Chapter: Elasticity

Elasticity:

→ The tendency to oppose any change in its shape, size by a body, is called elasticity.

→ The tendency of a body (molecules of object) to regain it's original configuration after removal of deforming force up to its elastic limit, is called elasticity.

e.g. Steel has more elasticity than rubber because steel regains its original configuration faster than rubber.

e.g. Fiber, metallic wire etc are elastic body.

Non-elastic body: The body which cannot regain its original shape and size after removable of deforming force is called non-elastic body. E.g. brick, Wood, papers etc

Deforming Force: which change the shape and size of body (i.e. external force needed)

Restoring force: This restores the shape and size of body

Elastic limit (or yield point): The maximum value of deforming force which a body can experience and still regain its original size and shape once the force has been removed is called elastic limit.

If the upper limit of deforming force is increased, the body loses its property of elasticity and gets permanently deformed.

Stress (σ): The internal restoring force per unit cross-sectional area is called stress.

Here: restoring force = Deforming force [$F_R = F_{external}$]

It is defined as the ratio of deforming force acting on a body to the cross-sectional are of the body.

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

$$\text{Stress } (\sigma) = \frac{\text{Deforming force (F)}}{\text{Cross−Section Are (A)}} \qquad \text{i.e. } (\sigma) = \frac{F}{A}$$

It's unit is N/m^2 and dimensional formula $[ML^{-1}T^{-2}]$

It is a scalar quantity (or Tensor quantity)

Types of stress

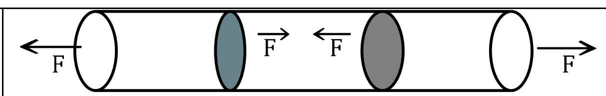
(I) Normal stress or longitudinal stress:

If deforming force applied normally to the surface of a body, the applied force per unit area is called normal stress.

i.e. Normal stress = $\frac{\text{normal force}}{\text{Area}}$

a) Tensile stress :

If there is extension of the body in the direction of applied force, the stress set up is called Tensile stress.



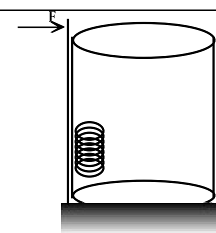
b) Compressive stress

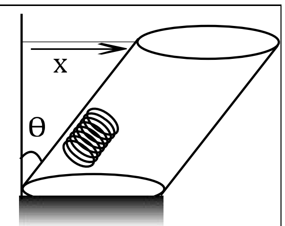
If there is compression of the body due to applied force, the stress set up is called Compressive stress.

(II) Shearing or Tangential stress:

→ When the external deforming forces acts tangentially to the surface of body, there is change in shape of body. The stress developed in body in such a case is called tangential or shearing stress.

i.e. Tangential stress = $\frac{\text{Tangential force}}{\text{Area}}$


attached to support



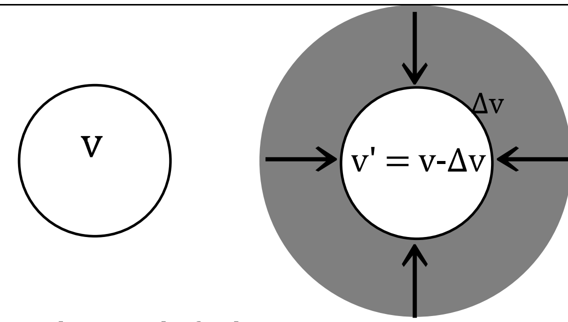
attached to support

(III) Value stress or Bulk stress or Hydraulic stress:

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

When a body is acted upon by normal deforming force over its entire surface such that there is change in volume, then restoring force developed per unit area called Bulk or Volume stress.

i.e. Bulk Stress = $\frac{\text{Force}}{\text{Area}}$



a) Body outside find b) Bulkstress

Strain (∈):

It is defined as the ratio of change in configuration to the original configuration.

Strain (∈) = $\frac{\text{Change in configuration}}{\text{original configuration}} = \frac{\text{change in shape}}{\text{original shape}}$

It is unit less and dimensionless.

Here, change in configuration may involve the change in length, volume and even shape. Hence, accordingly, strain can be classified as:

Types of strain:

(I) Longitudinal strain:

It is defined as the ratio of change in length of body to the original length.

i.e. Longitudinal strain = $\frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L} = \frac{e}{L}$

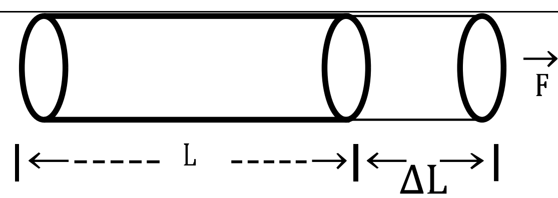


Fig (1) Longitudinal strain

Here, change in length = ΔL = e = extension

(II) Shear strain:

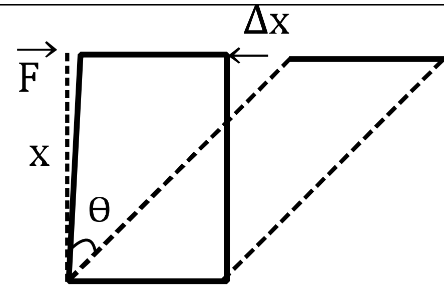
It is defined as the angular displacement produced in a body when tangential force acts.

$$\tan \tan\theta = \frac{\Delta x}{x}$$

For small θ, $\tan\theta \cong \theta$

$$\therefore \theta = \frac{\Delta x}{x} = \frac{\text{linear displacement}}{\text{height}}$$

Such shearing can happen only in case of solids

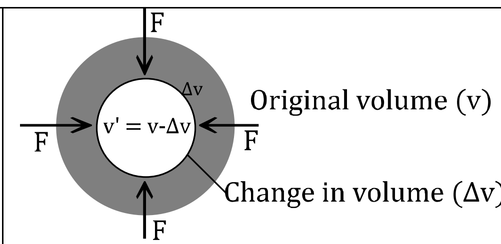


Shear strain

(III) Volumetric strain:

It is defined as the ratio change in volume of body to the original body.

$$\text{Volumetric strain} = \frac{\text{Chang in volume}}{\text{original volume}} = \frac{\Delta v}{v}$$



Hook's Law: 2075, 2056

Q. State Hook's law. How would you verify the experimentally?

Hook's Law: It states that "Within elastic limit, extension produced in a body is directly proportional to the force applied".

i.e. Force (F) $\propto$ extension (x)

or F = Kx

Where, k is proportionality constant known as spring constant or force constant.

It can also state as, "Within elastic limit, strain produced in a body is directly proportional to the stress applied".

i.e. stress $\propto$ strain

or, stress = E strain

or, $\frac{\text{stress}}{\text{strain}} = E$

Where, E is proportionality constant called Modulus of elasticity or Coefficient of elasticity.

Experimental verification of Hooke's law:

Consider a spring length 'L' suspended from a rigid support as in Fig(a). The spring carries a pan at its lower and so that it can be loaded. A pointer P slides over a scale fitted parallel to the spring, so that the position of the spring can be noted.

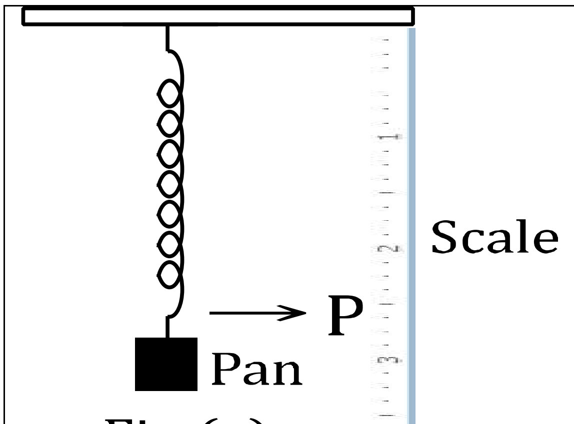


Fig (a)

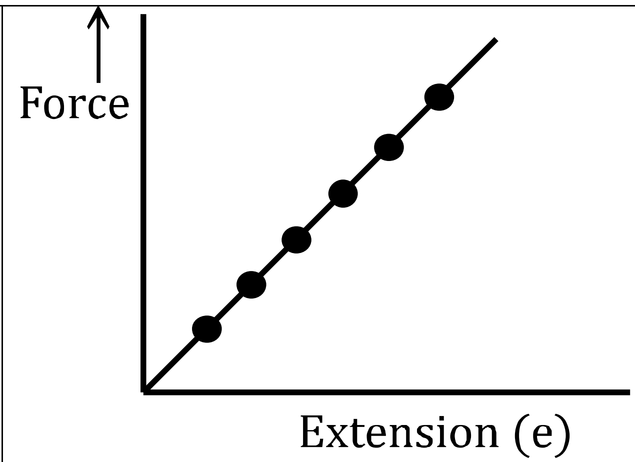


Fig (b)

The pan is loaded in steps of 0.5kg and corresponding pointer regarding are noted. The difference between each reading with first reading gives the extension produced in the spring by different load.

A graph between load 'F' and extension produced 'e' is drawn as shown in fig (b), which a straight line is passing through origin.

This indicates that

$F \propto x$; x = extension

$F = kx$ .. (1)

Dividing equation (1) by A (Cross sectional Area) on both sides and multiply RHS and dividing RHs with L.

$$\frac{F}{A} = \left(\frac{Kx}{A}\right)\frac{L}{L}$$

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

$$\frac{F}{A} = \left(\frac{KL}{A}\right)\frac{x}{L}$$

Since, Cross-sectional area A and original length L are constant so $\left(\frac{KL}{A}\right)$ is constant.

$$\therefore \frac{F}{A} \propto \frac{x}{L}$$

$$(\because \text{stress} = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}, \text{and Strain} = \frac{x}{L} = \frac{\text{extension}}{\text{original lenght}})$$

$$\text{or, Stress } \propto \text{stratin}$$

This verifies Hooke's law

Characteristics of Hooke's Law:

The plot of force (F) versus extension (x) or stress versus strain is called characteristics of Hooke's law. It is a straight line passing through origin as shown in fig. (1)

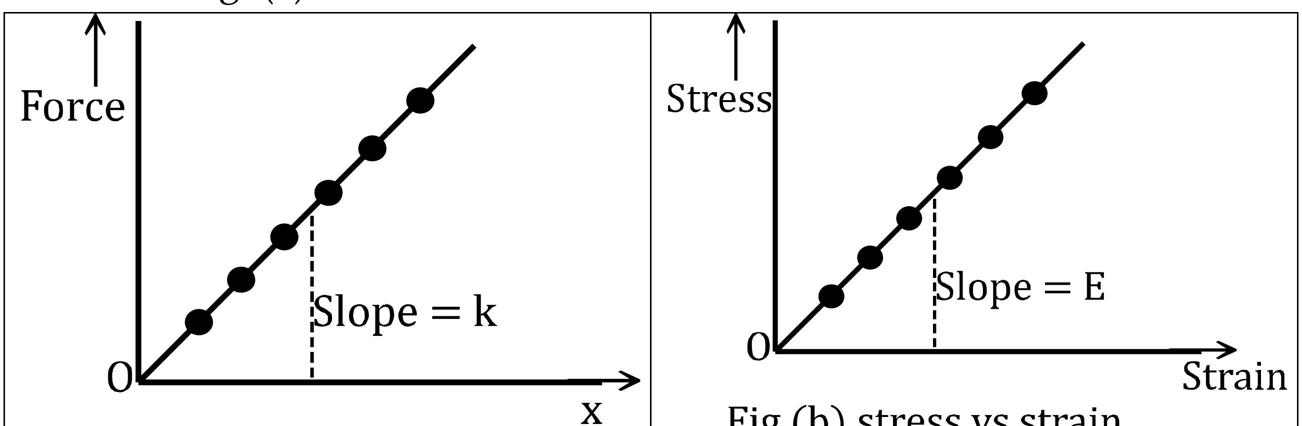


Fig (a) Force vs extension

Fig (b) stress vs strain

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

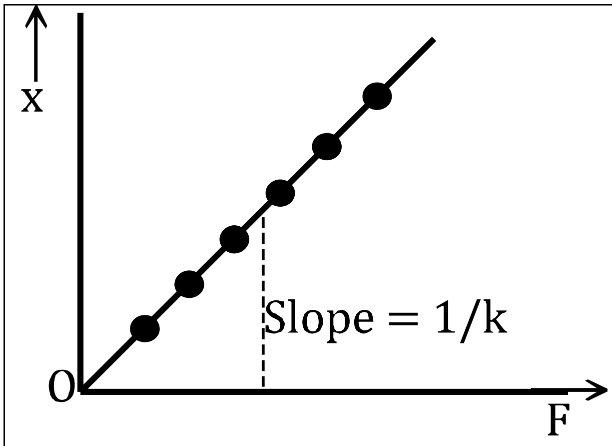


Fig (c) extension vs force

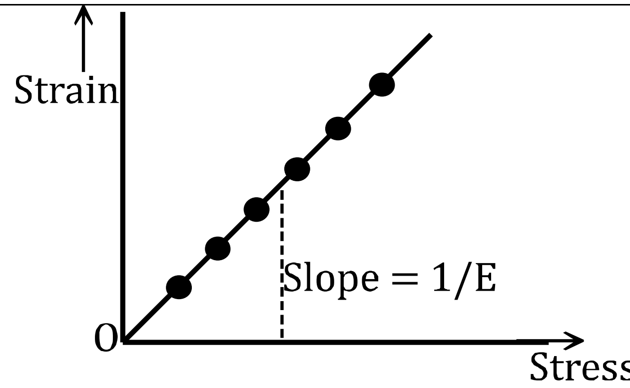
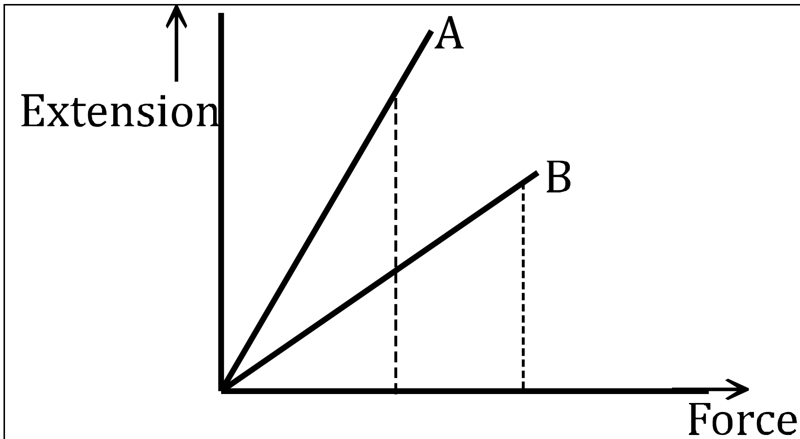


Fig (d)Strain vs stress

Q. The extension vs force for two material A and B is plotted as below. Which material has greater extension.



In this fig. extension vs force

We know, slope = $\frac{1}{k}$

Or, $k = \frac{1}{\text{slope}}$

From graph,

Slope of B < Slope of A

$\therefore K_B > K_A$, hence, B has greater extension.

Modulus of elasticity (E):-

It is defined as the ratio of stress applied on a body to strain produced.

i.e. Modulus of Elasticity (E) = $\frac{\text{stress}}{\text{strain}}$

It's unit is N/m^2 and dimensional formula is $[ML^{-1}T^{-2}]$ as same as that of stress or pressure.

Types of modulus of elasticity:-

(I) Young's Modulus of elasticity (Y):

It is defined as the ratio of Normal stress applied on a body to longitudinal strain produced.

It is denoted by N/m^2.

It is different for different materials.

i.e. Young's modulus (Y) = $\frac{\text{normal stress}}{\text{Longitudinal strain}} = \frac{\text{Longitudinal stress}}{\text{Longitudinal str}}$

Consider the wire of radius r and length L. Let a force F be applied on the wire along its length i.e. normal to surface of wire.

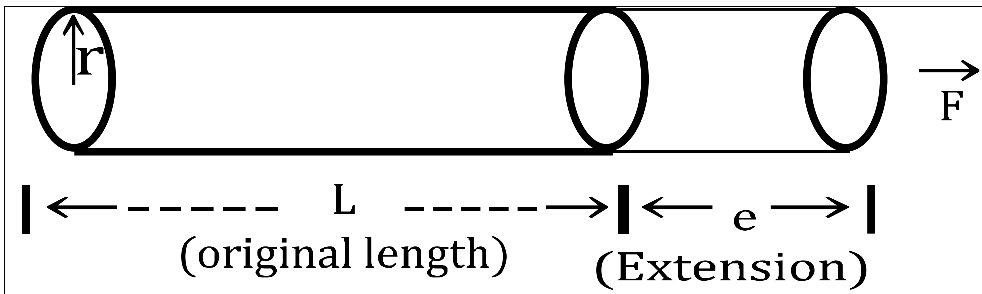


Fig (1) Longitudinal strain

Here, $e = \Delta L = \text{extension}$, $L = \text{Original length}$

If ΔL be change in length of wire then,

Longitudinal strain = $\frac{\Delta L}{L} = \frac{e}{L}$

Longitudinal stress = normal stress = $\frac{F}{A}$ Where A is corss section area of wire.

$$\therefore Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{e}{L}\right)} = \frac{FL}{Ae} = \frac{FL}{\pi r^2 e} \qquad \therefore A = \pi r^2 \text{, cross sectional area}$$

$$\therefore Y = \frac{FL}{\pi r^2 e}$$

(II) Bulk Modulus of Elasticity (K):

It is defined as the ratio of Normal stress to the volumetric strain, within the elastic limit.

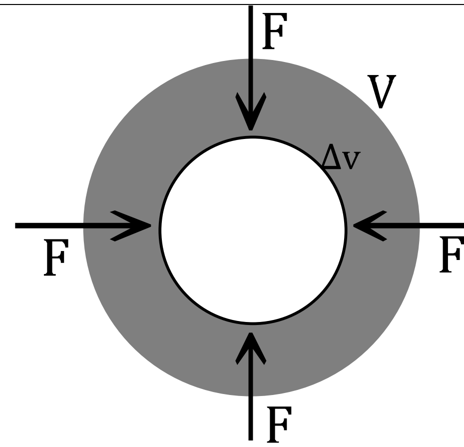
Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

i.e. Bulk Modulus $K = \frac{\text{Normal Stress}}{\text{volumetric strain}}$

$K = \frac{F/A}{-(\Delta V/V)}$

Here, -ve sign implies that, with increase in pressure, volume decrease.

$\therefore K = \frac{F/A}{-(\Delta V/V)} = -\frac{PV}{\Delta V}$



Compressibility (C):

The reciprocal of Bulk modulus of elasticity is called compressibility, i.e.

$$c = \frac{1}{k} = -\frac{\Delta V}{PV}$$

it's unit in SI is $N^{-1}m^2$ or $(\text{pascal})^{-1}$

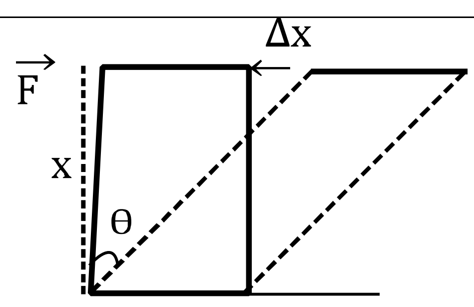
(III) Shear Modulus or Modulus of Rigidity (n):

It is defined as the ratio of the tangential stress to the tangential strain, within the elastic limit.

i.e. $n = \frac{\text{Tangential stress}}{\text{Tangential strain}} = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{\frac{F}{A}}{\frac{\Delta x}{x}}$

or, $n = \frac{F}{A\theta} = \frac{Fx}{A\Delta x} = \frac{FL}{Ae}$ $\therefore x = L = \text{original lenght}, e = \Delta x =$ extension

Shear modulus has significance only for solids. For the same solid material, the shear modulus is roughly one third the value of the young's modulus.



Shear strain

2061, 2058 Explain why steel is said more elastic than rubber.

The body which has More Modulus of elasticity is known as more elastic body.

In solid material, we compare the elasticity from modulus of elasticity.

From the definition of Young's modulus of elasticity,

$$Y = \frac{FL}{eA}$$

For constant force applied (F), same cross-section Area (A)

And same length (L) of steel and rubber, we have

$$Y \propto \frac{1}{e(\text{extension})}$$

(I) For steel: $Y_s = \frac{1}{e_s}$ …….. (i)

(II) For rubber: $Y_r = \frac{1}{e_r}$ …….. (ii)

Dividing equation (I) by (II), we get

$\frac{Y_s}{Y_r} = \frac{\frac{1}{e_s}}{\frac{1}{e_r}} = \frac{e_r}{e_s}$ …….(iii)

As we know, extension of rubber is greater than steel i.e. $e_r > e_s$

So, from (iii) ($Y_s > Y_r$

Hence, steel is more elastic than rubber.

Q. (1) Define

a) elastic limit	f) plasticity
b) elastic fatigue	g) elastomor
c) elastic after effect	h) ductile material
d) poisson ratio	i) Brittle material
e) breaking stress	j) Elastic Hysteresis

2062

* **Plasticity:** It is the property of a material by virtue of which it does not regain its original shape and size after removing deforming forces. The objects which obey plastic behavior are known as plastic objects. They are

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

not suitable to use in construction of building and bridges. Nobody in nature is perfectly plastic.

*Elastic after effect:

It is the temporary delay in regaining the original configuration by an elastic body after removal of a deforming force.

The elastic after effect is negligibly small for quartz fiber, phosphor-bronze, silver, gold etc while it is larger for glass fibre.

*Elastomor $\coloneqq$ A substance having a large value of strain without breaking is known as elastomer. E.g. rubber

Imp

*Proportional limit : In a stress strain graph, proportional limit is a point at which the stress is directly proportional to the strain.

Imp

*Breaking stress:

It is the ratio of maximum force[load] to which the wire is subjected to the original cross-sectional area.

i.e. Breaking stress $= \frac{\text{maximum load}}{\text{original cross}-\text{sectional area}}$

$$= \frac{\text{maximum force}}{\text{original cross}-\text{sectional area}}$$

i.e. Maximum load=Breaking stress *original cross sectional area

This shows that maximum load bearing capacity of a material is directly proportional to the cross-sectional area. As the elephant has heavier load of its body, it should have larger cross-sectional area of its legs.

Q. Explain terms breaking stress. Why elephant has thicker legs as compared to human beings. 2066, 2069

2069

* Elastic fatigue or Elastic tiredness

→ The loss of elastic behavior due to the regular elongation and contraction of materials is known as elastic fatigue or elastic tiredness. Since the material reaches on elastic fatigue, it may break at any time. Hence, bridges are declared unsafe after a few decades of use. (long use)

Similarly due to same reason, spring balance shows wrong reading after long use.

[Q. steel bridges are declared unsafe after a few decades of use. Why? 2069]

* Ductile Materials: Those materials which undergo plastic deformation beyond elastic limit are called Ductile materials.

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

*Brittle material : Those materials which show very small plastic deformation beyond elastic limit.

Imp (2072, 2057) Poission ratio (σ) :

The ratio of the lateral strain to the longitudinal strain produced in a wire, within the elastic limit is called poisson ratio.

i.e. poission ratio $(\sigma) = \frac{\text{lateral strain } (\beta)}{\text{longitudial strain } (\alpha)}$

or, $(\sigma) = \frac{(\beta)}{(\alpha)}$ where, lateral strain is the ratio of change in diameter of wire to its original diameter. i.e. $\beta = \frac{\Delta d}{d}$

It is unit less and dimensionless quantity.

It 's value usually lies between 0.2 to 0.4

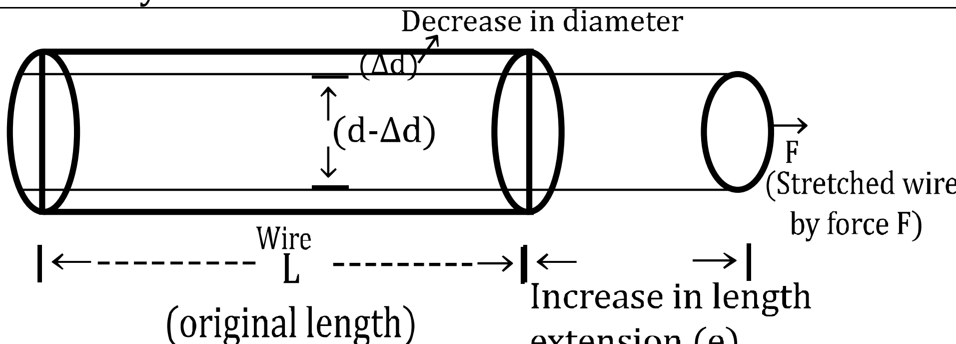


Fig (I) Extension of a wire demonstrating lateral and longitudinal strain

We know that, lateral strain is directly proportional to the longitudinal strain within elastic limit

i.e. $\beta \propto \alpha$

or, $\beta = \sigma\alpha$

or, $\sigma = \frac{\beta}{\alpha}$

Where, σ is the proportionality constant or elastic constant called as Poisson's ratio.

Now, Longitudinal strain $(\alpha) = \frac{\Delta L}{L} = \frac{e}{L}$

And lateral strain$(\beta) = -\frac{\Delta d}{d}$

$$\therefore \sigma = -\left(\frac{\Delta d}{d}\right)\frac{L}{e}$$

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

Here, -ve sign shows that if length of wire increases then diameter decreases.
Imp
* Elastic Hysteresis:
→ Hysteresis is the characteristic that something follows a return path that is different from the outgoing path.
The lagging of strain behind stress in a substance is called elastic hysteresis and the curve obtain by plot of stress versus strain is called elastic hysteresis curve.

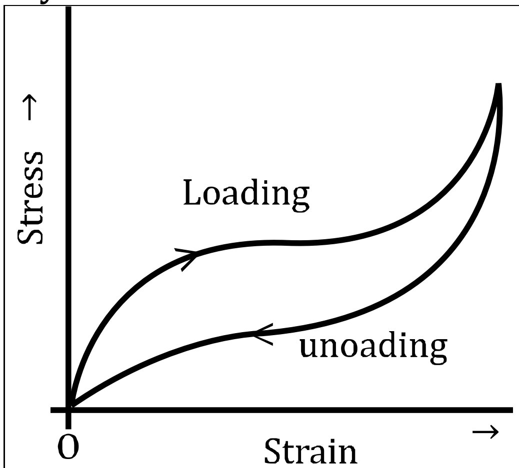


Fig (I) Elastic hysteresis for rubber

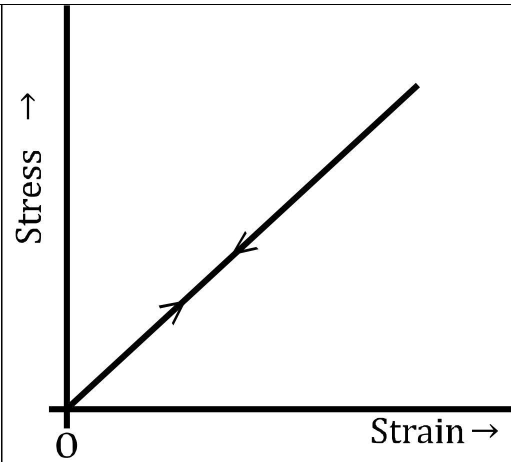


Fig (II) Elastic hysteresis for metallic wires

The work done by the material in returning to its original shape is less than the workdone by the deforming force. This difference of energy is absorbed by the material and appears as heat. This phenomenon is called elastic hysteresis.

Application of elastic hysteresis:
(I) car tyres are made with synthetic rubbers having small hysteresis loops because the tyre made of such rubber will not get excessively heated during the Journey.
In the case of rubber, it can be compressed or stretched easily due to its coiled form and area of elastic hysterias for rubber is increased, there is net energy stored for every cycle on application of deforming force and removal of it.
Due to this, it can store energy in the potential form and finally converts it into heat energy, thus rubber absorbs vibration K.E and release slowly heat energy. Hence, it is used as vibration absorber or shock wave.

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal

[Why rubbers are used as vibration absorber? [2069, 2063]

*** Energy stored in stretched wire:**
2057/2071/2073/2074/2075/2061/2067/2069

Q. Derive an expression for the energy stored in stretched wire? Define energy density of body under strain.
Q. Prove that elastic energy stored per unit volume of a stretched wire is equal to ½ x stress x strain.

→ Consider a wire of length 'L' and cross-sectional are 'A' , having Young's Modulus of elasticity 'Y' . Then let 'F' be the external force applied to stretch (expand) the wire and total extension produced is x, so we can write,

$Y = \frac{\text{stress}}{\text{strain}}$

$$Y = \frac{F/A}{x/L} = \frac{FL}{Ax}$$

Or, $F = \frac{YAx}{L}$, ... (1)

If dx be the small extension produced in a wire due to force F in the range o to x, then small workdone is given by

$dW = Fdx$

Or, $dW = \frac{YAx}{L}dx$........(2)

Now, integrating it under limit $x = 0$, to $x = x$, we get the total workdone to stretch the wire,

$$W = \int_0^x Fdx$$

$$= \int_0^x \frac{YAx}{L}dx$$

$$= \frac{YA}{L}\int_0^x xdx$$

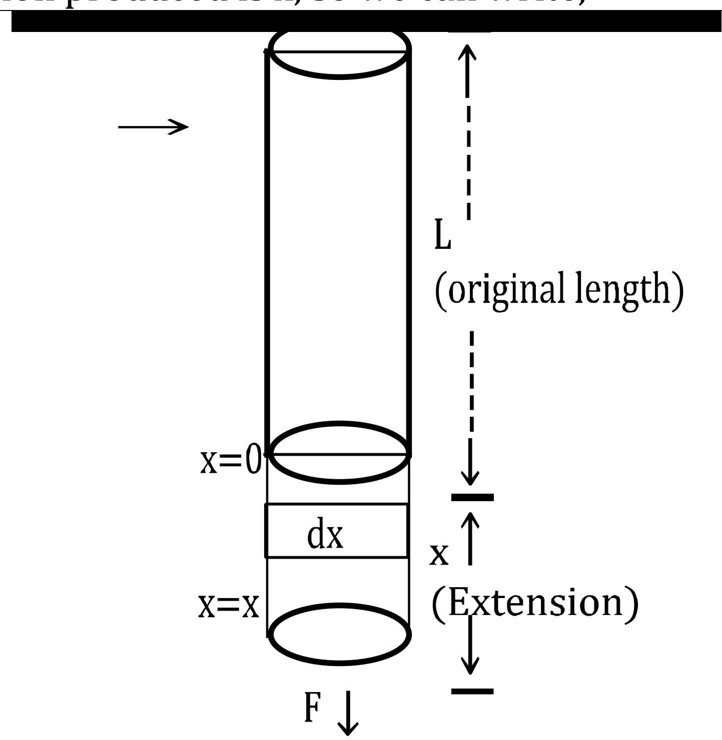


Fig (1) energy stored in wire due to strain

$= \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^x = \frac{YA}{2L}[x^2 - 0^2] =$ $\frac{1}{2}\frac{YA}{L}x^2$ $W = \frac{1}{2}\left(\frac{YAx}{L}\right)x = \frac{1}{2}Fx$ ....(3)	

Hence, Workdone (W) = ½ Fx $= \frac{1}{2} \times$ external force $\times$ extension .....(4)

When external force is applied to the wire then it extends and restoring force act on it. In order to extend further more, the work has to be done against this restoring force, which is called potential energy, stored in a wire.

$\therefore$ Energy stored in a stretched wire $= \frac{1}{2} \times$ stretching force $\times$ extension ..(5)

Also strain energy $= \frac{1}{2}Fx$

Energy density : It is the ratio of energy stored in a body to the unit volume in a stretched wire.

i.e. Energy density (U) $= \frac{\text{energy stored in a stretched wire}}{\text{volume}}$

$= \frac{\frac{1}{2}Fx}{(A\times L)} = \frac{\frac{1}{2}F}{A}.\frac{x}{L}$

$\therefore$ Energy density (U) $= \frac{1}{2}$ stress $\times$ strain ..... (6)

Brij Kumar Singh M.Sc Physics , CDP, TU, Kirtipur
Physics lecturer at Shree Secondary Technical & vocational school, Dharapani, Dhanusha Nepal
Former Physics Lecturer at Nepal Adarsha School/Campus Waling
Former secondary Science teacher at Everest Academy Lalbandi, Sarlahi Nepal