DELHI PUBLIC SCHOOL BANGALORE - EAST

MATHEMATICS- ANSWER KEY

POLYNOMIALS - WORKSHEET

1. d
2. b
3. b
4. c
5. d
6. $\frac{2}{5}$
7. Zero
8. -1
9. Since $(x^2 + kx - 3) = (x - 3)(x + 1)$ then (x +1)and (x- 3) are the factors ,

   ∴ it implies, g(x)=x+1=0
   ⇒ x = -1
   ⇒p(-1) =0
   ⇒ $(-1)^2$ + k(-1) -3 =0
   ⇒ k = -2

10.

Let $p(z) = az^3 + 4z^2 + 3z - 4$ and $q(z) = z^3 - 4z + a$

When $p(z)$ is divided by $z-3$ the remainder is given by,

$$p(3) = a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4 = 27a + 36 + 9 - 4$$

$$p(3) = 27a + 41 \qquad ...(i)$$

When $q(z)$ is divided by $z-3$ the remainder is given by,

$$q(3) = 3^3 - 4 \times 3 + a = 27 - 12 + a$$

$$q(3) = 15 + a \qquad ...(ii)$$

According to question, $p(3) = q(3)$

$\Rightarrow \quad 27a + 41 = 15 + a \quad \Rightarrow \quad 27a - a = -41 + 15$

$26a = -26$

$\Rightarrow \quad a = \frac{-26}{26} \quad \Rightarrow \quad a = -1$

11. $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$, $g(x) = x+1$ leaves a remainder 19,

$\Rightarrow p(-1) = 19 \Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7 = 19$

$\Rightarrow 1 + 2 + 3 + a + 3a - 7 = 19$

$\Rightarrow 6 + 4a - 7 = 19$

$\Rightarrow a = \frac{20}{4} \Rightarrow a = 5$ and hence $p(x) = x^4 - 2x^3 + 3x^2 - 5x + 3(5) - 7 \Rightarrow p(x) = x^4 - 2x^3 + 3x^2 - 5x + 15 - 7$

$\Rightarrow p(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$, so when divided by $x+2$

$\Rightarrow p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8 \Rightarrow p(-2) = 16 + 16 + 12 + 10 + 8 = 62$

12.

Let $f(x) = px^2 + 5x + r$

Since, $x-2$ is a factor of $f(x)$, then $f(2) = 0$

$\therefore \quad p(2)^2 + 5(2) + r = 0$

$\Rightarrow \quad 4p + 10 + r = 0 \qquad ...(i)$

Since, $x - \frac{1}{2}$ is a factor of $f(x)$, then $f\left(\frac{1}{2}\right) = 0$

$\therefore \quad p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$

$\Rightarrow \quad p \times \frac{1}{4} + \frac{5}{2} + r = 0$

$\Rightarrow \quad p + 10 + 4r = 0 \qquad ...(ii)$

Since, $x-2$ and $x - \frac{1}{2}$ are factors of $f(x) = px^2 + 5x + r$.

From Eqs. (i) and (ii), $4p + 10 + r = p + 10 + 4r \Rightarrow 3p = 3r$

$\therefore \quad p = r$

13. First of all,

x^2-3x+2

$\Rightarrow x^2-2x-1x+2$

$\Rightarrow x(x-2)-1(x-2)$

$\Rightarrow$ (x-2)(x-1)

Therefore,(x-2)(x-1)are the factors.

i) (x-2)

x-2=0

$\Rightarrow$x=2

So,p(x)=2

p(x)= $2x^4 - 5x^3 + 2x^2 - x + 2$

p(2)= $2(2)^4 - 5(2)^3 + 2(2)^2 - 2 + 2$

=32-40+8

= -40+40=0

Hence, it proves that (x-2) is a factor .

ii) (x-1)

$\Rightarrow$x-1=0

$\Rightarrow$x=1

So,p(x)=1

p(x)= $2x^4 - 5x^3 + 2x^2 - x + 2$

p(1)= $2(1)^4 - 5(1)^3 + 2(1)^2 - 1 + 2$

=2-5+2-1+2

=6-6=0

Hence, it proves that (x-1) is a factor.

14.

Using cubic identity,

$(a-b)^3 = a^3 + b^3 - 3ab(a-b)$

Applying identity in $(2x-5y)^3$

$(2x-5y)^3 = (2x)^3 - (5y)^3 - 3(2x)(5y)(2x-5y)$

Applying identity in $(2x+5y)^3$

$(2x+5y)^3 = (2x)^3 + (5y)^3 + 3(2x)(5y)(2x+5y)$

Substitute in expression,

$(2x-5y)^3 - (2x+5y)^3$

$= (2x)^3 - (5y)^3 - 3(2x)(5y)(2x-5y) - (2x)^3 - (5y)^3 - 3(2x)(5y)(2x+5y)$

$= -125y^3 - 30xy(2x-5y) - 125y^3 - 30xy(2x+5y)$

$= -250y^3 - 60x^2y + 150xy^2 - 60x^2y - 150xy^2$

$= -250y^3 - 120x^2y$

Therefore, $(2x-5y)^3 - (2x+5y)^3 = -250y^3 - 120x^2y$

15. $x^2 + 4y^2 + z^2 - 4xy - 2xz + 4yz$ Can be written as

$(-x)^2 + (2y)^2 + (z)^2 + 2(-x)(2y) + 2(2y)(z) + 2(z)(-x)$ which is in the form of

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$

$\Rightarrow (-x + 2y + z)^2$

16. If a, b, c are all non-zero and a + b+ c = 0, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

Multiplying by abc both sides

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (a+b)^3 - 3ab(a+b) + c^3 = 3abc$

as we know that a + b + c = 0

$\Rightarrow a + b = -c$

$\Rightarrow (-c)^3 - 3ab(-c) + c^3 = 3abc$

$\Rightarrow -c^3 + 3abc + c^3 = 3abc \Rightarrow 3abc = 3abc \Rightarrow$ LHS = RHS

17. $(a + b + c)^2 = (5)^2$

$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$

$\Rightarrow a^2 + b^2 + c^2 = 25 - 2(10)$ $(\because ab + bc + ca = 10)$

$\Rightarrow a^2 + b^2 + c^2 = 25 - 20$

$\Rightarrow a^2 + b^2 + c^2 = 5$

Also, $a^3 + b^3 + c^3 - 3abc = (a + b + c)\{(a^2 + b^2 + c^2 - (ab + bc + ca)\}$

$\therefore a^3 + b^3 + c^3 - 3abc = (5)(5 - 10)$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 5 \times -5$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = -25$ (Hence Proved)

18. $30^3 + 20^3 - 50^3$.

Using the conditional identity,

If a + b+ c = 0, then $a^3 + b^3 + c^3 = 3abc$

$\because 30 + 20 - 50 = 0$

$\because 30^3 + 20^3 - 50^3 = 3(30)(20)(-50) = -90000$

19. $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

Using the conditional identity,

If a + b+ c = 0, then $a^3 + b^3 + c^3 = 3abc$

∵ $x - 2y + 2y - 3z + 3z - x$

∵ $(x-2y)^3 + (2y-3z)^3 + (3z-x)^3 = 3(x-2y)(2y-3z)(3z-x)$

20. The length and breadth of the rectangle whose area is given by

$4a^2 + 4a - 3$.

Further, factorising $4a^2 + 4a - 3$.

$\Rightarrow 4a^2 + 6a - 2a - 3$

$\Rightarrow 2a(2a+3) - 1(2a+3)$

$\Rightarrow (2a-1)(2a+3)$

$\Rightarrow$ length= $(2a-1)$ and breadth = $(2a+3)$

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