Master Theorem:

$T(n) = aT(n/b) + f(n)$, where $a \ge 1$, $b > 1$

& $f(n) = \theta(n^k \log^p n)$

case 1: if $\log_b a > k$, Then, $T(n) = \theta(n^{\log_b a})$

case 2: if $\log_b a = k$, then

- 2.1 if $p > -1$, $T(n) = \theta(n^k \log^{p+1} n)$
- 2.2 if $p = -1$, $T(n) = \theta(n^k \log \log n)$
- 2.3 if $p < -1$, $T(n) = \theta(n^k)$

case 3: if $\log_b a < k$, then

- 3.1 if $p \ge 0$, $T(n) = \theta(n^k \log^p n)$
- 3.2 if $p < 0$, $T(n) = \theta(n^k)$

Examples

1. $T(n) = 2T(n/2) + 1$

$a = 2$, $b = 2$, $f(n) = 1 = (n^0 \log^0 n)$

$k = 0$, $p = 0$

$\log_2 2 = 1 > k$, Then

$T(n) = \theta(n^{\log_2 2}) = \theta(n) \leftarrow \boxed{\text{Case 1}}$

2. $T(n) = 9T(n/3) + n \rightarrow \boxed{f(n) = n = n^1 \log^0 n}$

$a = 9$, $b = 3$, $k = 1$, $p = 0$

$\log_b a = \log_3 3^2 = 2 > k$

$T(n) = \theta(n^2) \leftarrow \boxed{\text{case 1}}$

3. $T(n) = 2T(n/2) + n$

$a = 2$, $b = 2$, $k = 1$, $p = 0$

$\rightarrow \log_2 2 = 1 = k$

then, $p = 0$

$T(n) = \theta(n^1 \log^{0+1} n)$

$= \theta(n \log n)$

4. $T(n) = 8T(n/2) + n^3 \log n$

$\log_2 8 = 3 = k$, $p = 1$

$T(n) = \theta(n^k \log^{p+1} n) = \theta(n^3 \log^2 n)$

5. $T(n) = 2T(n/2) + \frac{n}{\log n} \rightarrow n \log^{-1} n$

$\log_2 2 = 1 = k$, $p = -1$

$T(n) = \theta(n^k \log \log n)$

$= \theta(n \log \log n)$

$\uparrow$ case 2.2

6. $T(n) = 2T(n/2) + \frac{n}{\log^2 n}$

$\log_2 2 = 1 = k$, $p = -2$

$T(n) = \theta(n^k) = \theta(n)$

$\boxed{\text{case 2.3}}$

7. $T(n) = T(n/2) + n^2 \log n$

$\log_2 1 = 1 < k$, $p = 1$

$T(n) = \theta(f(n)) = \theta(n^2 \log n)$

$\boxed{\text{case 3.1}}$

8. $T(n) = 4T(n/2) + \frac{n^3}{\log n}$

$\log_2 4 = 2 < k$, $p = -1$

$T(n) = \theta(n^k) = \theta(n^3)$

$\boxed{\text{case 3.2}}$