DELHI PUBLIC SCHOOL BANGALORE - EAST

MATHEMATICS

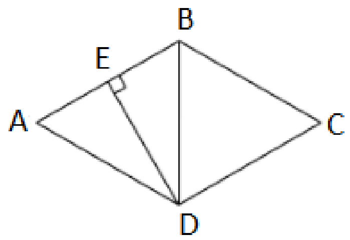
QUADRILATERALS- WORKSHEET

NAME:_______________ CLASS: IX SEC:_____ DATE: ___________

ANSWERS FOR QUADRILATERAL

1. (b) 20
2. (a) square
3. (c) 72
4. (c) rectangle
5. (c) 80
6. rectangle
7. square
8. 45

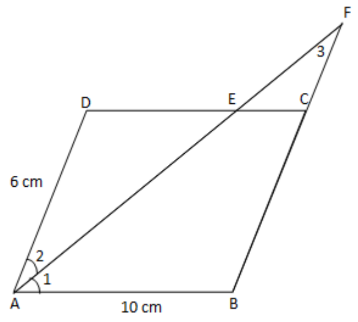
9) ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.



In ΔAED and ΔBED,
$DE = DE$ (common)
$\angle AED = \angle BED$ (each $90°$)
$AE = BE$ (given)
$\Rightarrow \Delta AED \cong \Delta BED$ (SAS congruence)
$\Rightarrow AD = BD$ (CPCT)
But, $AD = AB$ (sides of a rhombus are equal)
$\Rightarrow AD = AB = BD$
$\Rightarrow \Delta ABD$ is an equilateral triangle.
$\Rightarrow \angle A = 60°$
$\Rightarrow \angle A = \angle C = 60°$ (opposite angles of a rhombus aer equal)
Now, $\angle ABC + \angle BCD = 180°$ (adjacent angles are supplementary)
$\Rightarrow \angle ABC + 60° = 180°$
$\Rightarrow \angle ABC = 120°$
Now, $\angle ABC = \angle ADC = 120°$ (opposite angles of a rhombus are equal)
Thus, we have
$\angle A = 60°$ and $\angle B = 120°$

$\angle C = 60^0$ $\angle D = 120^0$

10)In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of $\angle A$ meets DC at E. AE and BC produced meet at F. Find the length of CF.



Since AE is the bisector of $\angle A$*, we have*

$\angle 1 = \angle 2$ ....(1)

ABCD is a parallelogram,

$\Rightarrow AD \parallel BC$ *i.e.,* $AD \parallel BF$

$\Rightarrow \angle 2 = \angle 3$ ....(*alternate angles*)

$\Rightarrow \angle 1 = \angle 3$ ....[*From* (1)]

In $\triangle ABF$, $\angle 1 = \angle 3$

$\Rightarrow AB = BF$ ....(*Sides opposite to equal angles are equal*)

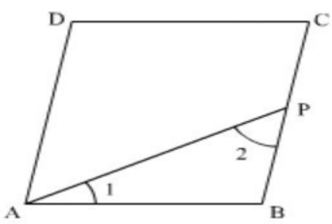
$\Rightarrow 10 = BC + CF$

$\Rightarrow 10 = AD + CF$ ....($AD = BC$ ....*Opposite sides of a parallelogram*)

$\Rightarrow 10 = 6 + CF$

$\Rightarrow CF = 10 - 6 = 4$ *cm*

11) In the adjoining figure, P is the midpoint of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$. Prove that AD = 2 CD.



Given In a parallelogram *ABCD*, *P* is a mid-point of *BC* such that $\angle BAP = \angle DAP$.

To prove $AD = 2CD$

Proof Since, *ABCD* is a parallelogram.

So, $AD \parallel BC$ and *AB* is transversal, then

$\angle A + \angle B = 180°$ [sum of cointerior angles is 180°]

$\Rightarrow \angle B = 180° - \angle A$ ...(i)

In ΔABP, $\angle PAB + \angle B + \angle BPA = 180°$ [by angle sum property of a triangle]

$\Rightarrow \frac{1}{2}\angle A + 180° - \angle A + \angle BPA = 180°$ [from Eq. (i)]

$\Rightarrow \angle BPA - \frac{\angle A}{2} = 0$

$\Rightarrow \angle BPA = \frac{\angle A}{2}$ ...(ii)

$\Rightarrow \angle BPA = \angle BAP$

$\Rightarrow AB = BP$ [opposite sides of equal angles are equal]

On multiplying both sides by 2, we get

$2AB = 2BP$

$\Rightarrow 2AB = BC$ [since *P* is the mid-point of *BC*]

$\Rightarrow 2CD = AD$

[since, *ABCD* is a parallelogram, then $AB = CD$ and $BC = AD$]

12)In a rectangle ABCD, the diagonals intersect at O. If $\angle OAB = 30^\circ$, find

a) $\angle ACB$ b) $\angle ABO$ c) $\angle COD$ d) $\angle BOC$.

. In $\triangle ABC$,

$\Rightarrow \angle CAB + \angle ABC + \angle ACB = 180^0$.

$\Rightarrow 30^0 + 90^0 + \angle ACB = 180^0$.

$\Rightarrow 120^0 + \angle ACB = 180^0$.

$\therefore \angle ACB = 60^0$

We know that, diagonals of rectangle are equal and bisect each other equally.

$\therefore AO = OC = BO = OD$

In $\triangle ABO$,

$\Rightarrow AO = BO$

$\Rightarrow \angle OAB = \angle ABO$ [Angle opposite to equal side are also equal]

$\Rightarrow \angle OAB = \angle ABO = 30^0$

$\Rightarrow \angle OAB + \angle ABO + \angle BOA = 180^0$

$\Rightarrow 30^0 + 30^0 + \angle BOA = 180^0$.

$\Rightarrow \angle BOA = 120^0$.

$\Rightarrow \angle BOA = \angle COD$ [Vertically opposite angle]

$\therefore \angle COD = 120^0$

$\Rightarrow \angle COD + \angle BOC = 180^0$ [Linear pair]

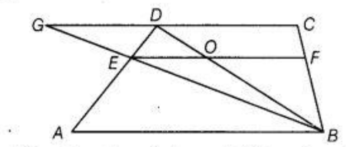
$\Rightarrow 120^0 + \angle BOC = 180^0$

$\therefore \angle BOC = 60^0$.

$\Rightarrow$ $\angle ACB=60^0, \angle ABO=30^0, \angle COD=120^0$ and $\angle BOC=60^0$.

13)E and F are the mid-points of non-parallel sides AD and BC of a trapezium ABCD. Prove that EF $\parallel$ AB and EF= ½ (AB + CD).

Given *ABCD* is a trapezium in which $AB \parallel CD$. Also, *E* and *F* are respectively the mid-points of sides *AD* and *BC*.



Construction Join *BE* and produce it to meet *CD* produced at *G*, also draw *BD* which intersects *EF* at *O*.

To prove $EF \parallel AB$ and $EF = \frac{1}{2}(AB + CD)$.

Proof In ΔGCB, *E* and *F* are respectively the mid-points of *BG* and *BC*, then by mid-point theorem,

$$EF \parallel GC$$

But $GC \parallel AB$ or $CD \parallel AB$ [given]

$\therefore$ $EF \parallel AB$

In ΔADB, $AB \parallel EO$ and *E* is the mid-point of *AD*.

Therefore by converse of mid-point theorem, *O* is mid-point of *BD*.

Also, $EO = \frac{1}{2} AB$ ...(i)

In ΔBDC, $OF \parallel CD$ and *O* is the mid-point of *BD*.

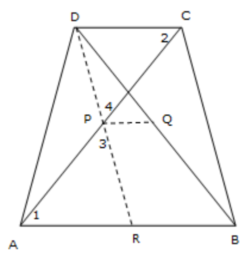
$\therefore$ $OF = \frac{1}{2} CD$ [by converse of mid-point theorem]...(ii)

On adding Eqs. (i) and (ii), we get

$$EO + OF = \frac{1}{2} AB + \frac{1}{2} CD$$

$\Rightarrow$ $EF = \frac{1}{2}(AB + CD)$ **Hence proved.**

14)Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium and is equal to half the difference of the parallel sides.



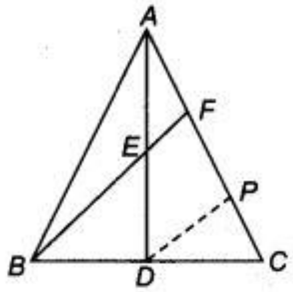
Given:
A trapezium ABCD in which AB $\parallel$ *DC and P and Q are the mid points of diagonals AC and BD respectively.*
To prove:
1)$PQ \parallel AB$ *or DC*
2) $PQ = \frac{1}{2}(AB - DC)$
Proof:
Since AB $\parallel$ *DC and transversal AC cuts them at A and C respectively,*
$\angle 1 = \angle 2$ ....(1)(*Alternate angles*)
Now, in traingles APR and DPC,
$\angle 1 = \angle 2$ ....(*From* (1))
$AP = CP$ ...(*P is mid − point of AC*)
$\angle 3 = \angle 4$ ...(*vertically opposite angles*)

$\Rightarrow \triangle APR \cong \triangle DPC$ by ASA criterion
$\Rightarrow AR = DC$ and $PR = DP$ ...(CPCT)
In $\triangle DRB$, P and Q are mid-points of sides DR and DB respectively.
$\Rightarrow PQ \parallel RB$
$\Rightarrow PQ \parallel AB$
$\Rightarrow PQ \parallel AB$ and DC
Agaian, P and Q are mid-points of sides DR and RB respectively in $\triangle DRB$.
$\Rightarrow PQ = \frac{1}{2} RB$
$\Rightarrow PQ = \frac{1}{2}(AB - AR)$
$\Rightarrow PQ = \frac{1}{2}(AB - DC)$

15) E is the mid-point of the median AD of a triangle ABC and BE is produced to meet AC at F. Show that AF= 1/3 AC

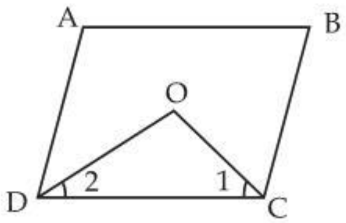
. According to question median AD of ΔABC and BE is produced to meet AC at

Given In a ΔABC, AD is a median and E is the mid-point of AD.
Construction Draw $DP \parallel EF$.
Proof In ΔADP, E is the mid-point of AD and $EF \parallel DP$.
So, F is mid-point of AP. [by converse of mid-point theorem]



In ΔFBC, D is mid-point of BC and $DP \parallel BF$.
So, P is mid-point of FC.
Thus, $AF = FP = PC$
$\therefore$ $AF = \frac{1}{3} AC$ **Hence proved.**

16) In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angle.



$\angle ADC + \angle BCD = 180^\circ$
$\frac{1}{2}\angle ADC + \frac{1}{2}\angle BCD = \frac{1}{2} \times 180^\circ$
$\Rightarrow \angle 1 + \angle 2 = 90^\circ$

In $\Delta ODC \rightarrow$
$\angle 1 + \angle 2 + \angle DOC = 180^\circ$
$\angle DOC = 90^\circ$

17) In the given figure (i),ΔABC is right angled at B. Given that AB=9cm,AC-=15cm and D aqnd E are the mid points of the sides AB and AC respectively, calculate (i) length of BC (ii) ar(ΔADE).

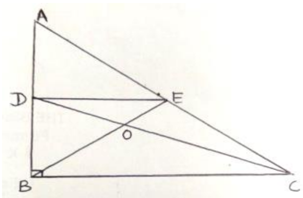


figure (i)

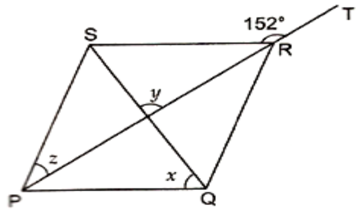


figure (ii)

In right ΔABC, $\angle B = 90°$

By using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad 15^2 = 9^2 + BC^2$

$\Rightarrow \quad BC = \sqrt{15^2 - 9^2}$

$\Rightarrow \quad BC = \sqrt{225 - 81}$

$\Rightarrow \quad BC = \sqrt{144}$

$= 12cm$

In ΔABC

D and E are midpoints of AB and AC

$\therefore DE \parallel BC$, $DE = \frac{1}{2} BC$ [By midpoint theorem]

AD = OB = $\frac{AB}{2} = \frac{9}{2}$ = 4 . 5cm [$\because$ D is the midpoint of AB]

DE = $\frac{BC}{2} = \frac{12}{2}$ = 6cm

Area of ΔADE = $\frac{1}{2} \times AD \times DE$

$= \frac{1}{2} \times 4.5 \times 6 = 13.5cm^2$

18) M,N and P are the mid points of AB,AC and BC respectively. If MN=3 cm,NP=3.5cm and MP=2.5 cm, calculate the perimeter of ΔABC.

In ΔABC

M and N are midpoints of AB and AC

$\therefore MN = \frac{1}{2} BC$, $MN \parallel BC$ [By midpoint theorem]

$\Rightarrow 3 = \frac{1}{2} BC$

$\Rightarrow 3 \times 2 = BC$

$\Rightarrow BC = 6cm$

Similarly

$AC = 2MP = 2\,(2.5) = 5cm$

$AB = 2NP = 2(3.5) = 7cm$

Perimeter ABC=18cm

19) In the given figure(ii), PQRS is a rhombus in which the diagonal PR is produced to T. If $\angle SRT = 152^\circ$, find x, y , z.

We know that in a rhombus, the diagonals bisect each other at right angle.

Therefore,

$y = 90^\circ$

Now,

$\angle 1 + \angle SRT = 180^\circ$

$\angle 1 + 152^\circ = 180^\circ$

$\angle 1 = 28^\circ$

In ΔSOR, by angle sum property of a triangle, we get:

$\angle 1 + y + \angle OSR = 180^\circ$

$28^\circ + 90^\circ + \angle OSR = 180^\circ$

$118^\circ + \angle OSR = 180^\circ$

$\angle OSR = 62^\circ$

Or, $\angle QSR = 62^\circ$ (Because *O* lies on *SQ*)

We have, SP || PQ .Thus the alternate interior opposite angles must be equal.

Therefore,

$x = \angle QSR$

$x = 62^\circ$

In ΔSPR,we have

Since opposite sides of a rhombus are equal.

Therefore,

PS = SR

Also,

Angles opposite to equal sides are equal.

Thus,

$z = \angle 1$

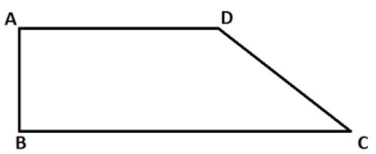
But $\angle 1 = 28^\circ$

Thus, $z = 20^\circ$

Hence the required values for *x,y* and *z* are $62^\circ, 90^\circ$ and 28° respectively.

20) In a trapezium ABCD,AD||BC, $\angle A = x + 20^\circ, \angle B = y, \angle C = 92^\circ, \angle D = 2x + 10^\circ$. Find x *and* y.

$\angle C + \angle D = 180^0$ (co-interior angles)



$2x + 10 + 92^0 = 180^0$ (co-interior angles)

$X = 39^0$

$\angle A + \angle B = 180^0$ (co-interior angles)

$x + 20^0 + y^0 = 180^0$

$y = 121^0$

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