Chapter 7: Gravitation

Newton's law of gravitation:

Statement:"Every particle in this universe attracts every other particle with the force of gravitation which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them."

If F be the magnitude of this force between two particles of mass m_1 and m_2 separated by a distance 'd' then according to above statement,

$F \propto m_1 m_2$ .... (I)

$F \propto 1/d^2$ .... (II)

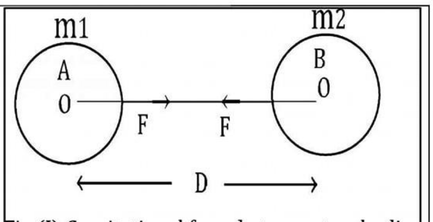

Fig (I) Gravitational force between two bodies

where G is the proportionality constant known as Universal gravitational constant. The value of $G = 6.67 \times 10^{-11} Nm^2/kg^2$ and remains constant throughout the universe.

Gravitational constant: We have $F = \frac{m_1 m_2}{d^2}$

If $m_1 = m_2 = 1kg$ and d= 1m then F=G

Henry Cavendish measured value of G in 1778A.D. with the help of highly sensitive Torsion balance. Gravitational constant is numerically equal to the force of attraction between two unity masses separated by unity distance.

Unit: $\frac{Nm^2}{kg^2} \rightarrow$ S. I unit. Value of G doesn't depend upon nature and size of bodies.

Acceleration due to gravity (g):

When a body is allowed to fall freely under gravity from a small height above the earth's surface, then it falls with uniform acceleration called the acceleration due to gravity. The value of g varies from place to place. However, its average value on the earth surface is 9.80m/s^2. **Note: Gravity**: It is the force of attraction exerted by the earth towards its centre. It is a special case of gravitation.

Relation between g and R: i.e. $g = \frac{GM}{R^2}$

Consider a small body of mass m on the earth surface at point A where the acceleration due to gravity is g, as shown in fig (1). Let M be the mass of the earth and R be the radius of the earth.

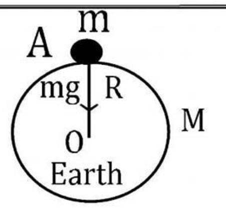

Fig (I) Freely falling body near Me

The weight mg of the body at point A is equal to the gravitational attraction of the earth on it. Thus, By Newton's law of gravitation, we can write

W= F

Or, $mg = \frac{GMm}{R^2}$ $\therefore g = \frac{GM}{R^2}$ ……(1) This is the required relation between g and R. This equation (1) gives the value of acceleration due to gravity at surface of the earth. If G and M are constants $\therefore g \propto \frac{1}{R^2}$ ……. (2)

i.e. Acceleration due to gravity at any place is inversely proportional to the radius of earth at the place.

Mass and Weight:

Mass of a body is the quantity of matter contained in it . Weight of a body is the gravitational pull on it due to earth. **i.e.**W=mg

Mass	Weight
I) It is the measure of inertia or gravitation.	(I) It is the measure of gravity.
(II) It is scalar quantity.	(II) It is vector quantity.
(III) It is constant quantity	(III) It may vary from place to place for the same body.
(IV) It cannot be zero of any material body.	(IV) It may be zero(centre of earth)
(V) It is essential property of material bodies.	(V) It is not an essential property.
(VI) Unit: kg or gram	(VI) Unit: Newton or Dyne
(VII) It is not affected by the presence of other bodies.	(VII) It may be affected by presence of other bodies.

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Q. Derive an expression for variation of g with altitude and explain its meaning.

Q. How does g vary with distance from centre of earth above and below its centre? (2061, 2064, 2068, 2056)

Variation of g due to altitude (Or above the earth's surface)

Let us consider the earth to be a perfect sphere of radius R and mass M. Let g be the value of acceleration due to gravity on the surface of earth, then

$g = \frac{GM}{(R)^2}$ ……… (1)

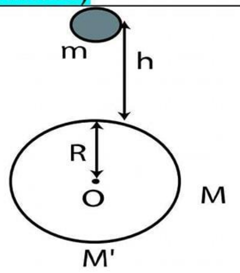


Fig (I) Value of g due to altitde

Let a body of mass 'm' is at point A at a height 'h' above the surface of earth and g' be the value of acceleration due to gravity at this position. Then

$g' = \frac{GM}{(R+h)^2}$ ……… (2)

Where$(R + h)$ = distancebetween centre of the body and earth

Dividing equation (2) by(1)

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{(R)^2}} = \frac{\frac{1}{(R+h)^2}}{\frac{1}{(R)^2}} = \frac{(R)^2}{(R+h)^2} = \frac{(R)^2}{R^2\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

$$\therefore \text{Binomialexpansion: } (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots]$$

$\therefore \frac{g'}{g} = \left(1 - \frac{2h}{R}\right) g$ (Since, $g' < g$)

Thus, value of g decreases as we move away from the surface of the earth.

Variation of g due to depth (below the earth surface):

Q. Discuss the variation of g below the earth's surface.(2053, 2060, 2064)

Let us consider a body of mass 'm' is lying at a depth 'x' from the surface of earth (fig I).

The acceleration due to gravity on the surface of earth is given by,

$$g = \frac{GM}{R^2} \ldots\ldots (1)$$

If ϱ be the density of earth then

$$M = V\varrho = \left(\frac{4}{3}\pi R^3\right)\varrho$$

$$\therefore g = \left(\frac{4}{3}\pi\varrho GR\right) \ldots\ldots\ (2)$$

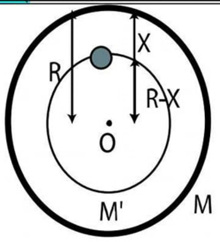


Fig (I) Acceleration due to gravity at depth

When the body is at depth x below the surface of earth, then acceleration g' on the body is due to the sphere of radius (R-x).

If M' be the mass of dotted sphere, then $g' = \frac{GM'}{(r-x)^2}$ ………….. (3)

But, $M' = \frac{4}{3}\pi(R-x)^3\varrho$ then, $g' = \frac{4}{3}\pi\varrho G(R-x)$ …….(4)

Dividingequation(4)by(2) ,

$$\frac{g'}{g} = \frac{\frac{4}{3}\pi\varrho G(R-x)}{\frac{4}{3}\pi\varrho GR} = (1-\frac{x}{R}) < 1 \qquad (\text{Since}, g' < g)$$

Thus, value of g decrease as we move towards the centre of the earth.

Thus, as we go towards the centre of the earth, the value of g decreases gradually. At the centre, x = R, so we get g=0. It means the weight of a body becomes zero i.e. weightlessness.

Note: Variation of 'g' inside and outside the earth.

It is seen that g varies linearly with distance from the centre of earth, attains maximum value at the surface and then decreases following inverse square law away from the surface of earth.

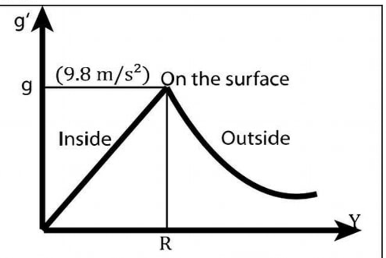


Fig (I) variation of g insde and outside of the ear

Variation of g due to Rotation of earth (i.e. with latitude) (2071)

Q. Obtain an expression for variation of g with rotation of earth. (4 marks)

Let us consider an object of mass m at a point p on the earth surface at latitude of angleθ.
The angle $\measuredangle POB$ made by the line joining the position of object to centre of earth with equatorial line is called Latitude angle. i.e. $\measuredangle POB = \theta$

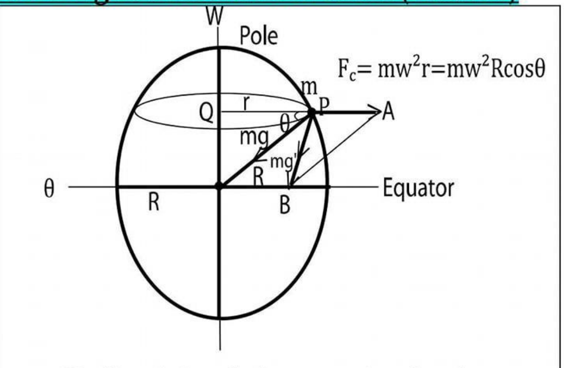


Fig (I) variation of g due to rotation of earth

When the earth is not rotating, its weight mg acts along the radius of the earth towards the centre(PO).
When the earth rotates with angular velocity ω on its axis, the object at P will also rotate about the centre Q of a circle of radius(r) where $r = R\cos\theta$ and R is radius of the earth. So the object move outward along PA will experience centrifugal force, $F_C = mr\omega^2 = m\omega^2R\cos\theta$ ....(1) $\left[PO = R\,, \cos\theta = \frac{r}{R}\right]$

And the object is under the action of two forces centrifugal force and its weight. The resultant of two forces $\overrightarrow{PA}$ and $\overrightarrow{PO}$ shown by $\overrightarrow{PB}$ in fig (I) ,which is the apparent weight mg' of the object.
To find the resultant, complete the parallelogram PABO and then apply parallelogram law of vector addition,

$$PB^2 = PO^2 + PA^2 + 2(PO)(PA)\cos(180° - \theta)$$
$$(mg')^2 = (mg)^2 + (F_C)^2 + 2(mg)(F_C).\cos(180° - \theta)$$
$$m^2g'^2 = m^2g^2 + m^2\omega^4R^2\cos\theta + 2mgm\omega^2R\cos\theta(-\cos\theta)$$

$$g'^2 = g^2 + \omega^4R^2\cos^2\theta + 2g\omega^2R\cos\theta(-\cos\theta)$$
$$g'^2 = g^2\left[1 + \frac{\omega^4R^2\cos^2\theta}{g^2} - \frac{2R\omega^2\cos^2\theta}{g}\right]$$
$$\therefore g' = g\sqrt{\left[1 + \frac{\omega^4R^2\cos^2\theta}{g^2} - \frac{2R\omega^2\cos^2\theta}{g}\right]} \text{ .... (2)}$$

Since $\frac{R\omega^2}{g}$ is a small quantity (i.e. $\frac{1}{279}$), the terms containing the factor $\frac{\omega^4R^2}{g^2}$ can be neglected. $g' = g\sqrt{\left[1 - \frac{2R\omega^2\cos^2\theta}{g}\right]}$

$$\left[\text{using Binomial Expansion } (1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \;\ldots\ldots\right]$$

$g' = g\left[1 - \frac{1}{2}\left(\frac{2R\omega^2\cos^2\theta}{g}\right) + \text{ higher order terms}\right]$ and higher power of $\left(\frac{2Rw^2\theta}{g}\right)$ is neglected $\quad \therefore g' = g\left[1 - \frac{R\omega^2\cos^2\theta}{g}\right]$

or, $\therefore g' = g - R\omega^2\cos^2\theta$ .... (3)

This is the required experssion for the variation of g due to rotation of earth.

Special case:

Case (I) At the equator, $\theta = 0°$, so $\cos 0° = 1$

$\therefore g' = g - R\omega^2$ .... (4)

Therefore, acceleration due to gravity is minimum at equator.

Case II At the poles, $\theta = 90°$ so, $\cos 90° = 0$

$\therefore g' = g$ .... (5) (Maximum value)

So, there is no effect on acceleration due to gravity of poles due to the rotation of the earth.

It Includes that acceleration due to gravity decreases due to the effect of latitude towards equator.

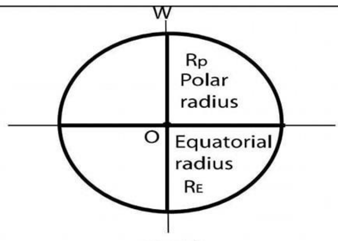
Case (III) If earth stops rotating, ω = 0 from (3).

$g' = g$ .... (6)

Variation of g due to the shape of the earth:

The value of acceleration due to gravity on the surface of the earth is given by,

$g = \frac{GM}{R^2}$, ...(1)

Since, G and M are constants, we have, $g \propto \frac{1}{R^2}$ .... (2) The earth is not a perfect sphere of radius R. It flattens at the poles and bulges out at the equator.	 Fig (I)
Since, $R_E > R_p$ so, from (2) $g_E < g_p$	R_E = Equatorial radius

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At equator region, $g_{Eq} = 9.78 m/s^2$ At polar region, $g_{pole} = 9.83 m/s^2$ On the average, $g = 9.8 m/s^2$	R_p = Polar radius g_E = acceleration due to gravity at equator g_p = acceleration due to gravity at pole

Gravitational Field:

The space around any material particle over which its gravitational influence can be felt is called gravitational field.

Gravitational field intensity (I or E)

The gravitational intensity at a point in the gravitational field is defined as the gravitational force per unit mass experienced by the small body kept at that point.

i.e. $I = \frac{F}{m}$, ……. (1)

Suppose, we have to find the gravitational field intensity I at point P, having distance r from the centre of the earth of mass M and radius R, (as shown in fig (I). For this, we consider a small body of mass 'm' at this point P. Then by Newton's law of gravitation,

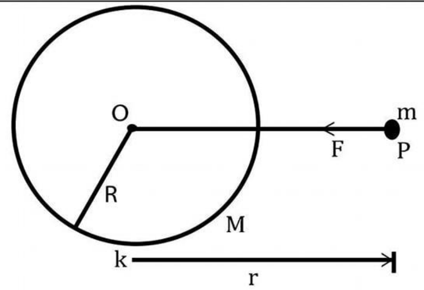


Fig (I) Intensity of gravitational field

$F = \frac{GMm}{r^2}$, ………. (2)

Thus, fromequation(1)and(2), we have

$$I = \frac{GM}{r^2} = g' \ldots\ldots\ldots (3)(\text{acceleration due to gravity at point P.})$$

If the test mass lies on the surface of earth then $r = R$.

$$\therefore I = \frac{GM}{r^2} = g \ldots\ldots\ldots (4)(\text{acceleration due to gravity on the earth's surface})$$

We Know, $G = 6.67 \times 10^{-11} Nm^2/kg^2$

$R = 6.4\times 10^6 m$ And $M = 6.0\times 10^{24} kg$

$$\therefore I = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times (6.0 \times 10^{24} kg)}{(6.4 \times 10^6 m)^2} = 9.8 N/kg$$

Gravitational potential energy (U or W):

The Gravitational potential Energy at a point in the gravitational field is defined as the amount of work-done while bringing a body from infinity to that point with constant velocity.

It is the energy associated with a body due to its position in the gravitational field of another body. It is a scalar quantity. $U = W = -\frac{GMm}{r}$

Gravitational potential (V):

The gravitational potential (V) is the energy associated with a unit mass and is defined as the amount of work-done in taking a unit mass from infinity to any specified position in the gravitational field of another body.

Simply, It is the gravitational potential energy per unit mass.

$$\text{i.e. } V = \frac{U}{m} \quad \text{or} \quad V = -\frac{GM}{r}$$

Expression for gravitational potential Energy (U) gravitational potential (V):

Q. Obtain an expression for gravitational potential energy and establish its dimension.

Q. Derive the relation for gravitational potential at a point due to a point mass.

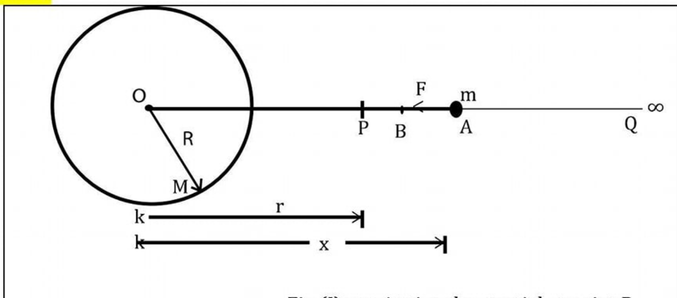


Fig (I) gravitational potential at point P

Here, in fig (I)
OP = r
OB = x
AB = dx

When the body is at infinity, its gravitational potential energy is zero.

Consider a point P at distance r from the centre of earth O. (R and M being Radius and mass of Earth respectively), where the gravitational potential energy (U) and gravitational potential (V) to be determined.

Let a body of mass 'm' initially at infinity is at point A, at any instant of time, in the gravitational field of earth (fig (1)). Let OA = x be its distance from centre of earth. So, the gravitational force on mass 'm' at point A due to earth is given by

$F = \frac{GMm}{x^2}$, ... (1)

Let the body is moved through a very small displacement dx from point A to B, then the small amount of workdone by the force is given by

$dW = Fdx$, ... (2)

from(1)and(2); $dW = \frac{GMm}{x^2}dx$ .... (3)

Thus, by definition, the GPE (U) of the body at point P, which is the work-done in moving the body from point Q at infinity to point P, is given by

$$U = \int_Q^P dW = \int_\infty^r \frac{GMm}{x^2}dx = GMm\int_\infty^r x^{-2}dx$$

$$= GMm\left[\frac{x^{-2+1}}{-2+1}\right]_\infty^r = GMm\left[\frac{x^{-1}}{-1}\right]_\infty^r = GMm\left[-\frac{1}{x}\right]_\infty^r$$

$= GMm\left\{-\left(\frac{1}{r}-\frac{1}{\infty}\right)\right\}$=$GMm\left\{-\frac{1}{r}-0\right\} = -\frac{GMm}{r}$

$\therefore U = W = -\frac{GMm}{r}$ .... (4)

This is the required expression for gravitational potential energy.

Here, -ve sign indicates the gravitational force is always attractive. As the distance r increases, gravitational potential increases as it becomes, less negative.

This equation (4) reveals that body's gravitational potential energy increases as it moves away from the earth and attains maximum value of 0 (zero) at r = ∞).

Note: If body be on earth surface then from (4)

$U = -$mgr where g = GM/R^2

Further, gravitational potential (V) = $\frac{U}{m} = -\frac{\frac{GMm}{r}}{m} = -\frac{GM}{r}$

$\therefore V = \frac{GM}{r}$ .... (6) *This is* the required expression for gravitational potential.

Note: **Relation between I and V:** (I = Gravitational field intensity, V= gravitational potential) so, I = $-\frac{dV}{dr}$

The gravitational field intensity at any point is equal to the negative gradient of gravitational potential.

Escape velocity (V_e): (2059, 2063, 2066, 2068, 2069, 2070, 2071, 2073, 2074)

Q.1 Define escape velocity of body on a planet. Derive an expression on for it.

Q.2 Suppose that a strong man can throw a stone so that it will never return to the surface of earth. How much work does he have to do in throwing up the stone of mass 'm' and find an expression for its minimum velocity?

Q.3 what is escape velocity? Prove that escape velocity of a body from earth's surface is $\sqrt{2gR} where$ g is acceleration due to gravity on the surface of the earth and R is radius of the earthQ.4 What is escape velocity? Derive it expression on the surface of earth.

Q5. Explain the meaning of escape velocity based on the concept of gravitational potential. Hence derive expression for escape velocity of a body thrown from the surface of the earth.

Escape velocity:- The minimum velocity with which a body must be thrown upwards in order that it may just escape the gravitational pull of the planet is called escape velocity. If a body is thrown by velocity equal to escape velocity, it never comes back to earth. It does not depend on angle of projection from the earth's surface.

Expression for escape velocity:

Consider a body of mass 'm' is one the surface of earth where R and M being Radius and mass of the earth respectively Let the body is projected with velocity V_e from surface of earth and it reaches to a point A of distance x from the centre of earth.

So, gravitational force between body and earth is : $F = \frac{GMm}{x^2}$ ……… (1)

Let the body is moved to point B from A such that AB =dx towards infinity. So the work-done dW on body against gravity due to loss in its K.E is given as, dW = Fds

$dW = \frac{GMm}{x^2}dx$, ... (2)

The total amount workdone in taking the body from surface to infinity is calculated by integrating equation (2) from limit x= R to x = ∞, we get

$$\int dW = \int_R^\infty \frac{GMm}{x^2}dx$$

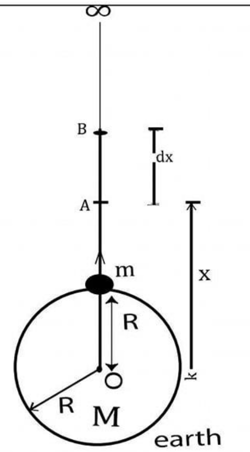


Fig (I) calculation of escape velocit

$$\therefore W = GMm\int_R^\infty x^{-2}dx = GMm\left[\frac{x^{-2+1}}{-2+1}\right]_R^\infty = GMm\left[\frac{x^{-1}}{-1}\right]_R^\infty$$

$$\therefore W = GMm\left\{-\left(\frac{1}{\infty}-\frac{1}{R}\right)\right\} = GMm\left\{-\left(0-\frac{1}{R}\right)\right\}$$

$\therefore W = \frac{GMm}{R}$ .... (3)

This is required workdone for escape velocity.

To Escape the object from earth's surface,

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}\quad\left[\therefore \text{work energy theorem } W = \Delta K.E. = \frac{1}{2}mv_e^2 - 0\right]$$

$or, V_e = \sqrt{2\frac{GM}{R}}$, ...(4) (Since, $g = \frac{GM}{R^2} \rightarrow gR = \frac{GM}{R}$

$\therefore V_e = \sqrt{2gR}$ .... (5) *This is required expression for escape velocity.*

using $g = 9.8 m/s^2$ and $R = 6400km = 6.4\times 10^6 m$

Then, $\boldsymbol{V_e = 11.2km/sec}$. This is escape velocity of earth.

The escape velocity for different planets is different because the mass and the radius of different planets are different.

Satellite:

A body which revolves around a planet in nearly circular orbit with uniform speed under the gravitational attraction of that planet alone is called a satellite of that planet. e.g. moon is the natural satellite of the earth. Since, 1957, several man made artificial satellites have been launched to revolve around the earth.

Orbital velocity of the satellite: (2069, 2056, 2065)

The minimum velocity required to put the satellite into its orbit around the earth is called orbital velocity of satellite.

Orbital velocity is the velocity required to keep the satellite into its orbit. The orbital velocity of a satellite is the velocity required to be given to the satellite in order to put it in an orbit around the earth.

Let a satellite of mass 'm' is orbiting in a circular orbit of radius 'r' with orbital velocity v_o. The necessary centripetal force required to keep the satellite in circular orbit is provided by the gravitational attraction between earth and satellite.

i.e. $\frac{mv_o^2}{r} = \frac{GMm}{R^2}$ where, G = universal gravitational constant & M = mass of earth

$\therefore v_o = \sqrt{\frac{GM}{r}}, \ldots\ldots (1)$

If h $be\ the\ height\ of satellite\ from\ the\ surface\ of\ earth, then$

r= R+h , Where R = radius of Earth

$\therefore v_o = \sqrt{\frac{GM}{(R+h)}}, \ldots\ldots\ldots\ldots (2)$

$If g\ the\ acceleration\ due$ to gravity on the earth's surface then

$$g = \frac{GM}{R^2}$$

or, $GM = gR^2$ ……….. (3)

$Then from (2) and (3)$.

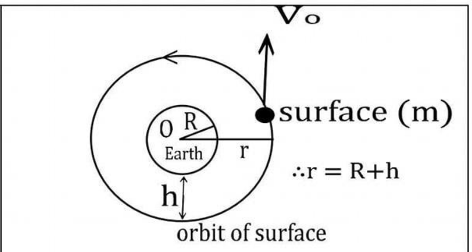


Fig (I) Calculation orbital velocity of surfa

$$v_o = \sqrt{\frac{gR^2}{(R+h)}},$$

$\therefore v_o = \sqrt{\frac{g}{(R+h)}} \ldots\ldots (4)$

This is the required expression for orbital velocity of the satellite.

Note: If satellite is very close to earth's surface then $r \cong R and\ h \cong 0$ so from (4)

$v_o = \sqrt{Rg} = \sqrt{6.4 \times 10^6 \times 9.8} = 7920 m/s \cong 8 km/sec.$

Time period of the satellite revolving around the earth (2072, 2069, 2065)

The time period T of a satellite is the total time taken by the satellite to complete one revolution around the earth.

We know; $w = 2\pi f = \frac{2\pi}{T}$

$\rightarrow T = \frac{2\pi}{w}$ .... (I)

$and\ , v_o = rw$ ..... (II)

$here, v = v_o = orbital\ velocity$.

$\therefore T = \frac{2\pi}{v_o} = \frac{2\pi r}{R\sqrt{\frac{g}{R+h}}}$ use equation (4)

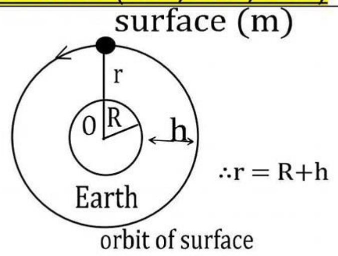


Fig (I) Satellite orbiting around the Earth

$\mathrm{T} = \frac{2\pi}{R}\sqrt{\frac{(R+h)}{g}}(R+h)$

$\therefore \mathrm{T} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$ .... (III)

$Also, \mathrm{T} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$ ...(IV)

This is the required expression for time period of satellite revolving around the earth.

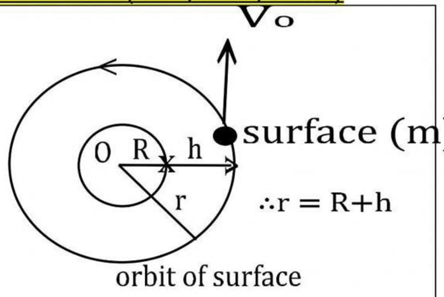
Thus, greater is the height of satellite, the greater will be time period and vice-versa.

Note: If the satellite is very close to the earth's surface, then from eqn (III),

$$T = \frac{2\pi}{R}\sqrt{\frac{R^3}{g}} = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4 \times 10^6}{9.8}} \cong 5000\ sec = 84\ minute$$

Total energy of a satellite orbiting round the earth(2071, 2069, 2067)

A satellite revolving round the earth has both K.E and P.E. Let a satellite of mass 'm' is orbiting in a circular orbit of radius 'r' with orbital velocity v_o. The necessary centripetal force required to keep the satellite in circular orbit is provided by gravitational attraction between earth and satellite.



Where, M = mass of earth
R= radius of earth

i.e. $\frac{mv_o^2}{r} = \frac{GMm}{r^2}$

$v_o = \sqrt{\frac{gm}{r}}$ ……….. (1)

$or, v_o = \sqrt{\frac{GM}{(R+h)}}$ ………….. (2)

$\therefore K.E = \frac{1}{2}mv_o^2 = \frac{1}{2}m\frac{GM}{R+h}$ … (3)

$P.E\ of\ satellite\ at$ height 'h' above the earth's surface is

P.E = $-\frac{GMm}{R+h}$ …… (4)

$$\therefore Total\ Energy(E) = K.E + P.E = \frac{GMm}{2(R+h)} - \frac{Gmm}{R+h} = -\frac{GMm}{2(R+h)}$$

$\therefore E = -\frac{GMm}{2(R+h)}$ ……. (5)

Here, -ve sign shows that satellite is bounded to the earth i.e. energy has to supply from outside to free a satellite from its orbit.

Height of satellite above earth surface(2072):

We know that:

The time period T of the satellite revolving around earth is give by

T = $\frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$ Squaring on both sides $T^2 = \frac{4\pi^2(R+h)^3}{R^2 g}$ $(R+h)^3 = \frac{gR^2T^2}{4\pi^2}$ (R+h) = $\left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3}$ $\therefore h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3} - $ R For, $R = 6.4 \times 10^6 m$ T = 26 hr = 24 × 60 × 60 sec. g = 9.8m/s Then h = 36,000 km	Also, T = $2\pi\sqrt{\frac{(R+h)^3}{gm}}$ Squaring on both sides $T^2 = \frac{4\pi^2(R+h)^3}{GM}$ $(R+h)^3 = \frac{GMT^2}{4\pi^2}$ (R+h) = $\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$ $\therefore h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$

Note: Radius of parking orbit, r = R+h = 6400 + 36000 = 42,400 km

$$\therefore r = 42{,}400 km = radiusofparkingorbit$$

Parking Orbit and Stationary (Geostationary) Satellite(2069, 2072, 2071)

Stationary Geostationary satellite:- The artificial satellite that always stays over the same place on earth while the earth rotation about its axis is called Stationary Geostationary satellite

A satellite of the earth that has time period equal to 24 hours appears stationary above the earth and is called Stationary or Geostationary satellite.

The satellite having time period equal to time period of earth (i.e. 24 hours) is called geostationary satellite or stationary satellite.

The orbit on which stationary satellite moves is called stationary orbit or parking orbit or synchronous orbit. The orbit of the geostationary satellite is called parking orbit.

For a satellite to appear stationary, following conditions should be satisfied:

a) It should revolve in an orbit concentric and coplanar with the equatorial plane i.e. in anticlockwise direction.
b) It's sense of rotation should be same as that of earth about its own axis.
c) It's period of revolution around the earth should be same as that of earth about its own axis i.e. exactly 24 hours.

For this, it should revolve at a height of 36000km above the surface of the earth.

Height of stationary satellite:

Time period of stationary satellite (T) = 24 hrs. = 86400 sec.

Radius of earth (R) = 6400km = 6.4×10^6 m

Acceleration due to gravity (g) = $9.8 m/s^2$

$$h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3} - R$$

$$\therefore h = \left(\frac{9.8 \times (6.4 \times 10^6)^2 (86400)^2}{4\pi^2}\right)^{1/3} - (6.4 \times 10^6)$$

$$\therefore h \cong 36000\ km = 3.6 \times 10^7 m$$

This is the height of parking orbit (h) from the surface of earth is nearly 36000 km

* Orbital velocity of stationary satellite:

$$v_o = R\sqrt{\frac{g}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}} = \sqrt{\frac{9.8 \times (6.4 \times 10^6)^2}{(6.4 \times 10^6 + 3.6 \times 10^7)}}$$

$$\therefore v_o = 7030 m/s \quad \cong 3.1 km/sec$$

Radius of parking (r):

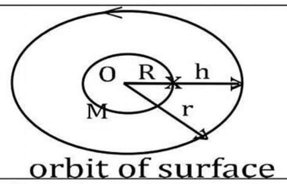
So,

r = R+h

= 6400km + 3600km

= 42400km

$$r = 4.24 \times 10^7 m$$



orbit of surface

These stationary satellites have been used to broadcast the radio and television news from one part to another part and also to forecast the weather.

Kepler's laws of planetary motion:

Statement:

(I) Each planet revolves around the sun in elliptical orbit, with sun remaining at one focus of the ellipse.

(II) The line joining the planet to sun sweeps equal areas in equal times i.e. areal velocity of the planet remains constant.

(III) If T (time period) of revolution of a planet and r is the mean distance of planet from the sun. i.e. $T^2 \propto r^3$

Q. 1 What do you mean by the terms Black Hole and Event Horizon) obtain an expression for the Schwarzschild radius. (2069 4 marks)

Black Hole:

A spherical non-rotating body having large gravity, large density nearly equal to density of the sun and radius 500 times greater than radius of the sun, through which even light cannot escape is called a black hole.

General relativity predicts that if a star of mass more than 5 solar masses has completely burned its nuclear fuel, it should collapse into configuration known as Black Hole.

Event Horizon: The surface of sphere with radius of Schwarzschild surrounding a black hole is called even horizon.

The surface of the black hole is called event horizon. This name is due to the fact that we cannot see events occurring inside the black hole since light cannot escape from a black hole.

If the sun collapsed to from a black hole, the orbits of the planets would be unaffected.

Expression for Schwarzschild radius:

The escape velocity of an object is given by,

$$v_e = \sqrt{\frac{2GM}{R}} \ldots\ldots\ldots (1)$$

If ϱ be average density of an object, V be its volume then

$M = \varrho V = \left(\frac{4}{3}\pi R^3\right)\varrho \ \ldots\ldots\ldots (2)$

$$\therefore V_e = \sqrt{\frac{2G}{R}\left(\frac{4}{3}\pi R^3\right)\varrho} = \sqrt{\frac{8\pi G\varrho R^2}{3}} \ldots\ldots\ldots (3)$$

$$V_e = \sqrt{\frac{8\pi G\varrho}{3}} R \ldots\ldots\ldots (4)$$

Here, $V_e \propto R$ but $\varrho \propto \frac{1}{R^2}$

Again, from equation (1), The radius R can be expressed in terms of escape velocity as,

$R_{star} = \frac{2GM}{v_e^2} \ \ldots\ldots\ldots\ldots (5)$

If the radius decreases, the escape velocity increases. At a certain radius, the escape velocity becomes equal to the velocity of light (C). This radius is called critical radius or Schwarzschild radius (Rs) , so

$R_s = \frac{2G}{C^2}$, ... (6)   Note : if $R = R_S$ then $V_e = C$

This is the required for Schwarzschild's radius.

In 1961, Karl Schwarzschild used Einstein's general theory of relativity to derive it.

Here, if $R_{star} > R_{schwazschild}$ then $V_{escape} > C$ and no things escape from surface of star. The star becomes dark or black and called black hole.

Q. Is there any effect on earth's orbit, if sun is collapsed to black hole?

If the sun is collapsed to Black Hole, there is no effect on the earth orbit because even though the gravitational force increases, the distance between black hole and earth also increases due to which no effect but the things around the sun has drastically and dramatically change.

Weightlessness:

Weightlessness is a situation in which the effective weight of the body becomes zero. Here, effective weigth of body will be zero if g = 0

The state of weightlessness can be observed in different situations:-

a) When the body is taken at the centre of the earth.

We know, at centre of earth, $g = 0\ m/s^2 \therefore W = m \times 0 = 0$.

b) When the body is taken at null points (i.e. those points where the gravitational forces due to different masses cancel out):

At null point, $g = 0\ m/s^2 \therefore W = 0$.

c) When a body is lying in a freely falling lift:

In this case a = g

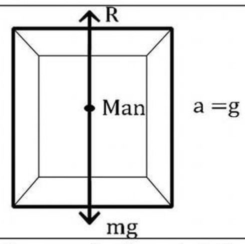
F = mg - R

ma = mg - R

$\therefore R = m(g - a)$

$\therefore R = 0$

$R =$ apparent weight



So, Weighting machine records no weight. So, we feel weightlessness.

d) When the astronaut and spaceship are continuously in the state of freefall towards earth, both of them will fall with same acceleration 'g' and hence astronaut exerts no force on spaceship and there is no reaction of spaceship on the astronaut. So, he feels weightlessness. Here, centripetal force balances gravitational force.

$a = \frac{v^2}{r}$ and $g = \frac{GM}{r^2}$ i.e. $\frac{mv^2}{r} = \frac{GMm}{r^2} \rightarrow a = g$

THE END

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