

# Bitwise Sieve

Status Array:

- 0th bit of status[0]  $\Rightarrow$  indicates primality of 0
- 1st bit of status[0]  $\Rightarrow$  indicates primality of 1
- 2nd bit of status[0]  $\Rightarrow$  indicates primality of 2
- 0th bit of status[1]  $\Rightarrow$  indicates primality of 32
- 1st bit of status[1]  $\Rightarrow$  indicates primality of 33

Bitwise Operations:

- $i \gg 5$  is equivalent to  $\frac{i}{2^5} = \frac{i}{32}$  which is much faster, it has brief description below<sup>1</sup>
- $j \& 32$  is equivalent to  $i \% 32$

```
#define mx 1000
int status[(mx/32)+2];
bool Check(int N,int pos){return (bool)(N & (1<<pos));}
int Set(int N,int pos){ return N=N | (1<<pos);}

void sieve()
{
    int i, j, sqrtN;
    sqrtN = int( sqrt( mx ) );
    for( i = 3; i <= sqrtN; i += 2 )
    {
        // in our code -> 0 means prime, 1 means not prime.
        if( Check(status[i>>5],i&31)==0)
        {
            for( j = i*i; j <= mx; j += (i<<1) )
            {
                status[j>>5]=Set(status[j>>5],j & 31)    ;
            }
        }
    }
}

bool isPrime(int n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    // in our code -> 0 means prime, 1 means not prime.
    return !Check(status[n >> 5], n&31);
}

int main() {
    sieve(); int n;
    n = 3; cout << n << ": " << (isPrime(n) ? " prime" : " NOT prime") << endl;
    n = 5; cout << n << ": " << (isPrime(n) ? " prime" : " NOT prime") << endl;
    n = 27; cout << n << ": " << (isPrime(n) ? " prime" : " NOT prime") << endl;
}
```

<sup>1</sup>Bitwise and in place of modulus operator:

$$x\%2^n \equiv x\&(2^{n-1})$$

$$x\%4 \equiv x\&3, x\%8 \equiv x\&7 \text{ and so on}$$

Though it's actually not accurate to say that  $x \% 2 == x \& 1$ . Simple counterexample:  $x = -1$ . In many languages, including Java,  $-1 \% 2 == -1$ . That is,  $\%$  is not necessarily the traditional mathematical definition of modulo. Java calls it the “**remainder operator**”, for example.