DELHI PUBLIC SCHOOL BANGALORE - EAST

MATHEMATICS

HERON'S FORMULA - WORKSHEET

NAME:_______________ CLASS: IX SEC:_____ DATE: ___________

Choose the correct answer in each of the following questions:

1. The area of a triangle is 150 cm^2 and its sides are in the ratio 3 : 4 : 5. What is its perimeter?
   a. 10 cm b. 30 cm c. 45 cmd. 60 cm

   Ans: d

2. What in the area of an equilateral triangle with side 2 cm?
   a. $\sqrt{6}cm^2$ b. $\sqrt{3}cm^2$ c. $\sqrt{8}cm^2$ d. $4cm^2$
   Ans: b

3. What is the length of each side of an equilateral triangle having an area of $4\sqrt{3}$ cm^2 ?
   a. 4cm b. 5cm c. 7cm d. 6cm
   Ans: a

4. Length of one of the equal sides of an isosceles triangle is 4 cm. If its base is 2 cm then what is its area?
   a. $\sqrt{15}cm^2$b. $\sqrt{13}cm^2$c. $\sqrt{12}cm^2$ d. $\sqrt{14}cm^2$
   Ans: a

5. The sides of a triangle are 15 cm, 17 cm and 8 cm. What is its area?
   a. $20cm^2$ b. $40cm^2$ c. $60cm^2$ d. $80cm^2$
   Ans : c

Fill in the blanks :

6. The sides of a triangle are in the ratio of 3 : 4 : 5. If its perimeter is 36 cm, then area is **54 cm^2** .

7. The cost of levelling the ground in the form of triangle having sides 51m,37m,20 m at the rate of Rs. 3 per m^2 is **Rs 918.**
8. If the perimeter of an equilateral triangle is 60 cm, then area is **$100\sqrt{3}$ cm^2.**

Solve the following:

9. The perimeter of a triangle is 240cm. If two of its sides are 78cm and 50cm. Find the length of perpendicular on the sides of length 50cm from opposite vertex.

   Here, a = 78 cm
   b = 50 cm
   c = ?
   perimeter = 240 cm
   so, s = 240/2 = 120
   also, perimeter = a + b + c
   or 78 + 50 + c = 240
   or c = 240 - 128 = 112 cm

so, area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{120\ (120\ -\ 78)\ (120\ -\ 50)(120\ -\ 112)}$

= $\sqrt{120 \text{ x } 42 \text{ x } 70 \text{ x } 8}$

= $\sqrt{6 \text{ x } 2 \text{ x } 10 \text{ x } 6 \text{ x } 7 \text{ x } 7 \text{ x } 10 \text{ x } 4 \text{ x } 2}$

= 6 x 2 x 10 x 7 x 2

= 1680 cm^2

again, area of the triangle = $\frac{1}{2}$ x base x height

or $\frac{1}{2}$ x 50 x H = 1680
or H = 1680 / 25 = 67.2 cm

10. The perimeter of a triangular field is 300 cm and its sides are in the ratio 5 : 12 : 13. Find the length of the perpendicular from the opposite vertex to the side whose length is 130 cm.

Given, perimeter of a triangle = 300 cm and ratio of the sides be 5 : 12 : 13.
Let the sides of the triangle are *x* cm.
∴ $5x + 12x + 13x = 300$
⇒ $30x = 300$
⇒ $x = 10$ cm
∴ Sides of the triangle are 50 cm, 120 cm and 130 cm.

So,semi-perimeter, $S = \frac{50+120+130}{2} = 150\text{cm}$

$$\begin{aligned} \text{Now, Area of}\Delta &= \sqrt{S(S-a)(S-b)(S-c)} \\ &= \sqrt{150(150-50)(150-120)(150-130)} \\ &= \sqrt{150\times100\times30\times20} \\ &= 3000 \text{ cm}^2 \end{aligned}$$

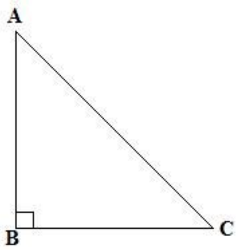
Also, Area of $\Delta = \frac{1}{2}\times\text{base}\times\text{height}$

$\Rightarrow 3000 = \frac{1}{2}\times130\times h$ [Base $=130$ cm, given]

$\Rightarrow h = \frac{3000\times2}{130} = \frac{600}{13}$ cm

Hence, the length of the perpendicular from the opposite vertex to the side whose length is 130 is $\frac{600}{13}$ cm. or 46.15 cm

11. The difference between the sides containing a right angle in a right angled triangle is 14cm.The area of a triangle is 120sq cm. Calculate the perimeter of the triangle.



Let the sides containing the right angle be x cm and (x - 14)cm

The, its area = [1/2 x (x - 14)]cm^2.

But, area = 120cm^2 [Given]

$$\therefore \quad \frac{1}{2} x (x - 14) = 120$$

$$\Rightarrow \quad x^2 - 14x - 240 = 0$$

$$\Rightarrow \quad x^2 - 24x + 10x - 240 = 0$$

$$\Rightarrow \quad x(x - 24) + 10 (x - 24) = 0$$

$$\Rightarrow \quad (x - 24) (x + 10) = 0$$

$$\Rightarrow \quad x = 24 \qquad [\text{Neglecting } x = -10]$$

∴ one side = 24cm, other side = (24 - 14)cm = 10cm

$$\text{Hypotenuse} \quad = \sqrt{(24)^2 + (10)^2} \text{ cm}$$

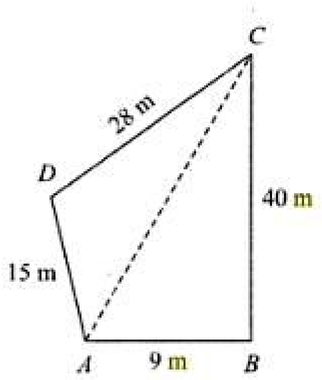
$$= \sqrt{576 + 100} \text{ cm}$$

$$= \sqrt{676} \text{ cm}$$

$$= 26 \text{ cm.}$$

∴ Perimeter of the triangle = (24 + 10 + 26)cm = 60cm.

12. Find the area of a quadrilateral whose sides in metres are 9,40,28 & 15respectively and the angle between first two sides is a right angle.



In quad. ABCD,
AB = 9m, BC = 40m,
CD= 28m, AD = 15m
$\angle ABC = 90°$

Now in ΔABC,

$$AC^2 = AB^2 + BC^2$$
$$= (9)^2 + (40)^2$$
$$= 81 + 1600$$
$$= 1681$$
$$AC = \sqrt{1681} = 41 \text{ m}$$

∴Area of quad.ABCD = Area of ΔABC+Area of ΔACD

$$= \frac{1}{2} \times \text{base} \times \text{height} \quad + \quad \sqrt{S(S-a)(S-b)(S-c)}$$
$$= \frac{1}{2} \times 9 \times 40 \quad + \quad \sqrt{42(42-15)(42-28)(42-41)}$$
$$= 180 \text{ m}^2 \quad + \quad \sqrt{42 \times 27 \times 14 \times 1}$$
$$= 180 \text{ m}^2 \quad + \quad 126 \text{ m}^2$$
$$= 306 \text{ m}^2$$

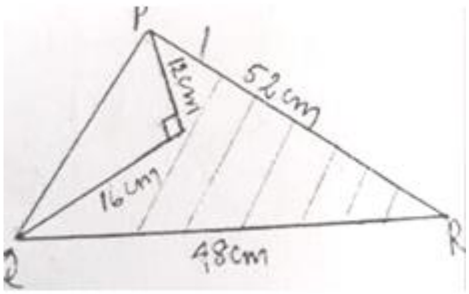
$\because$ Sides are 15m, 28 m and 41 m

$\therefore \quad S = \frac{15+28+41}{2}$

$= 42$ m

Thus, the area of quadrilateral ABCD$= 306 \text{ m}^2$

13. In the following figure calculate the area of the shaded portion.



In right triangle PSQ,
$PQ^2 = PS^2 + QS^2$

| By Pythagoras Theorem
$= (12)^2 + (16)^2$
$= 144 + 256 = 400$

$\Rightarrow \quad PQ = \sqrt{400} = 20$ cm

New, for $\triangle PQR$,

a = 20 cm
b = 48 cm
c = 52 cm

$\therefore \quad s = \dfrac{a+b+c}{2}$

$= \dfrac{20+48+52}{2} = 60$ cm

$\therefore$ Area of $\triangle PQR$

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-48)(60-52)}$

$= \sqrt{60(40)(12)(8)}$

$= \sqrt{(6\times10)(4\times10)(6\times2)(8)}$

$= 6\times10\times8 = 480\ cm^2$

Area of $\triangle PSQ = \dfrac{1}{2}$ x Base x Altitude

$= \dfrac{1}{2}$ x 16 x 12 = 96 cm^2

$\therefore$ Area of the shaded portion

= Area of ΔPQR – Area of ΔPSQ

= 480 – 96 = 384 cm^2.

14. The difference of semi-perimeter and the sides of a triangle are 8cm, 7cm & 5cm respectively.

Find the sides of the triangle.

Let the sides of the triangle be *a*, *b* and *c* and the semi perimeter (*s*) = $\dfrac{a+b+c}{2}$

Given : $s - a = 8$cm ...... (1)

$s - b = 7$cm ...... (2)

$s - c = 5$cm ...... (3)

Adding (1), (2), (3) we get

$3s - (a+b+c) = 20\text{cm}$

$\Rightarrow 3s - 2s = 20\text{cm}$

$\Rightarrow s = 20\text{cm}$

a= 12cm, b= 13cm and c= 15 cm

15. Find the area of a cyclic quadrilateral whose sides are 7 cm, 5 cm, 4 cm and 10 cm.

We use Brahmagupta's formula to find the area of the cyclic quadrilateral.

$A = \sqrt{(s-a)\cdot(s-b)\cdot(s-c)\cdot(s-d)}$ where semiperimeter $= s = \dfrac{a+b+c+d}{2}$,

and

a=5, b=4, c=7, and d=10

$s = \dfrac{a+b+c+d}{2} = \dfrac{5+4+7+10}{2} = \dfrac{26}{2} = 13$ }}}

$A = \sqrt{(s-a)\cdot(s-b)\cdot(s-c)\cdot(s-d)} = \sqrt{(13-5)\cdot(13-4)\cdot(13-7)\cdot(13-10)} = \sqrt{8\cdot9\cdot6\cdot3} = \sqrt{1296} = 36$ cm^2

16.The perimeter of a right triangle is 24 cm. If its hypotenuse is 10 cm, find the other two sides. Find its area by using the formula of area of a right triangle. Verify your result by using Heron’s formula.

Since hypotenuse is 10 cm . By using the concept of triplets we can guess the other two sides of right triangle i.e. 6 cm and 8cm

Checking if triplet 6,8 and 10 cm satisfies the Pythagoras theorem

$10^2 = 100 = 36 + 64 = 6^2 + 8^2$

Area of triangle $=\frac{1}{2}\times 6\times 8=24\text{cm}^2$

Verification

Perimeter=24

semi perimeter(s)= $\frac{24}{2}=12$

Area of triangle by heron's formula=$\sqrt{s(s-a)(s-b)(s-c)}$

$$=\sqrt{12(12-10)(12-8)(12-6)}$$

$$=\sqrt{12\times 2\times 4\times 6}$$

$$=24\text{cm}^2$$

17. Find the cost of turfing a triangular field at the rate of Rs. 5/m^2 having lengths of its sides as 40 m, 70 m and 90 m. (Take √20 = 4.47)

$Given\ sides\ of\ the\ triangle\ are\ 40m, 70m\ and\ 90m.$

$Perimeter=40+70+90=200m$

$\Rightarrow 2s=200$

$\Rightarrow s=100$

$Area\ of\ triangle=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow Area\ of\ triangle=\sqrt{100(100-40)(100-70)(100-90)}$

$\Rightarrow Area\ of\ triangle=\sqrt{100(60)(30)(10)}$

$\Rightarrow Area\ of\ triangle=\sqrt{10^2\times(10\times 3\times 2)\times(3\times 10)\times 10}=\sqrt{3^2\times 2\times 10^5}=3\times 2\times 10^2\sqrt{5}$

$\Rightarrow Area\ of\ triangle=600\sqrt{5}\ m^2$

$And,$

$Cost\ of\ turfing\ the\ field\ at\ Rs\ 5\ per\ m^2=(Area\ of\ triangle)\times(Rate\ per\ sq.m)$

$=600\sqrt{5}\times 5$

$=3000\times 2.236$

$=6708$

$Hence,\ Cost\ of\ turfing\ the\ field\ at\ Rs\ 5\ per\ m^2\ is\ Rs\ 6708$

18. Find the area of an isosceles triangle each of whose equal sides is 13 cm and whose base is 24 cm.

equal side = 13 cm

base= 24 cm

semi perimeter = 24 + 13+13 cm /2

=50 /2 cm

= 25cm

area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{25\ (25\ -\ 24)\ (25\ -\ 13)(25\ -\ 13)}$

$= \sqrt{25 \text{ x } 1 \text{ x } 12 \text{ x } 12}$

$= 5 \text{ x } 12 \text{ cm}^2$

$= 60 \text{ cm}^2$

19.The height of an equilateral triangle is 6cm. Find the area of the triangle. ($\sqrt{3}$ = 1.732)

Let each side of the given triangle be a cm.

Then, its height $= \left(\frac{\sqrt{3}}{2} \times a\right) cm.$

$\therefore \quad \frac{\sqrt{3}}{2} \times \text{a cm} = 6 \text{ cm} \Rightarrow a = \left(\frac{6 \times 2}{\sqrt{3}}\right)$

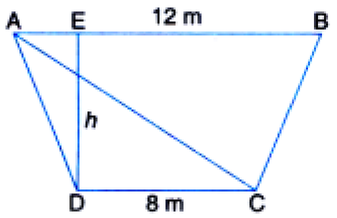
$\Rightarrow \quad \text{side} = 4\sqrt{3} \text{ cm.}$

$\therefore$ area of the given triangle

$= \left(\frac{\sqrt{3}}{4} \times a^2\right) \text{sq units} = \left\{\frac{\sqrt{3}}{4} \times (4\sqrt{3})^2\right\} cm^2$

$= \left(\frac{\sqrt{3}}{4} \times 48\right) cm^2 = 12\sqrt{3} cm^2.$

20.The cross-section of a canal is in the shape of a trapezium. If the canal is 12m wide at the top and 8m wide at the bottom and the area of its cross-section is 84 m^2, determine its depth.



$\Rightarrow$ Area of $\triangle ABC$+ Area of $\triangle ADC$= 84 m^2

$\Rightarrow \frac{1}{2}(AB)(DE) + \frac{1}{2}(DC)(DE) = 84$

$\Rightarrow \frac{1}{2}(12)(h) + \frac{1}{2}(8)(h) = 84$

$\Rightarrow 6h + 4h = 84$

$\Rightarrow 10h = 84$

$\Rightarrow h = \frac{84}{10} = 8.4$

Ans: height =8.4 m

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