = exEuclid(b, a % b);
exgcd bg = {sml.gcd, sml.y, sml.x - (a
    / b) * sml.y};
return bg;
}
ll gcd(ll a, ll b)
{
    while (b != 0)
    {
        ll temp = b;
        b = a % b;
        a = temp;
    }
    return a;
}
ll EuclidInverse(ll a, ll m)
{
    exgcd sml = exEuclid(a, m);
    if (sml.gcd != 1)
    {
        return -1;
    }
    return (sml.x % m + m) % m;
}

vector<ll> svp;//all primes
vector<ll> svl;//lowest prime
bitset<200005> check;
vector<ll> segmented_sieve(ll l, ll r)
{
    vector<ll> segpr;
    vector<bool> pr(r - l + 5, 1);
    if (l == 1)
    {
        pr[0] = false;
    }
    for (ll i = 0; svp[i] * svp[i] <= r;
        i++){
        ll cur = svp[i];
        ll base = cur * cur;
        if (base < l)
        {
            base = ((l + cur - 1) / cur)
                * cur;
        }
        for (ll j = base; j <= r; j +=
            cur)
        {
            pr[j - l] = false;
        }
    }
    for (ll i = 0; i <= r - l; i++){
        if (pr[i] == 1)
        {
            segpr.push_back(l + i);
        }
    }
    return segpr;
}
ll sumOfDivisors(ll n)
```

```
{
    vector<pair<ll, ll>> factors =
        primeFactorization(n);
    ll sum = 1;
    for (auto &factor : factors)
    {
        ll p = factor.first;
        ll a = factor.second;
        ll term = 1;
        for (ll i = 0; i <= a; i++)
        {
            term *= p;
        }
        sum += (term - 1) / (p - 1);
    }
    return sum;
}
ll CRT(vector<ll> nums, vector<ll> rems)
{
    ll prod = accumulate(nums.begin(),
        nums.end(), 1LL,
        multiplies<ll>());
    ll result = 0;
    for (size_t i = 0; i < nums.size();
        ++i)
    {
        ll pp = prod / nums[i];
        result += rems[i] *
            EuclidInverse(pp, nums[i]) *
            pp;
    }
    return result % prod;
}
};
```

### 11.3 Phi And Mobius [37 lines]

```
//all of (1-10^6) -> O(nlogn)
int phi[N];
void totient() {
    for (int i = 0; i < N; i++) phi[i] = i;
    for (int i = 2; i < N; i++) {
        if (phi[i] != i) continue;
        for (int j = i; j < N; j += i)
            phi[j] -= phi[j] / i;
    }
}
//10^16 range->O(sqrt(n))
int phiValue(int n)
{
    int ans=1;
    int q=sqrt(n);
    for(int i=2;i<=q;i++)
    {
        if(n%i==0)
        {
            int tem=1;
            while(n%i==0) tem*=i, n/=i;
            ans=ans*tem/i*(i-1);
            q=sqrt(n);
        }
    }
    if(n>1)ans=ans*(n-1);
    return ans;
}
```

```

//mobius O(nlogn)
int mob[N];
void mobius()
{
    for(int i=0;i<N;i++)mob[i]=0;
    mob[1]=1;
    for(int i=1;i<N;i++)
        for(int j=i+i;j<N;j+=i)mob[j]-=mob[i];
}

11.4 Pollard Rho [88 lines]
namespace PollardRho {
    mt19937 rnd(chrono::steady_clock::now().
        time_since_epoch().count());
    const int P = 1e6 + 9;
    ll seq[P];
    int primes[P], spf[P];
    inline ll add_mod(ll x, ll y, ll m) {
        return (x += y) < m ? x : x - m;
    }
    inline ll mul_mod(ll x, ll y, ll m) {
        ll res = __int128(x) * y % m;
        return res;
        // ll res = x * y - (ll)((long double)x * y
        // / m + 0.5) * m;
        // return res < 0 ? res + m : res;
    }
    inline ll pow_mod(ll x, ll n, ll m) {
        ll res = 1 % m;
        for (; n; n >>= 1) {
            if (n & 1) res = mul_mod(res, x, m);
            x = mul_mod(x, x, m);
        }
        return res;
    }
    // O(it * (logn)^3), it = number of rounds
    // performed
    inline bool miller_rabin(ll n) {
        if (n <= 2 || (n & 1 ^ 1)) return (n ==
            2);
        if (n < P) return spf[n] == n;
        ll c, d, s = 0, r = n - 1;
        for (; !(r & 1); r >>= 1, s++) {}
        // each iteration is a round
        for (int i = 0; primes[i] < n && primes[i]
            < 32; i++) {
            c = pow_mod(primes[i], r, n);
            for (int j = 0; j < s; j++) {
                d = mul_mod(c, c, n);
                if (d == 1 && c != 1 && c != (n - 1))
                    return false;
                c = d;
            }
            if (c != 1) return false;
        }
        return true;
    }
    //initialize just one time
    void init() {
        int cnt = 0;
        for (int i = 2; i < P; i++) {
            if (!spf[i]) primes[cnt++] = spf[i] = i;
            for (int j = 0, k; (k = i * primes[j]) <
                P; j++) {
                spf[k] = primes[j];
                if (spf[i] == spf[k]) break;
            }
        }
    }
}

```

```

    }
}
// returns  $O(n^{1/4})$ 
ll pollard_rho(ll n) {
    while (1) {
        ll x = rnd() % n, y = x, c = rnd() % n,
            u = 1, v, t = 0;
        ll *px = seq, *py = seq;
        while (1) {
            *py++ = y = add_mod(mul_mod(y, y, n),
                                c, n);
            *py++ = y = add_mod(mul_mod(y, y, n),
                                c, n);
            if ((x = *px++) == y) break;
            v = u;
            u = mul_mod(u, abs(y - x), n);
            if (!u) return __gcd(v, n);
            if (++t == 32) {
                t = 0;
                if ((u = __gcd(u, n)) > 1 && u < n)
                    return u;
            }
        }
        if (t && (u = __gcd(u, n)) > 1 && u < n)
            return u;
    }
}

vector<ll> factorize(ll n) {
    if (n == 1) return vector<ll>();
    if (miller_rabin(n)) return vector<ll>{n};
    vector<ll> v, w;
    while (n > 1 && n < P) {
        v.push_back(spf[n]);
        n /= spf[n];
    }
    if (n >= P) {
        ll x = pollard_rho(n);
        v = factorize(x);
        w = factorize(n / x);
        v.insert(v.end(), w.begin(), w.end());
    }
    return v;
}
}
}

12 String
12.1 2D Hashing [48 lines]
#include<bits/stdc++.h>
using namespace std;

const int N = 3e5 + 9;

struct Hashing {
    //just for string with characters >= a
    vector<vector<int>> hs;
    vector<int> PWX, PWY;
    int n, m;
    static const int PX = 3731, PY = 2999, mod
        = 998244353;
    Hashing() {}
    Hashing(vector<string>& s) {
        n = (int)s.size(), m = (int)s[0].size();
        hs.assign(n + 1, vector<int>(m + 1, 0));
        PWX.assign(n + 1, 1);

```

```

    PWY.assign(m + 1, 1);
    for (int i = 0; i < n; i++) PWX[i + 1] =
        1LL * PWX[i] * PX % mod;
    for (int i = 0; i < m; i++) PWY[i + 1] =
        1LL * PWY[i] * PY % mod;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++) {
            hs[i + 1][j + 1] = s[i][j] - 'a' + 1;
        }
    }
    for (int i = 0; i <= n; i++) {
        for (int j = 0; j < m; j++) {
            hs[i][j + 1] = (hs[i][j + 1] + 1LL *
                hs[i][j] * PY % mod) % mod;
        }
    }
    for (int i = 0; i < n; i++) {
        for (int j = 0; j <= m; j++) {
            hs[i + 1][j] = (hs[i + 1][j] + 1LL *
                hs[i][j] * PX % mod) % mod;
        }
    }
}

int get_hash(int x1, int y1, int x2, int y2)
{ // 1-indexed
    assert(1 <= x1 && x1 <= x2 && x2 <= n);
    assert(1 <= y1 && y1 <= y2 && y2 <= m);
    x1--;
    y1--;
    int dx = x2 - x1, dy = y2 - y1;
    return (1LL * (hs[x2][y2] - 1LL *
        hs[x2][y1] * PWY[dy] % mod + mod) %
        mod -
        1LL * (hs[x1][y2] - 1LL * hs[x1][y1]
            * PWY[dy] % mod + mod) % mod *
        PWX[dx] % mod + mod) % mod;
}

int get_hash() {
    return get_hash(1, 1, n, m);
}
};

```

---

## 12.2 Dynamic Aho Corrasic [148 lines]

```

#include <bits/stdc++.h>
using namespace std;

struct AhoCorasick
{
    struct Node
    {
        int children[26], go[26], fail;
        int patternCount; // Count
                           patterns ending here
        vector<int> patternIndices; //
                                   Stores pattern IDs

        Node()
        {
            memset(children, -1,
                sizeof(children));
            memset(go, -1, sizeof(go));
            fail = -1;
            patternCount = 0;
        }
    };
};

```

```

vector<Node> nodes;
int root;

AhoCorasick()
{
    root = 0;
    nodes.push_back(Node());
}

int charToIndex(char c)
{
    return c - 'a'; // assumes lowercase
                   'a' to 'z'
}

// Insert pattern, assign an ID
void insert(const string &s, int id)
{
    int curr = root;
    for (char c : s)
    {
        int i = charToIndex(c);
        if (nodes[curr].children[i] == -1)
        {
            nodes[curr].children[i] =
                nodes.size();
            nodes.push_back(Node());
        }
        curr = nodes[curr].children[i];
    }
    nodes[curr].patternCount++;
    nodes[curr].patternIndices.
        push_back(id);
}

void build()
{
    queue<int> q;
    nodes[root].fail = root;

    // Initialize go transitions for root
    for (int i = 0; i < 26; ++i)
    {
        if (nodes[root].children[i] != -1)
        {
            int child =
                nodes[root].children[i];
            nodes[child].fail = root;
            q.push(child);
            nodes[root].go[i] = child;
        }
        else
        {
            nodes[root].go[i] = root;
        }
    }

    while (!q.empty())
    {
        int curr = q.front();
        q.pop();

        for (int i = 0; i < 26; ++i)
        {

```

```

int child =
nodes[curr].children[i];
if (child != -1)
{
    int fail =
        nodes[curr].fail;
    while (fail != root &&
        nodes[fail].
        children[i] == -1)
        fail =
            nodes[fail].fail;

    if (nodes[fail].
        children[i] != -1)
        fail = nodes[fail].
            children[i];
    else
        fail = root;

    nodes[child].fail = fail;
    nodes[child].patternCount
        += nodes[fail].
            patternCount;

    // Merge pattern counts and
    // IDs
    for (int id :
        nodes[fail].
        patternIndices)
        nodes[child].
            patternIndices.
                push_back(id);

    q.push(child);

    nodes[curr].go[i] = child;
}
else
{
    nodes[curr].go[i] =
        nodes[nodes[curr].
            fail].go[i];
}
}
}

// Count total occurrences
int searchCount(const string &text)
{
    int curr = root;
    int count = 0;

    for (char c : text)
    {
        int i = charToIndex(c);
        curr = nodes[curr].go[i];
        count += nodes[curr].patternCount;
    }
    return count;
}

// Get matched pattern ids with position
vector<pair<int, int>>
searchWithIndex(const string &text)

```

```

{
    vector<pair<int, int>> matches;
    int curr = root;

    for (int j = 0; j < text.size(); ++j)
    {
        int i = charToIndex(text[j]);
        curr = nodes[curr].go[i];

        int temp = curr;
        while (temp != root)
        {
            for (int id : nodes[temp].
                patternIndices)
                matches.push_back({id,
                    j});
            temp = nodes[temp].fail;
        }
    }
    return matches;
}
}
};

```

### 12.3 KMP [54 lines]

```

vector<int> prefix_kmp(string &s)
{
    int n = s.size();
    vector<int> pi(n, 0);
    for (int i = 1; i < n; i++)
    {
        int j = pi[i - 1];
        while (j > 0 && s[i] != s[j]) j = pi[j]
            - 1;
        if (s[i] == s[j]) j++;
        pi[i] = j;
    }
    return pi;
}

vector<vector<int>> automation(string &s)
{
    vector<int> pre = prefix_kmp(s);
    int m = s.size();
    vector<vector<int>> nxt(m,
        vector<int>(26, 0));
    for (int st = 0; st < m; st++)
    {
        for (int ci = 0; ci < 26; ci++)
        {
            char ch = char('A' + ci);
            int j = st;
            while (j > 0 && s[j] != ch) j =
                pre[j - 1];
            if (s[j] == ch) j++;
            nxt[st][ci] = j;
        }
    }
    return nxt;
}

void kmp_base(string &demo, string &pattern)
{
    vector<int> prefix_arr =
        prefix_kmp(pattern);
    int i = 0, j = 0;
    vector<int> positions;

```

```

while (i < demo.size())
{
    if (demo[i] == pattern[j]) i++, j++;
    if (j == pattern.size())
    {
        int pos = i - j;
        positions.push_back(pos); // Store
            the start index of the match
        j = prefix_arr[j - 1]; //
            Continue searching for next
            occurrences
    }
    else if (i < demo.size() && demo[i]
        != pattern[j])
    {
        if (j != 0) j = prefix_arr[j - 1];
        else i++;
    }
}
}

```

### 12.4 Manacher [30 lines]

```

int p[2*N];
void manacher (string s)
{
    int n=s.size();
    char ss[2*n+2];
    ss[2*n+1]='\0';
    for(int i=0; i<2*n+1; i++)
    {
        p[i]=0;
        if(i%2==0)ss[i]='#';
        else ss[i]=s[i/2];
    }

    int l = 0, r = 0;
    for(int i = 0; i <=2*n; i++)
    {
        p[i] = max(0, min(r - i, p[l + (r -
            i)]));
        while((p[i]+i<=2*n)&&(i-p[i]>=
            0)&&(ss[i - p[i]] == ss[i +
            p[i]]))
        {
            p[i]++;
        }
        if(i + p[i] > r)
        {
            l = i - p[i], r = i + p[i];
        }
    }

    bool check(int l,int r)
    {
        return p[l+r+1]>r-l;
    }
}

```

### 12.5 Palindromic Tree [40 lines]

```

//just create one PTree for all the test cases
and reset everytime
struct PTree{
    /// N should be set so that N >=
    max_string_length + 5
    int S[N],nx[N];
    int head[N],nxt[N],ch[N];
    int link[N],len[N],cnt[N],lst,nd,n,e;
    ll total;

```

```

PTree(){reset();}
inline int newnode(int L){
    cnt[nd]=0,len[nd]=L,head[nd]=0;
    return nd++;
}
//O(1)
inline void reset(){
    total=e=nd=n=0,lst=1;
    newnode(0),newnode(-1);
    S[0]=-1,link[0]=1;
}
inline int getLink(int v){
    while(S[n-len[v]-1]!=S[n]) v=link[v];
    return v;
}
inline void add(int c){
    S[++n]=c;
    int cur=getLink(lst),i,j;
    for(i=head[cur];i;i=nxt[i]) if(nx[i]==c)
        break;
    if(!i){
        int now=newnode(len[cur]+2);
        int x=getLink(link[cur]);
        for(j=head[x];j;j=nxt[j]) if(nx[j]==c)
            break;
        if(j) link[now]=ch[j];else link[now]=0;
        nxt[++e]=head[cur];
        head[cur]=e;
        nx[e]=c;ch[e]=now;
        cnt[now]=cnt[link[now]]+1;
        lst=now;
    }else lst=ch[i];
        total+=cnt[lst];
    }
};

```

### 12.6 String Hashing [49 lines]

```

struct SimpleHash {
    long long len, base, mod;
    vector<int> P, H, R;
    SimpleHash() {}
    SimpleHash(string str, long long b, long
        long m) {
        base = b, mod = m, len = str.size();
        P.resize(len + 4, 1), H.resize(len +
            3, 0), R.resize(len + 3, 0);
        for (long long i = 1; i <= len + 3;
            i++)
            P[i] = ((1ll)P[i - 1] * base) %
                mod;
        for (long long i = 1; i <= len; i++)
            H[i] = ((1ll)H[i - 1] * base +
                str[i - 1]+1007) % mod;
        for (long long i = len; i >= 1; i--)
            R[i] = ((1ll)R[i + 1] * base +
                str[i - 1]+1007) % mod;
    }
    inline long long range_hash(long long l,
        long long r) {
        long long hashval = ((1ll)H[r + 1] -
            ((1ll)P[r - 1 + 1] * (1ll)H[l] %
                mod))%mod;
        return (hashval < 0 ? hashval + mod :
            hashval);
    }
}

```



```

}
inline long long reverse_hash(long long
1, long long r) {
    long long hashval = (1l)R[1 + 1] -
        ((1l)P[r - 1 + 1] * (1l)R[r + 2] %
        mod);
    return (hashval < 0 ? hashval + mod :
        hashval);
}
};
struct DoubleHash {
    SimpleHash sh1, sh2;
    DoubleHash() {}
    DoubleHash(string str) {
        sh1 = SimpleHash(str, 1949313259,
            2091573227);
        sh2 = SimpleHash(str, 1997293877,
            2117566807);
    }
    long long concat(DoubleHash& B, long long
        l1, long long r1, long long l2,
        long long r2) {
        long long len1 = r1 - l1 + 1, len2 = r2
            - l2 + 1;
        long long x1 = sh1.range_hash(l1, r1)
            ,
        x2 = B.sh1.range_hash(l2, r2);
        x1 = (x1 * B.sh1.P[len2]) %
            2091573227;
        long long newx1 = (x1 + x2) %
            2091573227;
        x1 = sh2.range_hash(l1, r1);
        x2 = B.sh2.range_hash(l2, r2);
        x1 = (x1 * B.sh2.P[len2]) %
            2117566807;
        long long newx2 = (x1 + x2) %
            2117566807;
        return (newx1 << 32) ^ newx2;
    }
    inline long long range_hash(long long l,
        long long r) {
        return (sh1.range_hash(l, r) << 32) ^
            sh2.range_hash(l, r);
    }
    inline long long reverse_hash(long long
        l, long long r) {
        return (sh1.reverse_hash(l, r) << 32)
            ^ sh2.reverse_hash(l, r);
    }
}
};

```

## 12.7 Suffix Array [105 lines]

```

vector<pair<int,int>>suflcp(vector<int>ss)
//string s -- vector<int>s (both works)
{
    int n=ss.size();
    //ranking elements
    auto s=ss;
    vector<pair<int,int>>rnk;
    //map<int,vector<int>>rnk;
    for(int i=0; i<n; i++)
        rnk.push_back({s[i],i});
    sort(all(rnk));
    int pre=-1,prev=-M;
    for(auto it:rnk)
    {

```

```

        if(it.first!=prev)pre++,prev=it.first;
        s[it.second]=pre;
    } //ranking ends
    int q=1,co=1;
    while(q<n)co++,q*=2;
    int tra[n][co];
    vector<pair<pair<int,int>,int>>str,tem[n+
        3];
    for(int i=0; i<n; i++)
        str.push_back({{s[i],-1},i});
    //sort(all(str));
    for(int i=0; i<n; i++) tem[str[i].first.
        second+1].push_back(str[i]);
    str.clear();
    for(int i=0; i<n+3; i++)
    {
        for(auto it:tem[i])str.push_back(it);
        tem[i].clear();
    }
    for(int i=0; i<n; i++) tem[str[i].first.
        first+1].push_back(str[i]);
    str.clear();
    for(int i=0; i<n+3; i++)
    {
        for(auto it:tem[i])str.push_back(it);
        tem[i].clear();
    }
    //sort end
    int k=-1;
    pair<int,int>p= {-inf,-inf};
    for(auto it:str)
    {
        if(it.first!=p)k++,p=it.first;
        tra[it.second][0]=k;
    }
    q=1;
    for(int i=1; i<co; i++)
    {
        str.clear();
        for(int j=0; j<n; j++)
        {
            if(j+q>n)
                str.push_back({{tra[j][i-1],-
                    1},j});
            else str.push_back({{tra[j][i-1],
                tra[j+q][i-1]},j});
        }
        //sort(all(str));
        for(int i=0; i<n; i++)
            tem[str[i].first.second+1].
                push_back(str[i]);
        str.clear();
        for(int i=0; i<n+3; i++)
        {
            for(auto
                it:tem[i])str.push_back(it);

```

```

            tem[i].clear();
        }
        //sort end
        int k=-1;
        pair<int,int>p= {-inf,-inf};
        for(auto it:str)
        {
            if(it.first!=p)k++,p=it.first;
            tra[it.second][i]=k;
        }
        q*=2;
    }
    vector<pair<int,int>>ans(n);
    for(int i=0; i<n; i++)
        ans[tra[i][co-1]].first=i;
    //lcp calculation
    vector<int> rank(n, 0);
    for (int i = 0; i < n;
        i++)rank[ans[i].first] = i;
    k = 0;
    //vector<int> lcp(n-1, 0);
    for (int i = 0; i < n; i++)
    {
        if (rank[i] == n - 1)
        {
            k = 0;
            continue;
        }
        int j = ans[rank[i] + 1].first;
        while (i + k < n && j + k < n &&
            s[i+k] == s[j+k]) k++;
        ans[rank[i]].second = k;
        if (k) k--;
    }
    return ans; //ans[n-1].second=0;
}
vector<pair<int,int>>suflcp(string ss)
{
    int n=ss.size();
    vector<int>sss;
    for(int i=0;i<n;i++)sss.push_back(ss[i]);
    return suflcp(sss);
}

```

## 12.8 Z Algorithm [14 lines]

```

vector<int> Zfunction(string &s)
{
    int n = s.size();
    vector<int> z(n, 0);
    int l = 0, r = 0;
    for (int i = 1; i < n; ++i)
    {
        if (i <= r) z[i] = min(r - i + 1, z[i
            - l]);
        while (i + z[i] < n && s[z[i]] == s[i
            + z[i]]) ++z[i];
        if (i + z[i] - 1 > r) l = i, r = i +
            z[i] - 1;
    }
    z[0] = n;
    return z;
}

```

## 13 Random

### 13.1 Combinatorics

$$\bullet \sum_{k=0}^n \binom{n-k}{k} = Fib_{n+1}$$

$$\bullet \binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\bullet k \binom{n}{k} = n \binom{n-1}{k-1}$$

• Number of binary sequences of length n such that no two 0's are adjacent =  $Fib_{n+1}$

• Number of non-negative solution of  $x_1 + x_2 + x_3 + \dots + x_k = n$  is  $\binom{n+k-1}{n}$

#### 13.1.1 Catalan Number

$$\bullet C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$\bullet C_0 = 1, C_1 = 1, C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}$$

$$\bullet 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58426, \dots$$

• Number of correct bracket sequences consisting of n opening brackets.

• Number of ways to completely parenthesize n+1 factors.

• The number of triangulations of a convex polygon with +2 sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).

• The number of ways to connect the 2n points on a circle to form n disjoint i.e. non-intersecting chords.

• The number of monotonic lattice paths from point (0,0) to point (n,n) in a square lattice of size  $n \times n$ , which do not pass above the main diagonal

• Number of permutation of length n that can be stack sorted.

• The number of non-crossing partitions of a set of n elements.

• The number of rooted full binary tree with n+1 leaves.



- The number of Dyck words of length  $2n$ . A string consisting of  $n$  X's and  $n$  Y's such that no string prefix has more Y's than X's.
  - Number of permutation of length  $n$  with no three-term increasing subsequence.
  - Number of ways to tile a staircase shape of height  $n$  with  $n$  rectangle.
  - $C_n^k = \frac{k+1}{n+1} \binom{2n-k}{n-k}$  denote the number of bracket sequences of size  $2n$  with the first  $k$  elements being  $($ .
  - $N(n, k) = \frac{1}{n} \binom{n}{k} \binom{n}{k-1}$
  - The number of expressions containing  $n$  pairs of correct parentheses, which contain  $k$  distinct nestings.  $N(4, 2) = 6$   $((()))$ ,  $((()()))$ ,  $((()()))$ ,  $((()()))$ ,  $((()()))$ ,  $((()()))$
  - The number of paths from  $(0,0)$  to  $(2n, 0)$  with steps only northeast and south-east, not staying below the  $x$ -axis with  $k$  peaks. And sum of all number of peaks is Catalan number.
- 13.1.2 Stirling Number of the First Kind**
- Count permutation according to their number of cycles.
  - $S(n, k)$  count the number of permutation of  $n$  elements with  $k$  disjoint cycles.
  - $S(n, k) = (n-1) \times S(n-1, k) + S(n-1, k-1)$ ,  $S(0, 0) = 1$ ,  $S(n, 0) = S(0, n) = 0$
  - $S(n, 1) = (n-1)!$
  - $S(n, n-1) = \binom{n}{2}$
  - $\sum_{k=0}^n S(n, k) = n!$
- 13.1.3 Stirling Numbers of the Second Kind**
- Number of ways to partition a set of  $n$  objects into  $k$  non-empty subsets.
  - $S(n, k) = k \times S(n-1, k) + S(n-1, k-1)$ ,  $S(0, 0) = 1$ ,  $S(n, 0) = S(0, n) = 0$
  - $S(n, 2) = 2^{n-1} - 1$

- $S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$
  - $S(n, k) * k! =$  number of ways to color  $n$  nodes using colors from 1 to  $k$  such that each color is used at least once.
- 13.1.4 Bell Number**
- Counts the number of partitions of a set.
  - $B_{n+1} = \sum_{k=0}^n \binom{n}{k} * B_k$
  - $B_n = \sum_{k=0}^n S(n, k)$ , where  $S$  is Stirling number of second kind.
  - The number of multiplicative partitions of a square free number with  $i$  prime factors is the  $i$ -th Bell number.
  - $B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$
  - If a deck is shuffled by removing and reinserting the top card  $n$  times, there are  $n^n$  possible shuffles. The number of shuffles that return the deck to its original order is  $B_n$ , so the probability of returning to the original order is  $B_n/n^n$ .
- 13.1.5 Lucas Theorem**
- If  $p$  is prime then  $\binom{p^a}{k} \equiv 0 \pmod{p}$
  - For non-negative integers  $m$  and  $n$  and a prime  $p$ :  

$$\binom{m}{n} = \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$$
 where  $m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0$   
 $n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$  are the base  $p$  expansion.
- 13.1.6 Derangement**
- A permutation such that no element appears in its original position.
  - $d(n) = (n-1) * (d(n-1) + d(n-2))$ ,  $d(0) = 1$ ,  $d(1) = 0$
  - $d(n) = nd(n-1) + (-1)^n = \lfloor \frac{n!}{e} \rfloor$ ,  $n \geq 1$

- 13.1.7 Burnside Lemma**
- Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals
- $$\frac{1}{|G|} \sum_{g \in G} |X^g|$$
- where  $X^g$  are the elements fixed by  $g(g.x = x)$ . If  $f(n)$  counts "configurations" of some sort of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get
- $$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k)$$
- 13.1.8 Eulerian Number**
- $E(n, k)$  is the number of permutations of the numbers 1 to  $n$  in which exactly  $k$  elements are greater than the previous element.
  - $E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$ ,  $E(n, 0) = E(n, n-1) = 1$
  - $E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$
  - $E(n, k) = E(n, n-1-k)$
  - $E(0, k) = [k=0]$
  - $E(n, 1) = 2^n - n - 1$
- 13.2 Number Theory**
- 13.2.1 Mobius Function and Inversion**
- define  $\mu(n)$  as the sum of the primitive  $n$ th roots of unity depending on the factorization of  $n$  into prime factors:
- $$\mu(x) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$
- Mobius Inversion:
- $$g(n) = \sum_{d|n} f(d) \leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$
- $\sum_{d|n} \mu(d) = [n=1]$

- $\phi(n) = \sum_{d|n} \mu(d) \cdot \frac{n}{d} = n \sum_{d|n} \frac{\mu(d)}{d} = \sum_{d|n} d \cdot \mu(\frac{n}{d})$
  - $a|b \rightarrow \phi(a)|\phi(b)$
  - $\phi(mn) = \phi(m) \cdot \phi(n) \cdot \frac{d}{\phi(d)}$  where  $d = \gcd(m, n)$
  - $\phi(n^m) = n^{m-1} \phi(n)$
  - $\sum_{i=1}^n [\gcd(i, n) = k] = \phi(\frac{n}{k})$
  - $\sum_{i=1}^n \gcd(i, n) = \sum_{d|n} d \cdot \phi(\frac{n}{d})$
  - $\sum_{i=1}^n \frac{1}{\gcd(i, n)} = \sum_{d|n} \frac{1}{d} \cdot \phi(\frac{n}{d}) = \frac{1}{n} \sum_{d|n} d \cdot \phi(d)$
  - $\sum_{i=1}^n \frac{i}{\gcd(i, n)} = \frac{n}{2} \cdot \sum_{d|n} \frac{1}{d} \cdot \phi(\frac{n}{d}) = \frac{n}{2} \cdot \frac{1}{n} \sum_{d|n} d \cdot \phi(d)$
  - $\sum_{i=1}^n \frac{n}{\gcd(i, n)} = 2 \cdot \sum_{i=1}^n \frac{i}{\gcd(i, n)} - 1$
- 13.2.2 GCD and LCM**
- $\gcd(a, b) = \gcd(b, a \bmod b)$
  - If  $a|b.c$ , and  $\gcd(a, b) = d$ , then  $(a/d)|c$ .
  - GCD is a multiplicative function.
  - $\gcd(a, \text{lcm}(b, c)) = \text{lcm}(\gcd(a, b), \gcd(a, c))$
  - $\gcd(n^a - 1, n^b - 1) = n^{\gcd(a, b)} - 1$
- 13.2.3 Gauss Circle Theorem**
- Determine the number of lattice points in a circle centered at the origin with radius  $r$ .
  - number of pairs  $(m, n)$  such that  $m^2 + n^2 \leq r^2$
  - $N(r) = 1 + 4 \sum_{i=0}^{\infty} (\lfloor \frac{r^2}{4i+1} \rfloor - \lfloor \frac{r^2}{4i+3} \rfloor)$

### 13.2.4 Pick's Theorem

According to Pick's Theorem We can calculate the area of any polygon by just counting the number of Interior and Boundary lattice points of that polygon. If number of interior points are I and number of boundary lattice points are B then Area (A) of polygon will be:

$$Area = I + B/2 - 1$$

where I is the number of points in the interior shape, B stands for the number of points on the boundary of the shape.

### 13.2.5 Formula Cheatsheet

- $\sum_{i=1}^n = \frac{1}{m+1}[(n+1)^{m+1} - 1 - \sum_{i=1}^n ((i+1)^{m+1} - i^{m+1} - (m+1)i^m)]$
- $\sum_{i=0}^n c^i = \frac{c^{n+1}-1}{c-1}, c \neq 1$
- $\sum_{i=0}^{\infty} c^i = \frac{1}{1-c}, \sum_{i=1}^{\infty} c^i = \frac{c}{1-c}, |c| < 1$
- $\sum_{i=0}^{n-1} ar^i = a \frac{1-r^n}{1-r}, r \neq 1$
- $\sum_{i=1}^n ir^{i-1} = \frac{1-(n+1)r^n + nr^{n+1}}{(1-r)^2}, r \neq 1$
- $\sum_{i=1}^n ir^i = \frac{r(1-(n+1)r^n + nr^{n+1})}{(1-r)^2}, r \neq 1$
- $1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}, |x| < 1$
- $\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x$
- $\sum_{i=0}^{\infty} (-1)^i \frac{x^i}{i!} = e^{-x}$
- $\sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!} = \sin x$
- $\sum_{i=0}^{\infty} (-1)^i \frac{x^{2i}}{(2i)!} = \cos x$
- $\sum_{i=0}^{\infty} \frac{x^{2i+1}}{(2i+1)!} = \sinh x$
- $\sum_{i=0}^{\infty} \frac{x^{2i}}{(2i)!} = \cosh x$

- $(1+x)^\alpha = \sum_{i=0}^{\infty} \binom{\alpha}{i} x^i, |x| < 1$
- $\frac{1}{1+x^2} = \sum_{i=0}^{\infty} (-1)^i x^{2i}, |x| < 1$
- $\arctan x = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{2i+1}, |x| \leq 1, x \neq \pm 1$
- $\frac{1}{1-x^2} = \sum_{i=0}^{\infty} x^{2i}, |x| < 1$
- $\frac{1}{(1-x)^2} = \sum_{i=1}^{\infty} ix^{i-1}, |x| < 1$
- $\frac{x}{(1-x)^2} = \sum_{i=1}^{\infty} ix^i, |x| < 1$
- $\ln(1+x) = \sum_{i=1}^{\infty} (-1)^{i+1} \frac{x^i}{i}, |x| < 1$
- $\ln\left(\frac{1+x}{1-x}\right) = 2 \sum_{i=0}^{\infty} \frac{x^{2i+1}}{2i+1}, |x| < 1$
- $H_n = \sum_{i=1}^n \frac{1}{i}, \sum_{i=1}^n iH_i = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}$
- $\sum_{k=0}^n \binom{r+k}{k} = \binom{r+n+1}{n}$
- $\sum_{i=r}^n \binom{i}{r} = \binom{n+1}{r+1}$
- $\sum_{i=0}^n i \binom{n}{i} = n2^{n-1}$
- $\sum_{i=0}^n i^2 \binom{n}{i} = n(n+1)2^{n-2}$
- $\sum_{i=0}^n k^i \binom{n}{i} = (k+1)^n$
- $\sum_{i=1}^n F_i = F_{n+2} - 1$
- $\sum_{i=1}^n F_{2i} = F_n F_{n+1}$
- $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$

### 13.2.6 3D Geometry

- Distance between points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$ :  $\frac{|AB|}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$
- Angle between vectors  $\vec{u}$  and  $\vec{v}$ :  $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$
- Area of triangle with edge vectors  $\vec{u}, \vec{v}$ :  $\frac{1}{2} \|\vec{u} \times \vec{v}\|$
- Volume of parallelepiped with generators  $\vec{u}, \vec{v}, \vec{w}$ :  $V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$
- Distance from point  $P(x_0, y_0, z_0)$  to plane  $ax + by + cz + d = 0$ :  $\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$
- Plane through three points  $A, B, C$ :  $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$ , equation  $\vec{n} \cdot (\vec{r} - \vec{r}_A) = 0$
- Plane from point  $P_0(x_0, y_0, z_0)$  and normal  $\vec{n} = (a, b, c)$ :  $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$
- Distance between parallel planes  $ax + by + cz + d_1 = 0$  and  $ax + by + cz + d_2 = 0$ :  $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$
- Distance from point  $P$  to line through  $A$  with direction  $\vec{d}$ :  $\frac{\|\overrightarrow{AP} \times \vec{d}\|}{\|\vec{d}\|}$
- Shortest distance between skew lines  $L_1: \vec{r} = \vec{a}_1 + s\vec{d}_1$  and  $L_2: \vec{r} = \vec{a}_2 + t\vec{d}_2$ :  $\frac{|(\vec{a}_2-\vec{a}_1) \cdot (\vec{d}_1 \times \vec{d}_2)|}{\|\vec{d}_1 \times \vec{d}_2\|}$
- Line through two points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$ :  $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$
- Plane through  $A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3)$ :  $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$
- Coplanarity of  $A, B, C, D$ :  $\begin{vmatrix} x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \\ x_D & y_D & z_D & 1 \end{vmatrix} = 0$

- Plane intercept form cutting axes at  $a, b, c$ :  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
- Angle between two planes with normals  $\vec{n}_1, \vec{n}_2$ :  $\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|}$
- Line-plane intersection for line  $\vec{p} = \vec{p}_0 + t\vec{d}$  and plane  $ax + by + cz + d = 0$ :  $t = -\frac{ax_0+by_0+cz_0+d}{ad_x+bd_y+cd_z}$  when denominator non-zero
- Parametric line from point  $P_0(x_0, y_0, z_0)$  with direction  $(a, b, c)$ :  $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$
- Line with direction ratios  $(l, m, n)$  through  $P_0$ : symmetric form  $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$ ; direction cosines  $(\cos \alpha, \cos \beta, \cos \gamma)$  satisfy  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
- Angle between line with direction ratios  $(l, m, n)$  and plane  $ax + by + cz + d = 0$ :  $\sin \theta = \frac{|al+bm+cn|}{\sqrt{l^2+m^2+n^2} \sqrt{a^2+b^2+c^2}}$
- Midpoint of segment joining  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$ :  $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
- Centroid of triangle with vertices  $A, B, C$ :  $G\left(\frac{x_A+x_B+x_C}{3}, \frac{y_A+y_B+y_C}{3}, \frac{z_A+z_B+z_C}{3}\right)$
- Centroid of tetrahedron with vertices  $A, B, C, D$ :  $\left(\frac{x_A+x_B+x_C+x_D}{4}, \frac{y_A+y_B+y_C+y_D}{4}, \frac{z_A+z_B+z_C+z_D}{4}\right)$
- Sphere of radius r: surface  $S = 4\pi r^2$ , volume  $V = \frac{4}{3}\pi r^3$
- Right circular cone (radius r, height h):  $V = \frac{1}{3}\pi r^2 h$ , lateral area  $\pi r \sqrt{r^2 + h^2}$

$C(x_3, y_3, z_3)$ :