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"Endogenous Gentrification and Housing Price Dynamics"

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Endogenous Gentrification and Housing Price Dynamics

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Very Preliminary

Abstract

In this paper, we explore differential changes in house prices across neighborhoods within a city to better understand the nature of house price dynamics across cities. Using a variety of different data sources, we show that, during city-level house price booms, neighborhoods with lower initial prices experience larger price growth. This occurs in spite of the fact that the supply of housing is more elastic in these neighborhoods. In particular, we show that low price neighborhoods that directly abut high price neighborhoods appreciate (depreciate) the most during city-wide housing price booms (busts). To explain these facts, we then present a spatial equilibrium model of a city with rich and poor agents. The key ingredient of our model is a positive neighborhood externality: agents like to live in areas where more rich agents live. Also, rich agents are the ones who benefit the most from this externality. In equilibrium there is full segregation: the rich are concentrated in the city center and the poor live in the periphery of the city. In response to a demand shock (e.g. a decrease in the interest rate, an increase in city-wide income), rich households expand into adjacent poor neighborhoods. This is what we term "endogenous gentrification". As the rich move into a poor neighborhood, the land value increases due to the externality, driving house prices up. In addition, the model predicts that the city-wide responsiveness of house prices to a given demand shock will depend on the income distribution of the city. Richer cities respond more to the shock, even if housing supply is perfectly elastic, because they experience a higher degree of gentrification. We also show that the data are consistent with many additional predictions of our model. In particular, we find that the neighborhoods that experience the highest price increases show strong evidence of gentrification, and that the initial level of income and changes in income explain cross city differences in house price appreciation rates.

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1   Introduction

According to the Case-Shiller index, real residential property prices appreciated by over 80 percent for the United States as a whole between the late 1990s and the mid 2000s. Since that time, they have dropped by roughly 40 percent nationwide. However, the national data mask a large degree of heterogeneity at more localized levels. For example, real property prices increased by over 100 percent in Washington DC, Miami, and Los Angeles between 2000 and 2006 while property prices appreciated by less than 10 percent in Charlotte, Denver, and Detroit during the same time period. Despite the importance of changing housing prices in determining household consumption (Mian and Sufi (2009)) and borrower default (Foote, Gerardi, and Willen (2008)), relatively little work has been done to try to explain the nature of housing price dynamics *across cities*.[1] In this paper, we explore differential changes in house prices *across neighborhoods within a city* to shed light on the nature of house price dynamics across cities in reaction to housing demand shocks.

We start the paper by documenting a series of new stylized facts about the movement of house prices across neighborhoods within a city during city-wide housing price booms and busts. As far as we can tell, no one has ever systematically explored the movement of house prices across neighborhoods within a city for a large cross section of cities during many different time periods.[2] Using a variety of different data sources including the zip code level data that underlies the Case-Shiller indices, U.S. Census data, and deed data that we have compiled for certain cities, we find that three stable relationships emerge.

First, we find that initially low priced neighborhoods systematically experience higher appreciation rates during city-wide housing booms than do initially higher priced neighborhoods. For example, during $2000 - 2006$, initially low priced neighborhoods in New York (e.g., Harlem) appreciated at three times the rate of initially high priced neighborhoods in New York (e.g., Mid-town Manhattan). During periods of housing price busts, the results are the opposite. Initially low housing price neighborhoods are the ones that depreciate the most during a city-wide

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[1]Notable exceptions include Topel and Rosen (1988), Case and Shiller (1989), Glaeser, Gyourko, and Saks (2005a), Gyourko (2009), and Saiz (2009). In these papers, cross city differences in housing price dynamics are achieved – in part – by focusing on constraints to adjusting housing supply. These constraints take the form of either standard convex adjustment costs which slow down the speed of building, regulations which prevent building, or physical land barriers which also prevent building (such as the degree to which land within the city is sloped or the proximity of the city to natural barriers such as oceans or lakes). The model we present below will yield similar long run predictions without relying on city-wide supply restrictions. In section 6, we discuss how our work relates to this and other relevant literatures.

[2]A few papers exist that examine zip code level price movements within a city for a given time period. For example, Case and Mayer (1996) examine price movements by neighborhood in Boston between 1982 and 1992 while Case and Marynchenko (2002) look at within city price movements for Boston, Chicago, and Los Angeles during the 1983 to 1993 period. We discuss these papers in greater depth in section 6.

property price bust. Our results show that it is low housing price neighborhoods - not high housing price neighborhoods - that are the most price elastic during city-wide property price booms and busts.

Second, we find that the extent of the difference in appreciation (depreciation) rates between low and high price neighborhoods within the city is greater the larger the price appreciation (depreciation) rate for the city as a whole. For example, while lower price neighborhoods appreciated at three times the rate of higher priced neighborhoods in New York between 2000 and 2006, less expensive neighborhoods in Chicago appreciated at only twice the rate of more expensive neighborhoods and there was no difference in appreciation rates between high and low price neighborhoods in Charlotte during that time period. Between 2000 and 2006, the New York metro area as a whole experienced a 70 percent real increases in housing prices, while Chicago and Charlotte experienced real housing price increases of roughly 40 percent and 10 percent, respectively.

Lastly, while low property price neighborhoods are likely to appreciate (depreciate) more on average, there is a large degree of heterogeneity in the price responsiveness during property price booms (busts) among the low price neighborhoods. For example, some low price neighborhoods appreciate by a substantial amount during a city-wide price boom while others only appreciate slightly. With respect to observables, these two types of neighborhoods are very similar. However, we show that along one dimension, there is a substantial difference between the low price neighborhoods that do and do not experience a rapid increase in housing prices during a city-wide housing boom. Specifically, we show that it is the low price neighborhoods that are in close proximity to the high price neighborhoods that are the most price elastic.

In this section, we also document that these relationships between city-wide housing booms and the convergence of neighborhoods within a city are prominent features of the data in all time periods that we analyzed. In particular, we show that such results held during the recent property price boom as well as local property price booms that occurred during the 1980s and the local property price busts that occurred during the1990s. Finally, we show the patterns are robust regardless of the measure of housing prices we use.

In the next part of the paper, we present a spatial equilibrium model of property price movements across neighborhoods within a city to explain the above facts. There are two key features of our model. First, we analyze a linear city (in the spirit of Mills (1967) and Muth (1969)). Such an assumption is common in most spatial equilibrium models of a city.[3] Second, we model

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[3]For an exception, see Robert E. Lucas and Rossi-Hansberg (2002).

within cities positive neighborhood externalities.[4] In particular, we assume that individual utility increases as the income of the neighbors increase. Although, we do not explicitly model the direct mechanism for the externality, we have many potential channels in mind. For example, crime rates are lower in rich neighborhoods. If households value low crime, they will prefer to live amongst wealthier neighbors as opposed to poorer neighborhoods. Likewise, the quality and extent of public goods may be correlated with the income of neighborhood residences. For example, school quality - via peer effects, parental monitoring or direct expenditures - tends to increase with neighborhood income. Additionally, if there are increasing returns to scale in the production of desired neighborhood amenities (number and variety of restaurants, easier access to service industries such as dry cleaners, movie theaters, etc.) such amenities will be more common as the richness of one's neighbors increase. Although we do not take an official stand on which mechanism is driving the externality, our preference structure is general enough to allow for any story that makes higher income neighborhoods more desirable. The importance of urban density in facilitating local consumption externalities has been recently emphasized in the work of Glaeser, Kolko, and Saiz (2001) and in Becker and Murphy (2003).[5] Our innovation is to embed these consumption externalities into a model of neighborhood development within a city and show how such preferences affect the reaction of house prices to housing demand shocks both across various types of neighborhoods and at the aggregate level.

We propose a linear city populated by two types of households: a continuum of identical rich households and a continuum of identical poor households. As noted above, our key assumption is that utility is increasing in the number of rich neighbors. We further assume that compared to low income households, high income households have a stronger preference for richer neighbors. On the supply side, we assume a continuum of perfectly competitive firms who can build houses in any neighborhood. In equilibrium households optimally choose consumption, housing services, and their neighborhood.

We show that there is an equilibrium with full segregation, where the rich are concentrated all together and the poor live at the periphery of the city. Given the externality, households are willing to pay more to live closer to rich neighbors. Poor households who benefit less

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[4]Our externality is a pure consumption externality, different from productive externalities that are usually emphasized in spatial equilibrium models.

[5]Our work is closest to Becker and Murphy (2003). In their work, Becker and Murphy present a theoretical model that generates residential segregation by income. The mechanism in their model is identical to ours. They posit that individuals prefer to live around richer neighbors (and that richer individuals have a greater preference for richer neighbors). Our work differs from theirs by showing how such a mechanism yields different within city and cross city predictions about house price dynamics when there are aggregate shocks to housing demand. In particular, we show that aggregate shocks to housing demand (low interest rates, increase in skill premiums, increase in aggregate TFP) will endogenously lead to neighborhood gentrification. Lastly, we present many empirical facts that are consistent with the model's predictions.

from the externality are less willing to pay high rents to live in the rich neighborhoods, so in equilibrium they live at the periphery of the city. House prices achieve their maximum in the rich neighborhoods and decline as we move away from them, to compensate for the lower level of the externality.[6] At the margin of the city, there is no externality and house prices are equal to the marginal cost of construction.

Our model predicts that permanent shocks to housing demand lead to permanent increases in house prices within the city as a whole – even when the size of the city is completely elastic. In other words, our model generates cross city differences in house prices and house price appreciation rates that do not rely on traditional supply constraints (regulation that prevents building, the steepness of the land gradient, natural barriers, etc.). The main implication of this simple model is that a housing demand shock will cause the rich to expand into areas previously occupied by the poor. We refer to this phenomenon as gentrification. As this happens, house prices in rich neighborhoods are not affected, while house prices in gentrified neighborhoods are driven up due to the neighborhood externality. This generates our first fact: within a city, low price neighborhoods appreciate at a faster rate than high price neighborhoods. Moreover, this mechanism directly generates also our third fact: the low income neighborhoods directly abutting the rich neighborhoods are the ones that are more price elastic. This is due to the fact that, after a demand shock, rich households will expand in locations as close as possible to the rich neighborhoods, in order to enjoy the highest level of externality.

Furthermore, our model shows that aggregate house prices can respond differently in different cities even if supply constraints are not binding. We show that average price growth is affected both by the size of the demand shock and by the particular shape of preferences and technology. This lead to our second fact: cities that experience higher average growth rate in house prices also features a higher degree of price convergence across neighborhoods. In particular, we show that our model can generate this fact with two alternative stories. First, it could be that two identical cities hit by a demand shock of different sizes (say a different income shock). Second, it could be that two cities that differ for preferences and/or technology (say a different stage of development) are hit by the same demand shock. In both cases, the city that experiences the higher aggregate housing boom also features more gentrification. For

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[6]This is a similar prediction as the models put forth by Mills (1967) and Muth (1969) which focused on transportation costs. The farther away from the center city one lived, the larger their commuting costs and, as a result, the lower the price they were willing to pay to live far away from the city. In this paper, we abstract from commuting costs completely. Commuting costs would not be able to explain some of the features of the data that we describe in the first and third parts of our paper. In particular, such a model is inconsistent with the fact that it is only the poor neighborhoods that abut the rich neighborhoods that have the most elastic prices and it does not match our results in Section 5 showing that these neighborhoods with high house price growth show signs of gentrification.

example, given a large housing demand shock, a city with a larger fraction of rich households will experience a larger average price response and more gentrification.

Lastly, we extend our model to include convex adjustment costs in construction. We assume that a larger expansion in housing supply in a given city increases the marginal cost of building in that period.[7] This simple variation to the baseline model yields rich price dynamics, that differ across locations. Rich neighborhoods exhibit a standard pattern with an initial spike in prices followed by a gradual return to the initial price level. Newly gentrified neighborhoods also experience overshooting, but prices are going to be permanently higher. Finally, new neighborhoods are going to be born and experience a gradual price increase. In our model, price dynamics are affected by the combination of adjustment cost and neighborhood externalities. As in a standard adjustment cost model with no externality, after a demand shock, the aggregate price level overshoots on impact and then gradually declines. However, in contrast to the standard model, in the long run the aggregate price level is going to be higher than the steady state level before the shock, thanks to the development of low price neighborhoods. Moreover, given the externality, our model also delivers a magnified short run response of prices to demand shocks.

Aside from our three documented facts, the model yields a host of additional predictions. In the next part of the paper, we empirically test four additional predictions of the model and show that the data are consistent with them. First, we show that the neighborhoods that experience the greatest price increase during a city-wide house price boom appear - along many dimensions - to have gentrified. For example, using census data from 1990-2000, we show that neighborhoods that experienced the greatest property price increases during the 1990s, had bigger increases in average neighborhood income, had larger reductions in neighborhood poverty rates, and had a larger decline in the black percentage in the neighborhood. Second, using data on building permits in Chicago, we show that the Chicago neighborhoods that experienced the biggest price increases during the 2000-2006 period had the largest increase in building activity. Third, using both within city data and cross city data, we show a strong relationship between the level of income and the average housing price. Other papers generate a strong income-house price relationship by appealing to production externalities and housing supply constraints. Our model generates a similar pattern, without relying on supply constraints. In the data, we find a strong cross city relationship between average income (growth in average income) and average housing prices (growth in average housing prices) even among a sample of cities where housing

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[7]Such adjustment costs are found in the models of housing put forth by Topel and Rosen (1988) and by Glaeser and Gyourko (2006).

supply is very elastic (city-wide density is really low). Lastly, we show evidence that much of the cross city variation in house price appreciation experienced between 1990 and 2000 and some of the one between 2000 and 2006 can be explained by the fraction of rich people in the city (or the level of income in the city). This results holds even after conditioning for traditional supply constraint measures.

In summary, our paper shows that the existence of neighborhood externalities has important implications for the nature of real estate prices dynamics across neighborhoods within a city and across cities. We conclude that such externalities need to be embedded in both theoretical and empirical models designed to explain both time series and cross sectional housing price dynamics.

2 Data

To examine house price appreciation across neighborhoods within a city during different time periods, we use a variety of different data sets. In particular, we use (1) zip code level housing price indices computed as part of the Case-Shiller index, (2) transaction level data on the universe (or near universe) of residential housing transactions for Chicago, New York, and Charlotte, and (3) micro data from the 1980, 1990 and 2000 U.S. Censuses. As we show in the following section, all three data sources yield similar results about how housing prices evolve within a city during a given time period.[8]

The bulk of our results use the Case-Shiller zip code-level price indices.[9] The Case-Shiller indices are calculated from data on repeat sales of single-family homes. The benefit of the Case-Shiller index is that it provides consistent constant-quality price indices for localized areas within a city or metropolitan area over long periods of time. Most of the zip code-level price indices go back in time through the late 1980s or the early 1990s. The data was provided to us at the quarterly frequency and our most recent data is for the fourth quarter of 2008. As a result, for each metro area, we have quarterly price indices on selected zip codes within selected metropolitan areas going back roughly 20 years.

There are four limitations to the Case-Shiller data. First, the Case-Shiller index only covers approximately 30 metropolitan areas.[10] Second, as noted above, for almost all cities the Case-

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[8]For a complete discussion of the data sources - including a link to all of our online documentation - see the Data Appendix.

[9]The zip code indices are not publicly available. Fiserv, the company overseeing the Case-Shiller index, provided them to us for the purpose of this research project. The data are the same as the data provided to other researchers studying very local movements in housing prices. See, for example, Mian and Sufi (2009).

[10]We were provided with zip code level data for at least 20 zip codes at the city level or at least 65 zip codes at the metro area level for the following cities/metropolitan areas: Atlanta, Baltimore, Boston, Charlotte, Chicago, Cincinnati, Columbus (OH), Dayton, Denver, Detroit, Hartford, Jacksonville, Las Vegas, Los Angeles, Memphis,

Shiller index does not start until late 1980s or early 1990s. This limits the analysis we can do for cities that experienced housing price booms in the early to mid 1980s. Third, the Case-Shiller index only covers single-family homes (as opposed to also including condos or multi-family buildings). To the extent that condos and multi-family buildings evolve differently during housing price booms and to the extent that different neighborhoods have different compositions of housing structures, the Case-Shiller data could yield a biased picture of within city house prices changes. Finally, given its focus on repeat sales of single-family homes, there is not enough data to compute reliable price indices for all zip codes within a metropolitan area. As a result, many zip codes - particularly those in urban city centers - have no reported Case-Shiller price indices. Appendix Figure A1 illustrates this point. In this Figure we show the zip codes within three cities: Chicago (panel A), New York City (panel B), and Charlotte (panel C). The darkened zip codes on the city maps are the ones for which a Case-Shiller index exists. Notice, that a Case-Shiller zip code index exists for less than 50 percent of the zip codes in Chicago, less than 10 percent of the zip codes in New York City, and essentially all of the zip codes in Charlotte.[11] Notice, for New York City proper, only some zip codes in Staten Island are included.

To overcome some of the shortcomings of the Case-Shiller zip code indices, we supplement our analysis using data from two additional sources. First, for three cities, we were able to get data for the universe (or near universe) of residential housing transactions. The three cities were Chicago, Charlotte, and New York. These data allow us to explore whether our results change using data that include condos and multi-family buildings (as well as single-family homes) and allows us to examine price behavior in all zip codes within a city. As we show below, the results in these more detailed data sets mirror the results from the Case-Shiller indices giving us confidence that these limitations of the Case-Shiller indices are not biasing our results.

For Chicago and Charlotte, we compiled the data on all residential real estate transactions ourselves. Using the Chicago Tribune website, we downloaded all residential real estate transaction data for the City of Chicago. The data we were able to download include all residential real estate transactions from 2000 through 2008 (inclusive). We believe this to be the universe of residential real estate transactions. We merged the data from the Chicago Tribune on Chicago

Miami, Minneapolis, New York, Orlando, Philadelphia, Phoenix, Portland (OR), Sacramento, San Bernardino, San Diego, San Francisco, San Jose, Seattle, Tampa, and Washington DC.

[11]Coverage in the metro areas are much higher given that most zip codes outside the center city have a sufficiently large number of single-family home transactions to compute reliable price indices. For example, there are a total of 384 zip codes covered in the Case-Shiller data from the New York metro area (compared on only 9 in New York City proper). The reason that zip codes in center cities tend to have lower coverage is that most homes sales are condos or multi-family buildings which are excluded from the Case-Shiller index.

real estate transactions with information from the Cook County Tax Assessor which included information on the age of the structure, building type, etc.[12]. For Charlotte, the procedure was much easier. We downloaded all real estate transactions back to 1990 using the Mecklenburg County Real Estate Lookup System. These data include an extensive list of structural characteristics. Like the data from the Chicago Tribune, we believe the Charlotte Deed data to be the universe of all residential real estate transactions.

Given that we had some attributes of the structure in the Chicago data that we merged in from the Cook County Tax Assessor, we made a simple price index for each Chicago neighborhood.[13] The real estate transactions in the Chicago Tribune are mapped to Chicago neighborhoods (all within the city of Chicago). Chicago neighborhoods are slightly smaller than Chicago zip codes. For example, there are 77 Chicago neighborhoods and only 60 Chicago zip codes. In Appendix Figure A2, we show the Chicago neighborhood map. We have kept the Chicago Tribune price data at the level of Chicago neighborhood (instead of converting them to zip codes) so as to match our building permit data - which is at the neighborhood level - discussed in Section 5. To make the simple price index, we regressed all Chicago residential real estate transaction (log) price data on dummies for building type (multi-family, single-family, or condo), dummies for the age of the building (1-5 year old, 6-10, 11-20, 21-30, etc.), and dummies for neighborhood interacted with year. We evaluated the estimating equation using the mean structural characteristics for the entire city and added this component to coefficients for the neighborhood-year variables to form the neighborhood price indices over time. We used the same procedure to construct a hedonic index for Charlotte zip codes over time. The only difference was that we had a much richer set of building characteristics for each property in Charlotte.

For New York City, we use the Furman Center repeat sales index which covers all of NYC. The Furman data uses NYC community districts as its level of aggregation.[14] There are 59 community districts in New York City which represent clusters of several neighborhoods. The Furman data for New York City extends back to 1974. The benefit of the Furman data is that it gives us extensive coverage of New York neighborhoods over a long time period and covers all residential real estate transactions in New York City (not just single-family homes). Additionally, this is a repeat sales index measuring the change in price for constant quality

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[12]In the Data Appendix, we discuss our methodology in much greater detail

[13]For Chicago, the only structural characteristics that are available for all property types are the age of the building and the property type. A much richer set of structural characteristics are available for all Charlotte properties. See the data appendix for further details.

[14]See `http://furmancenter.org/`.

housing units. A map of the NYC community districts can be found in the second panel of Appendix Figure A2.

To help examine neighborhood trends within a broad set of cities during the 1980s and the 1990s, we augment our analysis using data from the 1980, 1990 and 2000 U.S. census. We restricted our analysis of the U.S. Census data to census tracts within a metro area. So, when we use the census data, our unit of analysis - a census tract - is much smaller than zip code or community area. We only focus on census tracts whose boundaries remained constant between the two adjacent census years. For each census tract within the metro area, we computed the growth rate in median home prices. When computing the growth rate in median house prices by census tract, we did not hedonicly adjust the series for changing housing characteristics. As a result, using this data, we can explore changes in house prices within a neighborhood that are both due to the fact that the price of a constant-quality unit of housing may be changing and due to fact the quality of housing in the neighborhood may be changing. Again, the Data Appendix gives a complete description of all of our data sources and our sampling restrictions.

Lastly, when comparing differences in house price appreciation across cities, we use both the Case-Shiller metro level price indices and the OFHEO metro level price indices. The Case-Shiller metro indices are limited to only 30 cities while the OFHEO metro indices cover over 170 cities. As noted above, the Case-Shiller index is a repeat sales index that only includes single family homes regardless of the type of financing used to purchase the home. The OFHEO index is also a repeat sales index but it includes properties of all different types (single family homes, condos, town homes, etc.) but restricts the properties in their index to only ones that are purchased with conventional mortgages. Despite the difference in coverage, the time series path of the OFHEO and Case Shiller indices are nearly identical for the metro areas where both indices exist.[15]

3   Three Facts About Within City House Price Movements

In this section, we explore the nature of housing price movements within a city or metro area during different time periods. However, before exploring within city/metro area results, we present the relationship between the initial level of housing prices and housing price growth *across* metro areas. To do this, we restrict our analysis to the metro areas for which OFHEO provides a repeat sales house price index. Furthermore, we restrict our analysis to only those

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[15]See online documentation for a discussion of how the Case Shiller index is computed. See http://www.fhfa.gov/Default.aspx?Page=14 for a similar discussion for the OFHEO index. Lastly, see http://faculty.chicagobooth.edu/erik.hurst/research/ for a comparison of the Case Shiller index and the OFHEO index for various MSAs.

metro areas where the population in 2000 exceeded 150,000 people. These restrictions leave us with 154 metro areas. Figure 1 plots the log of the initial median level of house prices in each of these metro areas for the year 2000 (as measured by the 2000 U.S. census) on the x-axis against the real growth rate in housing prices for these metro areas between 2000 and 2006 (as measured by the OFHEO price indices).[16] Across cities, it is the high priced cities that appreciate the most. A simple regression line of housing price growth between 2000 and 2006 against the median value of house prices in 2000 yields a coefficient on the initial house price of 0.41 (with a standard error of 0.07) and with an R-squared of 0.16. The regression implies that a 100 percent increase in the initial level of house prices in 2000 was correlated with a 41 percentage point higher growth rate in prices between 2000 and 2006.

3.1 Within City Movements House Price Dynamics: Fact 1

We begin our analysis by estimating the relationship between the initial level of house prices across neighborhoods within a city and the subsequent growth rate in house prices for that neighborhood during periods of city-wide housing booms and busts. Specifically, we estimate the following relationship of within city house price dynamics:

$$\overset{g}{H}P_{t,t+k}^{i,j} = \alpha_{t,t+k}^{j} + \beta_{t,t+k}^{j} \ln(HP_t^{i,j}) + \epsilon_{t,t+k}^{i,j}$$

where $HP_t^{i,j}$ is the level of housing prices in neighborhood i, within city (metro area) j, in year t and $\overset{g}{H}P_{t,t+k}^{i,j}$ is the growth in housing prices in neighborhood i, within city (metro area) j, between years t and $t+k$. We estimate these relationships separately within each city (metro area). $\beta_{t,t+k}^{j}$ is an estimate of the relationship between the initial level of house prices (in logs) and the subsequent house price appreciation within city (metro area) j between t and $t+k$. We will examine neighborhood price movements within both cities and broader metro areas. As a result, sometimes j will index a city and sometimes it will index the broader metro area that contains that city.

Figure 2 shows the estimates of (1) where j = Chicago (Panel A), New York City (Panel B), and Charlotte (Panel C) over the 2000-2006 time period. Along with the actual estimated regression line, we plot the underlying neighborhood data. The house price growth data (on the y-axis) is the growth rate in the neighborhood level house prices indices (which were discussed in the previous section). The initial level of house prices (on the x-axis) is the median house price in the neighborhood as reported by 2000 census.[17] We chose these three metro areas

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[16]All prices in the paper are measured in year 2000 dollars, unless otherwise indicated.

[17]Figures with zip code-level observations use 2000 Census summary file (SF3) tabulations of median value

to start our analysis with for two reasons. First, as noted in the previous section, we have house price measures from multiple sources for these three cities. Second, the cities provide a nice contrast with each other given that the New York metro area as a whole experienced a substantial housing price boom between 2000 and 2006 (of 75 percent), while the Chicago metro area experienced a medium sized housing price boom during that period (of 40 percent) and the Charlotte metro area experienced a very small housing price boom during that period (of 8 percent).[18]

For each of the three cities in Figure 2, we provide three panels that shed light on the relationship between house price growth across neighborhoods and the initial level of prices in the neighborhood. In the left most panel, we use our alternate data sets for each city. For Chicago, we use the Chicago Tribune data. For New York, we use the Furman data for Manhattan. And for Charlotte, we use the universe of deed data. In the middle panel, we use the zip code level data from Case-Shiller. As noted above, for New York City, the Case-Shiller data only covers some zip codes in Staten Island (see Appendix Figure A1). The left and middle panels focus on only neighborhoods within the city of Chicago, New York or Charlotte. In the right most panel, we expand our analysis to include all the zip code data from Case-Shiller for the entire metro area.

Focusing on the Chicago panel of Figure 2 (Panel A), we see that both the Chicago Tribune data and the Case-Shiller data show that lower priced neighborhoods, on average, appreciated at much greater rates than higher price neighborhoods during the 2000-2006 period. Focusing on the Tribune data (which, as discussed in the previous section, only control for limited property characteristics in the hedonic relationship), we find that prices of transacted properties in the initially low priced Chicago neighborhoods grew at a rate that was 2 to 3 times higher than the prices of transacted properties in the initially high priced Chicago neighborhoods. For example, the average transacted property grew at about 25 percent or less in Chicago's high priced neighborhoods of Lincoln Park (community 7), Lakeview (community 6) and The Loop (community 32). Conversely, some of the neighborhoods that had initially low price levels experienced an increase of 75 percent or more for the average value of transacted property prices.

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of owner-occupied housing units by zip code, which are available from `http://factfinder.census.gov/`. The left-most figure of Panel B also uses 2000 Census tabulations of median value of owner-occupied housing units, but these are specially tabulated for NYC community districts and are are available from NYC's GIS department at `http://gis.nyc.gov`. Finally, the x-axis for the left-most figure of panel A uses the mean owner-occupied housing value calculated from census 2000 tabulations by census tract and described in the data appendix.

[18]To compute the change in the metro area price indices as a whole, we use the Case-Shiller metro area price indices.

The left hand panel of Figure 2A also includes estimates of (1). The estimated β from (1) for Chicago using the Chicago Tribune data during the 2000-2006 period is -0.33 with a robust standard error of 0.05. In other words, a 100 percent increase in the initial level of housing prices reduces the growth rate in housing prices by 33 percentage points. The simple R-squared of the scatter plot is 0.29.

The data in the left-hand panel of Figure 2A includes the universe of transacted properties within the city of Chicago during the 2000-2006 period. The drawback of this data is that it is not based on repeat sales transactions. To explore the movement of prices using a better neighborhood repeat sales price index, we use the Case-Shiller zip code data for Chicago. As noted above, the Case-Shiller data only cover about 45 percent of Chicago's 60 or so zip codes. Moreover, the Case-Shiller data only focus on the price movements of single-family homes. Despite the differences in coverage, the results in the middle panel of Figure 2A (using the Case-Shiller data) are very similar to the results in the left panel of Figure 2A (using the Chicago Tribune data). A one-hundred percent increase in initial housing prices reduces the growth rate in housing prices by 33 percentage points (with a standard error of 4 percentage points). The simple R-squared of the scatter plot is 0.78. The right hand panel of Figure 2A shows the results using the Case-Shiller zip code data for the entire Chicago metro area (as opposed to just the city of Chicago). Again, a similar pattern emerges. High price neighborhoods experienced lower appreciation rates than lower priced neighborhoods (β = -0.23 with a standard error or 0.04).

In Figure 2B, similar results are shown for New York. Using the Furman data for community districts in Manhattan (left-hand panel) or using Case-Shiller zip codes from Staten Island (middle panel), we find that a 100 percent increase in initial housing prices reduced the subsequent growth rate in housing prices between 2000 and 2006 by 39 percentage points or 32 percentage points, respectively. Both estimated slope coefficients are significant at the 1 percent level. For example, the Harlem area of Manhattan appreciated at twice the rate of midtown Manhattan. As seen in the right hand panel of Figure 2B, the results also hold broadly for the New York metro area as whole (β = -0.35 with a standard error of 0.02).

In Figure 2C, we see similar graphical representations for Charlotte. The results, however, for Charlotte are very different. No matter what level of aggregation and no matter what measure of housing prices, there is either no systematic relationship between the initial level of housing prices and the subsequent growth in housing prices or evidence in the opposite direction. Across the three figures, β equals 0.14, 0.07 and 0.08 - only the first coefficient is significant at standard levels. As we will discuss below, the results

Figure 3 shows similar patterns for different cities and across different time periods. In the top panel of Figure 3, we show the relationship between initial neighborhood level of prices and subsequent growth rate in prices for other metropolitan areas during the 2000-2006 period (using the Case-Shiller data). These metro areas include Boston, Los Angeles, San Francisco, and Washington DC. These pictures are analogous to the right hand panels of Figure 2. In every one of these metro areas, property prices as a whole increased by at least 50 percent throughout the metro area. Also, in every one of these metro areas, the neighborhoods with initial low levels of housing prices increased by a substantially higher amount than higher price neighborhoods. For example, the estimated β's for Boston, L.A., San Francisco, and Washington, D.C., were, respectively, -0.22, -0.40, -0.44 and -0.49 (all significant at the 1 percent level).

In the middle panel of Figure 3, we present similar plots for metro areas during the 1990-1997 period. We start in 1990 so we can anchor the initial house price to the data from the 1990 census. We stop in 1997 because that is considered the start of the next housing boom cycle. Again, we only use the Case Shiller data for our measures of within city price movements. The metro areas include in the middle panel are: Denver, Portland, San Francisco, and Boston. Denver and Portland both experienced substantial real housing price booms (between 30 and 40 percent) while San Francisco and Boston experienced non-trivial metro area real housing price busts (roughly 20 percent). In Denver and Portland we see that the poor neighborhoods appreciated much more the richer neighborhoods (estimated β's of -0.48 and -0.35, respectively - both statistically significant at the 1 percent level). In San Francisco and Boston we see that the poorer neighborhoods had property prices that declined much more dramatically compared to the property prices in the richer neighborhoods during the city-wide property price busts (estimated β's of 0.09 and 0.22, respectively - both statistically significant at the 1 percent level).

In the bottom panel of Figure 3, we examine the relationship between initial level and subsequent property price appreciation for neighborhoods within cities during the 1980s. We start by focusing on New York and Boston - two cities that experienced substantial housing price booms during the 1980s. Average real housing prices increased in these two cities between 1984 and 1989 by well over fifty percent. For New York City, we data from the Furman Center to explore the within city growth rates (left panel). For Boston, we use both the Case-Shiller data which, for Boston, extends back into the early 1980s (middle panel) and then U.S. Census data (right panel). We do this to show that the Census data yields similar patterns as the Case-Shiller data. As noted above, however, the Census data is at the level of the Census

tract. Like the results for the 1990s and 2000s, New York and Boston saw sharp convergence in housing prices across neighborhoods during their 1980s property price booms. The estimated β's from the three panels are -0.38, -0.13, and -1.43, respectively. All convergence estimates are significant at the 1 percent level. Given that the Census data is not holding the quality of the housing stock constant in the price estimates, the magnitude of the housing price increases are much higher. That is why the estimated β from the Census data is so much higher than the estimated β's from the other two specifications.

Figures 2 and 3 show our first fact. We find that neighborhoods with initially low housing prices are more price elastic than neighborhoods with initially high housing prices. Specifically, during city-wide housing price booms, housing prices appreciate more in initially low priced neighborhoods than in initially high priced neighborhoods. Conversely, during city-wide housing price declines, housing prices fall more in initially low priced neighborhoods than in initially high priced neighborhoods. These facts are robust across time periods and across measures of house price appreciation.

3.2 Fact 2: City Wide Housing Price Growth and Within City Differences

In the previous sub-section, we showed for a handful of cities/metro areas that, on average, lower initial housing price neighborhoods are more price elastic than higher initial price neighborhoods. In this sub-section, we take a more systematic approach. We show that for all cities for which we have within city/metro area price indices from Case-Shiller, the extent to which low price neighborhoods appreciate more than high price neighborhoods is related to the size of the city-wide housing price boom. Conversely, the larger the city-wide housing price bust, the more likely that initially low price neighborhoods depreciate more than initially high price neighborhoods.

Figure 4 formalizes the relationship between the level of housing price growth within the metro area as a whole and the amount of cross-neighborhood convergence that takes place within the city. In particular, we estimate the following:

$$\beta_{t,t+k}^{j} = \gamma_0 + \gamma_1 \overset{g}{H} \overset{j}{P}_{t,t+k} + \eta_{t,t+k}^{j}$$

where the β^{j}'s are estimated for each metro area as described in (1) and $\overset{g}{H}\overset{j}{P}_{t,t+k}$ is house price appreciation in the entire metro area j between t and $t+k$. For panels A and B of Figure 4, we restrict our analysis to the 30 metro areas where we have zip code level price indices from Case-Shiller when estimating (2). Panel A, restricts our analysis to the 2000-2006 period while

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Panel B restricts our analysis to the 1990-1997 period. In Panel C, we examine the 1980-1990 period using U.S. Census data.

In all the different time periods (shown in panels A, B, and C of Figure 4), we find that the larger the price increase (decrease) in the metropolitan area as a whole, the more that poor neighborhoods appreciate (depreciate) relative to richer neighborhoods within the metropolitan area. For example, in all time periods, high positive price appreciation at the metro level are correlated with a more negative β while larger metro wide price depreciations are associated with a more positive β. This latter result can be seen from Panels B and C of Figure 4. The simple correlation between metro area price changes and the estimated β's from equation (1) for the data shown in Panels A, B and C, respectively, are 0.50, 0.58, and 0.28.

3.3   Fact 3:   The Spatial Patterns of Price Movements

While, on average, low price neighborhood appreciates more during city-wide housing booms, there is a large amount of heterogeneity within low priced neighborhoods. Some low priced neighborhoods appreciate by a very large amount and other low price neighborhoods do not appreciate that much at all. This fact can be illustrated formally. In particular, we can take the absolute value of the residuals from our estimation of (1) and regress them on the initial level of log house prices in the neighborhood. Systematically, we get that the variance of the residuals is higher for initially low priced neighborhoods relative to high price neighborhoods. For example, using the Chicago Tribune Data (left hand panel of Figure 1A), we find that the absolute value of the residuals decline by 6.5 percentage points as the initial level of house prices increases by 100 percent during the 2000-2006 period. These results are consistent within all of the metro areas we studied. Even though low initial priced neighborhoods appreciated more on average, the results were not uniform across all low price neighborhoods. Some low price neighborhoods increased by a lot, while others increased by very little. For our last set of results, we examine the spatial nature of the poor neighborhoods that appreciated most during the city-wide house price booms.

The top panel of Figure 5 shows a map of the neighborhoods within the city of Chicago. The underlying map is the same as Panel A of Appendix Figure A2 (discussed above). Using the complete data from the Chicago Tribune, we identify two types of neighborhoods within the city of Chicago. The first set of neighborhoods are the high property price neighborhoods in year 2000. Specifically, these neighborhoods were in the top quartile of all Chicago neighborhoods with respect to average housing prices in year 2000 (as measured by the 2000 U.S. Census). These high initial priced neighborhoods are indexed in Panel A of Figure 5 with darker

shading. The second set of neighborhoods includes those neighborhoods that experienced the highest growth in property prices between 2000 and 2006 (as measured by our Chicago Tribune Price Index). Again, we highlight only those neighborhoods in the top quartile of all Chicago neighborhoods with respect to the growth in property prices. On Figure 5, these neighborhoods are shaded light grey. The single neighborhood that falls into both categories, Logan Square, is shaded in black.

The results in Panel A of Figure 5 show the spatial analog to the results shown in Panel A of Figure 2. In this figure, we show that the areas that grew fastest in Chicago were the poor neighborhoods that directly neighbored the initially rich neighborhoods. Notice, the extreme south side of Chicago (neighborhoods 50-55 from Figure A1) did not grow as fast as the neighborhoods directly to the west and south of the initially high priced areas. Yet, the neighborhoods on the extreme south side of Chicago (the non-shaded areas) were similar in terms of demographics in 2000 to the neighborhoods that experienced rapid price appreciation from 2000-2006 (the grey shaded areas). For example, using Census data, the mean income of the 32 south side neighborhoods that did not experience rapid growth from 2000 - 2006 was about $49,000 while the mean income of the 14 southside neighborhoods that did was about $36,000. High growth neighborhoods on the southside had an average poverty rate of 34%, while other neighborhoods on the southside had an average poverty rate of 19%. The high growth southside neighborhoods were on average 65% African American, while the other southside neighborhoods were on average 54% African American. In summary, the demographics of the southside neighborhoods that experienced rapid growth from 2000 to 2006 were not wildly different than the demographics of the southside neighborhoods that did not grow rapidly. If anything the neighborhoods that grew were a little bit poorer and more heavily African American. Collectively, for Chicago, our results show that the poor neighborhoods that were close to the rich neighborhoods were much more likely to experience house price gains than the equally poor (or just slightly wealthier) neighborhoods that were not close to the rich neighborhoods.

Our results for Chicago are not unique. The spatial proximity of high price growth neighborhoods to initially high price level neighborhoods is found in nearly all cities where convergence was pronounced. In Panel B of Figure 5, we show similar patterns for New York City (using the Furman Center data) while in Panel C of Figure 5 we show the same type of results for the city of Charlotte (using the detailed Charlotte transaction data). Similar to Chicago, New York City's and Charlotte's high housing price growth areas are adjacent to its initial high price areas.

Table 1 systematically summarizes the results of the spatial patterns in price appreciation across all cities. In particular, we estimate:

$$\overset{g}{HP}{}^{i,j}_{t,t+k} = \alpha^j_{t,t+k} + \beta^j_{t,t+k} \ln(HP^{i,j}_t) + \omega^j_{t,t+k} \ln(Dist^{i,j}_t) + \varphi^{i,j}_{t,t+k} X^{i,j}_t + \mu_j + \epsilon^{i,j}_{t,t+k}$$

where $\overset{g}{HP}{}^{i,j}_{t,t+k}$ and $HP^{i,j}_t$ are defined as above. $\ln(Dist^{i,j}_t)$ is the log of the distance of the midpoint of neighborhood i in city j (in miles) to the midpoint of the nearest neighborhood in j that is in the top quartile (out of all neighborhoods within j) with respect to average housing price during period t. X is a vector of neighborhood control variables including fraction Hispanic, fraction African-American, fraction that commute by car, fraction that commute by public transit, labor force participation rate, unemployment rate, log median household income, fraction of housing units that are owner-occupied, and an indicator for whether the median structure was built before 1940, and μ is a city fixed effect. For this regression, we restrict our analysis to the neighborhoods for which we have Case-Shiller zip code level price indices. Furthermore, we restrict our analysis to the 2000-2006 period and only focus on zip codes within the city (as opposed to studying the metro area broadly). Lastly, we only include zip codes that had average housing prices that placed them in the bottom third off the city-wide price distribution during 2000. The regression attempts to assess whether poorer neighborhoods within a city that were systematically closer to richer neighborhoods appreciated more during 2000-2006 period than equally poor neighborhoods that were further away from rich neighborhoods.

The results from this regression are shown in columns (1) - (3) of Table 1. Column (1) only includes the log of the initial house price in the zip code during 2000. Column (2) includes the initial house price plus the log of the distance to the nearest high price neighborhood. The third column replicates column (2) but also includes the vector of X controls. The results are the analog of the visual descriptions shown in Figure 5. Poorer neighborhoods in close proximity to richer neighborhoods are much more likely to experience housing price increases during the 2000-2006 period, all else equal. The regression estimates that a 100 percent increase in distance from the nearest high property price neighborhood in 2000 reduces the property price appreciation of lower property price neighborhoods by between 6 and 8 percentage points.

3.4 Summary of Fact

In this section we have shown that there is a tremendous amount of heterogeneity in house price movements within a city/metro area during housing price booms and busts. We document

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three additional new facts about within city property price movements. First, we show that initially low price neighborhoods are much more housing price elastic than initially high priced neighborhoods. During city-wide housing price booms, low price neighborhoods - on average - appreciate more and during city-wide housing price busts, low price neighborhoods - on average - depreciate more. Second, the difference in property price appreciation rates (depreciation rates) between low and high price neighborhoods is greatest when the city-wide property price boom (bust) is larger. Lastly, while low property price neighborhoods are likely to appreciate (depreciate) more on average, there is a large degree of heterogeneity among the low property price neighborhoods. We show that it is the low price neighborhoods that are in close proximity to the high price neighborhoods that are the most price elastic.

4   Model

In order to rationalize the facts just documented, in this Section we develop a spacial equilibrium model of housing prices across neighborhoods within a city. The key ingredient of the model is a positive neighborhood externality: people like to live next to rich people. This externality can be motivated in terms of increasing returns to scale in the production of amenities, in the spirit of Glaeser, Kolko, and Saiz (2001) and Becker and Murphy (2003), so that the more rich people are around, the greater is the number of restaurants, museums, coffee shops, dry cleaners, etc. Moreover, we assume that this external effect is stronger for rich agent themselves.

4.1   Baseline model

4.1.1   Set up

Time is discrete and runs forever. We consider a city populated by two types of infinitely-lived households: a continuum of rich households of measure N^R and a continuum of poor households of measure N^P.

The city is represented by the real line and each point on the line $i \in (-\infty, +\infty)$ is a different location. Agents are fully mobile and can choose to live in any location i. Denote by $n_t^s(i)$ the measure of households of type s, for $s = R, P$, who live in location i at time t and by $h_t^s(i)$ the size of the house they choose. In each location, there is a maximum space that can be occupied by houses normalized to 1, that is,

$$n_t^R(i) h_t^R(i) + n_t^P(i) h_t^P(i) \leq 1 \text{ for all } i, t.$$

Moreover, market clearing requires

$$\int_{-\infty}^{+\infty} n_t^s(i) \, di = N^s \text{ for } s = R, P. \tag{1}$$

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The key ingredient of the model is that there is a positive location externality: households like to live in areas where more rich households live. Each location i has an associated neighborhood, given by the interval centered at i of radius γ. Let $H_t(i)$ denote the total space occupied by houses of rich households in the neighborhood around location i, that is,

$$H_t(i) = \int_{i-\gamma}^{i+\gamma} h_t^R(j) n_t^R(j)\, dj. \tag{2}$$

Households have separable utility in non-durable consumption c and housing services h. The location externality is captured by the fact that households enjoy more to live in locations with higher $H_t(i)$. The utility of an household of type s located in location i at time t is given by

$$u(c) + v^s(h, H_t(i)),$$

where $u(.)$ and $v^s(.)$ are weakly concave functions. For tractability, we assume that u is linear, so that we abstract from wealth effects, and that v^s takes the following functional form: $v^s(h, H) = \phi^s h^\alpha (A + H)^{\beta^s}$, where ϕ^s, α^s, and β^s are non negative scalar. The parameter ϕ^s captures the willingness of the households to pay for housing services and we let $\phi^P < \phi^R$. The fact that $\phi^P < \phi^R$ implies that poor agents have smaller willingness to pay for housing than rich ones, or, equivalently, that they have a smaller marginal value of money.[19] Moreover, we assume that that $\beta^R \geq \beta^P$, so that the rich households who generate the externality are also the ones who benefit the most from it. Also, each period households of type s receive an exogenous endowment of consumption goods equal to y^s, with $y^P < y^R$.

On the supply side, there is a representative firm who can build housing in any location $i \in (-\infty, +\infty)$. There are two types of housing: rich houses (type R) and poor houses (type P). Each type of household only demands houses of his own type. The marginal cost of building houses of type s is equal to C^s, with $C^R \geq C^P$. If the firm wants to convert houses of type $\tilde{s}$ into houses of type s, he has to pay $C^s - C^{\tilde{s}}$.

The (per square foot) price of a house for household of type s in location i at time t is equal to $p_t^s(i)$. Hence there is going to be construction in any empty location i as long as $p_t^s(i) \geq C^s$. Moreover, if the firm wants to construct a house of type s in a location occupied by a house of type $\tilde{s}$, he has to pay the converting cost and the additional cost of convincing households of type $\tilde{s}$ to leave. Hence, there is going to be construction of houses of type s in any location occupied by agents of type $\tilde{s}$ if $p_t^s(i) \geq C^s - C^{\tilde{s}} + p_t^{\tilde{s}}(i)$.

Finally, there is a continuum of competitive intermediaries who own the houses and rent them to the households. The intermediaries are only introduced for ease of exposition and

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[19]This would be an endogenous outcome due to $y^P < y^R$ if utility from consumption was strictly concave.

nothing would change if we allowed the households to own.[20] The (per square foot) rent for a house of type s in location i at time t is denoted by $R_t^s(i)$. As long as the rent in location i at time t is positive, the intermediaries find it optimal to rent all the houses in that location. Also, for simplicity, assume that houses do not depreciate. Competition among intermediaries requires that for each location i the following arbitrage equation holds:

$$p_t^s(i) = R_t^s(i) + \left(\frac{1}{1+r}\right) p_{t+1}^s(i) \text{ for all } t, i, s. \tag{3}$$

4.1.2 Equilibrium

An equilibrium is a sequence of rent and price schedules $\left\{ R_t^R(i), R_t^P(i), p_t^R(i), p_t^P(i) \right\}_{i \in R}$ and of allocations $\left\{ n_t^R(i), n_t^P(i), h_t^R(i), h_t^P(i) \right\}_{i \in R}$ such that households maximize utility, the representative firm maximizes profits, intermediaries maximize profits, and markets clear.

At each time, households decide their non-durable consumption, in which location to live, and the size of the house they want to live in, taking as given the rental price and the neighborhoods characteristics. Because of full mobility, the household's maximization problem reduces to a series of static problems.

The problem of a household of type s at time t is simply

$$\max_{c,h,i \in \mathcal{I}_t^s} c + \phi^s h^\alpha \left[A + H_t(i) \right]^{\beta^s},$$
$$s.t. \ c + h R_t^s(i) \leq y^s,$$

where $\mathcal{I}_t^s$ is the set of locations where houses for poor households are available and the household takes as given the function $H_t(i)$ and the rent schedule $R_t^R(i)$. Hence, conditional on choosing to live in location i at time t, the optimal house size is

$$h_t^s(i) = \left(\frac{\alpha \phi^s \left(A + H_t(i) \right)^{\beta_s}}{R_t^s(i)} \right)^{\frac{1}{1-\alpha}}. \tag{4}$$

Households choose to live in bigger houses in neighborhoods where the rental price is lower and the externality is stronger. Moreover, conditional on a location, richer households choose bigger houses both because of their higher willingness to pay for them and because they benefit more from the externality. Given that households are fully mobile, it must be that at each point in time, the equilibrium rents in different locations make them indifferent. In particular, agents of type s have to be indifferent among living in different locations where houses of their type are

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[20]This is thanks to the assumption of linear utility in consumption.

available at time t, that is, in all $i \in \mathcal{I}_t^s$.[21] Then it must be that

$$U^s(i) \equiv y^s - (\alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}}) \left(\frac{\phi^s (A + H_t(i))^{\beta_s}}{R_t^s(i)^\alpha} \right)^{\frac{1}{1-\alpha}} = \bar{U}^s \text{ for all } i \in \mathcal{I}_t^s. \tag{5}$$

This, in turns, requires that

$$R_t^s(i) = K^s [A + H_t(i)]^{\frac{\beta_s}{\alpha}} \text{ for all } i \in \mathcal{I}_t^s, \tag{6}$$

for some constant K^s. This expression is intuitive, as rents must be higher in locations with a stronger externality. Moreover, rich households who are more affected by the location externality, given $\beta^R \geq \beta^P$, are willing to pay higher rents for the same location.

Proposition 1. *If $\beta^R \geq \beta^P$, there exists an equilibrium with full segregation.*

Let us construct an equilibrium with full segregation, where the rich households are concentrated in the city center, while the poor households live at the periphery of the city. As a normalization, let us choose point 0 as the center of the city. It follows that $\mathcal{I}_t^R = [-I_t, I_t]$ and $\mathcal{I}_t^P = [-\bar{I}_t, -I_t) \cup (I_t, \bar{I}_t]$, for some $\bar{I}_t > I_t > 0$. In this model, both the size of rich neighborhoods, I_t, and the size of the city, $\bar{I}_t$, are equilibrium objects. Given that such an equilibrium is symmetric in i, from now on, we can restrict attention to $i \geq 0$.

Given that rich households live in locations where there are no poor, it must be that $h_t^R(i) n_t^R(i)$ is either equal to 1 or to 0 and is equal to 1 for all $i \in [0, I_t]$. Then, we can easily derive the function $H_t(.)$ as follows:

$$H_t(i) = \begin{cases} 2\gamma & \text{for } i \in [0, I_t - \gamma] \\ \max\{\gamma + I_t - i, 0\} & \text{for } i \in (I_t - \gamma, \bar{I}_t] \end{cases}, \tag{7}$$

so that the neighborhoods close to the city center are fully developed and enjoy the maximum externality degree, while the farther a location is from the center the smaller the strength of the externality. If $\bar{I}_t > I_t + \gamma$, there are going to be location at the margin of the city where the externality has zero effect. From now on, we assume that the measure of poor households, N^P, is sufficiently large so that $\bar{I}_t > I_t + \gamma$. Using (6), we obtain

$$K^R = R_t^R(I_t)(A + \gamma)^{-\frac{\beta_R}{\alpha}} \text{ and } K^P = R_t^P(\bar{I}) A^{-\frac{\beta_P}{\alpha}}, \tag{8}$$

so that we can rewrite the rent schedules as

$$R_t^R(i) = R_t^R(I_t) \left(1 + \frac{\min\{\gamma, I_t - i\}}{A + \gamma} \right)^{\frac{\beta_R}{\alpha}} \text{ for } i \in [0, I_t], \tag{9}$$

$$R_t^P(i) = R_t^P(\bar{I}_t) \left(1 + \frac{\max\{\gamma + I_t - i, 0\}}{A} \right)^{\frac{\beta_P}{\alpha}} \text{ for } i \in (I_t, \bar{I}_t]. \tag{10}$$

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[21]If there was a location with construction of type s and no type s households living there, the intermediaries would be willing to decrease the rent to 0 inducing households of type s to move into that location.

From the optimizing behavior of the representative firm, it must be that the price of a poor house at the boundary of the city is equal to the marginal cost C^P, while the price of a rich house at the boundary of the rich neighborhoods must be equal to the price of a poor house, which is the compensation needed to vacate poor households living there, plus the additional cost of transforming a poor house in a rich one. This implies that

$$p_t^P\left(\bar{I}_t\right) = C^P \text{ and } p_t^R\left(I_t\right) = p_t^P\left(I_t\right) + C^R - C^P.$$

In equilibrium prices are constant over time and hence arbitrage conditions (3) require that for each location $i \in \mathcal{I}_t^s$ prices satisfy

$$p_t^s(i) = \frac{1+r}{r}R_t^s(i) \text{ for all } t,i,s. \tag{11}$$

Combining this conditions we obtain

$$R_t^P\left(\bar{I}_t\right) = \frac{r}{1+r}C^P \text{ and } R_t^R\left(I_t\right) = R_t^P\left(I_t\right) + \frac{r}{1+r}\left(C^R - C^P\right), \tag{12}$$

where, from (6) and (8), we have

$$R_t^P\left(I_t\right) = \frac{r}{1+r}C^P\left(\frac{A+\gamma}{A+\max\left\{\gamma+I_t-\bar{I}_t,0\right\}}\right)^{\frac{\beta_P}{\alpha}}. \tag{13}$$

Combining these last two expressions with (9), (10), and (11) allows us to determine the rent and the price schedules as a function of I_t and $\bar{I}_t$ only.

To complete the characterization of the equilibrium, we need to determine the size of the city, $\bar{I}_t$, and the size of the rich neighborhoods, I_t. Using market clearing (1) together with the optimal housing size (4) and the fact that $\mathcal{I}_t^R = [0, I_t]$ and $\mathcal{I}_t^P = \left[I_t, \bar{I}_t\right]$, we obtain the following expressions for I_t and $\bar{I}_t$:

$$I_t = \gamma + (A+2\gamma)^{-\frac{\beta_R}{\alpha}}\left\{\left(\frac{\phi^R\alpha}{K_R}\right)^{\frac{1}{1-\alpha}}\frac{N^R}{2} - \frac{\alpha}{\alpha+\beta_R}\left[(A+2\gamma)^{\frac{\alpha+\beta_R}{\alpha}} - (A+\gamma)^{\frac{\alpha+\beta_R}{\alpha}}\right]\right\} \tag{14}$$

$$\bar{I}_t = I_t + \gamma + A^{-\frac{\beta_P}{\alpha}}\left\{\left(\frac{\phi^P\alpha}{K_P}\right)^{\frac{1}{1-\alpha}}\frac{N^P}{2} - \frac{\alpha}{\alpha+\beta_P}\left[(A+\gamma)^{\frac{\alpha+\beta_P}{\alpha}} - A^{\frac{\alpha+\beta_P}{\alpha}}\right]\right\}. \tag{15}$$

As intuition suggests, the rich neighborhoods are more developed when there are more rich households N^R and when the marginal cost of construction C^R or the interest rate r are lower. Moreover, the city overall is bigger when the rich neighborhoods are more developed, when there are more poor households, lower N^P, and when the marginal cost of construction C^P or the interest rate are lower.

Finally, to complete the construction of the equilibrium, we have to check that the households choose their location optimality, that is, we have to check that the rich would not prefer to move to a poor neighborhood and viceversa. In particular, we need to prove that

$$U^R (i) \leq \bar{U}^R \text{ for all } i \in \left[I_t, \bar{I}_t \right]$$

$$U^P (i) \leq \bar{U}^P \text{ for all } i \in [0, I_t]$$

where $U^s (i)$ is defined in expression (6). In the Appendix, we show that both these conditions are satisfied if $\beta^R \geq \beta^P$, completing the proof of the Proposition.

In our full segregation equilibrium, the rich households are concentrated in the city center, while the poor are located at the boundary of the city. Moreover, equilibrium prices reflect the fact that locations that are further away from the city center and closer to space occupied by poor households are less appealing. In particular, prices are the highest in the city center where the rich neighborhoods are fully developed and there is the maximum concentration of rich households. As we move away from the center, prices start declining because the space in the neighborhood occupied by rich households goes down. This segregation equilibrium is sustained by the fact that the externality is more beneficial for the rich who are the ones who create the externality itself.

4.1.3 Demand shock

We are now interested in analyzing how house prices, both at an aggregate and at a disaggregate level, react to shocks to the demand for housing. We will do so, by focusing on the equilibrium with full segregation we have constructed in the previous section.

In equilibrium, the aggregate price level is given by

$$P_t = \frac{2}{I_t} \int_0^{I_t} p_t^R (i) \, di + \frac{2}{\bar{I}_t - I_t} \int_{I_t}^{\bar{I}_t} p_t^P (i) \, di,$$

where, from the analysis in the previous section,

$$p_t^R (i) = \left[C^P \left(1 + \frac{\gamma}{A} \right)^{\frac{\beta_P}{\alpha}} + C^R - C^P \right] \left(1 + \frac{\min \{ \gamma, I_t - i \}}{A + \gamma} \right)^{\frac{\beta_R}{\alpha}} \text{ for } i \in [0, I_t] , \quad (16)$$

$$p_t^P (i) = C^P \left(1 + \frac{\max \{ \gamma + I_t - i, 0 \}}{A} \right)^{\frac{\beta_P}{\alpha}} \text{ for } i \in (I_t, \bar{I}_t], \quad (17)$$

with I_t and $\bar{I}_t$ given by (14) and (15).

For concreteness, we analyze the economy's reaction to an interest rate shock. Let us assume that at time t the interest rate is $r_t = r^H$. At $t + 1$ the economy is hit by an unexpected and permanent decrease in the interest rate so that $r_{t+1} = r^L < r^H$. We now show that in reaction

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to this positive demand shock, the aggregate level of house prices permanently increases and prices in locations with higher initial price level typically react less than prices in locations where houses are cheaper to start with. This implies that our model is consistent with our first stylized fact. The mechanism is driven by the externality at the core of our model. As a demand shock hits the economy, the city starts expanding, that is, $\bar{I}_t$ increases, and the rich households start expanding in poor neighborhoods, that is, I_t increases. This is what we refer to as endogenous gentrification. The house prices in gentrified neighborhood are driven up due to our externality. This generates the counterintuitive effect that prices react more in neighborhoods where the house supply is more elastic.

Let us define the function $g\left(.\right) : \left[C^P, \bar{p}\right] \mapsto [1, \infty)$, where $g\left(p\right)$ denotes the average gross growth rate in locations where the initial price is equal to p, that is,

$$g\left(p\right) = E_{t+1}\left[\frac{p_{t+1}\left(i\right)}{p_t\left(i\right)} | p_t\left(i\right) = p\right].$$

Next proposition shows that after an unexpected permanent demand shock, the aggregate price level permanently increases and the price growth rate is higher in locations with initial level of prices, whenever prices are higher than the minimum level. This is consistent with our first fact.

Proposition 2. *If at time $t+1$ the economy is hit by an unexpected and permanent decrease in r, then there is a permanent increase in the aggregate price level P_t, and*

$$E_{t+1}\left[\frac{p_{t+1}\left(i\right)}{p_t\left(i\right)} | p_t\left(i\right) = \bar{p}\right] < E_{t+1}\left[\frac{p_{t+1}\left(i\right)}{p_t\left(i\right)} | p_t\left(i\right) < \bar{p}\right].$$

Moreover, if the shock is large enough, $g\left(p\right)$ is non-increasing in p for all $p > C^P$.

At the initial equilibrium, the city size $\bar{I}^H$, the rich neighborhood size I^H, and the price schedule for rich and poor households are given by conditions (14), (15), (16), and (17) with $r_t = r^H$. When the economy is hit by the shock, it immediately reaches a new equilibrium, with I^L and $\bar{I}^L$ and the price schedules given by the same conditions with $r_{t+1} = r^L$. From these conditions, it is easy to see that: (1) the city expands, $\bar{I}^L > \bar{I}^H$, (2) the rich neighborhoods expand, $I^L > I^H$, (3) prices remain constant in the central locations, $i \in \left[0, I^H - \gamma\right]$, (3) prices strictly increase at the boundary with the rich neighborhoods, $i \in \left[I^H - \gamma, \min\left\{I^L + \gamma, \bar{I}^H\right\}\right]$, (4) prices may remain constant far enough from the center, $i \in \left[I^L + \gamma, \bar{I}^H\right]$ if $\bar{I}^H > I^L + \gamma$. Notice that initial prices are not well defined for locations $i \in \left[\bar{I}^H, \bar{I}^L\right]$, given that these locations were not developed before the shock, hence we drop these locations from the calculation of average price growth. Clearly, the aggregate level of prices is going to increase permanently.

Figure 6 illustrates the reaction of house prices to a positive demand shock (a decrease in the interest rate) in different locations. Given that the city is symmetric, the picture shows only the positive portion of the real line. Figure 7 shows the price growth rate as a function of the initial price level in different locations together with the OLS regression that corresponds to the regressions run in the data. Our model typically delivers a negative slope of the regression line as in our first stylized fact.[22]

Next, we want to use our simple model to explore our second stylized fact, that is, that, across cities, the slope of this regression line is steeper the higher is the average growth rate of prices. We consider two different stories which can rationalize this relationship within our model. First, it could be that different cities are hit by demand shocks of different size. Second, it could be that different cities are hit by a common demand shock, but differ in preferences and/or technology (e.g. different income). In both cases, our model is able to generate higher gentrification associated with stronger price boom.

Proposition 3. *If at time $t+1$ the economy is hit by an unexpected and permanent decrease in r, then the growth rate in the aggregate price level is larger the larger is the decrease in r and, if the shock is large enough,*

$$\frac{d^2 g\left(p\right)}{dpdr} \geq 0$$

for all $p > C^P$ where the derivative is well-defined.

This proposition shows that if two identical cities are hit by demand shocks of different sizes, the one hit by the larger shock is going to feature both higher aggregate price growth rate and more price convergence due to a higher degree of gentrification. Figure 8 shows the reaction of house prices in different neighborhoods after demand shocks of different sizes. In particular, the figure shows the house price growth rate in different locations after a decrease of the interest rate from 3.5% to 3% (blue dots) and from 3.5% to 2.8% (red dots). The figure also shows the corresponding OLS regression lines (solid lines). After a larger demand shock average prices increase more and, at the same time, the regression coefficient is larger, consistently with our second stylized fact.

Proposition 4. *Consider two cities, A and B, where $\phi_R^A \geq \phi_R^B$ and $\phi_P^A \geq \phi_P^B$ with at least one strict inequality. If at time $t+1$ they are both hit by an unexpected and permanent decrease in r of the same size and large enough, then the growth rate in the aggregate price level is larger in city A and $g_A'\left(p\right) \leq g_B'\left(p\right)$ for all $p > C^P$ where the derivative is well-defined.*

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[22]There is a lot of volatility in the growth rate for locations where the initial price is equal to C^P. In theory, this may revert the slope of the regression line when the measure of locations in poor neighborhoods, where prices do not change, is large enough.

This proposition shows that in two cities at a different stage of development hit by the same demand shock, house prices react differently. In particular, if the shock is large enough, the more developed city is the one that features both higher aggregate price growth rate and higher convergence. Figure 9 shows the reaction of house prices to the same demand shock in cities with different preferences. In particular, the figure shows the house price growth rate in different location after a decrease of the interest rate from 3.5% to 3% in a city with $\phi_R = 1$ (blue dots) and in a city with $\phi_R = 1.1$ (red dots). Clearly the city with a lower ϕ_R is less developed for the same initial interest rate. The figure also shows the associated OLS regression lines (solid lines), showing that the regression line is steeper in the city where the externality is stronger (higher ϕ_R), which is also the city where prices grow more on average, delivering again our second empirical fact.

Both these stories may be relevant for different cross sections in different time periods. However, we believe the second story may be more reasonable to understand, for example, why Charlotte did not react as Chicago or Boston did to the recent aggregate demand shock.

4.2 Model with Adjustment Costs

In this section, we extend the previous model, by introducing standard adjustment costs on the supply side. This extension is interesting for two reasons. First, adjustment costs make the dynamics of the model richer. Second, we can compare our model to a standard adjustment cost model and show that the aggregate implications are substantially different.

4.2.1 Equilibrium

The set up of the model is exactly the same as in Section (4.1.1), except that the representative firm faces now a construction cost that increases with the amount of construction at each time t. Also, we make two simplifying assumptions. First, we assume that $\beta_P = 0 < \beta_R \equiv \beta$. Second, in order to get rid of an extra state variable, we assume that the houses for rich and poor are the same, that is, $C^R = C^P = C$. Then, by market clearing, the amount of construction of new houses at time t needs to be equal to the increase in the size of the city $\bar{I}_t - \bar{I}_{t-1}$. Then, the construction cost is a convex function of $\bar{I}_t - \bar{I}_{t-1}$.

The analysis in Section (4.1.2) still goes through, except that the price of the rich houses in the marginal location I_t is now equal to the the price of poor houses and

$$p_t^R (I_t) = p_t^P = c' \left(\bar{I}_t - \bar{I}_{t-1} \right). \tag{18}$$

Also, the equalization of rents for poor households and for rich households living in the marginal

location I_t implicitly defines I_t as a function of $\bar{I}_t$ as follows:

$$\left(\bar{I}_t - I_t\right)^{1-\alpha} \gamma^\beta \frac{\phi^R}{\phi^P} \left(\frac{N^R}{N^P}\right)^{1-\alpha} = \left[2^{\frac{\beta}{\alpha}}\left(I_t - \gamma\right) + \frac{\alpha\gamma}{\alpha+\beta}\left(2^{\frac{\alpha+\beta}{\alpha}} - 1\right)\right]^{1-\alpha}. \tag{19}$$

In our numerical examples, we set the cost function to be $c(x) = C_1 x + C_2 x^\psi/\psi$. Moreover, the arbitrage equation (3) together with the expression for the rents (9) for any location $i \leq I_t$ yields

$$p_t^R(i) = \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j R_t^R\left(I_{t+j}\right) \min\left\{\left(1 + \frac{I_{t+j} - i}{\gamma}\right)^{\frac{\beta}{\alpha}}, 2^{\frac{\beta}{\alpha}}\right\}, \tag{20}$$

where, combining the market clearing condition (1) with the optimality condition (4) and the expressions (7) and (9), one obtains

$$R_t^R\left(I_{t+j}\right) = \rho\left(I_{t+j}\right) \equiv \gamma^\beta \phi^R \alpha \left(\frac{N^R}{2}\right)^{1-\alpha} \left[2^{\frac{\beta}{\alpha}}\left(I_{t+j} - \gamma\right) + \frac{\alpha\gamma}{\alpha+\beta}\left(2^{\frac{\alpha+\beta}{\alpha}} - 1\right)\right]^{\alpha-1}.$$

Letting $\tilde{p}_t$ denote the price at the boundary location at time t, $p_t(I_t)$, equation (18) and equation (20) evaluated at $i = I_t$ together with expression (19) give us a dynamic system that characterizes the equilibrium sequence $\left\{\bar{I}_t, I_t, \tilde{p}_t\right\}_{t=0}^{\infty}$. Once we have solved for the sequence $\left\{\bar{I}_t, I_t, \tilde{p}_t\right\}_{t=0}^{\infty}$, we can solve for the sequence of rents at any location i, $\{R(i)\}_{t=0}^{\infty}$, using expression (6), and for the sequence of prices at any location i, $\{p_t(i)\}_{t=0}^{\infty}$, using the same equation (20).

4.2.2 Demand shock

We now analyze the same interest rate shock studied in the baseline model. Imagine the economy starts with an interest rate $r_t = r^H$ and at time $t+1$ is hit by an unexpected permanent decrease in the interest rate $r_{t+1} = r^L < r^H$. We show that, after the shock, the city starts growing slowly and the aggregate price overshoots in the short run and then, in contrast with the standard adjustment cost model, converges to a permanently higher level.

In our model, price dynamics are affected by the combination of adjustment costs and gentrification. The price in the marginal location I_t, which is changing over time as the city grows, behaves similarly to the standard adjustment cost model. It overshoots on impact and then declines back to the marginal cost C, once the steady state is reached and the city does not grow anymore. However, price dynamics in each given location are richer as Figure 10 shows.

Figure 10 illustrates the price behavior in different locations over time. Panel (a) shows a sample of locations, including some developed after the shock, as the city expanded. The dynamics differ across neighborhoods. Developed neighborhoods experience price dynamics that are qualitatively similar to the standard adjustment cost model, with an initial spike in

prices followed by a gradual return to the initial price level. Neighborhoods that develop late in the process do not experience overshooting at all, and their prices display a gradual increasing path towards their long run level. Finally, intermediate neighborhoods show hump-shaped dynamics, with a gradual increase followed by a gradual decrease towards the long run level. Initially these neighborhoods grow and the local externality makes them more desirable, but eventually the city expands reducing the pressure of housing demand and driving prices down. Panel (b) focuses on locations which used to exist already in the initial steady state, and shows that prices in neighborhoods originally father from the city center react more to the shock than prices in neighborhoods already fully developed. This is due to our gentrification mechanism and it is what drives the amplification result. Also, this implies that, after a city-wide demand shock, richer neighborhoods experience a more cyclical behavior than poorer neighborhoods which feature a trend increase.

Let us now compare the aggregate house price behavior in our model with a standard adjustment cost model with no externalities. In the benchmark adjustment cost model, where $\beta = 0$, the rent is going to be the same in all locations and equal to $\alpha \left(N/2 \right)^{1-\alpha} I_t^{\alpha-1}$. The equilibrium is then simply characterized by equations (18) together with

$$p_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j \alpha \left(\frac{N}{2} \right)^{1-\alpha} I_t^{\alpha-1}.$$

After a decrease in the interest rate, construction slowly increases to the new steady state level and aggregate price increase on impact and then decline to the same constant initial level. In our model, instead, the price in the marginal neighborhood goes back to the marginal cost $c'(0)$, but the aggregate price level of prices is going to be permanently higher. This is due to the fact that, after the shock, the city starts expanding, so that neighborhoods that were not fully developed are going to grow, reaching permanently higher price levels. In the short run, prices in the less developed neighborhoods grow faster than in the standard model, because agents living there expect their neighborhood to grow and the neighborhood externality to be stronger. This generates the amplification effect.

Figure 11 shows the behavior of aggregate prices, after a decline in the interest rate from 3.5% to 3%, both in our model and in the standard adjustment cost model, showing both the level effect and the amplification effect on aggregate prices.

5 Additional Empirical Evidence

The model in the previous section makes additional predictions about both the within city and cross city housing price dynamics. In particular, the model suggests that the level of

rich individuals are important drivers of within city and cross city house price movements. In this section, we test four additional predictions. First, we show that the neighborhoods that experience rapid price growth show strong evidence of neighborhood gentrification. Second, we show that for Chicago (where we have detailed building data) neighborhoods that experienced rapid house price growth during the 2000-2006 period also experienced a dramatic increase in new building activity. Third, we show that both within cities and across cities, higher levels of income (larger change in income) is associated with higher levels of land prices (larger change in land prices). These patterns even hold in cities where housing supply is fairly elastic. Lastly, we show that part of the differences in the 2000-2006 house price appreciation across cities can be explained by differences in initial income between the cities.

5.1 Neighborhood Gentrification

Within the model, the mechanism by which initially lower priced neighborhoods experienced a rapid increase in home prices is through gentrification. A demand shock for housing induces richer households to move outward to the fringe of the high priced neighborhoods thereby expanding the high priced neighborhoods into the lower price neighborhoods. As the rich people move into the formerly poorer neighborhoods, the positive externality from living in that neighborhood increases, driving up the equilibrium price of land in that neighborhoods.

To test for evidence of this version of the model, we examine whether the neighborhoods that experienced rapidly growing home prices actually gentrified. To answer this question, we need very detailed demographic and income data for neighborhoods within a city. We use two types of data to perform this analysis. For earlier periods, we use data from the 1980, 1990, and 2000 U.S. Census. The Census data is ideal given that it provides a variety of demographic and income measures at low levels of aggregation. For the 2000-2006 period, we use statistics on average income by zip code compiled from tax records by the IRS.[23] The IRS data provides consistent measures of the following income variables for all zip codes in the country for the years 1998, 2001, 2002, and 2004 - 2006: the number of returns filed from people living in the zip code, the sum of total adjusted gross income (AGI) for the entire zip code, and the number of filers in the zip code with AGI under $10,000, between $10,000 and $25,000, between $25,000 and $50,000 and over $50,000.[24] When measuring potential gentrification during the 2000-2006 period, we use the IRS data from 1998 - 2006 period. We start in 1998 because it is the first

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[23]See the Data Appendix for details of the data.

[24]The IRS data also includes information about salaries and wages, charitable contributions, and the number of Schedule C, F, and A returns. We do not use such measures in our analysis.

year of data we can analyze prior to 2000.[25]

To analyze whether a potential neighborhoods experiences signs of gentrification, we estimate the following regression:

$$y_{t,t+k}^{i,j} = \omega_0 + \omega_1 \overset{g}{HP}_{t,t+k}^{i,j} + \delta X_t^{i,j} + \mu^j + \nu_{t,t+k}^{i,j}$$

where $y_{t,t+k}^{i,j}$ is some measure of gentrification in neighborhood i of city/metro area j during the period t to $t+k$, $\overset{g}{HP}_{t,t+k}^{i,j}$ is the growth rate in house prices within the neighborhoods during t and $t+k$ (as defined as in section 3), $X_t^{i,j}$ is a vector of controls for neighborhood i of city/metro area j during period t and μ^j is a vector of city/metro area fixed effects. In this sub-section, we explore whether various measures of gentrification are related to changes in house prices during the 1980-1990 period, the 1990-2000 period, and the 2000-2006 period. The key to this analysis is defining potential measures of gentrification.

In Table 2, we estimate (3) on potentially gentrifying neighborhoods from the 1990-2000 period and from the 1980-1990 period. The benefit of using the earlier periods is that we can then use more diverse and more localized measures of gentrification. The four gentrification measures explored in Table 2 (i.e., our measures of $y_{t,t+k}^{i,j}$) are: (1) the percentage change in median income, (2) the percentage point change in the poverty rate, (3) the percentage point change in the fraction Black, and (4) the percentage point change in the vacancy rate. In rows (1), our measure of $\overset{g}{HP}_{t,t+k}^{i,j}$ is the percentage change in the Case-Shiller index between 1990 and 2000. As a result, we again restrict our analysis to only those zip codes covered by Case-Shiller. For this regression, we examine all metro areas covered by Case-Shiller during the 1990s. In particular, we estimate (3) pooling together all the metro areas and including metro area fixed effects. Consistent with the model's predictions, neighborhoods that appreciated the fastest seemed to gentrify. Relative to other areas within the city, high house price appreciation neighborhoods experienced higher growth in median income, a bigger decline in the poverty rate, a fall in the fraction of neighborhood populated by Blacks, and a fall in the vacancy rate.

Rows (2)-(3) are analogous to row (1) except our measure of $\overset{g}{HP}_{t,t+k}^{i,j}$ is now the percentage change in median census tract house prices (using Census data). In row (2), we examine the relationship between our measures of gentrification and the change in median housing prices between 1990 and 2000 within census tracts of a metropolitan area. In row (3), we restrict our analysis to the 1980-1990 period. As with the results in Figure 4, we only include metro areas that have at least 50 consistently measured census tracts between 1980 (1990) and 1990 (2000) with non-missing house price information. Again, we find that neighborhood price

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[25]Our results are unchanged if we focus on the 2001 - 2006 period.

appreciation within a city is correlated with observable measures of gentrification. For all results in Table 2, we exclude the vector of additional controls (X). However, we re-estimated all the specification including the initial level of the variable being examined as a control. The results were quantitatively similar.

Table 3 shows the results of the estimation of (3) for the pooled sample of metropolitan areas covered by the Case-Shiller data for the 2000-2006 period. For the results of in Table 3, we use the IRS data to define our measures of gentrification.[26] For our measure of gentrification, we focus both on the change in the fraction of low income households living in the zip code between 1998 and 2006 (columns 1 and 2) and the change in the fraction of high income households living in the zip code between 1998 and 2006 (column 3). We define the fraction of "low income" households in a zip code in two ways: the fraction of households who reported having their AGI being below \$10,000 (column 1) and the fraction who reported having their AGI being below \$25,000 (column 2). We define the fraction of "high income" households as being the fraction of households who reported having their AGI being above \$50,000. We use these cutoffs because they are the cutoffs given to us by the IRS. Our measure of $\overset{g\ i,j}{HP_{t,t+k}}$ is the change in house prices at the zip code level as measured by the Case-Shiller index.

In the first row of Table 3, we estimate (3 omitting the vector of additional controls. The results show that neighborhoods that experienced substantial price appreciation between 2000 and 2006 also experienced a dramatic decline in the fraction of poorer households living in the neighborhood. In particular, a 100 percent increase in house prices within a neighborhood was associated with a 4.9 percentage point decline in the fraction of households who had AGI less that \$10,000 per year and was associated with a 7.4 percentage point decline in the fraction of households who had AGI less than \$25,000 per year. For the pooled data across the metropolitan areas covered by Case-Shiller, roughly 20 percent of all respondents had AGI less than \$10,000 and roughly 45 percent had AGI less than \$25,000. Also seen from Table 3, neighborhoods that experienced rapid house price growth experienced a slight increase in the fraction of households in the neighborhood that had AGI above \$50,000 per year (column 3).

In rows 2 and 3 of Table 3, we include controls to proxy for initial differences in average AGI across the different neighborhoods. Specifically, in row 2, we include controls for the initial share of the variable in question. For example, in column (1), we include the share of households with AGI less than \$10,000 in neighborhood i in 1998. Similarly for columns (2) and (3), we include the fraction of households with AGI less than \$25,000 and the fraction of households

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[26] We start in 1998 given that was the first year that the IRS had income data prior to 2000.

with AGI more than \$50,000, respectively. In row 3, we include 4 separate 1998 neighborhood level controls in each regression: the fraction of households with AGI less than \$10,000, the fraction of households with AGI less than \$25,000, the fraction of households with AGI above \$50,000 and the mean level of AGI in the neighborhood. As seen from these results, even controlling for initial differences in the level of income in these neighborhoods, neighborhoods that experienced high growth rate in house prices experienced a non-trivial decline in the fraction of poor households living in the neighborhood.

In column 4 of Table 3, we tried a different measure of gentrification: the percentage increase in average income within the zip code. As seen from the table, this measure is less correlated with changes in house prices. In hindsight, this is not surprising given the large divergence in income experienced between wealthier households and either median or poorer households from the mid-1990s through the mid-2000s[27]. The wealthy households who lived in the initially high priced areas in 2000 got much richer during the 2000-2006 period. So, even if some of those people moved into neighboring areas, the increase in average income in the high priced neighborhoods could still be large relative to the increase in income in the gentrifying neighborhoods.

5.2 Building Within Gentrifying Areas

In this sub-section, we explore building patterns within gentrifying neighborhoods. There is no systematic collection of building behavior at the neighborhood level across multiple cities for a long time period. As a result, in this sub-section, we restrict our analysis to the neighborhoods within Chicago during the 2000-2004 period. We use this time period because the city of Chicago maintained a database with all Chicago building permits issued during that time period. For each permit in the database, it was recorded whether or not the permit was for residential housing (as opposed to non-residential). For residential properties, the number of units in the proposed building was also tracked. Lastly, we only focus on permits for "new construction' and permits for "building demolition". At this point we ignore all permits for additions, alterations, and repairs. Each permit is associated with an address. We use the addresses to map the permits into the 77 Chicago neighborhoods. (These are the same neighborhoods as shown in Panel A of Appendix Table A2).

Using the permit data, we make measures of gross and net building activity within each neighborhood for each year. The gross number of units added to a neighborhood in each year is defined as the total units planned to be built from permits issued in a given year using the information from the "new construction permits". The net building activity is gross building

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[27]See, for example, Piketty and Saez (2007) and Kaplan and Rauh (2009)

activity less the total number of building units planned to be demolished during a given year. See the Data Appendix for a complete discussion of the Chicago permit data and for our construction of gross and net new units added by neighborhood.[28]

In Figure 12, we show the scatter plot of housing price growth by neighborhood between 2000 and 2006 (x-axis) against the change in the gross building activity (in units) between 2000 and 2004 (circles on the scatter plot) and the change in net building activity (in units) between 1998 and 2004 (triangles on the scatter plot). We fit two regression lines to the scatter plot. The solid line is the regression line for the net building activity while the dashed line is the regression line for the gross building activity. As predicted by the model, there is a large correlation between building activity and neighborhood price appreciation. For the neighborhoods that experienced a rapid growth in prices, there was a rapid expansion of new net housing units in those areas - relative to 2000 housing investment levels.

It should be noted that the Chicago permit data that we have access to extends back to 1993. Between 1993 and 1998, there was essentially no change in the yearly addition to net housing units within the areas that experienced high property growth rates during the late 1990s and the early 2000s. All the increase in building activity only started when house prices actually started rising.

6   Discussion

The model and empirical results have implications for many related literatures. In this section, we briefly discuss the relationship of our results to these literatures.

There is a well developed literature using spatial equilibrium models to explain differences in house prices within a metropolitan area. See, for example, Alonso (1964), Mills (1967) and Muth (1969). These models focus on the trade-off between transportation costs (measured in distance to the center city business district) and house prices. A separate literature using spatial equilibrium models developed to explain price differentials across metropolitan areas. See, for example, Rosen (1979) and Roback (1982). In these models, places are distinguished by the amenities they provide and by the wages paid to workers. In high amenity cities (e.g., warm weather cities), wages are low or housing prices are high.[29]

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[28]For ease of exposition, we are going to refer to the amount of planned units built from the permit data as being actual building behavior. We are well aware of the possibility that not all building permits will be executed. However, such differences between building and actual construction are typically small. For example, for the U.S. as whole between the 1999 and 2004 period, the U.S. Census Bureau estimated that 97.5 percent of all residential building permits resulted in a housing start (i.e., construction started) and 94 percent of permits resulted in a housing completion (i.e., construction was completed). See `http://www.census.gov/const/www/nrcdatarelationships.html` for more details.

[29]Glaeser (2007) offers a thorough survey of this literature.

We contribute to this broad literature by introducing neighborhood externalities into a spatial equilibrium model. The importance of neighborhood externalities has emphasized recently. For example, Glaeser, Kolko, and Saiz (2001) present evidence showing that cities are centers of consumption and that bigger cities offer more consumption opportunities. They conclude that "the role of urban density in facilitating consumption is extremely important and understudied." Rossi-Hansberg, Sartre, and III (2009) use data from revitalization programs in Richmond to show that residential externalities are important. They show that the neighborhoods adjacent to the areas that were targeted by the program experienced considerable increases in land value relative to similar sites in a control neighborhood. Becker and Murphy (2003) present a model of neighborhood sorting with two types of individuals in two fixed size neighborhoods. In the Becker-Murphy model, all individuals prefer to live around one of the types. In this set up, neighborhoods provide an externality to individuals to the extent that there is a greater share of good types residing in the neighborhood.

The innovation of our paper is to embed these neighborhood externalities into a standard spatial equilibrium model and show how such preference affect the evolution of house prices across various types of neighborhoods with respect to shocks to housing demand. Given recent housing price dynamics, there has been a renewed interest in understanding why house prices cycle and why the size of the cycle differs across different locations. A prominent explanation to explain why housing prices increased more in one area relative to another for a similar sized demand shock is that different areas have different short-to-medium run housing supply elasticities. In areas where supply can adjust easily, the effect of demand shocks on prices are mitigated via increased building. In areas where it is harder or more costly to adjust supply, increased demand shocks will raise the equilibrium price of housing. If long run housing supply elasticities are bigger than short run housing supply elasticities, an increase in housing demand followed by the subsequent housing supply adjustment will lead to equilibrium housing price cycles.[30]

What are these potential short run supply constraints and why do they differ across locations? Through a series of papers, Glaeser shows that strict regulations on new building is an important deterrent to housing supply adjustments.[31]. In a recent paper, Saiz (2009) shows that water barriers and the steepness of land gradients, in addition to regulation, serve as binding barriers to the adjustment of housing supply. Gyourko (2009) provides a detailed survey

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[30]For a formalization of these concepts, see Topel and Rosen (1988) and Glaeser and Gyourko (2006).

[31]See, for example, Glaeser and Gyourko (2003), Glaeser, Gyourko, and Saks (2005b), Glaeser, Gyourko, and Saks (2005a), and Glaeser and Ward (2009)

of barriers to supply adjustment. Our paper adds to the literature of why housing prices can evolve differentially within a given area for a given demand shock. The ability to expand neighborhoods and still maintain neighborhood externalities acts like a barrier to adjusting supply. As seen from section 4, even in a model with no adjustment costs, housing demand shocks can have a sustained effect on prices in a neighborhood depending on the concentration of poor people in neighborhoods adjacent to the rich neighborhoods. Additionally, we show that the existence of positive neighborhood externalities can amplify housing price dynamics relative to a similar model with standard adjustment costs.

In a separate literature, Gyourko, Mayer, and Sinai (2006) develop a model to explain the phenomenon of "superstar cities" - cities that had house price appreciation rates well above the national average. The cornerstone of their model is there are certain locations (in fixed supply) that have amenities that are more desirable than other the amenities in other locations. Given these high amenity locations are in fixed supply, rising national incomes raise the prices in these locations as high income individuals compete to live in these locations. Our model suggests another reason for such superstar cities (which could complement the results in Gyourko, Mayer, and Sinai (2006)). Our model predicts that the amenities in these superstar cities will actually grow over time due to the neighborhood externalities. As more rich people move into the superstar cities, the amenities provided from living around more rich people could put additional upward pressure on housing prices in these cities. Even though the discussion in our paper focused on explaining cross neighborhood housing prices within a city over time, the model could be augmented naturally to explain cross city differences in housing prices over time.

Our paper also speaks to the large literature on the gentrification of urban areas. Recent work has discussed the role of the following in explaining gentrification: the increased consumption benefits from living in a city (Glaeser, Kolko, and Saiz (2001)), the age of a city's housing stock (Rosenthal (2008)) and Brueckner and Rosenthal (2008)) and direct public policy initiatives via community redevelopment programs (Busso and Kline (2007), Rossi-Hansberg, Sartre, and III (2009)).[32]. Our work adds to this literature by showing how generic shocks to housing demand within a city can result in the gentrification of neighborhoods.

Despite the broad literature on gentrification, very little work emphasizes the importance of spatial dependence - either theoretically or empirically - in predicting the spatial patterns of gentrification. There are two notable exceptions. Brueckner (1977) finds that urban neighborhoods in the 1960s that were in close proximity to rich neighborhoods got relatively poorer

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[32]For a recent survey of the theoretical and empirical work on gentrification, see Kolko (2007)

between 1960 and 1970 (as measured by income growth). Conversely, Kolko (2007) finds the opposite pattern. Kolko finds that 1990 neighborhoods bordering richer neighborhoods grew faster between 1990 and 2000 than 1990 neighborhoods bordering poorer neighborhoods.

Our model reconciles both of these results. During periods of declining housing demand in urban areas (like the suburbanization movement during the 1960s), neighborhoods within an urban area will diverge - with the divergence being most pronounced for the areas neighboring the high priced areas. Conversely, during urban renewals (like what was witnessed during the 1990s), neighborhoods within the urban area will converge - with the convergence being most pronounced for areas neighboring the high priced areas. In short, our model makes very specific predictions about the spatial patterns of gentrification when urban centers are faced with either positive or negative housing demand shocks.

Very few existing studies have examined within city (or metropolitan area) price movements. Of those that exist, most perform their within city analysis by comparing the appreciation rates of "high end" properties to the appreciation rate of "low end" properties. Moreover, these studies tend to focus their analysis on a given market (or a very small set of markets) during a given time period.[33] Two papers, however, look specifically at differences in house prices appreciation across zip codes within a metropolitan area. Case and Mayer (1996) look at differential movements in prices within cities of the Boston metro area between 1982 and 1992 while Case and Marynchenko (2002) look at differential trends in prices across different zip codes within the Boston, Chicago, and Los Angeles metro areas during the 1983 to 1993 period. No systematic relationships emerged from these studies.

Recently, Mian and Sufi (2009) documented that the zip codes within the U.S. that had increased access to subprime mortgages during the recent period also experienced higher levels of property price appreciation during this period. They took this finding as additional evidence that the housing demand shock experienced during 2000 and 2006 was larger for poorer (subprime) households than it was for richer households. As poor households suddenly received access to credit, they bid up the price of properties in poorer neighborhoods (where they lived) relative to rich neighborhoods. While our paper does not speak directly to the differential demand shock experienced between rich and poor households during recent periods, our spatial equilibrium model with neighborhood externalities shows that different prices movements between rich and poor neighborhoods cannot - by itself - be used as evidence of different demand

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[33]See, for example, Mayer (1993) and Poterba (1991) who analyzed Oakland, Dallas, Chicago, and Atlanta between 1970 and the mid 1980s, Case and Shiller (1994) who analyzed Boston and Los Angeles during the 1980s, and Smith and Tesarek (1991) who analyzed Houston during the 1970s and 1980s.

shocks for rich and poor households. In terms of our model, we show that even in a world with a common housing demand shock (across people with different incomes), initially poor neighborhoods will appreciate at a higher rate as richer households expand out of their original neighborhoods into newer neighborhoods. Given all the empirical work we present in this paper, it is likely that the reason prices appreciated more in initially low priced neighborhoods during the recent period was do, at least in part, to the mechanisms presented in our paper.

7   Conclusion

In this paper, we have focused our attention on within city price movements to learn about housing price dynamics across cities. We start by using a variety of data sources to show a new empirical finding: during housing price booms (busts), the neighborhoods with lower initial house prices experience larger appreciation (depreciation) than neighborhoods with higher initial prices. This fact is robust to the use of different housing price series and across different time periods. Also, the low price neighborhoods which appreciate the most are the ones closer to the high price neighborhoods. Lastly, we find that the difference in appreciation rates between low and high price neighborhoods is larger when the city experiences a larger average housing boom. In other words, larger city-wide housing booms are associated with a greater convergence of prices across neighborhoods within the city. We also find that larger city-wide housing busts are associated with a greater divergence of neighborhoods within the city.

We then have presented a spatial equilibrium model of property price movements across neighborhoods within a city to explain these facts. There are two key features of our model. First, we analyze a linear city with rich and poor households. Second, we assume that there is a positive neighborhood externality: households like to live closer to rich neighbors. Also, the rich households are the ones who most benefit from the externality that they generate. One story behind this assumptions is that the more rich people live in a neighborhood, the greater is the number and the variation in services and public goods (restaurants, museums, coffee shops, dry cleaners, etc.) that are going to be provided. The importance of urban density in facilitating neighborhood externalities has been recently emphasized in the work of Glaeser, Kolko, and Saiz (2001) and in Becker and Murphy (2003). Our innovation is to embed these externalities into a model of neighborhood development within a city and show how they affect the evolution of house prices across neighborhoods in reaction to housing demand shocks.

The existence of such externalities affects the nature of housing price dynamics in important ways. In equilibrium, there is full segregation: the rich households live in the city center and the

poor in the periphery of the city. Hence, after a demand shock, there is going to be endogenous gentrification. The rich households are going to expand in the adjacent poor neighborhoods, driving the house prices up, due to the externality. Our model predicts that, even in the presence of an elastic housing supply, house prices increase permanently in reaction to an unexpected and permanent demand shock. Moreover, our model puts some structure on which factors may drive different house price dynamics across different cities. In particular, we show that richer cities gentrify more in reaction to the same shock, and hence experience higher average price increase. Also, higher gentrification leads to higher convergence of prices across neighborhoods. This suggests not only that different cities may react differently to a common demand shock, but also that the same city, at different stages of its development, may experience differential housing price responses to housing demand shocks of similar size.

In the last part of the paper, we test a number of additional predictions of our spatial equilibrium model. All empirical tests support the predictions of the model. In particular, we show three sets of facts. First, the neighborhoods which experience higher appreciation show evidence of gentrification. Second, during a housing boom, there is more construction in the neighborhoods which experience higher appreciation. Third, the initial income level of a city explain part of the cross city house price appreciation, irrespective of the density of the city (housing supply constraints).

One area on which our paper is silent is the welfare implications of our model. A full model – potentially incorporating commuting costs – could be used to quantitatively assess the welfare effects of a housing demand shock. Although we did not attempt such an analysis in this paper, we feel this is an important area for future research.

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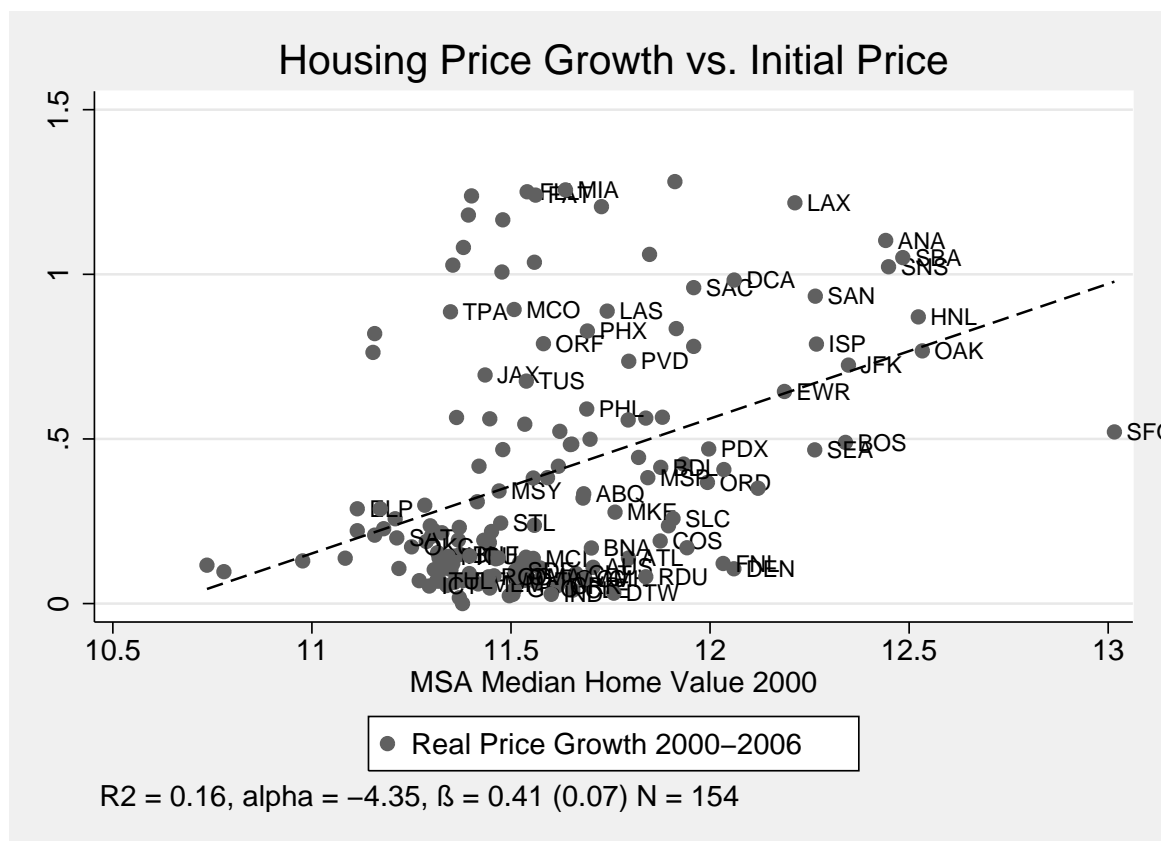
Figure 1: Price Growth Across MSAs

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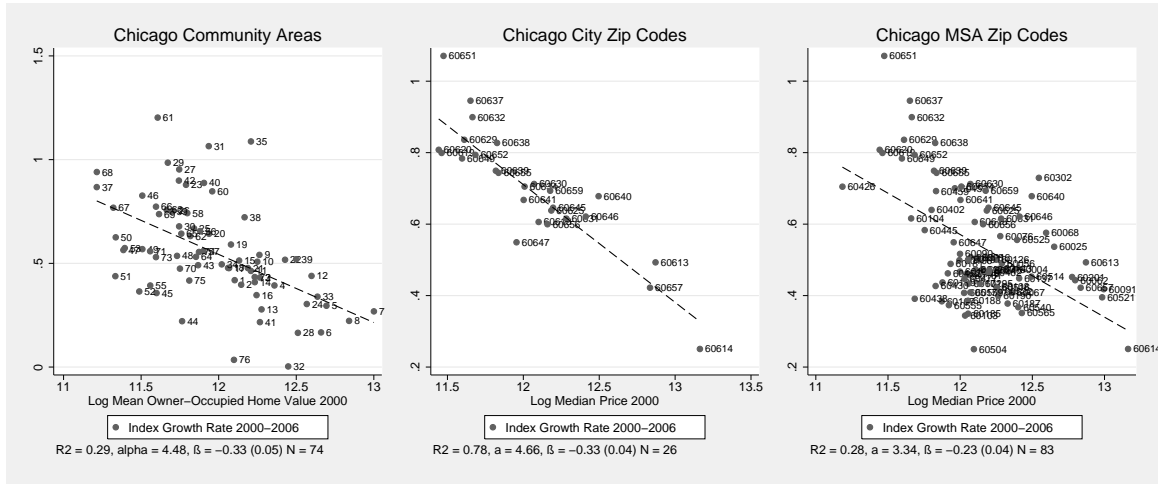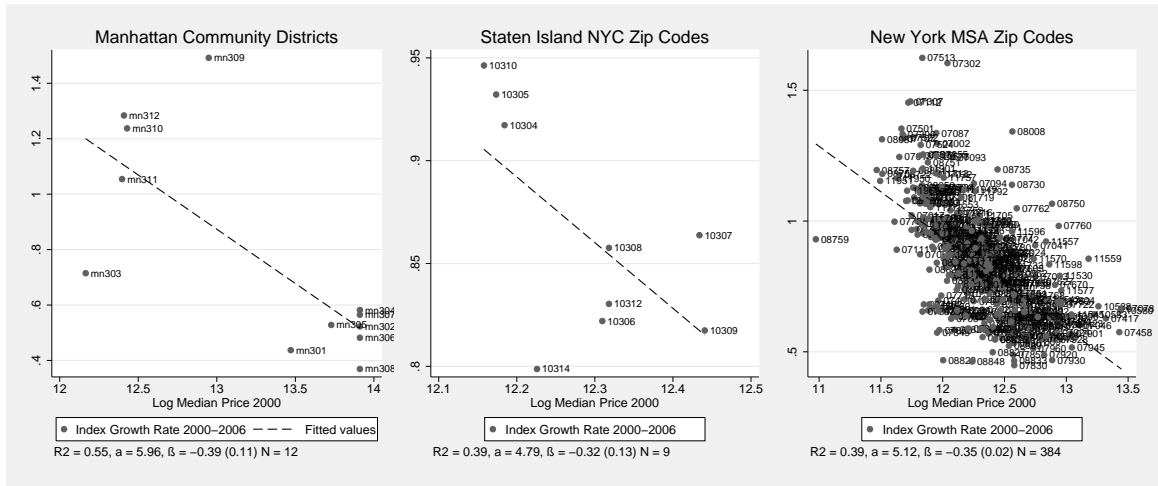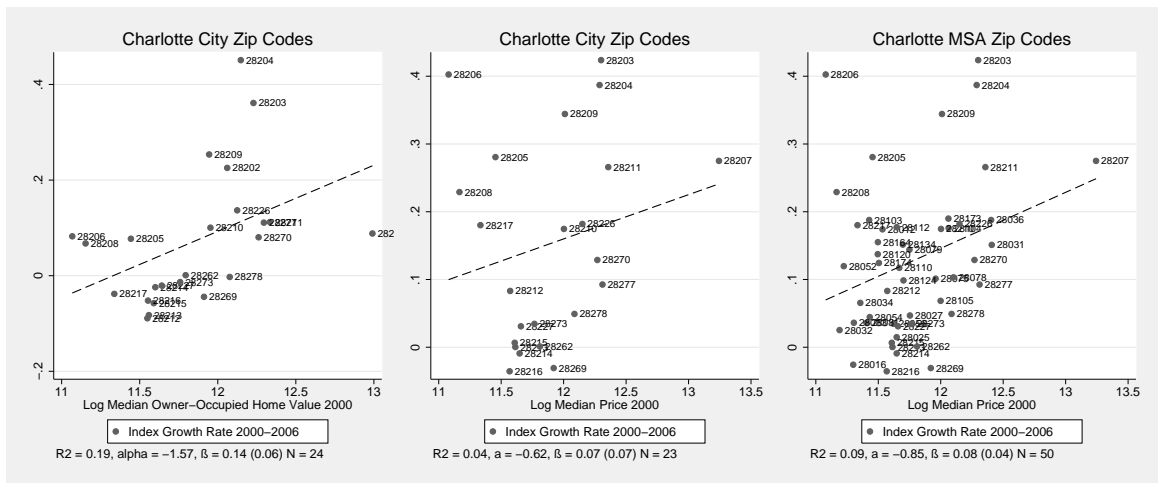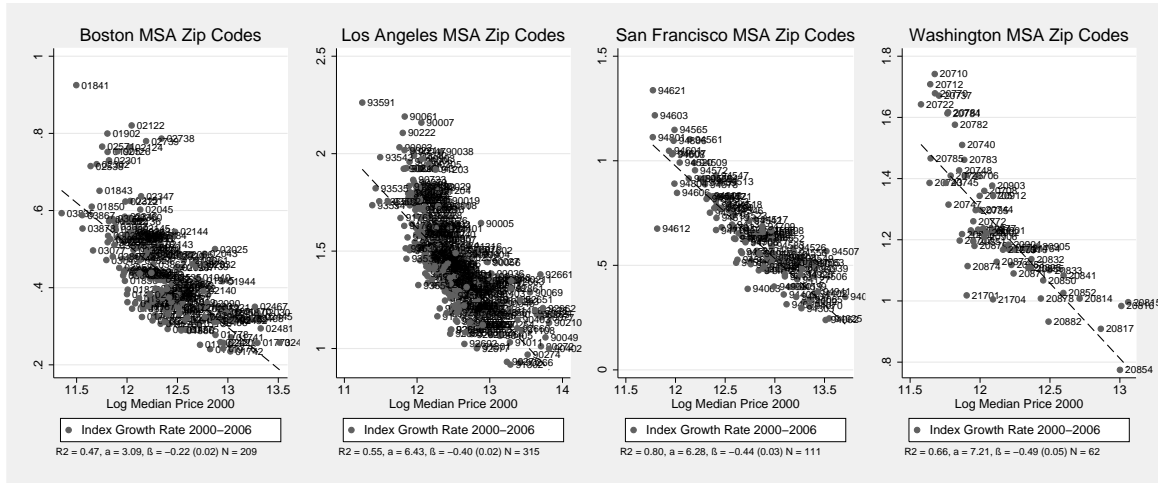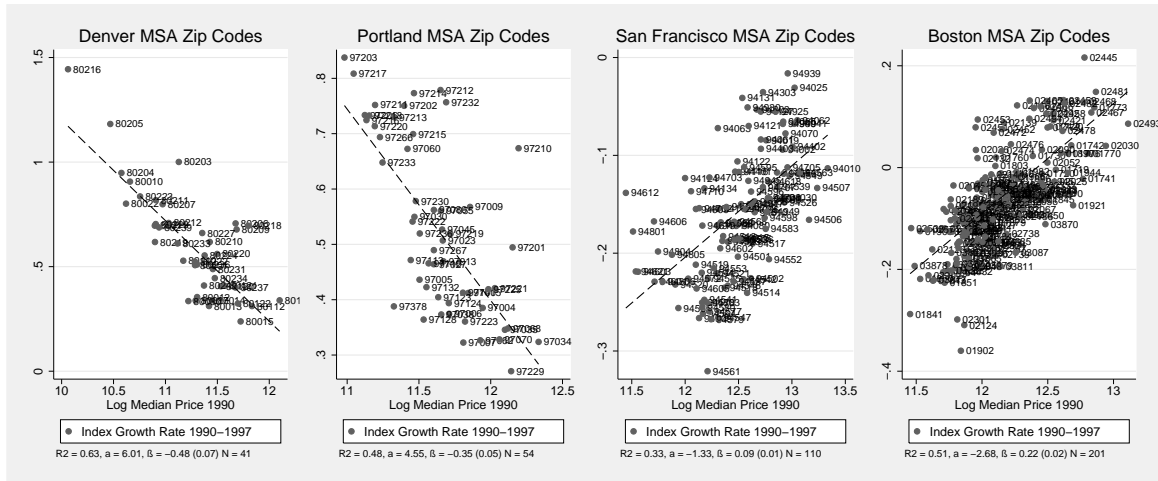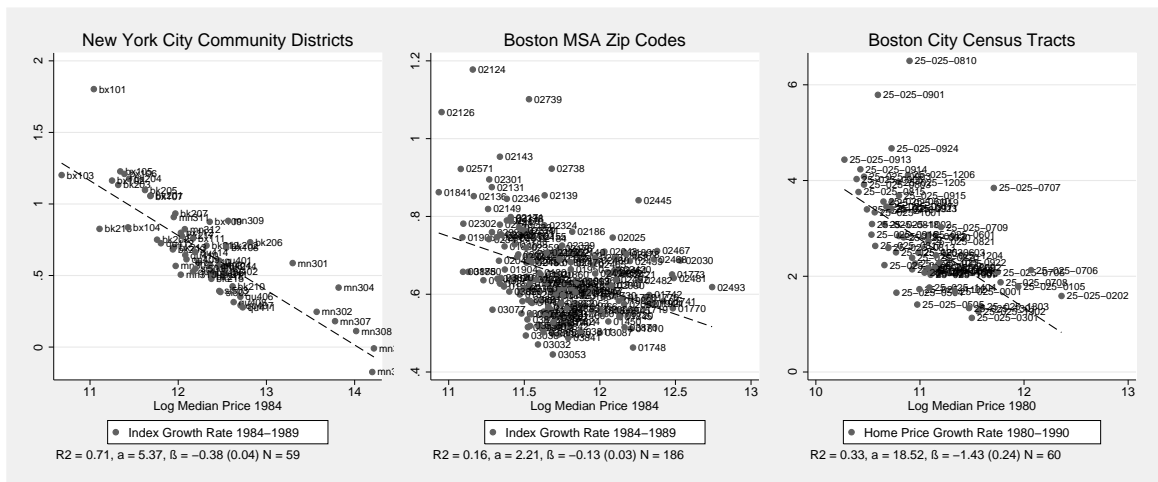
Figure 2: Price Growth vs. Initial Price 2000-2006 for Chicago, NYC, and Charlotte.
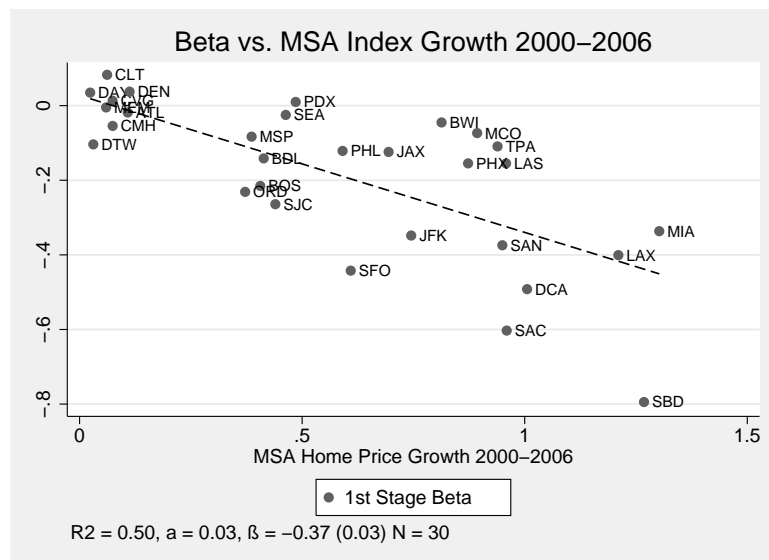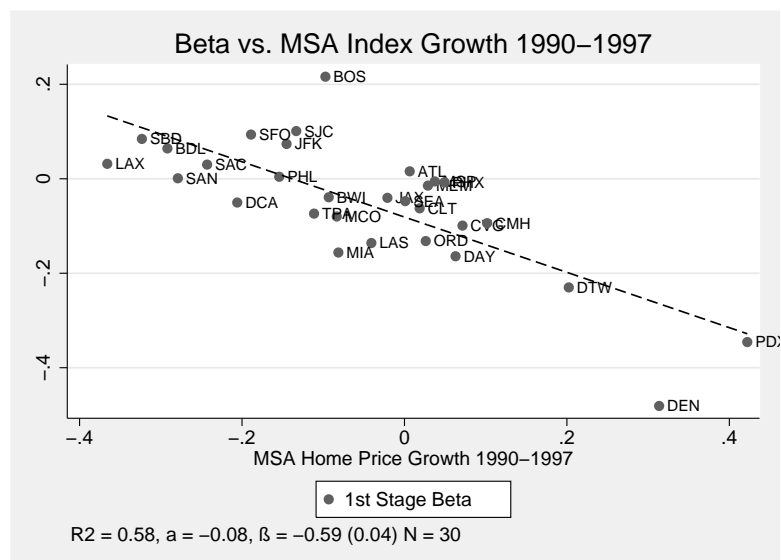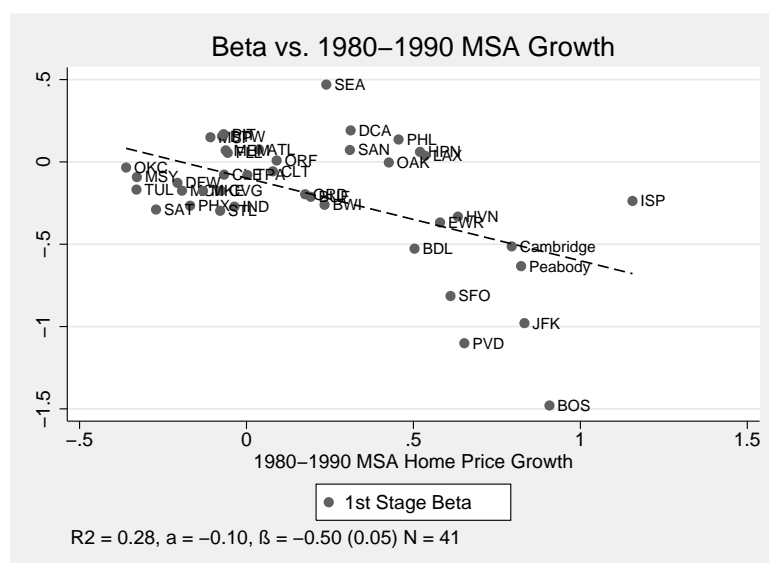
A: Other cities 2000-2006.

B: Other cities 1990-1997.

C: Other cities 1984-1989.

Figure 3: Price Growth vs. Initial Price during several periods for several other cities.

43

A: 2000-2006.

B: 1990-1997.

C: 1980-1990.

Figure 4: Beta vs. MSA Price Growth

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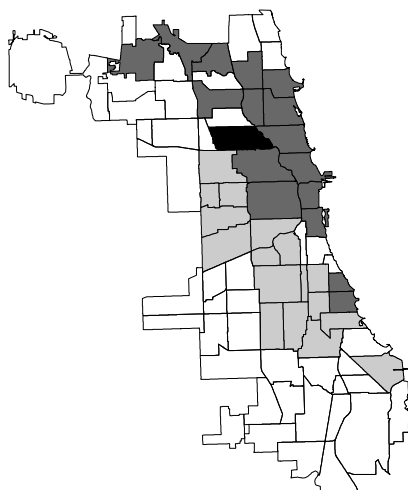
A: Chicago City
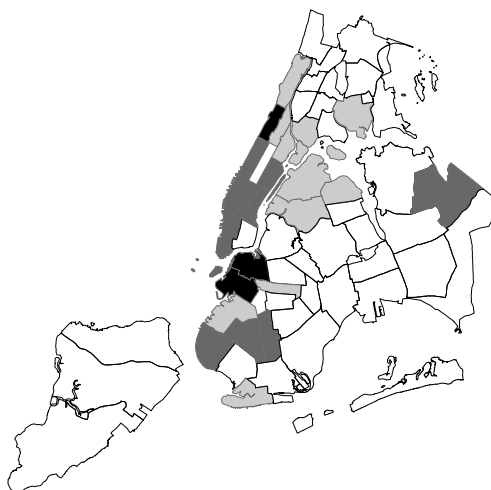
B: New York City
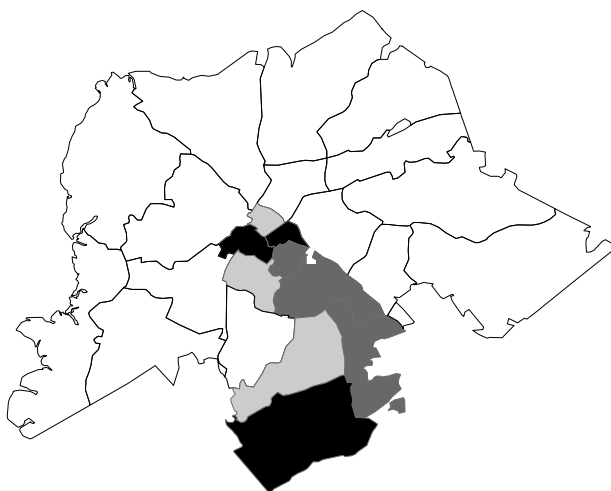
C: Charlotte City

Figure 5: Diffusion Maps. Dark shaded areas are high price in 2000, light shaded ares are high growth 2000 - 2006, black regions are both.
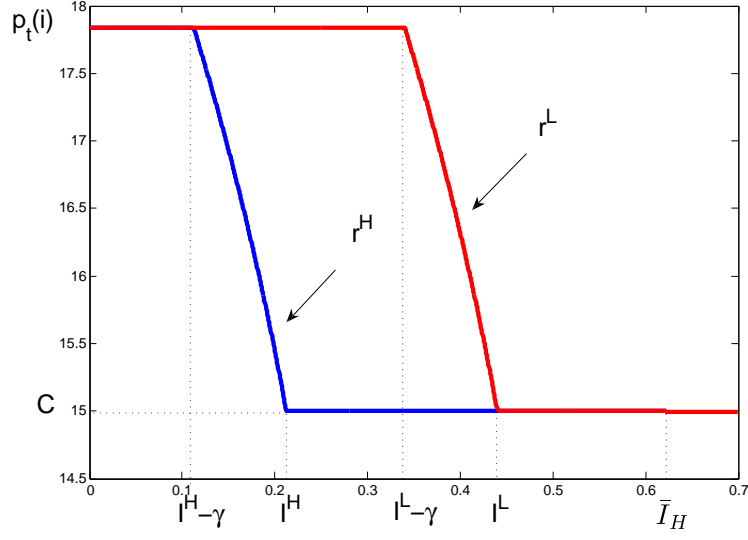
Figure 6: Reaction of the price schedule to a decline in the interest rate. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$, $r^L = .03$, and $r^H = .035$.
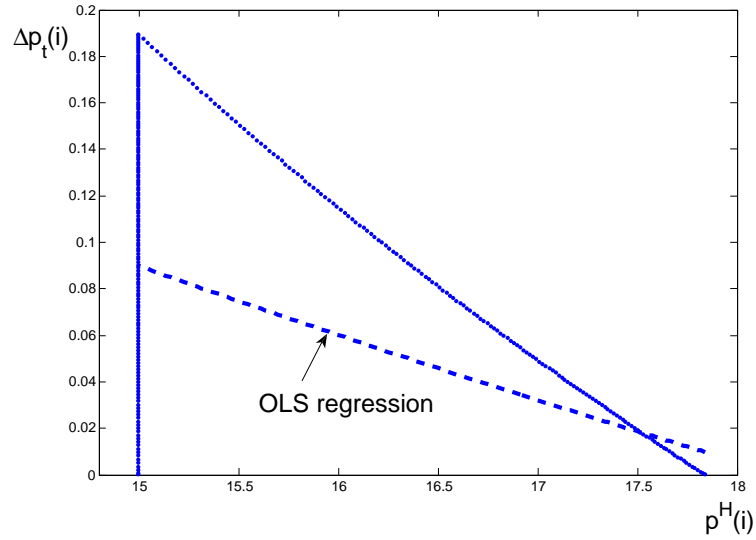

Figure 7: Price growth rate across locations as a function of the initial price level and OLS regression. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$, $r^L = .03$, and $r^H = .035$.
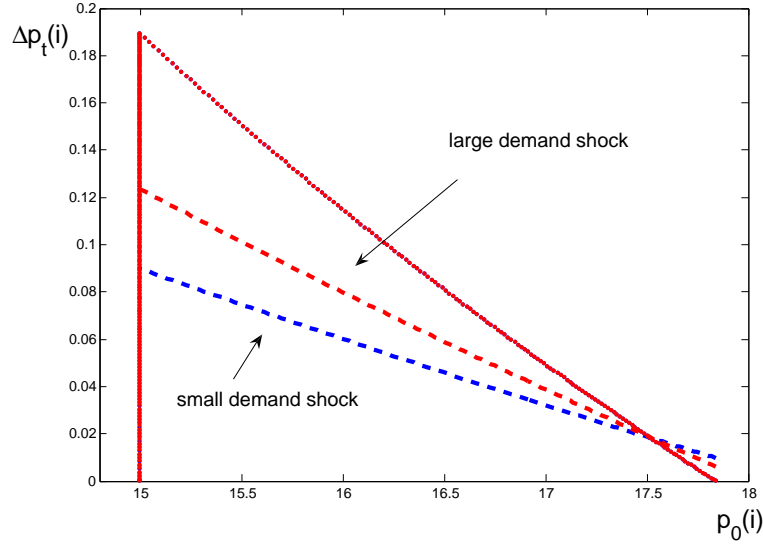
Figure 8: Price growth rate across locations as a function of the initial price level in reaction to a decline of the interest rate from .035 to .03 (blue dots) and from .035 to .028 (red dots). The associated solid lines represent the OLS regressions. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$.
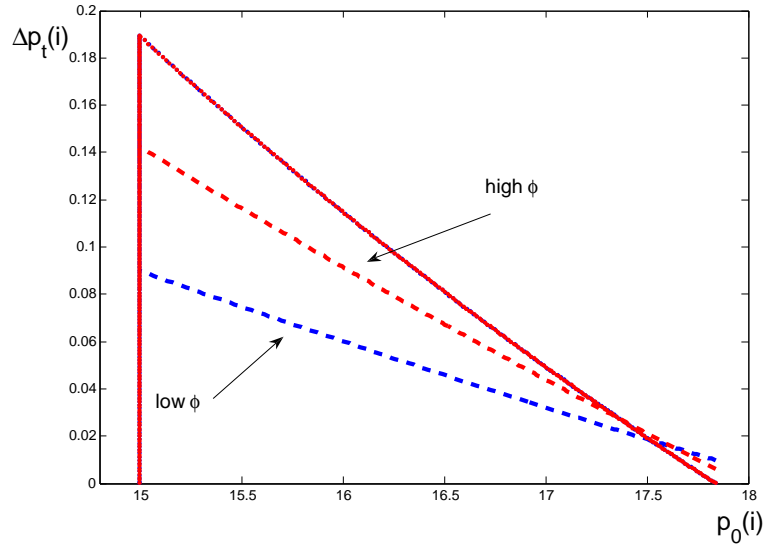

Figure 9: Price growth rate across locations as a function of the initial price level in reaction to a decline of the interest rate from .035 to .03 in a city with $\phi^R = 1$ (blue dots) and one with $\phi^R = 1.1$ (red dots). The associated solid lines represent the OLS regressions. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$, $r^L = .03$, and $r^H = .035$.
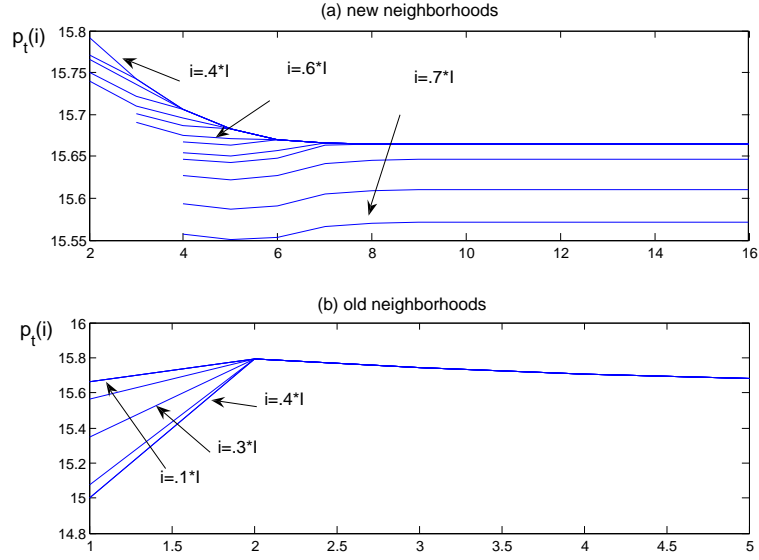
Figure 10: The figure shows the price behavior in different locations in reaction to a decline in the interest rate. Panel (a) shows neighborhoods that are born as a result of the shock, while panel (b) shows neighborhoods that existed at the initial steady state. We set $\alpha = .8$, $\beta = .05$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C_1^= 15$, $C_2 = 2$, $\psi = 2$, $r^L = .03$, and $r^H = .035$.
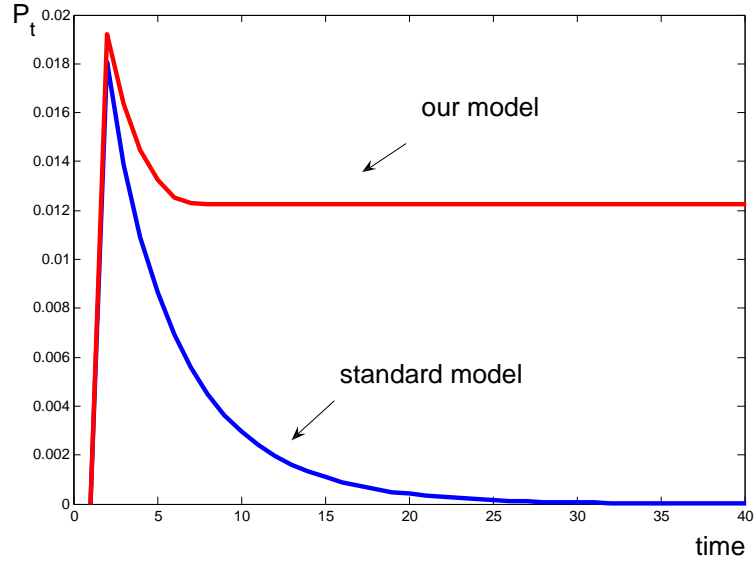

Figure 11: The figure shows the behavior of aggregate house prices after a positive demand shock in our model (red line) and in the standard adjustment cost model (blue line). We set $\alpha = .8$, $\beta = .05$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C_1^= 15$, $C_2 = 2$, $\psi = 2$, $r^L = .03$, and $r^H = .035$. In the standard adjustment cost model we set all the same common parameters.
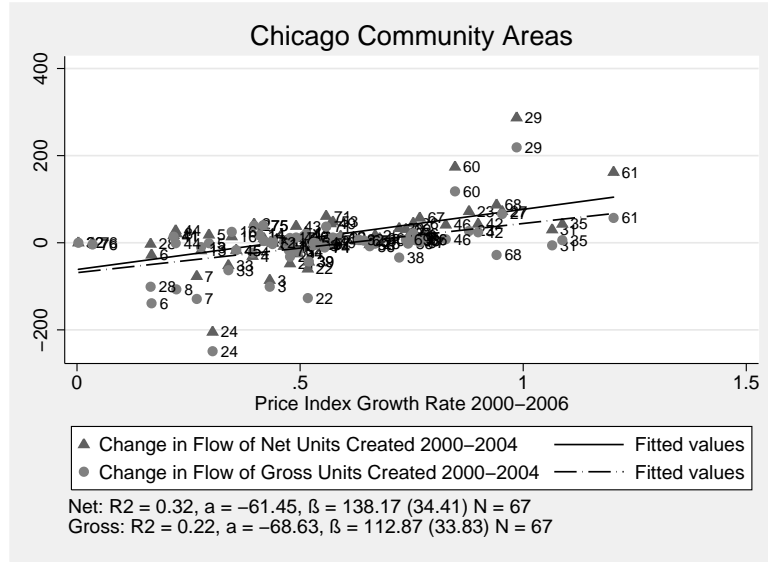
Figure 12: Change in Construction Flows and Housing Prices 1997-2004/6

Table 1: Correlation Between Zip Code Home Price Growth and Distance to Nearest High Price Zip Code

	(1) $g\stackrel{i,j}{HP}_{t,t+k}$	(2) $g\stackrel{i,j}{HP}_{t,t+k}$	(3) $g\stackrel{i,j}{HP}_{t,t+k}$
log $HP_t^{i,j}$	-0.440***	-0.425***	-0.350***
	(0.107)	(0.098)	(0.134)
log distance to nearest Zip Codes in top price tercile		-0.081***	-0.058**
		(0.026)	(0.029)
Controls	No	No	Yes
N	192	192	188
R^2	0.96	0.96	0.96

Note: Sample includes zip codes in the bottom price tercile of each city in 2000. The dependent variable is zip code housing price growth from 2000-2006. All specifications include city effects. The specification in column 3 includes zip code-level demographic controls from the 2000 census: fraction Hispanic, fraction African-American, fraction that commute by car, fraction that commute by public transit, labor force participation rate, unemployment rate, log median household income, fraction of housing units that are owner-occupied, an indicator for whether the median structure was built before 1940. Eicker-White standard errors in parentheses.

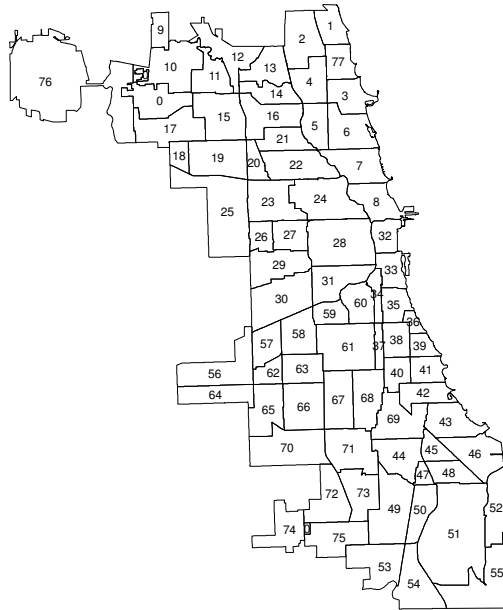## Table 2: Correlation Between Home Price and other Gentrification Measures

	(1) % Change in Med. Inc.	(2) Δ Poverty Rate	(3) Δ Frac. Afr. Amer.	(4) Δ Vac. Rate
All C-S MSA Zip Codes **1990-2000**				
$\overset{g}{HP}{}_{t,t+k}^{i,j}$	0.209***	-0.056***	-0.078***	0.014*
	(0.026)	(0.006)	(0.012)	(0.008)
	<< 0.064 >>	<< 0.008 >>	<< 0.019 >>	<< −0.018 >>
N	1,896	1,896	1,896	1,896
R^2	0.30	0.22	0.13	0.25
All Census MSA Tracts **1990-2000**				
$\overset{g}{HP}{}_{t,t+k}^{i,j}$	0.149***	-0.027***	-0.023***	-0.004**
	(0.019)	(0.003)	(0.003)	(0.002)
	<< 0.131 >>	<< 0.003 >>	<< 0.024 >>	<< −0.009 >>
N	12,077	12,081	12,081	12,081
R^2	0.07	0.11	0.07	0.18
All Census MSA Tracts **1980-1990**				
$\overset{g}{HP}{}_{t,t+k}^{i,j}$	0.080***	-0.010***	-0.007***	-0.005**
	(0.012)	(0.002)	(0.002)	(0.002)
	<< 0.219 >>	<< 0.016 >>	<< 0.027 >>	<< 0.016 >>
N	6,474	6,476	6,476	6,476
R^2	0.25	0.16	0.09	0.15

Note: MSA effects included. Eicker-White standard errors in parentheses. Mean of dependent variable in <<>>.

Table 3: Correlation Between Zip Code Home Price and Income Growth

	(1) Δ Share < 10K	(2) Δ Share < 25K	(3) Δ Share > 50K	(4) Income Growth
$\overset{g}{HP}{}_{t,t+k}^{i,j}$	-0.049***	-0.074***	0.006*	-0.027*
	(0.002)	(0.004)	(0.003)	(0.015)
R^2	0.30	0.31	0.19	0.18
$\overset{g}{HP}{}_{t,t+k}^{i,j}$	-0.018***	-0.010**	-0.001	-0.022
	(0.002)	(0.004)	(0.005)	(0.022)
Init. Share/log Wage	Y	Y	Y	Y
R^2	0.61	0.52	0.19	0.18
$\overset{g}{HP}{}_{t,t+k}^{i,j}$	-0.012***	-0.011***	0.003	-0.019
	(0.002)	(0.004)	(0.005)	(0.020)
All Thresholds, Mean	Y	Y	Y	Y
R^2	0.63	0.52	0.23	0.23

Note: N = 2,738. Initial share of households below or above threshold income in 1998 or log mean wage in 1998 included as controls in middle panel specifications. All thresholds and log mean included in bottom panel. Eicker-White standard errors in parentheses.
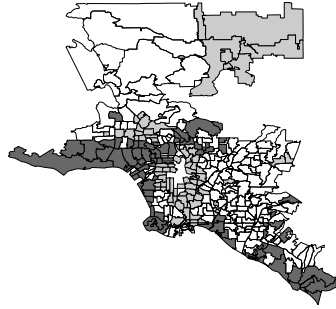
A: Chicago

B: New York City

C: Charlotte

Figure A1: Shaded Zip codes are Covered by Case-Shiller Indices in 2005
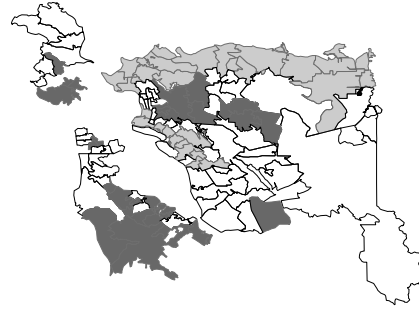
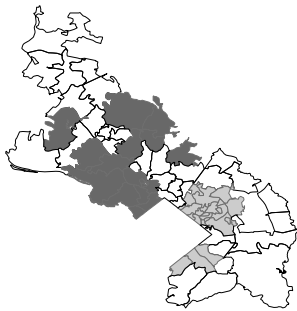A: Chicago

B: New York City

Figure A2: Chicago Community Areas and NYC Community Districts
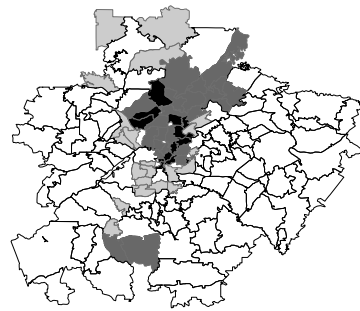
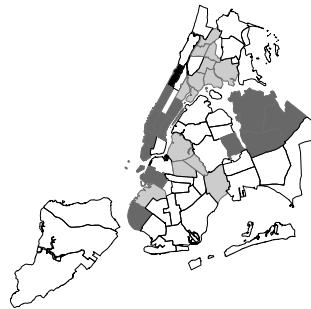A: Los Angeles Metro 2000 - 2006   B: San Francisco Metro 2000 - 2006

C: Washington DC Metro (MD) 2000 - 2006   D: Atlanta Metro 2000 - 2006

E: New York City 1984-1989

Figure A3: Diffusion Maps: dark shaded areas are high price in initial year, light shaded ares are high growth over period listed, black regions are both.

D1 Appendix

D1.1 Proof of Proposition 1

Most of the proof of Proposition 1 is in the text. As we argue in the text, we are left to check only that

$$
\begin{aligned}
U^R (i) &\leq \bar{U}^R \text{ for all } i \in \left[I_t, \bar{I}_t \right], \\
U^P (i) &\leq \bar{U}^P \text{ for all } i \in [0, I_t],
\end{aligned}
$$

where $U^s (i)$ is defined in expression (5). Using expression (6), these two conditions can be rewritten as

$$
K_R (A + H_t (i))^{\frac{\beta_R}{\alpha}} \leq K_P^{\frac{\beta_P}{\alpha}} (A + H_t (i)) + \frac{r}{1+r} \left(C^R - C^P \right) \text{ for all } i \in \left[I_t, \bar{I}_t \right], \quad (21)
$$

$$
K_P^{\frac{\beta_P}{\alpha}} (A + H_t (i)) \leq K_R^{\frac{\beta_R}{\alpha}} (A + H_t (i)) - \frac{r}{1+r} \left(C^R - C^P \right) \text{ for all } i \in [0, I_t]. \quad (22)
$$

Combining (8) with (12) and (13) we obtain

$$
\begin{aligned}
K_P &= \frac{r}{1+r} C^P A^{-\frac{\beta_P}{\alpha}}, \\
K_R &= \frac{r}{1+r} \left[C^P \left(\frac{A}{A+\gamma} \right)^{-\frac{\beta_P}{\alpha}} + \left(C^R - C^P \right) \right] (A+\gamma)^{-\frac{\beta_R}{\alpha}}.
\end{aligned}
$$

Using these expressions, condition (21) can be rewritten as

$$
\left[\frac{A + H_t (i)}{A + \gamma} \right]^{\frac{\beta_R - \beta_P}{\alpha}} \leq \frac{1 + \left(\frac{C^R - C^P}{C^P} \right) \left(\frac{A}{A + H_t(i)} \right)^{\frac{\beta_P}{\alpha}}}{1 + \left(\frac{C^R - C^P}{C^P} \right) \left(\frac{A}{A+\gamma} \right)^{\frac{\beta_P}{\alpha}}}.
$$

for all $i \in \left[I_t, \bar{I}_t \right]$. This implies that $H_t (i) < \gamma$ and hence the RHS is not smaller than 1 and that, if $\beta_R \geq \beta_P$, the LHS is not bigger than 1. Hence, $\beta_R \geq \beta_P$ is a sufficient condition for this condition to be satisfied. Notice that if $C^R = C^P$, this is also a necessary condition.

Next, condition (22) can be rewritten as

$$
\left[\frac{A + H (i)}{A + \gamma} \right]^{\frac{\beta_P - \beta_R}{\alpha}} \leq 1 + \frac{C^R - C^P}{C^P} \left(\frac{A}{A + \gamma} \right)^{\frac{\beta_P}{\alpha}} \left[1 - \left(\frac{A + \gamma}{A + H_t (i)} \right)^{\frac{\beta_R}{\alpha}} \right]
$$

for all $i \in [0, I_t]$. In these locations, by construction, $H_t (i) > \gamma$, which implies that the RHS is not smaller than 1 and that, if $\beta_R \geq \beta_P$ the LHS is not bigger than 1. Hence, $\beta_R \geq \beta_P$ is also a sufficient condition for this equation to hold. Again, it is also a necessary condition if $C^R = C^P$. Hence, this completes the proof that a fully segregated equilibrium exists if $\beta^P \leq \beta^R$.

D1.2 Proof of Proposition 2

Deonte by r^H the initial level of r, and by r^L the level of r after the shock, with $r^L > r^H$. The price schedule before $(s = H)$ and after the shock $(s = L)$ are:

$$p^s(i) = \begin{cases} \left[C^P\left(1+\frac{\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + C^R - C^P\right]\left(1+\frac{\gamma}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}} & \text{for } i \in [0, I^s - \gamma] \\ \left[C^P\left(1+\frac{\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + C^R - C^P\right]\left(1+\frac{I^s-i}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}} & \text{for } i \in [0, I^s] \\ C^P\left(1+\frac{\gamma+I^s-i}{A}\right)^{\frac{\beta_P}{\alpha}} & \text{for } i \in [I^s, I^s + \gamma] \\ C^P & \text{for } i \in [I^s + \gamma, \bar{I}^s] \end{cases} \qquad (23)$$

First, notice that if $i \geq I^H + \gamma$, then $p^H(i) = C^P$, and if $i < I^H + \gamma$, then $p^H(i) > C^P$. Also, if $i < I^H - \gamma$, then $p^H(i) = \bar{p}$, where

$$\bar{p} \equiv \left[C^P\left(1+\frac{\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + C^R - C^P\right]\left(1+\frac{\gamma}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}.$$

Next, we obtain

$$\frac{p^L(i)}{p^H(i)} = \begin{cases} \dfrac{\left(1+\frac{\min\{\gamma, I^L-i\}}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}}{\left(1+\frac{\min\{\gamma, I^H-i\}}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}} & \text{for } i \in [0, I^H] \\ \left[\left(\frac{A+\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + \frac{C^R - C^P}{C^P}\right]\dfrac{\left(1+\frac{\min\{\gamma, I^L-i\}}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}}{\left(1+\frac{\max\{\gamma+I^H-i, 0\}}{A}\right)^{\frac{\beta_P}{\alpha}}} & \text{for } i \in [I^H, I^L] \\ \dfrac{\left(1+\frac{\max\{\gamma+I^L-i, 0\}}{A}\right)^{\frac{\beta_P}{\alpha}}}{\left(1+\frac{\max\{\gamma+I^H-i, 0\}}{A}\right)^{\frac{\beta_P}{\alpha}}} & \text{for } i \in [I^L, \bar{I}^H] \end{cases} \qquad (24)$$

Also, from equations (14) and (15), we obtain $I^L > I^H$ and $\bar{I}^L > \bar{I}^H$. Then, if $i < I^H - \gamma$, it must be that $p^L(i)/p^H(i) = 1$, which implies that

$$E_{t+1}\left[\frac{p_{t+1}(i)}{p_t(i)}|p_t(i) = \bar{p}\right] = 1.$$

Moreover, $I^L > I^H$, together with expression (24), immediately implies that $p^L(i)/p^H(i) \geq 1$ for $i > I^H - \gamma$, and hence

$$E_{t+1}\left[\frac{p_{t+1}(i)}{p_t(i)}|p_t(i) < \bar{p}\right] > 1,$$

which proves the first statement or the proposition.

We now want to prove the second statement of the proposition, that is, that the price ratio $p_{t+1}(i)/p_t(i)$ is non-increasing in $p_t(i)$, where $p_t(i) = p^H(i)$ and $p_t(i) = p^L(i)$. First, notice that $p^H(i)$ is non-increasing in i, so proving that $p^L(i)/p^H(i)$ is non-increasing in $p^H(i)$ is

equivalent to prove that $p^L(i)/p^H(i)$ is non-decreasing in i. The ratio $p^L(i)/p^H(i)$ is continuous and differentiable except at a finite number of points. Hence, in order to prove that it is non-decreasing in i, it is enough to show that $d\left[p^L(i)/p^H(i)\right]/di$ is non-negative, for all i where this derivative exists. Let us show that.

For $i \in \left[0, I^H - \gamma\right]$, $p^L(i)/p^H(i) = 1$ and hence $p^L(i)/p^H(i)$ is constant in i. For $i \in \left[I^H - \gamma, I^H\right]$, we have that

1. if $I^H - \gamma < i < I^L - \gamma$, then

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\beta_R}{\alpha(A+\gamma)}\left(\frac{A+2\gamma}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}\left(\frac{A+\gamma+I^H-i}{A+\gamma}\right)^{-\frac{\beta_R}{\alpha}-1} > 0 \qquad (25)$$

2. if $I^H - \gamma < i < \min\left\{I^L - \gamma, I^H\right\}$, then

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\beta_R}{\alpha}\left(\frac{A+\gamma+I^L-i}{A+\gamma+I^H-i}\right)^{\frac{\beta_R}{\alpha}}\left[\frac{1}{A+\gamma+I^H-i} - \frac{1}{A+\gamma+I^L-i}\right] > 0 \qquad (26)$$

given that $I^H < I^L$.

For $i \in \left[I^H, I^H + \gamma\right]$ we have that

1. if $I^H < i < \min\left\{I^H + \gamma, I^L - \gamma\right\}$

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\tilde{C}}{A}\left(\frac{A+2\gamma}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}\left(\frac{A+\gamma+I^H-i}{A}\right)^{-\frac{\beta_P}{\alpha}-1} > 0 \qquad (27)$$

where

$$\tilde{C} \equiv \left[\left(\frac{A+\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + \frac{C^R - C^P}{C^P}\right]$$

2. if $\max\left\{I^H, I^L - \gamma\right\} < i < \min\left\{I^L, I^H - \gamma\right\}$

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\tilde{C}}{\alpha}\frac{\left(\frac{A+\gamma+I^L-i}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}}{\left(\frac{A+\gamma+I^H-i}{A}\right)^{\frac{\beta_P}{\alpha}}}\left[\frac{\beta_P}{A+\gamma+I^H-i} - \frac{\beta_R}{A+\gamma+I^L-i}\right] \qquad (28)$$

hence

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} > 0 \text{ iff } \frac{\beta_R}{\beta_P} < \frac{A+\gamma+I^L-i}{A+\gamma+I^H-i},$$

which is true if the shock is big enough and $I^L - I^H$ is big enough;

3. if $\max\left\{I^H, I^L\right\} < i < I^H + \gamma$

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\beta_P}{\alpha}\left(\frac{A+\gamma+I^L-i}{A+\gamma+I^H-i}\right)^{\frac{\beta_P}{\alpha}}\left[\frac{1}{A+\gamma+I^H-i} - \frac{1}{A+\gamma+I^L-i}\right] > 0. \qquad (29)$$

This proves that, if the shock is big enough, the second statement of the proposition holds.

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D1.3 Proof of Proposition 3

First, notice that at time t, each location i may lie in four possible intervals that implies different pricing behavior: $[0, I_t - \gamma]$, $[I_t - \gamma, I_t]$, $[I_t, I_t + \gamma]$, and $[I_t, \bar{I}_t]$. From expression (23), it is immediate that prices at time $t+1$ in each location i are weakly increasing in I_{t+1}, whenever i is in the same type of interval at t and $t+1$. From expression (14), I_{t+1} is non-decreasing in r and hence prices are weakly decreasing in r for all i which remain in the same type of interval. Let us consider any $r_{t+1}^A < r_{t+1}^B < r_t$, with $I_{t+1}^A > I_{t+1}^B$. Then all $i \in [0, I_{t+1}^B - \gamma]$ are also in $[0, I_{t+1}^B - \gamma]$, but some $i \in [I_{t+1}^B - \gamma, I_{t+1}^B + \gamma]$ may be in $[0, I_{t+1}^A - \gamma]$ or some $i \in [I_{t+1}^B, I_{t+1}^B + \gamma]$ may be in $[I_{t+1}^A - \gamma, I_{t+1}^A]$. Given that, from inspection of expression (23), $p_{t+1}(i)$ is non-increasing in i, this implies that aggregate prices P_{t+1} must be non-increasing in r. Hence, if at time $t + 1$ the economy is hit by an unexpected and permanent decrease in r, then P_{t+1} is going to be higher, the larger is the decrease in r. Given that P_t is given, this immediately proves the first statement of the proposition that the percentage increase in aggregate price is higher the larger is the decrease in r.

Second, we want to prove the second statement of the proposition, that

$$\frac{d^2 \left(p_{t+1}(i) / p_t(i)\right)}{dp_t(i) \, dr} \geq 0$$

for all $p_t(i) > C^P$ where the derivative is well-defined. Equations (25)-(29) in the proof of Proposition (2) define $d\left[p_{t+1}(i) / p_t(i)\right] / di$ for all i where this derivative is well-defined and $p_t(i) > C^P$. If the decrease in r is big enough, $d\left[p_{t+1}(i) / p_t(i)\right] / di > 0$ for all $p_t(i) > C^P$. Moreover, by inspection, it is easy to see that $d\left[p_{t+1}(i) / p_t(i)\right] / di$ is increasing in I_L, and hence decreasing in r, whenever i is in the same type of interval after a small or a large shock, say r^A or r^B. Moreover, given that $I_A^L > I_B^L$, i may lie in different types of interval in the two cases. In particular, it could be that $\min \{I_B^L - \gamma, I^H\} < i < I^H$ but $I^H - \gamma < i < \min \{I_A^L - \gamma, I^H\}$, or that $\max \{I^H, I_B^L - \gamma\} < i < I^H + \gamma$ and $I^H < i < \min \{I^H + \gamma, I_A^L - \gamma\}$, or that $I_B^L < i < I^H + \gamma$ but $\max \{I^H, I_A^L - \gamma\} < i < I^H + \gamma$. It is easy to see that expression (25) is not smaller than expression (26) and that expression (27) is not smaller than expression (28). Finally expression (28) is bigger than expression (29) iff

$$\left(\frac{A + \gamma + I^L - i}{A + \gamma}\right)^{\frac{\beta_R - \beta_P}{\alpha}} \left[1 - \frac{(\beta_R - \beta_P)\left(A + \gamma + I^H - i\right)}{\beta_P \left(I^L - I^H\right)}\right] > 1,$$

which is true if the shock is large enough so that $I^L - I^H$ is big enough, as we assumed. This proves that $d^2 \left[p_{t+1}(i) / p_t(i)\right] / di \, dr$ is positive for all i such that the derivative exists and $p_t(i) > C^P$. Given that $p_t(i)$ is non-increasing in i, this completes the proof of the second claim of the proposition.

D1.4 Proof of Proposition 4

Consider two cities, A and B, with both $\phi_R^A > \phi_R^B$ and $\phi_P^A > \phi_P^B$. First, we want to prove the claim that if at time $t+1$ they are both hit by an unexpected and permanent decrease in r of the same size, with $r_t = r^H > r^L = r_{t+1}$, the percentage increase in the aggregate price level P_t is larger in city A. The two cities are exactly the same except for the size of the city and of the rich neighborhoods, which, from expressions (14) and (15), are so that $I_A^h > I_B^h$ and $\bar{I}_A^h > \bar{I}_B^h$ for both $h = H$ and $h = L$. Hence city A has a larger center and a larger size overall. After the decrease in r, prices do not change for all $i < I_j^H - \gamma$ and for all $i > I_j^L + \gamma$ if $I_j^L + \gamma < \bar{I}_j^H$, for both $j = A$ and $j = B$. Moreover, given that $I_A^L > I_B^L$, the growth rate $p_A^L(i)/p_A^H(i)$ is weakly higher than $p_B^L\left(i + I_B^H - I_A^H\right)/p_B^H\left(i + I_B^H - I_A^H\right)$ for all $i \in \left[I_A^H - \gamma, \bar{I}_A^H\right]$. This implies that the gross growth rate in aggregate prices is also higher in city A than in city B if the shock is big enough that the higher expansion in city A dominates the zero growth rate in the rich neighborhoods in the center.

To prove the second claim notice that the price in city A at time t for $i \in \left[I_A^H - \gamma, I_A^H + \gamma\right]$ are exactly the same as in city B for $i \in \left[I_B^H - \gamma, I_B^H + \gamma\right]$. However, $I_A^H > I_B^H$, so that the interval $\left[0, I_A^H\right]$ is larger than $\left[0, I_B^H\right]$. When a decrease in r hits both cities, expressions (14) and (15) give that both I and $\bar{I}$ increase more in city A. Hence, the expression for $d(p_{t+1}(i)/p_t(i))/di$ in city A for all $i \in \left[I_A^H - \gamma, I_A^H + \gamma\right]$ is equivalent to $d(p_{t+1}(i)/p_t(i))/di$ in city B for all $i \in \left[I_B^H - \gamma, I_B^H + \gamma\right]$ if city B was facing a larger decrease in r, and, in particular, if r_{t+1} was equal to

$$\tilde{r}^L = \left[\left(1 + \frac{1}{r^L}\right)\frac{\phi_R^A}{\phi_R^B} - 1\right]^{-1} < r^L.$$

From the proof of Proposition 3, this immediately implies that $d(p_{t+1}^A(i)/p_t^A(i))/di > d(p_{t+1}^B\left(i + I_A^H - I_B^H\right)/p_t^B$ that is, $g_A'(p) > g_B'(p)$ for all $p \in \left(\bar{p}, C^P\right)$. Finally, the gross growth rate of prices in all locations where the initial price was $\bar{p}$ is everywhere equal to 1 so that is not sensitive to i and $g_A'(\bar{p}) = g_B'(\bar{p})$. This completes the proof that $g_A'(p) \geq g_B'(p)$ for all $p > C^P$ whenever this derivative is well defined.