DELHI PUBLIC SCHOOL BANGALORE - EAST

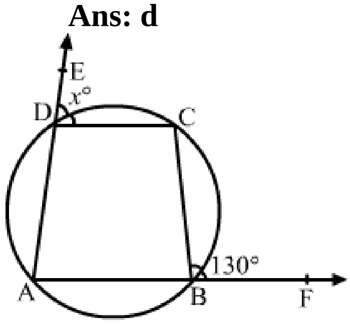
MATHEMATICS-ANSWER KEY

CIRCLES - WORKSHEET

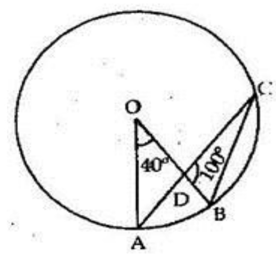
NAME:______________ CLASS: IX SEC:_____ DATE: ___________

Choose the correct answer in each of the following :

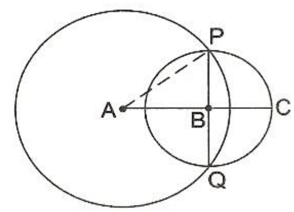
1. In the given figure(i) , sides AD and AB of cyclic quadrilateral ABCD are produced to E and F respectively. If$\angle CBF=130$ and $\angle CDE=x$, then x= ?
   a) $100°$ b) $80°$ c) $130°$ d) $50°$
   Ans: d



Figure(i)



Figure(ii)



Figure(iii)

2. In the given figure(ii) , O is the centre of a circle, $\angle AOB=40°$ and $\angle BDC=100°$,find $\angle OBC$.
   a)$60°$ b) $80°$ c) $130°$ d) $50°$

Ans: a

3.In the given figure(iii), A and B are the centres of two circles having radii 5cm and 3cm respectively and intersecting at point P and Q respectively. If AB=4cm then the length of the common chord PQ is

a) 3cm b) 6cm c) 7.5cm d) 9cm

Ans: b

4.The radius of a circle is 13cm and the length of one of its chords is 10cm.The distance of the chord from the centre is

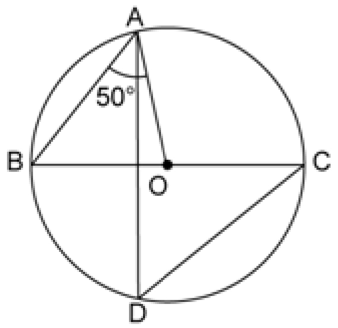
a) 11.5cm b) 12cm c) $\sqrt{69}$ cm d) 23cm

Ans: b

5.In the given figure, O is the centre of a circle and $\angle OAB = 50°$. Then, $\angle CDA$ =?

a) 40° b) 50° c) 75° d) 25°

Ans: b



Fill in the blanks:

6. A circle divides the plane , on which it lies ,in **3** parts.

7.The longest chord of circle is **diameter**

8.The centre of a circle lies in **interior** of a circle.

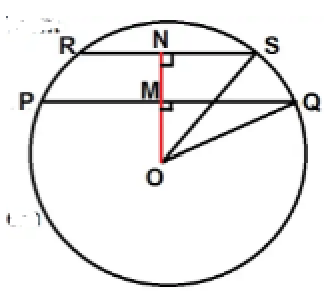
Solve the following:

9.PQ and RS are the two parallel chords of a circle on the same side of the centre O and radius is 10cm. If PQ= 16cm and RS = 12cm , find the distance between the chords.

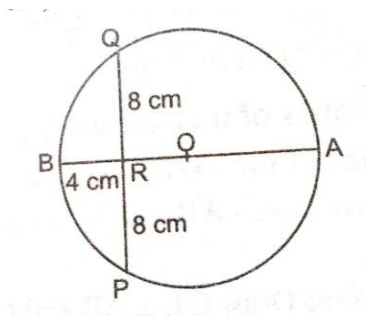
So, by Pythagoras theorem, we get $ON = \sqrt{OS^2 - SN^2} = \sqrt{10^2 - 6^2}$ cm $= 8$ cm and $OM = \sqrt{OQ^2 - QM^2} = \sqrt{10^2 - 8^2}$ cm $= 6$ cm.

($\because$ P Q & RS are to the same side of the centre O.

Here $MN = ON - OM = (8 - 6)$ cm= 2 cm.



10. In the given figure(iv) , diameter AB of circle with centre O bisects the chord PQ. If PR = QR = 8cm and RB = 4cm, find the radius of the circle.



Given: AB is the diameter and PQ is the chord of the circle with centre O and AB bisects PQ at R also
PR = RQ = 8 cm and RB = 4 cm

we know that the line from the centre of the circle which bisects the chord is perpendicular to the chord
$\Rightarrow AB \perp PQ$

Let the radius of the circle be *r*
$\Rightarrow OP = OB = r$
$\Rightarrow OR + RB = r$
$\Rightarrow OR = r - RB = r - 4$

Now in right Δ ORP
$OR^2 + PR^2 = OP^2$
$\Rightarrow (r-4)^2 + 8^2 = r^2$
$\Rightarrow r^2 + 4^2 - 2 \times 4 \times r + 64 = r^2$
$\Rightarrow 16 - 8r + 64 = 0$
$\Rightarrow 80 - 8r = 0$
$\Rightarrow 8r = 80$
$\Rightarrow r = \frac{80}{8} = 10 \text{ cm}$

Hence the radius of the given circle is 10 cm.

11.In the given figure(v), O is the centre of the circle and $\angle AOB = 110°$,find the value of x, y and z.

y = (1/2)$\angle$AOB (angle subtended by chord AB at centre & arc)

=> y= (1/2)110°

=> y = 55°

BD is Diameter

=> x + y = 90°

=> 55° + x = 90°

=> x = 35°

x + z = 180° (sum of opposite angles of cyclic quadrilateral)

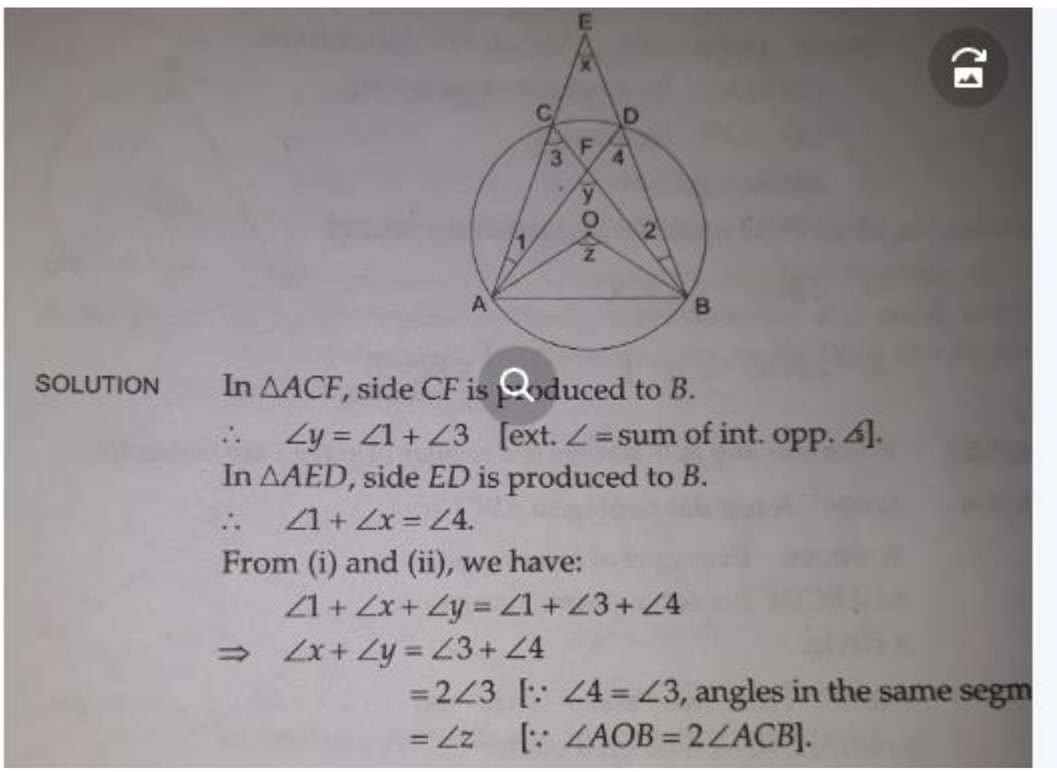
=> 55° + z = 180°

=> z = 125°

12.In the given figure(vi) .B and E are points on the line segments AC and DF respectively. how that AD || CF.

As ABED is a cyclic quadrilateral .
So , $\angle BAD + \angle BED = 180^\circ \quad \ldots(1)$
(Opposite angles of cyclic quadrilateral are supplementary)
As CBEF is a cyclic quadrilateral
$\angle BCF + \angle BEF = 180^\circ \quad \ldots(2)$ (Opposite angles of cyclic quadrilateral are supplementary)
Adding (1) and (2) , we get
$\angle BAD + \angle BED + \angle BCF + \angle BEF = 360^\circ$
$\angle BAD + \angle BCF + (\angle BED + \angle BEF) = 360^\circ$
$\angle BAD + \angle BCF + 180^\circ = 360^\circ$ (Linear Pair)
$\angle BAD + \angle BCF = 180^\circ$
$\Rightarrow \angle CAD + \angle ACF = 180^\circ$
$AD \parallel CF$ (By converse of consecutive interior angle theorem)

13. In the given figure(vii) , O is the centre of the circle. Prove that ∠x + ∠ y = ∠ z



SOLUTION In $\triangle ACF$, side CF is produced to B.
$\therefore \quad \angle y = \angle 1 + \angle 3$ [ext. $\angle$ = sum of int. opp. $\angle$s].
In $\triangle AED$, side ED is produced to B.
$\therefore \quad \angle 1 + \angle x = \angle 4$.
From (i) and (ii), we have:
$\angle 1 + \angle x + \angle y = \angle 1 + \angle 3 + \angle 4$
$\Rightarrow \quad \angle x + \angle y = \angle 3 + \angle 4$
$= 2\angle 3$ [$\because \angle 4 = \angle 3$, angles in the same segm
$= \angle z$ [$\because \angle AOB = 2\angle ACB$].

14.In the given figure , determine a, b and c.

Now, $\angle ACE = 43°$ and

$\angle CAF = 62°$ [given]

In ΔAEC

$\therefore \angle ACE + \angle CAE + \angle AEC = 180°$

$\Rightarrow 43° + 62° + \angle AEC = 180°$

$\Rightarrow 105° + \angle AEC = 180°$

$\Rightarrow \angle AEC = 180° - 105° = 75°$

Now, $\angle ABD + \angle AED = 180°$

[Opposite angles of a cyclic quad and $\angle AED = \angle AEC$]

$\Rightarrow a + 75° = 180°$

$\Rightarrow a = 180° - 75°$

$\Rightarrow a = 105°$

$\angle EDF = \angle BAF$

$\therefore c = 62°$ [Angles in the alternate segments]

In $\Delta BAF, a + 62° + b = 180°$

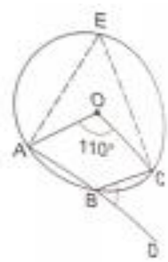
$\Rightarrow 105° + 62° + b = 180°$

$\Rightarrow 167° + b = 180°$

$\Rightarrow b = 180° - 167° = 13°$

Hence, $a = 105°$, $b = 13°$ and $c = 62°$

15. If O is the centre of the circle as shown in figure(viii), find ∠CBD.



Take a point E on the remaining part of the circumference. Join EA and EC.

We know that the angle subtended by an arc at the centre of a circle is twice the angle subtended at a point on the remaining part of the circumference.

$\therefore \quad \angle AEC = \frac{1}{2}\angle AOC = \left(\frac{1}{2} \times 110°\right) = 55°.$

Now, $ABCE$ is a cyclic quadrilateral whose side AB has been produced to a point D.

But, the exterior angle of a cyclic quadrilateral is equal to its interior opposite angle.

$\therefore \quad \angle CBD = \angle AEC = 55°.$

Hence, $\angle CBD = 55°$.

16.In the given figure(ix), O is the centre of the circle and arc ABC subtends an angle of 130°at the centre . If AB is extended to P, find ∠PBC .

Reflex $\angle AOC + \angle AOC = 360°$
$\Rightarrow$ Reflex $\angle AOC + 130° + x = 360°$
$\Rightarrow$ Reflex $\angle AOC = 360° - 130°$
$\Rightarrow$ Reflex $\angle AOC = 230°$

We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by it on the remaining part of the circle.

Here, arc AC subtends reflex $\angle AOC$ at the centre and $\angle ABC$ at B on the circle.

$\therefore \angle AOC = 2\angle ABC$
$\Rightarrow \angle ABC = \frac{230^\circ}{2} = 115^\circ$ ...(1)

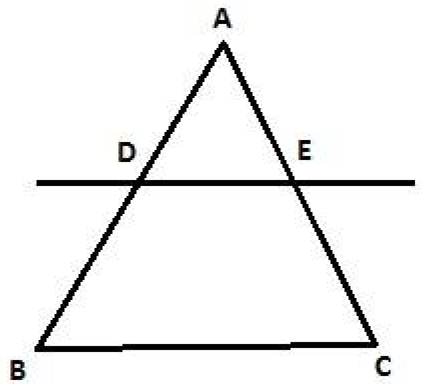
Since ABP is a straight line, $\angle ABC + \angle PBC = 180°$
$\Rightarrow \angle PBC = 180° - 115°$
$\Rightarrow \angle PBC = 65°$ ...(2)

Hence, $\angle PBC = 65°$.

17.In the given figure, O is the centre of the circle and ∠DAB=50°. Calculate the vales of x and y.

O is the centre of the circle and $\angle DAB = 50°$.
$OA = OB$ (Radii of a circle)
$\Rightarrow \angle OBA = \angle OAB = 50°$
In ΔOAB, we have:
$\angle OAB + \angle OBA + \angle AOB = 180°$
$\Rightarrow 50° + 50° + \angle AOB = 180°$
$\Rightarrow \angle AOB = (180° - 100°) = 80°$
Since AOD is a straight line, we have:
$\therefore x = 180° - \angle AOB$
$= (180° - 80°) = 100°$
i.e., $x = 100°$
The opposite angles of a cyclic quadrilateral are supplementary.
$ABCD$ is a cyclic quadrilateral.
Thus, $\angle DAB + \angle BCD = 180°$
$\angle BCD = (180° - 50°) = 130°$
$\therefore y = 130°$
Hence, $x = 100°$ and $y = 130°$

18.If a line drawn parallel to the base of an isosceles triangle to intersect its equal sides ,prove that quadrilateral so formed is cyclic.



Since ABC is an isosceles triangle with AB = AC and DE is parallel to BC.
So,

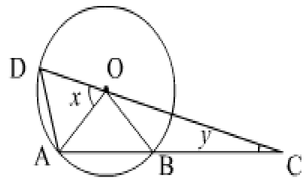
$\angle ADE = \angle ABC \quad \{corresponding\ angles\}$
$\angle ABC = \angle ACB \quad \{opp.\ angle\ of\ isosceles\ triangle\}$
$\Rightarrow \angle ADE = \angle ACB$

Now,

$\angle ADE + \angle EDB = 180^0$
$\angle ACB + \angle EDB = 180^0$

Thus the opposite angles of DECB are supplementary.
Hence DECB is a cylclic quadrilateral

19.In the given figure(x) , AB is a chord of a circle with centre O and AB is produced to C such that BC=OB. Also CO is a joined and produced to meet the circle in D. If ∠ACD= y° and ∠AOD =x°, prove that x=3y.



O is the centre of the circle. AB is the chord of the circle. AB is produced to C such that OB = BC. CO produced intersects the circle in D.

$\angle OCB = y$ and $\angle AOD = x$.

In ΔOBC,

OB = BC (Given)

$\therefore \angle OCB = \angle BOC$ (Equal sides have equal angles opposite to them)

$\Rightarrow \angle BOC = y$

$\angle OBA = \angle BOC + \angle OCB$ (Exterior angle of a triangle is equal to sum of its interior opposite angles)

$\therefore \angle OBA = y + y = 2y$ ...(1)

In ΔAOB,

OA = OB (Radius of the circle)

$\therefore \angle OBA = \angle OAB$ (Equal sides have equal angles opposite to them)

$\Rightarrow \angle OAB = 2y$ [Using (1)]

In ΔAOC,

$\angle AOD = \angle OAC + \angle OCA$ (Exterior angle of a triangle is equal to sum of its interior opposite angles)

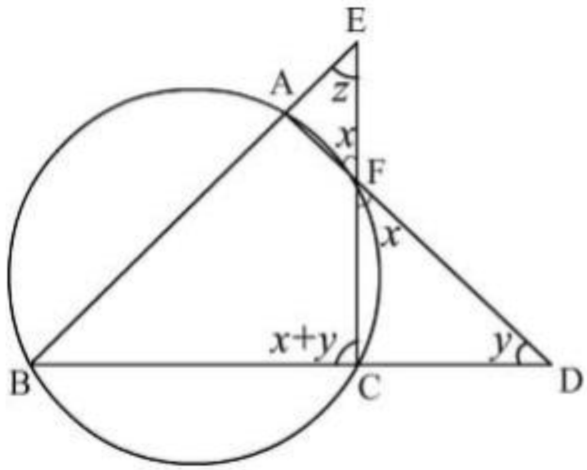
$\therefore x = 2y + y$ [

$\because$

$\angle OAC = \angle OAB$]

$\Rightarrow x = 3y$

20.In the given figure(xi) ,if y = 32° and z = 40° , determine x. If y+z =90° , prove that x=45°.



$\angle FCB$ is the exterior angle of ΔFCD

$\therefore \angle FCB = x + y \quad ...(i)$ $\left[\begin{array}{l}\text{Exterior angle is equal to the} \\ \text{sum of remote interior angles}\end{array}\right]$

$\angle CFD = \angle AFE = x \quad ...(ii)$ [Vertically opposite angles]

$\angle FAB$ is the exterior angle of ΔEAF

$\therefore \angle FAB = z + x \quad ...(iii)$ [From (ii)]

Quadrilateral ABCF is cyclic.

$\therefore \angle FCB + \angle FAB = 180^\circ$ $\left[\begin{array}{l}\text{Opposite angles of a cyclic} \\ \text{quadrilateral are supplementary}\end{array}\right]$

$\therefore (x + y) + (z + x) = 180^\circ$

$\therefore 2x + y + z = 180^\circ \quad ...(iv)$

If y =32° and z= 40° then

$$2x + 32 + 40 = 180$$

$$\therefore x = 54^\circ$$

(ii) $y + z = 90^\circ$

$\Rightarrow 2x + 90^\circ = 180^\circ \quad ...$[From (iv)]

$\therefore 2x = 180^\circ - 90^\circ$

$\therefore 2x = 90^\circ$

$\therefore x = 45^\circ$

Hence proved