Date: ....

Page: ....

Chapter: 2: THERMAL EXPANSION

<u>Thermal expansion</u>: The change in the size of the material due to the change / increase in temperature is called thermal expansion.

It takes place due to the molecular vibration of the molecules with the gain in energy.

Examples:

(i): Roads expand as they expand on the heat.

(ii): On bridge and other sensitive sections, expansion panels allow it to flex.

<u>Bimetallic Thermostat</u>: The strip made ~~of~~ up of two strips of different metals which is used to control higher temperature and as an electric contact breaker in an electrical heating circuit is called bimetallic thermostat.

We used the bimetallic thermostat for controlling the temperature of laundry irons, hot water ~~stro~~ storage tanks, etc.

For it's construction, we take two strips of metal that have different coefficients of linear expansion. The two strips are ~~re~~ welded and riveted together. When the bimetallic strip gets heated, the metal with higher expancivity expands more and bends into an arc. This helps to break the electric contact.

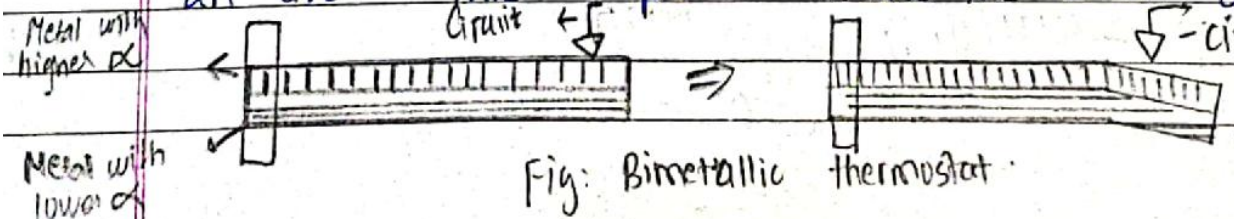


Fig: Bimetallic thermostat

Date :
Page:

Solid expancion

The change or increase in size of a solid material due to the supplied heat is called solid expansion.

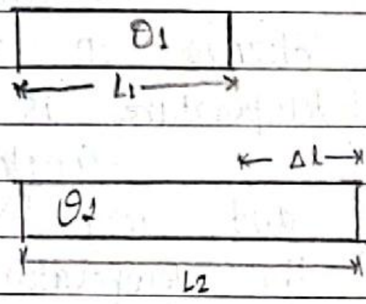
As the temperature increases, the atoms vibrate more vigourously and inter-molecular space increases. Hence, the size of the solid increases.

<i>: Linear Expansion:

The change in length or one dimensional change of a material due to the increase in temperature is called linear expansion.

Consider a metal rod having initial length 'l_1' at the temperature 'θ_1'. On heating, it's temperature increases to 'θ_2' and the final length is 'l_2'. The change in temperature ($\Delta\theta$) is $\theta_2 - \theta_1$ and the change in length (Δl) is $l_2 - l_1$.

Experimentally, it is found that



$$\Delta l \propto \Delta\theta \quad \text{—— (i)}$$

$$\Delta l \propto l_1 \quad \text{—— (ii)}$$

Combining eqⁿ (i) and (ii) we get

$$\Delta l \propto l_1 \cdot \Delta\theta$$

or, $$\Delta l = \alpha\, l_1 \cdot \Delta\theta \quad \text{—— (iii)}$$

where,

α is the coefficient of linear expansion.

Date :

Page:

or, $l_2 - l_1 = \alpha\, l_1\, \Delta\theta$

or, $\frac{l_2 - l_1}{l_1 \cdot \Delta\theta} = \alpha$

$$\boxed{\therefore \alpha = \frac{l_2 - l_1}{l_1\, \Delta\theta}}$$

Hence, coefficient of linear expansion can be defined as the ratio of change in length to the original length per degree rise in temperature.

It's unit is K^{-1} or $°C^{-1}$.

from eq^n (iii),

$l_2 - l_1 = l_1\, \Delta\theta$

or, $l_2 = l_1 + l_1\, \Delta\theta$

$\therefore l_2 = l_1 [1 + \Delta\theta]$

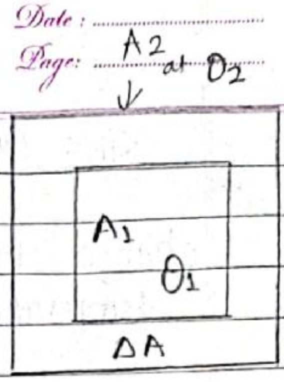
(ii): Superficial expansion:

The change in area or two dimensional change in a material due to the change in temperature is called superficial expansion.

Consider a square metal plate of length 'l_1' and area 'A_1' at temperature 'θ_1'. On heating the temperature changes to 'θ_2', the length increases to 'l_2' and the area increases to 'A_2'. The change in temperature ($\Delta\theta$) is $\theta_2 - \theta_1$ and the change in area (ΔA) is $A_2 - A_1$.

Date:

Page:



Experimentally, it is found that

$\Delta A \propto \Delta\theta$ —— (i)

$\Delta A \propto A_1$ —— (ii)

Combining eq^n (i) and (ii),

$\Delta A \propto A_1 \Delta\theta$

or, $\Delta A = \beta A_1 \Delta\theta$ —— (iii)

where,

β is the superficial expansivity of the metal.

or, $A_2 - A_1 = \beta A_1 \Delta\theta$

or, $\frac{A_2 - A_1}{A_1 \cdot \Delta\theta} = \beta$

$$\boxed{\therefore \beta = \frac{A_2 - A_1}{A_1 \Delta\theta}}$$

Hence, the coefficient of superficial expansion is defined as the ratio of change in area to the original area per degree rise in temperature.

~~Wri~~ from eq^n (iii),

$\Delta A = \beta A_1 \Delta\theta$

or, $A_2 - A_1 = \beta \cdot A_1 \Delta\theta$

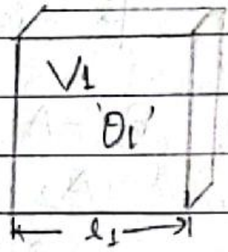
or, $A_2 = A_1 + A_1 \beta \Delta\theta$

$$\boxed{\therefore A_2 = A_1 [1 + \beta\Delta\theta]}$$

(iii): Cubical Expansion:

The change in volume or three-dimensional change in a material due to the change in temperature is called cubical expansion.

Consider a cubical metal box of length 'l_1' with volume 'V_1' at temperature 'θ_1'. On heating, the temperature increases to 'θ_2', the length increases to 'l_2' and volume increases to 'V_2'.

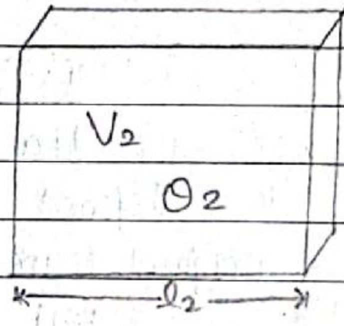


Experimentally, it is found that,

$\Delta V \propto \Delta\theta$ —— (i)

$\Delta V \propto V_1$ —— (ii)

Combining eq^n (i) and (ii),



$\Delta V \propto V_1 \cdot \Delta\theta$

or, $\Delta V = V_1 \cdot \gamma\, \Delta\theta$ —— (iii)

where γ is the cubical expansion of solid.

or $V_2 - V_1 = V_1 \cdot \gamma \cdot \Delta\theta$

or, $\dfrac{V_2 - V_1}{V_1\,\Delta\theta} = \gamma$

$$\boxed{\therefore \gamma = \frac{V_2 - V_1}{V_1 \cdot \Delta\theta}}$$

Hence the coefficient of cubical expansion is defined as the ratio of change in volume to the original volume per degree rise in temperature.

From eq^n (iii),

$$\Delta V = V_1 \cdot \gamma \, \Delta\theta$$

or, $V_2 - V_1 = V_1 \cdot \gamma \cdot \Delta\theta$

or, $V_2 = V_1 + V_1 \gamma \Delta\theta$

$$\boxed{\therefore V_2 = V_1 [1 + \gamma \Delta\theta]}$$

Bimetallic Thermostat:

Thermostat is a bi-metallic device which is used to maintain a desired / constant temperature in a system. It works on the principle of thermal expansion of solid. They are widely used in numerous appliances such as refrigerator, air-conditioner, laundry iron, etc.

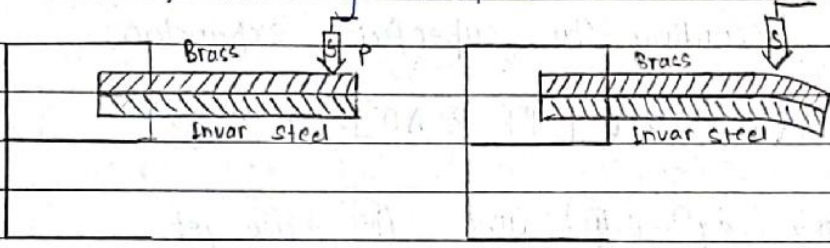
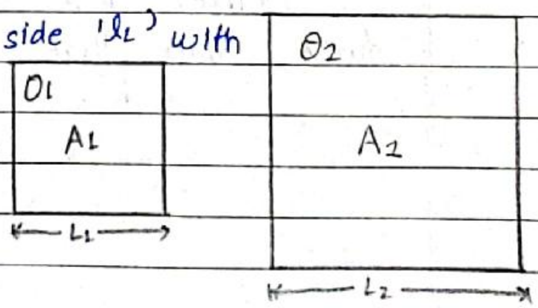


Fig: Bimetallic thermostat.

Relation between α and β:

Consider a square metal plate of side 'l_1' with area 'A_1' at θ_1°C. On heating, the temperature increased to θ_2°C. and the length increased to 'l_2' ~~and~~ with area 'A_2'.



Date: ....
Page: ....

We know,

Initial area of metal $(A_1) = (l_1)^2$ — (i)

According to linear expansion,

$$l_2 = l_1 [1 + \alpha \Delta\theta] \text{ — (ii)}$$

Now,

final area of metal $(A_2) = (l_2)^2$

or, $A_2 = (l_2)^2$

or, $A_2 = \{l_1 (1 + \alpha \Delta\theta)\}^2$ [∵ from eq^n (ii)]

or, $A_2 = (l_1)^2 [1 + 2\alpha\Delta\theta + \alpha^2\Delta\theta^2]$

Since, α is a small valued physical constant, hence, the higher powered values can be neglected.

or $A_2 = (l_1)^2 [1 + 2\alpha\Delta\theta]$

or $A_2 = A_1 [1 + 2\alpha\Delta\theta]$ — (iii)

~~from~~ According to superficial expansion,

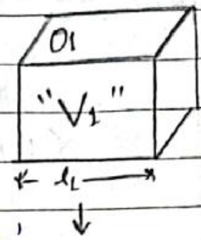
$$A_2 = A_1 [1 + \beta \Delta\theta] \text{ — (iv)}$$

Comparing eq^n (iii) and (iv), we get

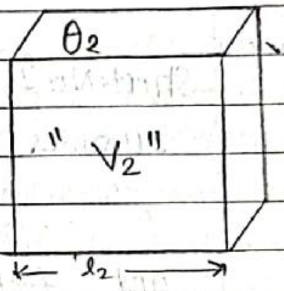
$\boxed{\beta = 2\alpha}$ or, $\boxed{\alpha : \beta = 1:2}$ → (a)

Hence, the superficial ~~tension~~ expansition is twice than that of linear expansion.

Relation of α and γ



Consider a cubical metal box with length 'l_1' ~~to~~ having volume 'V_1' at θ_1 °C. On heating, the temperature increases to θ_2 °C with length being 'l_2' ~~and~~ having volume "V_2".



We know,

Initial volume of metal $(V_1) = (l_1)^3$ —— (i)

According to linear expansion,

$$l_2 = l_1 [1 + \alpha \Delta\theta] \quad \text{—— (ii)}$$

Now,

final ~~a~~ volume of metal $(V_2) = (l_2)^3$

or, $V_2 = (l_2)^3$

or, $V_2 = [l_1(1 + \alpha\Delta\theta)]^3$ [∵ from eq^n (ii)]

or, $V_2 = (l_1)^3 (1 + \alpha\Delta\theta)^3$

or, $V_2 = (l_1)^3 [1 + \cancel{3\alpha\Delta\theta}\ 3 \cdot 1^2 \cdot \alpha\Delta\theta + 3 \cdot 1 \cdot \alpha^2 \Delta\theta^2 + \alpha^3 \Delta\theta^3]$

or $V_2 = V_1 [1 + 3\alpha\Delta\theta + 3\alpha^2 \Delta\theta^2 + \alpha^3 \cdot \Delta\theta^3]$

Since α is a small valued physical constant, its highered powered values can be neglected.

on $V_2 = V_1 [1 + 3\alpha\Delta\theta]$ —— (iii)

According to cubical expansion,

$$V_2 = V_1 [1 + \gamma \Delta\theta] \quad \text{—— (iv)}$$

Comparing eq^n (iii) and (iv), we get.

$\boxed{\gamma = 3\alpha}$ or, $\boxed{\alpha : \gamma = 1 : 3}$ → (b)

Hence, the cubical expansion is thrice than that of linear expansion.

Date :
Page:

Relation of α, β and γ:

From eq^n (a) and (b), we get

$\alpha : \beta = 1 : 2$ $\alpha : \gamma = 1 : 3$

Hence,

$\boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$

Short No: 1: A pendulum clocks is slower in summer and fast in winter, why?

Answer:

A pendulum clock is slower in summer and fast in winter because we know that

$T = 2\pi\sqrt{\frac{l}{g}}$ T = Time period.

where, $T \propto l$ i.e, length of the pendulum when temperature increases, the length of the pendulum increases and since length is directly proportional to the time period, the time period for the pendulum increases, making the clocks slower in summer.

When the temperature, decreases, the length of the pendulum also decreases and since length is directly proportional to time period of the pendulum, the time also decreases making the clock faster during winter season.

Hence, a pendulum clock is slower in summer and fast in winter.

Date :
Page:

Short. Qn. 2: Does the linear expansion coefficient depend upon it's initial length? Explain.

Answer:

The ~~linear expansion~~ of coefficient of linear expansion depends on the nation of materials. We know,

$$\alpha = \frac{\Delta l}{l_1 \Delta \theta}$$

For any material the ratio of change in length to original length $(\Delta l / l)$ remains constant. So, we can conclude that the linear expansion coefficient depend upon its initial length.

Short. Q. 3: Why do solids expand on heating?

Answer:

Solid expand on heating. This is because when we heat a solid, the kinetic energy between the molecules of the solid increases which leads to increase in the inter-molecular space between the molecules.

Hence, the solid expand on heating.

Short. Q. 4: Glass windows are possible to be cracked during winter, why?

Answer.

Glass windows are possible to be cracked during winter. It is because ~~the~~ inside the window, the temperature is warm leading to expansion of the glass. But the temperature outside

Date:
Page:

is cold during winter, due to which the glass contracts. Since there is uneven expansion of ~~water~~ glass, the glass ~~breaks during~~ cracks during winter.

Short. Q.5: A square metal ~~plate~~ sheet with a hole at it's centre. what will happen to the size of the hole, when it is heated?

Answer:

The size of the hole increases because according to analogy of thermal expansion, "Dimension of the object undergoes the same fractional change". Hence, when steel is heated, all dimensions increases expanding the hole.

Num.No.1: The pendulum clock made up of brass having linear expansivity $1.9 \times 10^{-5}\ K^{-1}$ give the correct time at $10°C$. How many seconds per day will it gain or lose at $30°C$?

Soln:

Let 'T_1' and 'T_2' be the time period of the pendulum clock at $10°C$ to $30°C$ respectively.

Since,

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$$

$$\text{or, } T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} \quad \text{--- (i)}$$

Similarly,

$$T_{30} = 2\pi\sqrt{\frac{l_{30}}{g}} \quad \text{--- (ii)}$$

Scanned with CamScanner

Dividing eq^n (ii) by (i),

$$\frac{T_{30}}{T_{10}} = \frac{2\pi\sqrt{l_{30}/g}}{2\pi\sqrt{l_{10}/g}}$$

or $$\frac{T_{30}}{T_{10}} = \sqrt{\frac{l_{30}}{l_{10}}}$$

We know, $T_{10} = 2$ sec [Time period of pendulum = 2s]

$$\frac{T_{30}}{2} = \sqrt{\frac{l_{10}(1+\alpha\Delta\theta)}{l_{10}}}$$

or, $$\frac{T_{30}}{2} = \sqrt{1+\alpha\Delta\theta}$$

or $T_{30} = 2\sqrt{1+\alpha\Delta\theta}$

or $T_{30} = 2\sqrt{1+(\cancel{1\times g\times 10}\ 1.9\times10^{-5}\times20)}$

$\therefore T_{30} = 2.000379964$.

Time lost in two seconds $= (2.000379964 - 2)$
$= 3.79963\times10^{-4}$

$\therefore$ Time lost in 1 second $= 1.899\times10^{-4}$

~~Let the~~ Time period of second pendulum = 2 second.
Hence, $T_{10} = 2$ sec.

Dividing eq^n (ii) by (i),

$$\frac{T_{30}}{T_{10}} = \frac{2\pi\sqrt{\frac{l_{30}}{g}}}{2\pi\sqrt{\frac{l_{10}}{g}}}$$

or, $$\frac{T_{30}}{T_{10}} = \sqrt{\frac{l_{30}}{l_{10}}}$$

Date: ....
Page: ....

or, $\frac{T_{30}}{T_{10}} = \sqrt{\frac{\cancel{l_{10}}(1+\alpha\Delta\theta)}{\cancel{l_{10}}}}$

or, $\frac{T_{30}}{T_{10}} = \sqrt{1+(\cancel{1\times 9}\ 1.9\times10^{-5})\times(20)}$ $[\because \Delta\theta = 30-10 = 20]$

~~or $T_{30} = 2\sqrt{1+3.8\times10^{-4}}$~~

~~or $\therefore T_{30} = 1.000189$~~

or, $T_{30} = 2\sqrt{\cancel{4.8\times10^{-4}}\ 1+3.8\times10^{-4}}$

$\therefore T_{30} = 2.0003799$

Time lost in two second $= (2.0003799-2) = 0.0003799$

$\therefore$ Time lost in one second $= 0.0001899$ sec.

$\therefore$ Time lost in a day $= (0.0001899 \times 86400)$ s
$= 16.407$ sec.

$\therefore$ The time lost in a day is 16.407 sec.

Num. No: 2: A pendulum having linear expansivity $1.9\times10^{-5}\ K^{-1}$ gives the correct time at 20°C. How many seconds per day will it gain or lose, when temperature falls to 15°C.

Soln:

Given,

Linear expansivity of pendulum $(\alpha) = 1.9\times10^{-5}\ K^{-1}$

Let 'T_{20}' and 'T_{15}' be the time period of pendulum at 20°C and 15°C respectively.

Scanned with CamScanner

Date :
Page:

We know
Time period of second pendulum = 2 seconds
Since the pendulum clock is correct at $20^\circ c$,
$\therefore T_{20} = 2$ sec.

Here,
Difference in temperature $(\Delta\theta) = \cancel{20^\circ C - 15^\circ C}\ 15^\circ C - 20^\circ C$
$= -5^\circ C$.

Now,

$$T_{20} = 2\pi\sqrt{\frac{l_{20}}{g}} \quad \text{---} \quad (i) \quad \left[\because T = 2\pi\sqrt{\frac{l}{g}}\right]$$

or $$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}} \quad \text{---} \quad (ii) \quad [\because \text{As above}]$$

Dividing eq^n (i) by (ii), we get.

$$\frac{T_{20}}{T_{15}} = \frac{\cancel{2\pi}\sqrt{\frac{l_{20}}{g}}}{\cancel{2\pi}\sqrt{\frac{l_{15}}{g}}}$$

or, $$\frac{2}{T_{15}} = \sqrt{\frac{l_{20}}{\cancel{g}} \times \frac{\cancel{g}}{l_{15}}} \quad \left[\because T_{20} = 2s\right]$$

or, $$\frac{2}{T_{15}} = \sqrt{\frac{l_{20}}{l_{15}}}$$

or $$\frac{2}{T_{15}} = \sqrt{\frac{\cancel{l_{15}}(1+\alpha\Delta\theta)}{\cancel{l_{15}}}}$$

Scanned with CamScanner

Date :
Page:

Or, $T_{15} = \frac{2}{\sqrt{1+(1.9\times10^{-5}\times5)}}$

$\therefore T_{15} = 1.999905007$ s

Here,

Time ~~lost~~ gained in 2 seconds = $(2 - 1.999905007)$
$= 0.0000949932$.

Time gained in 1 second = $\frac{0.0000949932}{2}$
$= 0.0000474966$ s.

$\therefore$ Time gained in 1 day $= (0.0000474966 \times 86400)$ s
$= 4.103$ s.

$\therefore$ The time gained by the pendulum in a day is 4.103 seconds.

Num. No. 3: At 25°C, the length of the glass is 50 cm. After heated, the final length of the glass is 50.09 cm ~~Describe~~. Determine the final temperature of the glass. [$\alpha_g = 9\times10^{-6}\ °C^{-1}$]

Solⁿ:

Given,

Initial temperature $(\theta_1) = 25°C$ Final temperature $(\theta_2) = ?$

Length at 25°C $(l_{25}) = 50$ cm

Length at θ_2 °C $(l_{\theta_2}) = 50.09$ cm

Linear expanisivity of glass $(\alpha_g) = 9\times10^{-6}\ °C^{-1}$

Scanned with CamScanner

Date :
Page:

We know,

According to linear expansion,

$$l_2 = l_1 [1 + \alpha \Delta\theta]$$

or, $l_2 = l_1 [1 + \alpha(\theta_2 - \theta_1)]$

or, $l_{\theta_2} = l_{25} [1 + (9\times10^{-6})(\theta_2 - 25)]$

or $50.09 = 50 [1 + (9\times10^{-6})(\theta_2 - 25)]$

or $1.0018 = 1 + (9\times10^{-6})(\theta_2 - 25)$

or $\dfrac{0.0018}{9\times10^{-6}} = \theta_2 - 25$

or $200 = \theta_2 - 25$

$\therefore \theta_2 = 225°C.$

$\therefore$ The final temperature is 225°C.

Force/Tension Produced in Solid Betn Two Rigid Support

We know,

Young modulus (Y) = Stress / Strain —— (i)

Now,

Stress $= \dfrac{F}{A}$

$\therefore$ Young Modulus $= \dfrac{F}{A\alpha\Delta\theta}$

Strain $\epsilon = \Delta l / l_1$

$= \dfrac{l_2 - l_1}{l_1}$

$\boxed{\therefore \text{Tencion} = F = Y A \alpha \Delta\theta}$

Here,

A = cross-sectional area.

Also, $\Delta\theta = \dfrac{\cancel{l_1} + l_1\alpha\Delta\theta - \cancel{l_1}}{l_1}$

$= \alpha\Delta\theta$

Date :

Page:

Short Q.No.6: Two rods are made up of same metal but one has a cylindrical cross-sectional area whereas another has a rectangular cross-sectional area. Which has the higher ~~ex~~ linear expansion. Explain.

Answer:

~~The~~ Both the rods will have the same linear expansion because the linear expansion of a solid depends upon the nature of the material and doesn't depend upon the cross-sectional area.

Num.No.4: A brass wire of 8m long and 4mm diameter is fixed in two rigid supports. Calculate the increase in tension) force, when temperature falls by 10°C. ($\alpha_b = 12\times10^{-6}\ K^{-1}$ & $Y_B = 2\times10^{11}\ N/m^2$)

Soln:

Given fact,

Initial length of brass (l_1) = 8 m

Diameter of brass (d) = 4 mm = 4×10^{-3} m

Linear expansivity of brass (α_B) = $12\times10^{-6}\ K^{-1}$

Young's modulus of brass (Y_B) = $2\times10^{11}\ N/m^2$

Change in temperature ($\Delta\theta$) = 10°C.

Increase in tension (T) = ?

We know,

$$T = Y A \alpha \Delta\theta$$

$$= 2\times10^{11} \times \left(\frac{22}{7}\times(4\times10^{-3})^2\times\frac{1}{4}\right)\times 12\times10^{-6}\times 10$$

Scanned with CamScanner

Date : ....

Page: ....

$\therefore T = 301.44$ N

The increase in tension is 301.44 N.

Num. No. 5: A steel rod when measured with a zinc scale both being being at 25°C appear to be a metre long. If the zinc scale is correct at 0°C, what is the actual length of rod at 25°C. What will be the length of the rod at 0°C?

$[\alpha_s = 12 \times 10^{-6} K^{-1}, \quad \alpha_z = 26 \times 10^{-6} K^{-1}]$

Soln

~~Given~~, for zinc scale,

length of ~~zi~~ scale at 0°C $(l_0) = 1$ m

Initial temperature $(\theta_1) = 0°C$

final temperature $(\theta_2) = 25°C$

$\therefore \Delta\theta = 25°C$

According to linear expansion,

$l_2 = l_1 [1 + \alpha \Delta\theta]$

$\therefore l_{25} = l_0 [1 + \alpha \Delta\theta]$

$= 1 [1 + (26 \times 10^{-6} \times 25)]$

$\therefore l_{25} = 1.00065$ m.

Since both zinc scale and steel rod appear to be 1 m long at 25°C,

steel length at 25°C $(l_{25}) = 1.00065$ m

Now,

initial temperature $(\theta_1) = 0°C$ length at 0°C $(l_0) = ?$

final temperature $(\theta_2) = 25°C$

$\therefore \Delta\theta = 25°C$

length of rod at 25°C $(l_{25}) = 1.00065$ m

Scanned with CamScanner

We know,

$$l_2 = l_1 [1 + \alpha \Delta\theta]$$

or $l_{25} = l_{0} [1 + \alpha \Delta\theta]$

or $\frac{\cancel{l_{25}}\ 1.00065}{[1+(12\times10^{-6}\times25)]} = l_0$

$\therefore l_0 = 1.0035$ m

$\therefore$ The length of rod at $0°C$ is 1.0035 m.

Pullinger's Apparatus Method (α of a solid rod):

The coefficient of linear expansion of given solid can be determined by Pullinger's apparatus. It consist of a metallic rod whose ~~specific~~ linear expansion coefficient ~~heat capacity~~ is to be determined. The rod with measuring its ~~length~~ initial length is placed inside a hollow jacket which has inlet (I) and outlet (O) to enter and escape the steam. A thermometer is fixed with the help of a rubber cork to measure temperature.

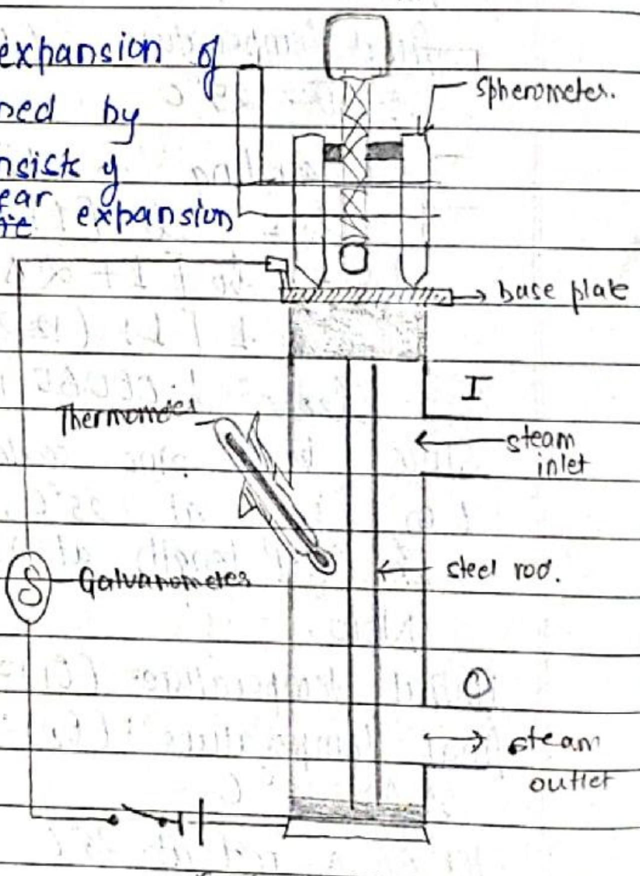


Fig. Pullinger's Apparatus

Date : ....

Page: ....

A spherometer with which we can measure the length of the rod. An electric circuit containing battery and galvanometer is connected to the two terminals of the arrangement.

The initial temperature of the rod is noted then inserted into the arrangement. spherometer is turned forward till it touches the rod and we observe a deflection in the galvanometer. The initial spherometer reading is taken and turned backward to provide room for expansion. We pass steam until we reach a steady temperature by is recorded by the thermometer then noted. The spherometer is again turned forward till it touches expanded end B of the rod then final spherometer reading is taken again.

Let 'L' be the original length of the rod, 'θ_1' be the initial temperature of the rod, 'R_1' be the initial reading in the spherometer, 'θ_2' be the final temperature, 'R_2' be the final reading in the spherometer, '~~θ~~ α' be the coefficient of linear expansion and 'Δl' be the increase in length of rod.

$$\therefore \Delta l = (R_2 - R_1)$$

From definition, we know,

coefficient of linear expansion is defined as the ratio of change in length to the original length per degree rise in temperature.

Scanned with CamScanner

Date:
Page:

$$\alpha = \frac{\text{Increase in length}}{\text{initial length} \times \text{change of temperature}}$$

$$= \frac{\Delta l}{l_1 \times \Delta\theta}$$

Thus,

$$\alpha = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$$

Hence, knowing the values of R.H.S. terms, the linear expancivity of the solid can ~~be th~~ be determined.

Assignments

Num. No. 6: A pendulum having linear expansivity $1.9 \times 10^{-5}\ K^{-1}$ gives the correct time at $30°C$. How seconds per day will it gain or loss if when the temperature falls below $10°C$?

Soln:

Given,

Linear expancivity $(\alpha) = 1.9 \times 10^{-5}\ K^{-1}$.

Initial temperature $(\theta_1) = 30°C$.

Final temperature $(\theta_2) = 10°C$.

∴ Difference in temperature $(\Delta\theta) = \theta_2 - \theta_1$

$= 10°C - 30°C$

$\therefore \Delta\theta = -20°C$.

(Since the temperature decreases, the length of the pendulum decreases, reducing time period which gains time.)

Scanned with CamScanner

Date :
Page:

Let 'T_{30}' and 'T_{10}' be the representation of time period of pendulum at $30°C$ and $10°c$ respectively.

We know,

Time period of second pendulum = ~~30~~ 2 sec.

Since the pendulum gives correct time at $30°C$,

$$T_{30} = 2 \text{ sec}$$

Now,

We know,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So,

$$T_{30} = 2\pi\sqrt{\frac{l_{30}}{g}} \quad \text{--- (i)}$$

$$T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} \quad \text{--- (ii)}$$

Dividing eq^n (i) by (ii), we get.

$$\frac{T_{30}}{T_{10}} = \frac{\cancel{2\pi}\sqrt{\frac{l_{30}}{g}}}{\cancel{2\pi}\sqrt{\frac{l_{10}}{g}}}$$

or, $$\frac{2}{T_{10}} = \sqrt{\frac{l_{30}}{\cancel{g}} \times \frac{\cancel{g}}{l_{10}}}$$

or $$\frac{2}{T_{10}} = \sqrt{\frac{l_{30}}{l_{10}}}$$

or $$\frac{2}{T_{10}} = \sqrt{\frac{\cancel{l_{10}}(1+\alpha\Delta\theta)}{\cancel{l_{10}}}} \quad [\because l_2 = l_1[1+\alpha\Delta\theta]]$$

or, $\dfrac{2}{T_{10}} = \sqrt{1+\{(1.9\times10^{-5})\times 20\}}$ $[\because 30^\circ C - 10^\circ C = 20^\circ C]$

or $\dfrac{2}{T_{10}} = \sqrt{1+3.8\times10^{-4}}$

or $\dfrac{2}{T_{10}} = \sqrt{1.00038}$

or $\cancel{\dfrac{2}{T_{10}} \neq 1.00038}$ $\quad T_{10} = \dfrac{2}{\sqrt{1.00038}}$

$\therefore$ ~~$T_{10} = 1.99924$ sec.~~ $\quad \therefore T_{10} = 1.99962$ s

~~Here,~~

~~Time gained in two seconds $= (2-1.99924)\,s = 7.6\times10^{-4}\,s$~~

~~i.e., $= 0.00076$ s~~

~~Time gained in one second $= \left(\dfrac{0.00076}{2}\right) s$~~

~~$\therefore$ Time gained in one day $= \left(\dfrac{0.00076}{2}\right)\times 86400$~~

Here,

Time gained in two seconds $= (2-1.99962)\,s$

$= 0.000379$ s.

Time gained in one second $= \left(\dfrac{0.000379}{2}\right) s$

$\therefore$ Time gained in one day $= \left\{\left(\dfrac{0.000379}{2}\right)\times 86400\right\} s$

$= 16.42$ sec.

Date :
Page:

Num. No 7: A steel wire of 2mm radius is stretched between two rigid supports at 20°C. What would be the tension produced in the wire when temperature drops to 10°C?

Soln:

Given,

Diameter of cross-section $(d) = 2\,mm = (2\times10^{-3})\,m$

initial temperature $(\theta_2) = 20°C$

final temperature $(\theta_2) = 10°C$

$\therefore$ Difference in temperature $(\Delta\theta) = 10°C - 20°C = -10°C$.

Young's modulus for steel $= 2\times10^{11}\,N/m^2$ ~~= 10°C~~ (Y_s)

Linear expansivity of steel $(\alpha_s) = 1.2\times10^{-5}\,K^{-1}$

Force (Tension) (F) = ?

We know,

$$F = Y \cdot A \cdot \alpha \cdot \Delta\theta$$

$$= 2\times10^{11} \times \left(3.14 \times \frac{(2\times10^{-3})^2}{4}\right) \times 1.2\times10^{-5} \times 10$$

$\therefore F = 75.36\ N$

$\therefore$ The tension produced in the wire is 75.36 N.

Scanned with CamScanner

Date: ....
Page: ....

Num. No. 8: The marking on an aluminium rules and brass rules are ~~prefe~~ perfectly aligned at 0°C. How far apart will the 20 cm marks be on the two rulers at 100°C, if precise alignment of the left hand ends of the rulers is maintained? [$\alpha_B = 12 \times 10^{6}\ K^{-1}$, $\alpha_Z = 26 \times 10^{6}\ K^{-1}$]

Soln:

For ~~steel~~ aluminium ~~rod~~, rules,

Coefficient of linear expansion (α_{al}) = $12 \times 10^{-6}\ K^{-1}$.
initial temperature (θ_1) = 0°C.
final temperature (θ_2) = 100°C.
initial length (l_1^{al}) = 20 cm
final length (l_2^{al}) = ?

According to linear expansion,

$$l_2 = l_1 [1 + \alpha \Delta\theta]$$

So,

$$l_2^{al} = l_1^{al} [1 + \alpha_{al} \Delta\theta]$$

$$= 20 [1 + (12 \times 10^{-6}) \times 100] \quad [\because \Delta\theta = 100°C]$$

$$\therefore l_2^{al} = 20.024 \text{ cm.}$$

For brass rules,

Coefficient of linear expansion (α_B) = $26 \times 10^{-6}\ K^{-1}$
initial temperature (θ_1) = 0°C.
final temperature (θ_2) = 100°C
initial length (l_1^{br}) = 20 cm
final length (l_2^{br}) = ?

Scanned with CamScanner

Date :
Page:

According to linear expansion,

$l_2 = l_1 [1 + \alpha \Delta\theta]$

so,

$l_2^{br} = l_1^{al} [1 + \alpha_B \Delta\theta]$

or, $= 20 [1 + (26 \times 10^{-6} \times 100)]$

$\therefore l_2^{br} = 20.052$ cm

$\therefore$ Difference in reading $= l_2^{br} - l_2^{al}$

$= 20.052 - 20.024$

$= 0.028$ cm.

~~θ θ~~ 20 cm is 0.028 cm far apart when the temperature is 100°C.

Num. No. 9: An aluminium rod when measured with a steel scale both being at 25°C appear to be 1 m long. If the scale is correct at 0°C, what will be the length of the rod at 0°C?

$[\alpha_s = 12 \times 10^{-6}, \alpha_{az} = 26 \times 10^{-6}]$

Soln:

For steel scale,

initial length $(l_0) =$ ~~12×10^{-6} K^{-1}~~ 1 m

initial temperature $(\theta_1) = 0°C$

final temperature $(\theta_2) = 25°C$

coefficient of linear expansion $(\alpha_s) = 12 \times 10^{-6}$ K^{-1}.

length at 25°C $(l_{25}) = ?$

According to linear expansion,

$$l_2 = l_1 [1 + \alpha \Delta\theta]$$

$$\therefore l_{25} = l_0 [1 + \alpha_s \Delta\theta]$$

$$= \cancel{3}\ 1\ [1 + (12 \times 10^{-6}) \times 25]$$

$$\therefore l_{25} = 1.0003 \text{ m.}$$

We know,

length of steel at 25°C = length of aluminium at 25°C
= 1·0003 ~~cm~~ m

Now,
for aluminium rod,

$l_{25} = 1.0003 \text{m}$

$l_0 = ?$

$\Delta\theta = 25°C$

coefficient of linear expansion $(\alpha_{al}) = 26 \times 10^{-6}\ K^{-1}$

~~Now~~ We know,

$$l_{25} = l_0 [1 + \alpha \Delta\theta]$$

or, $$\frac{l_{25}}{[1 + \alpha\Delta\theta]} = l_0$$

$$= \frac{1.0003}{[1 + 26 \times 10^{-6} \times 25]}$$

$$\therefore l_0 = 0.999 \text{ m.}$$

$\therefore$ The length of aluminium rod is 0.999 cm.

Date: ....
Page: ....

Differential Expansion: The difference in expansion of two different materials when they are heated through the same range of temperature is called differential expansion.

Let 'l_A' and 'l_B' be the initial length of two different material having linear expansion coefficients 'α_A' and 'α_B' respectively. Now, initial difference,

$$d_1 = l_A - l_B \cdots \text{(i)} \quad [\because l_A > l_B]$$

When they are heated by the same temperature $(\Delta\theta)$ then their length becomes 'l'_A' and 'l'_B'.
Then,

$$d_2 = l'_A - l'_B \cdots \text{(ii)}$$

From eq^n (ii),

$$\boxed{d_2 = [l_A(1+\alpha_A \Delta\theta)] - [l_B(1+\alpha_B\Delta\theta)]}$$

$$\cancel{\text{or, } d_2 = [l_A - l_B][(1+\alpha_A\Delta\theta) -}$$

$$\cancel{\text{or, } d_2 = [(l_A - l_B) + \Delta\theta(l_A\theta_A - l_B\theta}$$

$$\text{or, } d_2 = [(l_A - l_B) + \Delta\theta(l_A\alpha_A - l_B\alpha_B)]$$

$$\text{or, } d_2 - d_1 = (l_A\alpha_A - l_B\alpha_B)\cdot\Delta\theta \cdots \text{(iii)}$$

where, $(d_2 - d_1)$ is called differential expansion.

Now,

$d_2 = d_1$ if $(l_A\alpha_A - l_B\alpha_B) = 0$ i.e, $\dfrac{l_A}{l_B} = \dfrac{\alpha_B}{\alpha_A}$

Date: ....
Page: ....

The difference between two different material is same / constant at all temperature when length is invercely proportional to their expansion coefficient.

Short. Q.7: Why invar is used in a pendulum clock?

Answer:

Invar is used in a pendulum clock. It is because invar has very less linear expansivity which prevents the ~~time from~~ clock from getting slow in summer and going quicker during winter.

Hence, invar is used in a pendulum clock.

Num. No. 10: The difference in a brass rod & a steel rod is 12cm at all temperatures. Find their length at $0°C$? [$\alpha_b = 18\times10^{-6}\ °C^{-1}$, $\alpha_s = 12\times10^{-6}\ °C^{-1}$]

Sol^n:

Given,

Linear expansivity of brass $(\alpha_b) = 18\times10^{-6}\ °C^{-1}$.

Linear expansivity of steel $(\alpha_s) = 12\times10^{-6}\ °C^{-1}$

For brass,

Initial length ~~($L_{1/b}$)~~ $= L_{1/b}$

Final length $= L_{2/b} = L_{1/b}[1+\alpha_b\Delta\theta]$ —— (i)

For steel,

Initial length $= L_{1/s}$

Final length $= L_{2/s} = L_{1/s}[1+\alpha_s\Delta\theta]$ —— (ii)

We know,

the difference in a brass rod and a steel rod is 12 cm i.e.

$$L_{11s} - L_{11b} = 12 \text{ cm} \quad \text{—— (iii)}$$

$$L_{21s} - L_{21b} = 12 \text{ cm}$$

or, $[L_{11s}(1+\alpha_s \Delta\theta)] - [L_{11b}(1+\alpha_b \Delta\theta)] = 12$

Let the length of brass and steel at 0°C be '$L_{B(0)}$' and '$L_{S(0)}$' respectively. When heated to temperature θ, the length of brass and steel becomes '$L_{B(\theta)}$' and '$L_{S(\theta)}$' respectively.

According to question,

(i): $L_{S(0)} - L_{B(0)} = 12$

or, $L_{S(0)} = 12 + L_{B(0)}$ —— (i)

(ii): $L_{S(\theta)} - L_{B(\theta)} = 12$ —— (ii)

~~From~~ Using eq^n (ii),

$L_{S(\theta)} - L_{B(\theta)} = 12$

or, $[L_{S(0)}(1+\alpha_s \cdot \Delta\theta)] - [L_{B(0)}(1+\alpha_B \cdot \Delta\theta)] = 12$

$[\because l_2 = l_1(1+\alpha\Delta\theta)]$

or, $L_{S(0)} + L_{S(0)}\alpha_s \cdot \Delta\theta - L_{B(0)} - L_{B(0)}\alpha_B \cdot \Delta\theta = 12$

or, $12 + L_{B(0)} + (12 + L_{B(0)})\alpha_s \Delta\theta - L_{B(0)} - L_{B(0)}\alpha_B \cdot \Delta\theta = 12$

or, $\cancel{12} + \cancel{L_{B(0)}} + 12\alpha_s\Delta\theta + L_{B(0)}\alpha_s\Delta\theta - \cancel{L_{B(0)}} - L_{B(0)}\alpha_B\Delta\theta = \cancel{12}$

Date: ....
Page: ....

$$L_{B(0)}\,\alpha_B\,\Delta\theta - L_{B(0)}\,\alpha_S\,\Delta\theta$$

or, $12\,\alpha_S\,\Delta\theta = L_{B(0)}\,\alpha_B\,\Delta\theta - L_{B(0)}\,\alpha_S\,\Delta\theta$

or, $12\,\alpha_S\,\Delta\theta = L_{B(0)}\cdot\Delta\theta\,(\alpha_B - \alpha_S)$

or, $12\,\alpha_S = L_{B(0)}\cdot(\alpha_B - \alpha_S)$

or, $\dfrac{12 \times 12\times10^{-6}}{(18\times10^{6} - 12\times10^{6})} = L_{B(0)}$

$\therefore L_{B(0)} = 24$ cm

Putting the value of $L_{B(0)}$ in eq^n (i),

$L_{S(0)} = (12 + 24)$ cm

$\therefore L_{S(0)} = 36$ cm.

$\therefore$ The length of brass and steel at $0°C$ is 24 cm and 36 cm respectively.

Num. No. 11: A brass and steel rod are 4 m and 4.01 m respectively at 20°C. At what temperature, will the rods have the same length?
$[\alpha_B = 18.9\times10^{-6}\ °C^{-1}, \quad \alpha_S = 11.9\times10^{6}\ °C^{-1}]$

Sol^n:

Let the length of brass ~~of~~ and steel rod at 20°C be ~~4m~~ 'L_{1b}' and 'L_{1s}' respectively.

From question,

$L_{1b} = 4$ m $\qquad \alpha_B = 18.9\times10^{-6}\ °C^{-1}$

$L_{1s} = 4.01$ m $\qquad \alpha_S = 11.9\times10^{-6}\ °C^{-1}$

Say, the rods have equal length when heated to temperature θ °C.

Date :
Page:

For brass,
Let 'L_{2lb}' be the length of brass at θ°C.
We know,
According to linear expansion,

$$l_2 = l_1 [1 + \alpha (\theta_2 - \theta_1)]$$

$$\therefore L_{2lb} = L_{1lB} [1 + \alpha_B (\theta - 20)] \quad \text{--- (i)}$$

for steel.,
Let L_{2ls} be the length of steel at θ°c.
We know,
According to linear expansion,

$$l_2 = l_1 [1 + \alpha (\theta_2 - \theta_1)]$$

$$\therefore L_{2ls} = l_{1ls} [1 + \alpha_s (\theta - 20)] \quad \text{--- (ii)}$$

Equating eq^n (i) and (ii),

$$L_{1lB} [1 + \alpha_B (\theta - 20)] = L_{1ls} [1 + \alpha_s (\theta - 20)]$$

or, $4 [1 + 18.9 \times 10^{-6} (\theta - 20)] = 4.01 [1 + 11.9 \times 10^{-6} (\theta - 20)]$

or, $4 + 7.56 \times 10^{-5} (\theta - 20) = 4.01 + 4.7719 \times 10^{-5} (\theta - 20)$

or, $7.56 \times 10^{-5} (\theta - 20) - 4.7719 \times 10^{-5} (\theta - 20) = 4.01 - 4$

or, $2.7881 \times 10^{-5} (\theta - 20) = 0.01$

or, $\theta - 20 = \dfrac{0.01}{2.7881 \times 10^{-5}}$ or, $\theta - 20 = 358.67$

$$\boxed{\therefore \theta = 378.67\text{°C}}$$

The rods will have same length at 378.67 °c.

Scanned with CamScanner

Date : ....
Page: ....

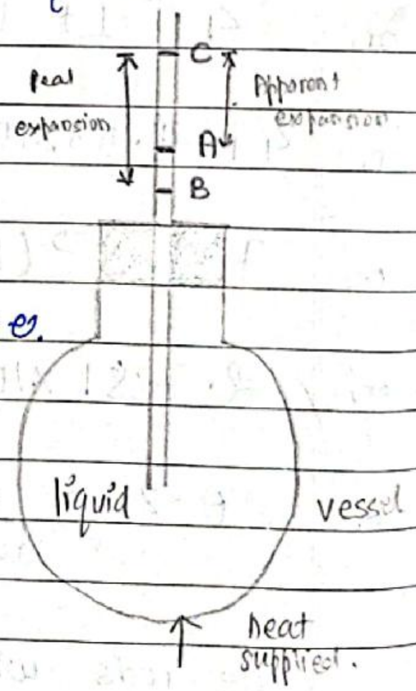
Liquid Expansion: The increase in the volume of a liquid on heating is called liquid expansion.

Coefficient of Liquid Expansion:

Liquid don't have a fixed shape and size but it has fixed volume, so it has only the cubical expansion.

When liquid is heated in a container, heat flows through the container to the liquid. The vessel expands first so the liquid level decreased / falls initially which is considered as expansion of vessel. When the liquid gets heated, it expands more and beyond its initial level. The observed expansion of the liquid is known as the apparent expansion of the liquid. The expansion of the vessel and the liquid expansion i.e, total expansion of the liquid is called real or absolute expansion of liquid.

In the diagram,



AB = expansion of vessel
AC = apparent expansion of vessel
BC = Real / absolute expansion of vessel.

Thus,

BC = AB + AC — (i)

<•>: Coefficient of Real Expansion: The ratio of real increase in volume of the liquid to the original volume per degree rise in temperature is called coefficient of real expansion.

Mathematically,

$$\gamma_r = \frac{(\Delta V)_r}{V \cdot \Delta\theta}$$ where,

γ_r = coefficient of real expansion.
$(\Delta V)_r$ = real increase in liquid volume
V = original volume
$\Delta\theta$ = temperature difference.

<•>: Coefficient of Apparent Expansion: The ratio of apparent increase in volume of the liquid to the original volume per degree rise in temperature is called coefficient of apparent expansion.

Mathematically,

$$\gamma_a = \frac{(\Delta V)_a}{V \cdot \Delta\theta}$$ where,

γ_a = coefficient of apparent expansion
$(\Delta V)_a$ = apparent increase in volume
V = original volume
$\Delta\theta$ = temperature difference.

<•>: Expansion coefficient of Vessel: The ratio of increase in volume of vessel to the original volume per degree rise in temperature is called expansion coefficient of vessel. Mathematically,

Date: ....
Page: ....

$$\gamma_v = \frac{(\Delta V)_v}{V \cdot \Delta\theta}$$

where,

γ_v = coefficient expansion of vessel.

$(\Delta V)_v$ = increase in volume of vessel.

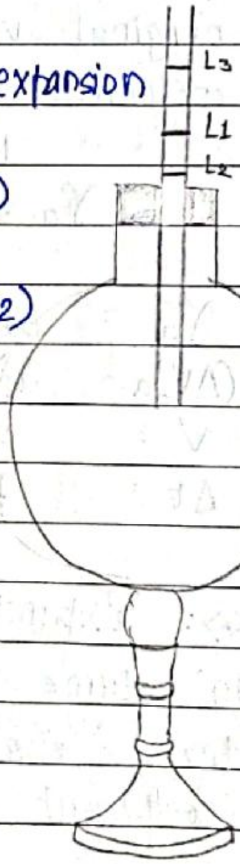
V = original volume

$\Delta\theta$ = temperature difference.

Relation between Real and Apparent Expansion Coefficient of liquid

Consider a glass vessel of volume 'V' filled with some liquid at temperature 'θ_1' °C. when heated to 'θ_2' °C, then both glass and the liquid expands

We know,



Real expansion = Expansion of vessel + Apparent expansion

or, $(\Delta V)_r = (\Delta V)_a + (\Delta V)_g$ —— (1)

According to liquid expansion,

Real increase in volume $(\Delta V)_r = \gamma_r V \Delta\theta$ —(2)

Similarly,

Apparent increase in volume $(\Delta V)_a = \gamma_a V \Delta\theta$ (3)

Expansion of glass $(\Delta V)_g = \gamma_g V \Delta\theta$ — (4)

Substituting (2), (3), (4) in eq^n (1)

$$\gamma_r V\Delta\theta = \gamma_a V\Delta\theta + \gamma_g V\Delta\theta$$

or, $\gamma_r = \gamma_a + \gamma_g$ —— (5)

Scanned with CamScanner

Date :
Page:

Also,

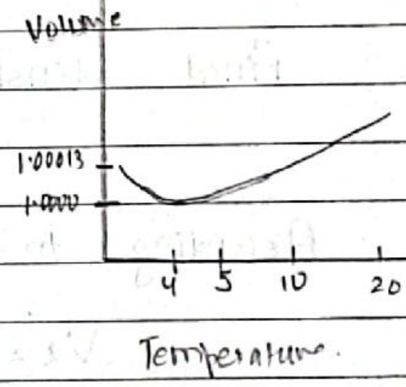
$$\gamma_r = \gamma_a + 3\alpha_g \quad \text{—— (6)}$$

Hence, real expansion coefficient is equal to the sum of the expansion coefficient of vessel and the apparent expansion coefficient of liquid.

Anomalous Expansion of Water and its Significance.

Most of the liquid expands on heating, although water shows the different behaviour. When temperature is rises from 0°C to 4°C instead of expansion, water contracts. This special property is called anomalous expansion of water. Therefore, at 4°C the volume of water is lowest and corresponding density is maximum.

Eg: Survival of aquatic animals.



Scanned with CamScanner

Date :
Page:

Effect of temperature on density of the substance,

When temperature of a substance is increased, its volume increases. The density is inversely proportional to its volume. So, the density of an substance decreases with increase in temperature.

Let the initial mass be 'M_1', volume be 'V_1' and density be 'ρ_1' and final mass be 'M_2' volume be 'V_2' and density be 'ρ_2' when temperature increase.

We know,

$$\rho = \frac{M}{V}$$

So,

Initial density $(\rho_1) = \frac{M_1}{V_1}$ — (i)

final density $(\rho_2) = \frac{M_1}{V_2}$ — (ii) $\left[M_1 = M_2 \text{, mass is constant} \right]$

According to cubical ~~solid~~ thermal expansion,

$$V_2 = V_1 [1 + \gamma \Delta\theta]$$

So,

$$\rho_2 = \frac{M_1}{V_1 [1 + \gamma \Delta\theta]}$$

or $\rho_2 = \frac{M_1}{V_1} \cdot \frac{1}{[1 + \gamma \Delta\theta]}$ $\therefore \rho_2 = \frac{\rho_1}{[1 + \gamma \Delta\theta]}$

[From eq^n (i)]

Hence, density of substance decreases with increase in temperature.

Date: ....

Page: ....

Short Question:

<1>: Liquid has only the cubical expansion. Explain:

Answer: Liquid has only the cubical expansion. It is because liquid doesn't have fixed shape and size but has fixed volume. So, liquid has only the cubical expansion.

<2> : The level of liquid initially falls in a vessel when it is heated, why?

Answer: The level of liquid falls in a vessel and then rises when it is heated because the vessel acquires heat earlier than the liquid due to which the vessel expands making the water level up which again rises after the liquid acquires heat. Therefore, the level of liquid initially falls in a vessel and then rises when heated.

<3>: Frozen water pipes often burst. Why?

Answer: Frozen water pipes often burst because the water inside the pipe expands when temperature decreases from 4°C to 0°C but the pipe contracts when temperature moves from 4°C to 0°C. Due to this, it creates uneven expansion making the pipe burst. Therefore, the ~~to~~ frozen water pipes often burst.

Date: ....
Page: ....

<4>: So frozen water pipes often burst. Will an alcohol thermometer break if the temperature drops below the freezing point of alcohol?

Answer: Though frozen water pipes burst in winter, an alcohol thermometer will not break if the temperature drops below the freezing point of alcohol because when cooled, ~~the~~ alcohol also contracts along with the thermometer and there is no uneven expansion as alcohol doesn't have the property for anomalous expansion.

<5>. A beaker is completed filled with water at 4°C. Whether the temperature is increased or decreased, there is overflow of water. Why?

Answer: There is overflow of water when a beaker completed filled with it at 4°C either with increases or decreases of the temperature because due to ~~anam~~ anomalous expansion of water, it expands when cooled from 4°C to 0°C and also expands when the temperature increases from 4°C. Therefore, a beaker with completely filled water at 4°C has water overflow when temperature is increased or decreased.

<6>: Two bodies of the same material with same ~~extra~~ external dimensions but one is solid and other is hollow. When they are heated, through same ~~the~~ temperature difference is the overall volume expansion, same or different?

Scanned with CamScanner

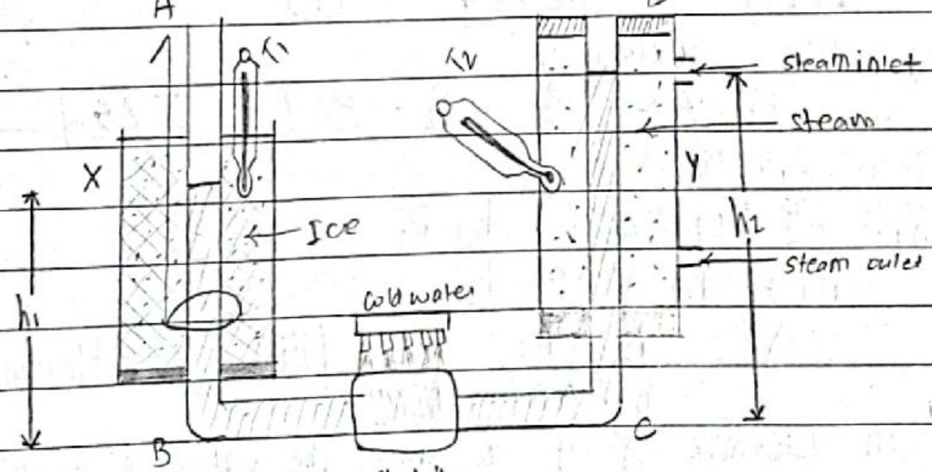
Answer: Two bodies of the same material and which same external dimensions but one being hollow and others being solid will have the same overall volume expansion through the same dimension difference because the cubical expansion of the two ~~identical~~ materials depends upon their volume and not their masses.

Q7: Fishes stay alive in frozen pond in winters. Why?

Answer: Fishes stay alive in frozen pond in winter.

We know that the density of water is at highest at 4°C. When cooled from 4°C to 0°C, the density decreases as the volume increases. So, when the temperature reaches 4°C, the water settles down creating a water convection until ice is formed on the surface. Hence, fishes stay alive in frozen pond in winter.

Dulong's and Petit's Method (Determination of real/absolute expansion coefficient of liquid.)



Real expansivity of a liquid is determined by Dulong and Petit's method as shown in the diagram. It is based on the hydrostatics principle which states "Height of two liquid column which produces equal precsure are inversely proportional to their dencities."

It consists of a U-shaped tube (ABCD) surrounded by a hollow jacket. The liquid whose real expansion coefficient is to be determined is taken in the tube. At one limb (AB), cold water is circulated and at another limb (CD), steam is circulated and is filled ~~which~~ with thermometer to record ~~the~~ temperatures. Horizontal section (BC) is wrapped with wet cloth to prevent the direct heat flow from one end to another.

Let 'θ_1' and 'θ_2' be the temperature at two limbs/seconds having liquid level of height 'h_1' and 'h_2' respectively. Also, 'ρ_1' and 'ρ_2' be the density of liquid at the two ends.

At the equilibrium of liquid pressure,

$$h_1 \rho_1 g = h_2 \rho_2 g \quad \text{— (i)}$$

~~or,~~ we know,

$$\cancel{\rho_2 \cdot \rho_2 = \rho_1} \quad \rho_1 = \rho_2 [1 + \gamma \Delta\theta] \quad \text{— (ii)}$$

From eq^n (ii),

$$h_1 \cdot \cancel{\rho_2} [1 + \gamma \Delta\theta] = h_2 \cancel{\rho_2}$$

or, $$h_1 + h_1 \gamma \Delta\theta = h_2$$

$$\therefore \gamma = \frac{h_2 - h_1}{h_1 \cdot \Delta\theta (\theta_2 - \theta_1)} \quad \text{— (iii)}$$

Hence, proved.

We can determine γ if we know the values of $h_2, h_1, \theta_2, \theta_1$.

Date:
Page:

Numericals:

Q: A glass flask whose volume is exactly $1000 cm^3$ at $0°C$ is filled full of mercury. When the flask and mercury are heated to $100°C$, $15.2 cm^3$ mercury overflows. Find the linear expansion coefficient of the glass. [$\gamma_{Hg} = 0.000182 °C^{-1}$]

Soln:

Given,

Initial volume of flask at $0°C$ $(V_1) = 1000 cm^3$ = Also of mercury.

Initial temperature $(\theta_1) = 0°C$

Final temperature $(\theta_2) = 100°C$

Coefficient of expansion of glass $(\alpha_g) = ?$

Coefficient of cubical expansion of mercury $(\gamma_m) = 1.82 \times 10^{-4} °C^{-1}$

Now,

Final volume of mercury $(V_{2(m)}) = V_1 (1 + \gamma_m \Delta\theta)$

$= 1000 \times (1 + 1.82 \times 10^{-4} \times 100)\ cm^3$

Final volume of flask $(V_{2(g)}) = V_1 (1 + \gamma_g \Delta\theta)$

$= 1000 \times (1 + 3\alpha_g \times 100)\ cm^3$

$[\because \gamma = 3\alpha]$

We know,

Volume of mercury overflow $= 15.2 cm^3$

or, $V_{2(m)} - V_{2(g)} = 15.2$

or $1000 \times (1 + 1.82 \times 10^{-4} \times 100) - 1000 (1 + 3\alpha_g \times 100) = 15.2$

or, $1018.2 - 1000 - 100000 \times 3\alpha_g = 15.2$

or $18.2 - 100000 \times 3\alpha_g = 15.2$

$\therefore \alpha_g = 1.0 \times 10^{-5}\ °C^{-1}$

Date : ....
Page: ....

Q2: Using the following data at $0°C$, determine the temperature at which wood just sink in ~~de~~ benzene. (Density of benzene at $0°C$ $(\rho_1)_b = 9\times10^2\ kg/m^3$, Dencity of wood at $0°C$ $(\rho_1)_w = 8.8\times10^2\ kg/m^3$ and $\gamma_b = 1.2\times10^{-3}\ K^{-1}$ and $\gamma_w = 1.5\times10^{-4}\ K^{-1}$)

Sol^n:

Given,

~~ρ_{1b}~~ dencity of benzene at $0°C$ $(\rho_1)_b = 9\times10^2\ kg/m^3$
density of wood at $0°C$ $(\rho_1)_w = 8.8\times10^2\ kg/m^3$
Cubical expansion of benzene $(\gamma_b) = 1.2\times10^{-3}\ K^{-1}$.
Cubical expansion of wood $(\gamma_w) = 1.5\times10^{-4}\ K^{-1}$

Let 'θ' be the temperature at which the wood just sinks in the benzene.

When the wood just sinks in benzene,

density of benzene at $\theta°C$ = density of wood at $\theta°C$.

or, $(\rho_2)_b$ at $\theta°C$ = $(\rho_2)_w$ at $\theta°C$.

or, $\dfrac{(\rho_1)_b}{1+\gamma_b\Delta\theta} = \dfrac{(\rho_2)_w}{1+\gamma_w\Delta\theta}$ $\left[\because \rho_2 = \dfrac{\rho_1}{1+\gamma\Delta\theta}\ ~~and~~\right]$

~~or, $\dfrac{9\times10^2}{1+1.2\times10^{-3}\times\theta} = \dfrac{8.8\times10^2}{1+1.5\times10^{-4}\times\theta}$ $[\because \Delta\theta = \theta - 0 = \theta]$~~

~~or $9 + 1.35\times10^{-3}.\theta = 8.8 + 0.01056\ \theta$~~ 10.56×10^{-3}

Date :

Page:

or, $\dfrac{9\times10^{-2}}{1+1.2\times10^{-3}\cdot\theta} = \dfrac{8.8\times10^{-2}}{1+1.5\times10^{-4}\cdot\theta}$

or, $\dfrac{900}{1+1.2\times10^{-3}\cdot\theta} = \dfrac{880}{1+1.5\times10^{-4}\cdot\theta}$

or, $900 + 0.135\,\theta = 880 + 1.056\,\theta$

or, $20 = 0.921\,\theta$

$\boxed{\therefore \theta = 21.71\ ^\circ C}$

Q31: A silica bulb holds 340 gm of mercury at 0°C when full. Some steel balls introduced the remaining space is occupied at 0°C by 255 gm of mercury. On heating the bulb and its contents to 100°C, 4.8 gm of mercury overflow. Calculate the linear expancivity of the steel.
$(\gamma_{Hg} = 180\times10^{-6}\ K^{-1})$.

Soln:

When the silica bulb is full of mercury at 0°C.

Volume of mercury = volume of silica bulb = $(V_b) = \dfrac{m}{d} = \dfrac{340}{13.6}$

$= 25\ cm^3$

After the steel balls are introduced, the ~~volur~~ mass of mercury is 255 gm,

Volume of mercury at steel ~~states~~ balls were introduced at 0°C.

$(V_1)_m = \dfrac{255}{13.6} = 18.75\ cm^3$

Date :

Page:

~~Volume~~ Soln:

Volume of mercury when full in silica bulb at $0°C$

$$= \frac{340}{13.6} = 25\ cm^3$$

Volume of mercury when steel balls are introduced at $0°C$

$$(V_1)_m = \frac{255}{13.6} = 18.75\ cm^3$$

Volume of steel balls at $0°C$ $(V_1)_s = (25 - 18.75)\ cm^3$

$$= 6.25\ cm^3$$

Since the temperature is increased to $100°C$,

$\therefore \Delta\theta = 100°C - 0°C = 100°C$

Mass of mercury overflowed at $100°C$ = 4.8 gm

Density of mercury at $100°C$ $(\rho_2) = \frac{\rho_1}{1 + \gamma_{Hg}\Delta\theta}$

$$= \frac{13.6}{1 + 180 \times 10^{-6} \times 100} = 13.359\ g/cm^3$$

Volume of overflowed mercury $= \frac{4.8}{13.359} = 0.3593\ cm^3$

Volume of mercury at $100°C$ $(V_2)_m = (V_1)_m[1 + \gamma_{Hg}\Delta\theta]$

$$= 18.75\ [1 + 180 \times 10^{-6} \times 100]$$

$$= 19.0875\ cm^3$$

Difference in volume of mercury $= (19.0875 - 18.75)\ cm^3$

$$= 0.3375\ cm^3$$

Date :
Page:

Volume of steel balls at $100^\circ C$ $(V_2)_s = (V_1)_s [1+ \gamma_s \Delta\theta]$

$= 6.25 [1+ \gamma_s \cdot 100]$

$= 6.25 + 625 \gamma_s$

Difference of volume of steel balls $= (V_2)_s - (V_1)_s$

$= \cancel{6.25} + 625 \gamma_s - \cancel{6.25}$

$= 625 \gamma_s \ cm^3$

We know,

Volume of mercury overflowed = ~~Volume of mercury at +~~ Difference in volume of mercury + Difference in volume of steel

or, $0.3593 \ cm^3 = 0.3375 \ cm^3 + 625 \gamma_s$

or, $0.0218 \ cm^3 = 1875 \alpha_s$

$$\boxed{\therefore \alpha_s = 1.162 \times 10^{-5} \ ^\circ C^{-1}}$$

Assignment

(1): A copper vessel with a volume of exactly $1.8 m^3$ at $20^\circ C$ is filled with glycerine. If the temperature rises to $30^\circ C$, how much glycerine will spill out overflows.
[Cubical expansivity of glycerine $(\gamma_g) = 5.3 \times 10^{-4} \ ^\circ C^{-1}$
& Linear expandivity of copper $(\alpha_{cu}) = 1.67 \times 10^{-5} \ ^\circ C^{-1}$]

Soln:

Given,
Initial temperature $(\theta_1) = 20^\circ C$ Final temperature $(\theta_2) = 30^\circ C$
$\therefore$ Difference in temperature $(\Delta\theta) = 30^\circ C - 20^\circ C = 10^\circ C$
Initial volume of copper vessel $(V_1)_{cu}$ = Initial volume of glycerine $(V_1)_g = 1.8 \ cm^3$.

Date: ....
Page: ....

Now,

final volume of glycerine $(V_2)_g = (V_1)_g[1+\gamma_g \cdot \Delta\theta]$

$= 1.8 \times [1+5.3\times10^{-4}\times100]$

$= \cancel{1.8954\ m^3}$ $= 1.80954\ cm^3$

final volume of copper $(V_2)_{cu} = (V_1)_{cu}[1+\gamma_{cu}\Delta\theta]$

$= 1.8\times[1+(3\times1.67\times10^{-5}\times100)]$

$= \cancel{1.809018\ cm^3}$ $1.8009018\ cm^3$

Now,

Volume of overflowed glycerine $= (V_2)_g - (V_2)_{cu}$

$= (1.80954 - 1.8009018)\ cm^3$ $\cancel{= 1.8954 - 1.809018}$

$= 8.6382\times10^{-3}\ cm^3$ $\cancel{= 0.086382\ cm^3}$

$\therefore$ 8.6382×10^{-3} $\cancel{0.086382\ cm^3}$ of glycerine overflowed.

27: The density of a certain oil at 20°C is 1020 kg/m³. If the oil does not mix with ~~a~~ water, find the temperature at which drops of the oil will just float on water. ($\gamma_{oil} = 8.5\times10^{-4}\ °C^{-1}$ and $\gamma_w = 4.5\times10^{-4}\ °C^{-1}$)

Soln:

Date: ....

Page: ....

<3>: A glass vessel is completely filled with 500 gm of mercury at 0°C. What weight of mercury will overflow when it is heated at 80°C. [$\gamma_{Hg} = 18.2 \times 10^{-5}$ °C^{-1} and $\alpha_g = 1 \times 10^{-5}$ °C^{-1}]

Soln:

Mass of mercury in completely filled vessel = 500 gm

Volume of mercury in glass vessel at 0°C $(V_1)_{gm}$ = $\left(\frac{500}{13.6}\right)$ cm^3

= 36.76 cm^3

Volume of glass at 0°C $(V_1)_g$ = 36.76 cm^3 [glass completely filled]

When the glass and mercury is heated to 80°C,

$\Delta\theta = (80° - 0°)°C = 80°C.$

Volume of mercury at 80°C $(V_2)_m = (V_1)_m [1 + \gamma_{Hg} \Delta\theta]$

$= 36.76 [1 + 18.2 \times 10^{-5} \times 80]$

$= 37.29$ cm^3

Volume of glass at 80°C $(V_2)_g = (V_1)_g [1 + 3\alpha_g \Delta\theta]$

$= 36.76 [1 + 3 \times 1 \times 10^{-5} \times 80]$

$= 36.84$ cm^3

Volume of mercury overflow = $(37.29 - 36.84)$ cm^3

$= 0.45$ cm^3

Density of mercury at 80°C $(\rho_2)_m$ $= \frac{(\rho_1)_m}{1 + \gamma_{Hg} \Delta\theta} = \frac{13.6}{1 + 18.2 \times 10^{-5} \times 80}$

$= 13.4$ g/cm^3

Date : ....
Page: ....

Mass g mercury at $80^\circ C$ = (13.4×0.45) ~~Kg~~ gm
= 6.03 gm