Study of spin polarisation in simulated high energy pp and heavy ion collisions

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to the

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Bhubaneswar

April 23, 2017

Contents

Acknowledgement

It is a strange thing, but when you are dreading something, and would give anything to slow down time, it has a disobliging habit of speeding up.

— J.K.Rowling, *Harry Potter and the Goblet of Fire*

This quote sums up in entirety how I feel about finishing this work. This project has been one of my best experiences and has shaped my further interest in research in physics. As such, this project will be only half done if I do not begin this report by thanking the people who endured my stupidity at every step, eventually pushing me over the line.

I start by thanking *Sourav Kundu* who had initiated me to the lengthy derivations and made them graspable for me. I thank all the lab members of the Experimental High Energy Physics Group who sat through my presentations and poured in their suggestions. I extend my heartfelt gratitude to *Ajay Kumar Dash* without whose immense patience, I would still be debugging my code on submission date.

I would also like to reach out to my friends in Physics Batch 14 who have often been tremendous supports, knowingly or unknowingly. Knowing that your fellow comrades are as miserable as you is a peace and motivation of another kind, I have realised.

Finally, it is only done when I thank *Dr. Bedangadas Mohanty*, my guide who has given me this opportunity to work with this wonderful group and appreciate research like never before.

Chapter 1

Introduction

In physics, you don't have to go around making trouble for yourself - nature does it for you.

— Frank Wilczek

In this report, we try to understand the orientation of the spin of the constituent quarks of the produced vector mesons in pp and heavy ion collisions using a simple model. We argue that this orientation can be due to a relativistic effect called Thomas precession. In high energy physics, a central collision refers to a head-on collision. In such cases, the perpendicular distance between the momentum vector of one nucleus and the center of mass of the other nucleus, known as the impact parameter is 0 fm. For non-central collisions, the impact parameter is non-zero. In case of non-central heavy ion collisions, it has been shown in [1] and [2] that there is a large initial angular momentum which results in polarisation of the finally produced hadrons. However, the same is not expected for pp collisions as they are central collisions and hence, there is no initial angular momentum. A study at STAR collaboration has shown the dependence of the spin density matrix element, $\rho_{0,0}$ on the transverse momentum for Φ and K^* produced in pp and $Au - Au$ collisions. For the heavy ion collision [3], it has been reported in reference [4] that the $\rho_{0,0}$ element deviates significantly from the 'no-polarisation' value of $\frac{1}{3}$. However, for the pp collision, $\rho_{0,0}$ assumes the value of $\frac{1}{3}$.
In this report, we have analysed simulated pp collisions at $\sqrt{s} = 13$ TeV. The data is generated using a Monte-Carlo event generator PYTHIA(v. 8186)[5] and [6].We have derived an expression for angular distribution of the produced vector mesons. We have also reproduced the derivation of polarisation using Thomas precession[4]. Finally, it has been shown from the angular distribution of the produced Φ that the $\rho_{0,0}$ for Φ is $\frac{1}{3}$.

Chapter 2

Angular distribution of Vector Meson decay products

"I accept no principles of physics which are not also accepted in mathematics."

— René Descartes

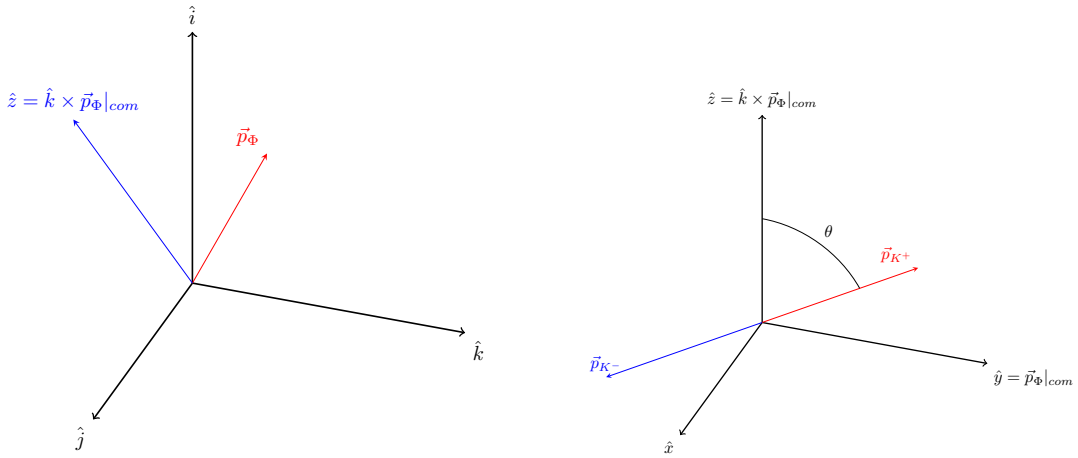
Introduction

- The reaction we are dealing with is the decay of Φ to K^+ and K^- produced in a symmetric collision. It is a two particle decay and we want to find the angular distribution of the produced particles.
- We will work in the rest frame of Φ .
- Since, we are in the rest frame of Φ , the initial angular momentum $\vec{L}$ is 0. Φ is a spin 1 particle, therefore, total angular momentum $\vec{J} = \vec{L} + \vec{S} = 1$.
- Further, since this is a decay, the total angular momentum will be conserved in the Φ rest frame.

Coordinate System

- We consider a Cartesian coordinate and define the z-axis perpendicular to the plane containing the beam direction vector and the velocity vector of the Φ produced in the collision. (This plane is known as the **production plane**.) It is defined for each Φ .
- Let y-axis be along the velocity vector of Φ . This fixes our coordinate system.

- We define θ as the angle the velocity vector of K^+ makes with the z-axis and ϕ as its azimuthal angle. We also note that K^+ and K^- will move opposite to each other.
- For completely determining the final state, we also need the helicities of the daughter particles. Let they be λ_1 for K^+ and λ_2 for K^- .



(a) (b)

Figure 2.1: (a) This system is to determine the production plane and is in the COM frame. (b) In the rest frame of Φ .

Therefore, the final state is given by $|f\rangle = |\theta, \phi, \lambda_1, \lambda_2\rangle$.
We need the angular distribution of K^+ i.e. $\frac{dN}{d\cos\theta d\phi}$. Since, it's a two particle decay, N for $K^+ = N$ for Φ .

$$\frac{dN}{d\cos\theta d\phi} = \langle f|R|f\rangle \tag{2.1}$$

where $R = M\rho M^\dagger$ and M is the decay amplitude of Φ and $\rho(K^*)$ is the density matrix of Φ .

$$\frac{dN}{d\cos\theta d\phi} = \left\langle \theta, \phi, \lambda_1, \lambda_2 | M\rho M^\dagger | \theta, \phi, \lambda_1, \lambda_2 \right\rangle \tag{2.2}$$

$$= \sum_{\lambda_V} \sum_{\lambda_{V'}} \langle \theta, \phi, \lambda_1, \lambda_2 | M | \lambda_V \rangle \langle \lambda_V | \rho | \lambda_{V'} \rangle \left\langle \lambda_{V'} | M^\dagger | \theta, \phi, \lambda_1, \lambda_2 \right\rangle \tag{2.3}$$

Equation 2.3 has been expanded the RHS in the helicity basis of the Φ .
Now, total helicity of the decay product is 0 because they move opposite to each other($\therefore \vec{s}.\vec{p}$ is equal in magnitude and opposite in sign).
Since we are looking at two particle decay, $|f\rangle = |\theta, \phi, \lambda_1 - \lambda_2\rangle = |\theta, \phi, 0\rangle$. Since, we are in the rest frame of Φ and total angular momentum is conserved, we can write the final state as $|f\rangle = |1, 0\rangle$ while being cautious that the θ and ϕ dependence is still intact in $|f\rangle$.
Also, instead of the helicity, we now use the spin as it differs only up to a constant and that can be absorbed in the normalisation constant. Therefore, $|\lambda_V\rangle = |1, m_1\rangle$ and $|\lambda_{V'}\rangle = |1, m_2\rangle$.

$$\frac{dN}{d\cos\theta d\phi} = \sum_{m_1}\sum_{m_2} \langle 1,0|M|1,m_1\rangle \langle 1,m_1|\rho|1,m_2\rangle \left\langle 1,m_2|M^\dagger|1,0\right\rangle \tag{2.4}$$

We need to calculate the terms $\langle 1,0|M|1,m_1\rangle$ and $\left\langle 1,m_2|M^\dagger|1,0\right\rangle$ for which we will use Wigner-D matrices.
Now, in the final state, only the m_z component is different than the initial state. Therefore, effectively the operator M acts as a rotation operator which aligns the z-axis along the velocity vector of K^+ so that the z-component of spin in the final state is 0.

$$\text{Hence, } \langle 1,0|M|1,m_1\rangle = C \times D^1_{m_1,0}(\phi,\theta,-\phi) \times \sqrt{\tfrac{2J+1}{4\pi}}$$
$$\text{and } \left\langle 1,m_2|M^\dagger|1,0\right\rangle = C^* \times D^{*1}_{m_2,0}(\phi,\theta,-\phi) \times \sqrt{\tfrac{2J+1}{4\pi}}$$

Therefore, Equation 2.4 reduces to

$$\frac{dN}{d\cos\theta d\phi} = |C|^2 \times \frac{3}{4\pi} \sum_{m_1,m_2} D^1_{m_1,0}(\phi,\theta,-\phi) D^{*1}_{m_2,0}(\phi,\theta,-\phi) \rho_{m_1,m_2} \tag{2.5}$$

where $\rho_{m_1,m_2} = \langle 1,m_1|\rho|1,m_2\rangle$ are the elements of the density matrix and $J = 1$.

Now, we express $D^l_{m,0}(\alpha,\beta,\gamma)$ in terms of the spherical harmonics. This is given by

$$D^l_{m,0}(\alpha,\beta,\gamma) = \sqrt{\frac{4\pi}{2l+1}} Y^*_{lm}(\beta,\alpha) \tag{2.6}$$

Therefore, from Equation 2.5, we have 9 terms which we need to calculate using Equation 2.6. $l = 1$ for all the 9 terms, $\therefore$ the constant term in Equation 2.6 has a value of $\sqrt{\frac{4\pi}{3}}$.

$$\frac{dN}{d\cos\theta d\phi} = |C|^2 \times \sum_{m_1,m_2} Y^*_{1,m_1}(\theta,\phi) Y_{1,m_2}(\theta,\phi)\rho_{m_1,m_2} \tag{2.7}$$

$$\begin{aligned}
Y^*_{1,-1}Y_{1,-1} &= \frac{1}{4}\times\frac{3}{2\pi}\sin^2\theta \\
Y^*_{1,-1}Y_{1,0} &= \sqrt{2}\frac{1}{4}\times\frac{3}{2\pi}\sin\theta\cos\theta e^{i\phi} \\
Y^*_{1,-1}Y_{1,1} &= -\frac{1}{4}\times\frac{3}{2\pi}\sin^2\theta e^{2i\phi} \\
Y^*_{1,0}Y_{1,-1} &= \sqrt{2}\frac{1}{4}\times\frac{3}{2\pi}\sin\theta\cos\theta e^{-i\phi} \\
Y^*_{1,0}Y_{1,0} &= \frac{1}{4}\frac{3}{\pi}\cos^2\theta
\end{aligned}
\qquad
\begin{aligned}
Y^*_{1,0}Y_{1,1} &= -\sqrt{2}\frac{1}{4}\times\frac{3}{2\pi}\sin\theta\cos\theta e^{i\phi} \\
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Y^*_{1,1}Y_{1,0} &= -\sqrt{2}\frac{1}{4}\times\frac{3}{2\pi}\sin\theta\cos\theta e^{-i\phi} \\
Y^*_{1,1}Y_{1,1} &= \frac{1}{4}\times\frac{3}{2\pi}\sin^2\theta
\end{aligned} \tag{2.8}$$

Before putting the values from Equation 2.8 in Equation 2.7, we calculate the ϕ integral for all the terms in Equation 2.8.

$$\because \int_0^{2\pi} e^{i\phi} = 0 \quad , \int_0^{2\pi} e^{2i\phi} = 0 \tag{2.9}$$

therefore, when we perform the ϕ integral on Equation 2.7, all the terms containing ϕ become 0. Therefore, Equation 2.7 reduces to

$$\begin{aligned}
\frac{dN}{d\cos\theta} &= |C|^2 \times \frac{3}{8\pi}\left[sin^2\theta\rho_{-1,-1} + 2\cos^2\theta\rho_{0,0} + \sin^2\theta\rho_{1,1}\right]\times 2\pi \\
&= |C|^2 \times \frac{3}{4}\left[\sin^2\theta\left(\rho_{-1,-1}+\rho_{1,1}\right) + 2\cos^2\theta\rho_{0,0}\right]
\end{aligned} \tag{2.10}$$

Now, from the property of a normalised density matrix, we know that its trace is 1. Therefore, $\rho_{-1,-1}+\rho_{0,0}+\rho_{1,1}=1$. Using this in Equation 2.10, we have

$$\begin{aligned}
\frac{dN}{d\cos\theta} &= |C|^2 \times \frac{3}{4}\left[\sin^2\theta\left(1-\rho_{0,0}\right) + 2\cos^2\theta\rho_{0,0}\right] \\
&= |C|^2 \times \frac{3}{4}\left[1-\rho_{0,0}+\cos^2\theta\left(3\rho_{0,0}-1\right)\right] \\
\implies \frac{dN}{d\cos\theta} &= N_0\left[1-\rho_{0,0}+\cos^2\theta\left(3\rho_{0,0}-1\right)\right] \;\; N_0 \text{ is a normalisation.}
\end{aligned} \tag{2.11}$$

Equation 2.11 gives the angular distribution of the produced particles as a function of θ. If there is no polarisation i.e. no preferred direction, $\rho_{0,0} = \frac{1}{3}$. Then, the angular distribution is a constant (no θ dependence).
The next part of the work is to relate the $\rho_{0,0}$ element to the polarisation of the produced hadrons and compare it to data.

Chapter 3

Thomas Precession

> *"I claim that relativity and the rest of modern physics is not complicated. It can be explained very simply. It is only unusual or, put another way, it is contrary to common sense."*
>
> — Edward Teller, *Conversation of the Dark Secrets*

Thomas Precession is a semi-classical effect originating due to relativistic kinematics. It can simultaneously explain the anomalous Zeeman effect and the fine structure splitting. The complete explanation for this however comes for Dirac's relativistic electron theory which is not explored here.
Consider an electron(e^-) with a spin angular momentum $\vec{s}$ which can take on quantized values of $\pm\frac{\hbar}{2}$. The magnetic moment of the electron is $\vec{\mu} = \frac{ge}{2mc}\vec{s}$ where g is the Lande's g-factor and is equal to 2. The e^- is moving with a velocity $\vec{v}$ in external $\vec{E}$ and $\vec{B}$. In the rest frame of the e^-, the equation of motion is given by

$$\left(\frac{d\vec{s}}{dt}\right)_{\text{rest frame}} = \vec{\mu} \times \vec{B'} \tag{3.1}$$

This is essentially the torque-equivalent on the system.
Now, $\vec{B'} = \vec{B} - \frac{\vec{v}}{c} \times \vec{E}$ when we consider up to 1st order in $\frac{\vec{v}}{c}$. Thus the interaction energy becomes

$$\therefore U' = -\vec{\mu}.\left(\vec{B} - \frac{\vec{v}}{c} \times \vec{E}\right) \tag{3.2}$$

Now, for an electron in an atom, we can estimate the magnitude of the electric force as the gradient of a radial potential energy $V(r)$. For a single electron system, this is

an exact description.

$$\therefore e\vec{E} = -\frac{\vec{r}}{r}\frac{dV}{dr} \tag{3.3}$$

Therefore, the final expression for the spin-interaction energy reduces to

$$U' = -\frac{ge}{2mc}\vec{s}.\vec{B} + \frac{g}{2m^2c^2}(\vec{s}.\vec{L})\frac{1}{r}\frac{dV}{dr} \tag{3.4}$$

Here $\vec{L}$ is the electron's orbital angular momentum. Now, we have anomalous Zeeman effect for this case, so $g = 2$. But experimentally, it has been seen that the resulting spin-orbit interaction term is twice as large.
Thomas precession explains this error by accounting for the rotation of the rest frame of the electron coordinate system with respect to the lab frame. Here we will prove an well-known result which connects the rate of change of any vector in a non-rotating frame to that in a rotating frame.
Consider any vector $\vec{G}$. Consider two coordinate systems, one fixed $(\hat{i}, \hat{j}, \hat{k})$ and one rotating $(\hat{i'}, \hat{j'}, \hat{k'})$ having the same origin.

$$\begin{aligned}
\vec{G}|_{fixed} &= \vec{G}|_{rotating} \\
\implies \frac{d\vec{G}}{dt}|_{fixed} &= \frac{d\vec{G}}{dt}|_{rotating} \\
&= \frac{d}{dt}\left(G'_1\hat{i'} + G'_2\hat{j'} + G'_3\hat{k'}\right) \\
&= \left(\frac{dG'_1}{dt}\hat{i'} + \frac{dG'_2}{dt}\hat{j'} + \frac{dG'_3}{dt}\hat{k'}\right) + \left(G'_1\frac{d\hat{i'}}{dt} + G'_2\frac{d\hat{j'}}{dt} + G'_3\frac{d\hat{k'}}{dt}\right) \\
\implies \frac{d\vec{G}}{dt}|_{fixed} &= \frac{d\vec{G}_{rotating}}{dt} + \left(G'_1\frac{d\hat{i'}}{dt} + G'_2\frac{d\hat{j'}}{dt} + G'_3\frac{d\hat{k'}}{dt}\right)
\end{aligned}$$

Now, for infinitesimal rotations, we can write $\delta\vec{A} = (\delta\theta)\,\hat{n} \times \vec{A}$. Therefore, $\left(\frac{d\vec{A}}{dt}\right) = \left(\frac{d\theta}{dt}\right)\hat{n} \times \vec{A} = \vec{\omega} \times \vec{A}$.

$$\frac{d\hat{i'}}{dt} = \vec{\omega} \times \hat{i'} \quad \Big| \quad \frac{d\hat{j'}}{dt} = \vec{\omega} \times \hat{j'} \quad \Big| \quad \frac{d\hat{k'}}{dt} = \vec{\omega} \times \hat{k'} \tag{3.5}$$

$$\therefore \left(\frac{d\vec{G}}{dt}\right)_{fixed} = \left(\frac{d\vec{G}_{rotating}}{dt}\right) + \vec{\omega} \times \vec{G} \tag{3.6}$$

Using equation 3.6, we have

$$\left(\frac{d\vec{s}}{dt}\right)_{non-rot} = \left(\frac{d\vec{s}_{rest}}{dt}\right) + \vec{\omega} \times \vec{s}$$

$$\implies \left(\frac{d\vec{s}}{dt}\right)_{non-rot} = \vec{s} \times \left(\frac{ge}{2mc}\vec{B'} - \omega_T\right) \tag{3.7}$$

The corresponding energy of interaction is given by

$$U = U' + \vec{s}.\vec{\omega_T} \tag{3.8}$$

Consider an electron moving with a velocity $\vec{v}(t)$ with respect to the laboratory frame. The rest frame of the electron is a sequence of co-moving coordinates whose successive origins move with the velocity of the electron.
Now $\vec{v}(t) = c\vec{\beta}$ at time t and $\vec{v}(t+\delta t) = c(\vec{\beta} + \delta\vec{\beta})$.
Let x' be the coordinate of the electron in its rest frame at time t and x'' at $t + \delta t$. The Lorentz boost matrix is represented by A_b further.

$$x' = A_b(\vec{\beta})x \quad \Big| \; x'' = A_b(\vec{\beta} + \delta\vec{\beta})x \tag{3.9}$$

From equation 3.9, we get $x'' = A_T x'$ where $A_T = A_b(\vec{\beta} + \delta\vec{\beta})A_b(-\vec{\beta})$.

$$A_T = \begin{bmatrix} 1 & -\gamma^2\delta\beta_1 & -\gamma\delta\beta_2 & 0 \\ -\gamma^2\delta\beta_1 & 1 & \left(\frac{\gamma-1}{\beta}\right)\delta\beta_2 & 0 \\ -\gamma\delta\beta_2 & \left(\frac{\gamma-1}{\beta}\right)\delta\beta_2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\delta\beta_1, \delta\beta_2, \delta\beta_3$ are components of the vector $\delta\vec{\beta}$ and we have used the Lorentz transformation matrices for boosts without rotation.
A_T essentially represents an infinitesimal Lorentz transformation. We will write them in terms of the generators of Lorentz transform given by $\vec{S}$ and $\vec{K}$ matrices.
Using the definitions of $\vec{S}$ and $\vec{K}$, we have

$$A_T = I - \left(\frac{\gamma-1}{\beta^2}\right)\left(\vec{\beta} \times \delta\vec{\beta}\right).\vec{S} - \left(\gamma^2\delta\vec{\beta}_{\|} + \gamma\delta\vec{\beta}_{\perp}\right).\vec{K} \tag{3.10}$$

Here $\delta\vec{\beta}_{\|}$ and $\delta\vec{\beta}_{\perp}$ are components parallel and perpendicular to $\vec{\beta}$ respectively.
Up to 1^{st} order in $\delta\vec{\beta}$, equation 3.10 can be written as $A_T = A_b(\Delta\vec{\beta})R(\Delta\vec{\Omega})$ where $A_b(\Delta\vec{\beta}) = I - \Delta\vec{\beta}.\vec{K}$ and $R(\Delta\vec{\Omega}) = I - \Delta\vec{\Omega}.\vec{S}$.

$$\Delta\vec{\beta} = \gamma^2\delta\beta_{\parallel} + \gamma\delta\beta_{\perp} \quad \Big| \quad \Delta\vec{\Omega} = \left(\frac{\gamma-1}{\beta^2}\right)\vec{\beta}\times\delta\vec{\beta} = \frac{\gamma^2}{\gamma+1}\vec{\beta}\times\delta\vec{\beta} \tag{3.11}$$

These are commuting infinitesimal boosts and rotations equivalent to a single Lorentz boost to a frame with velocity $c(\vec{\beta}+\delta\vec{\beta})$.

$$\therefore A_b(\vec{\beta}+\delta\vec{\beta}) = R(\Delta\vec{\Omega})A_b(\Delta\vec{\vec{\beta}})A_b(\vec{\beta}) \tag{3.12}$$

We would now consider the coordinate frame (denoted by x''') generated by $A_b(\Delta\vec{\vec{\beta}})A_b(\vec{\beta})$ i.e. without the rotation. Clearly $x''' = A_b(\Delta\vec{\beta})x$ which reduces to $x''' = R(-\Delta\vec{\Omega})x''$ using equation 3.9.
Therefore, the non-rotating rest system of coordinates defined by x''' precesses by an angle $-\Delta\vec{\Omega}$ relative to the boosted lab axes x''. From equation 3.7, we therefore have

$$\vec{\omega_T} = -\lim_{\delta t\to 0}\frac{\Delta\vec{\Omega}}{\delta t} = \frac{\gamma^2}{\gamma+1}\frac{\vec{a}\times\vec{v}}{c^2} \tag{3.13}$$

ω_T is known as the **Thomas frequency** and in natural units, it is given by

$$\vec{\omega_T} = \frac{\gamma}{\gamma+1}\frac{\vec{F}\times\vec{\beta}}{m_0} \tag{3.14}$$

Here m_0 is the rest mass of the electron. For electrons in atoms from our initial discussion, we have

$$\omega_T \approx -\frac{1}{2m^2c^2}\vec{L}\frac{1}{r}\frac{dV}{dr} \tag{3.15}$$

Putting it in equation 3.8, we get

$$U = -\frac{ge}{2mc}\vec{s}.\vec{B} + \frac{g-1}{2m^2c^2}(\vec{s}.\vec{L})\frac{1}{r}\frac{dV}{dr} \tag{3.16}$$

which is consistent for $g = 2$ and gives both anomalous Zeeman effect and the proper spin orbit interaction.

Chapter 4

Scattering, Resonances and Breit Wigner Function

> *"In fact, the mere act of opening the box will determine the state of the cat, although in this case there were three determinate states the cat could be in: these being Alive, Dead, and Bloody Furious."*
>
> — Terry Pratchett, *Lords and Ladies*

We want to look at the scattering of an incident beam off a central target and find the reaction cross section and the scattering amplitude. We will relate this scattering amplitude to the polarisation of the produced hadrons.
For this, we first consider a plane wave e^{ikz} with momentum $p = \hbar k$ along the z-axis as our incident beam [10]. To make things simple, we assume a central nuclear potential and express the incident wave as a superposition of incident waves.

$$\psi_{inc} = Ae^{ikz} = A\sum_{l=0}^{\infty} i^l(2l+1)j_l(kr)P_l(\cos\theta) \tag{4.1}$$

Here, the $j_l(kr)$ are the *spherical Bessel functions* and are solutions to the radial part of the Schrödinger equation. $P_l(\cos\theta)$ are the Legendre polynomials and are solutions to the angular part of the Schrödinger equation. This expansion of the incident wave is called the *partial wave equation*, with each partial wave corresponding to a specific angular momentum l.

When the wave is far from the nucleus, we can expand $j_l(kr)$ as

$$j_l(kr) \approx \frac{\sin(kr - l\pi/2)}{kr} \; (kr >> l) \tag{4.2}$$

$$= i\frac{e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)}}{2kr} \tag{4.3}$$

Then the incident wave becomes

$$\psi_{inc} = \frac{A}{2kr}\sum_{l=0}^{\infty} i^{l+1}(2l+1)\left[e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)}\right] P_l(\cos\theta) \tag{4.4}$$

When the scatterer is absent, we can analyse the plane wave as the superposition of spherically incoming waves involving e^{-ikr}/r and spherically outgoing waves involving e^{ikr}/r.
Scattering only affects the outgoing waves by changing its phase or amplitude or both. Change in amplitude is suggestive of fewer particles coming out than going in. Since the incident wave represents particles with momentum $p = \hbar k$ only, for an inelastic scattering, the energy of the particles will change and hence, change in amplitude is expected.
For the l^{th} outgoing partial wave, the changes can be incorporated by introducing a complex coefficient η_l into the outgoing term (e^{ikr}).

$$\psi = \frac{A}{2kr}\sum_{l=0}^{\infty} i^{l+1}(2l+1)\left[e^{-i(kr-l\pi/2)} - \eta_l e^{i(kr-l\pi/2)}\right] P_l(\cos\theta) \tag{4.5}$$

This wave is a superposition of the incident and the scattered waves $\psi = \psi_{inc} + \psi_{sc}$. Therefore,

$$\psi_{sc} = \frac{A}{2kr}\sum_{l=0}^{\infty} i^{l+1}(2l+1)(1-\eta_l)e^{i(kr-l\pi/2)}P_l(\cos\theta) \tag{4.6}$$

$$= \frac{A}{2k}\frac{e^{ikr}}{r}\sum_{l=0}^{\infty} i(2l+1)(1-\eta_l)P_l(\cos\theta) \tag{4.7}$$

Because we have accounted for partial waves with wavenumber k only, this represents only *elastic scattering*. To find the differential cross section, we have

$$d\sigma = \frac{j_{sc}r^2 d\Omega}{j_{inc}} \tag{4.8}$$

Now, the scattered current density is given by

$$j_{sc} = \frac{\hbar}{2mi}\left(\psi_{sc}^{*}\frac{\partial\psi_{sc}}{\partial r} - \psi_{sc}\frac{\partial\psi_{sc}^{*}}{\partial r}\right) \tag{4.9}$$

$$= |A|^2\frac{\hbar}{4mkr^2}\left|\sum_{l=0}^{\infty} i(2l+1)(1-\eta_l)P_l(\cos\theta)\right|^2 \tag{4.10}$$

Incident current density is given by $j_{inc} = \frac{\hbar k}{m}|A|^2$. Therefore, the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2}\left|\sum_{l=0}^{\infty} i(2l+1)(1-\eta_l)P_l(\cos\theta)\right|^2 \tag{4.11}$$

Total cross section is given by

$$\sigma_{sc} = \int \frac{d\sigma}{d\Omega} = \sum_{l=0}^{\infty}\frac{\lambda^2}{4\pi}(2l+1)|1-\eta_l|^2 \tag{4.12}$$

If elastic scattering were the only process that could occur, then $|\eta_l| = 1$ and hence we can write $\eta_l = e^{2i\delta_l}$ where δ_l is the *phase shift* of the l^{th} partial wave. Now, $|1-\eta_l|^2 = 4\sin^2\delta_l$. Therefore

$$\sigma_{sc} = \sum_{l=0}^{\infty}\lambda^2(2l+1)\sin^2\delta_l \tag{4.13}$$

For inelastic scattering processes, Eq. 4.13 is not valid as $|\eta_l| < 1$. To find the cross section σ_r due to all such processes, we need to find the rate at which particles disappear from a wave with wave number k. This is given by $|j_{in}| - |j_{out}|$ where j_{in} and j_{out} are obtained from the 1^{st} and the 2^{nd} terms of Eq. 4.5 respectively.

$$|j_{in}| - |j_{out}| = \frac{|A|^2\hbar}{4mkr^2}\left|\sum_{l=0}^{\infty} i^{l+1}(2l+1)e^{il\pi/2}P_l(\cos\theta)\right|^2 - \left|\sum_{l=0}^{\infty} i^{l+1}(2l+1)\eta_l e^{-il\pi/2}P_l(\cos\theta)\right|^2 \tag{4.14}$$

$$\sigma_r = \sum_{l=0}^{\infty}\frac{\lambda^2}{4\pi}(2l+1)(1-|\eta_l|^2) \tag{4.15}$$

Therefore, total cross section involving all processes is

$$\sigma_{tot} = \sigma_{sc} + \sigma_r \tag{4.16}$$

$$= \sum_{l=0}^{\infty} \frac{\lambda^2}{2}(2l+1)(1 - Re(\eta_l)) \tag{4.17}$$

4.1 Resonance

A resonance is the peak located around a certain energy found in differential cross sections of scattering experiments. The resonance will occur when the total cross section in Eq. 4.17 is maximum. In a single, isolated energy E_R and width Γ, the energy profile will be similar to that for any decaying state of lifetime $\tau = \hbar/\Gamma$.
Assuming only one partial wave l is important for the resonant state, there will be a scattering resonance where $\eta_l = -1$ for $\delta_l = \pi/2$. To obtain the shape of the resonance, the cotangent of the phase shift can be expanded in a Taylor series about $\delta_l = \pi/2$. From this we get

$$\cot \delta_l = \frac{E - E_R}{\Gamma/2} \tag{4.18}$$

where $\Gamma = 2\left(\frac{\partial \delta_l}{\partial E}\right)^{-1}_{E=E_R}$ From Eq. 4.18, we have

$$\sin \delta_l = \frac{\Gamma/2}{[(E - E_R)^2 + \Gamma^2/4]^{1/2}} \tag{4.19}$$

Therefore, the scattering cross section becomes

$$\sigma_{sc} = \frac{\pi}{k^2}(2l+1)\frac{\Gamma/2}{(E - E_R)^2 + \Gamma^2/4} \tag{4.20}$$

Eq. 4.20 is called the *Breit-Wigner formula* for the shape of a single, isolated resonance for scattering [10].
The scattering amplitude is a probability amplitude and the scattering cross section is the square of the scattering amplitude.
Therefore, from Eq. 4.20, we find that the scattering amplitude A is inversely proportional to the energy difference between the initial and the final states.

$$A \propto \frac{1}{\Delta E} \tag{4.21}$$

We will use this to find out the polarisation asymmetry in pp-collisions due to Thomas precession.

Chapter 5

Polarisation due to Thomas Precession and Asymmetry

"Nothing happens until something moves."

— Albert Einstein

When spin effects are present in a system, we have shown the presence of Thomas precession which contributes to the difference in energy between the initial and the final states. Therefore, the scattering amplitude in its presence is given by

$$A = \frac{1}{\Delta E_0 + \vec{\omega_T}.\vec{s}} \tag{5.1}$$

where ΔE_0 is the difference in energy among the initial and final states in absence of spin. If we choose our axis of quantisation to be along the normal to the scattering plane, polarisation asymmetry is given by

$$P = \frac{|A_+|^2 - |A_-|^2}{|A_+|^2 + |A_-|^2} \tag{5.2}$$

where $A_+ = \frac{1}{\Delta E_0 + \frac{1}{2}\omega_T}$ and $A_- = \frac{1}{\Delta E_0 - \frac{1}{2}\omega_T}$. Keeping only up to the 1^{st} order terms in $\frac{\omega_T}{2\Delta E_0}$, we have

$$P = -\frac{\omega_T}{\Delta E_0} \tag{5.3}$$

For hadron formation, one of the main ways is through recombination of a slow and a fast quark. In this process, the resulting hadron is polarised. The semi-classical picture which accounts for the polarisation is the Thomas precession produced by

the accelerating force that pulls a slow moving quark(q^s) [4, 7] or by the decelerating force that pulls a fast moving quark(q^f) to form a hadron. However, the signs of polarisation will be opposite for the two cases. Therefore

$$P^{s,f} = \mp \frac{\omega_T}{\Delta E_0} \tag{5.4}$$

Before we derive the relevant quantities to describe the spin alignment of vector meson Φ in pp collisions, we need to make a few assumptions. These are given below:

1. The fast quark (q^f) has a large transverse momentum $p^f_\perp$ and zero longitudinal momentum ($p^f_\parallel = 0$).

2. The slow quark (q^s) moves in the longitudinal direction with momentum $p^s_\parallel$ and zero transverse momentum ($p^s_\perp = 0$).

3. The rapidity of q^s is within the rapidity of hadron's one to enhance recombination probability.

From Eq 3.14, we have $\omega_T = \left(\frac{\gamma}{1+\gamma}\right) \frac{\vec{F}\times\vec{\beta}}{m}$ where γ is the Lorentz gamma-factor. For high energy relativistic collisions, we can consider $\frac{\gamma}{\gamma+1} \approx 1$.
The pulling force $\vec{F}$ is equal to the change in momentum $\Delta\vec{p}$ of the given quark in time Δt when the recombination happens. Therefore $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$. ω_T for the given quark is given by the average over Δt time interval.

$$\begin{aligned} \omega_T^{s,f} &\approx \frac{\Delta p^{s,f}}{\Delta t} \beta^{s,f} \left(\int_{\Delta t} dt \sin\theta^{s,f}/\Delta t \right) && (5.5) \\ &= \langle \sin\theta \rangle^{s,f} && (5.6) \end{aligned}$$

Now, we calculate the change in momenta. For the slow quark q^s, we have

$$\Delta p^s = \sqrt{(p^{s/H}_\parallel - p^s_\parallel)^2 + (p^{s/H}_\perp)^2} \tag{5.7}$$

From the definition of rapidity, we have $y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \ln\left(\frac{E+p_z}{M_\perp}\right)$. From here, we can derive that $\sinh y = \frac{p_\parallel}{M_\perp}$.
Using assumption 3, we can therefore write that $p^s_\parallel = m^s_\perp \sinh y^H = \frac{m^s_\perp}{m^H_\perp} p^H_\parallel = \frac{m^s}{m^H_\perp} p^H_\parallel$ where $m^s_\perp = m^s$ as $p^s_\perp = 0$.
We denote the momentum of the slow and the fast quark in the hadron by $p^{s/H}$ and $p^{f/H}$ respectively. So $p^{s/H}_\parallel$ represents the longitudinal component of momentum of the slow quark in the hadron and so on.

Therefore,

$$\begin{aligned}
\Delta p_s &= \sqrt{\left(p_\parallel^{s/H} - p_\parallel^s\right)^2 + (p_\perp^{s/H})^2} && (5.8)\\
&= \sqrt{\left(p_\parallel^{s/H} - \frac{m^s}{m_\perp^H} p_\parallel^H\right)^2 + (p_\perp^{s/H})^2} && (5.9)\\
&= \sqrt{\left(\frac{p_\parallel^{s/H}}{p_\parallel^H} - \frac{m^s}{m_\perp^H}\right)^2 \left(p_\parallel^H\right)^2 + \left(\frac{p_\perp^{s/H}}{p_\perp^H}\right)^2 p_\perp^2} && (5.10)\\
&= \sqrt{\left(x_\parallel - \frac{m^s}{m_\perp^H}\right)^2 \left(p_\parallel^H\right)^2 + (x_\perp p_\perp^H)^2} && (5.11)\\
\implies \Delta p_s &\approx x_\perp p_\perp^H && (5.12)
\end{aligned}$$

Here $x_\perp = \frac{p_\parallel^{s/H}}{p_\parallel^H}$ and $x_\parallel = \frac{p_\perp^{s/H}}{p_\perp^H}$. The last step is justified because the major component of the hadron momentum is the transverse momentum.
For the fast quark q^f, the change in momentum is similarly given by

$$\Delta\vec{p}^f = \vec{p}_\parallel^{f/H} - \vec{p}_\perp^{f/H} - \vec{p}_\perp^f \tag{5.13}$$

Now β^f initially points along the perpendicular direction. Although q^f picks up a longitudinal component following recombination, the dominant component is the transverse momentum $p_\perp^f$. Therefore we get

$$\begin{aligned}
\Delta\vec{p}^f \times \vec{\beta}^f &= p_\parallel^{f/H} \beta^f \langle \sin\theta \rangle^f && (5.14)\\
&= (1 - x_\parallel) p_\parallel^H \langle \sin\theta \rangle^f && (5.15)
\end{aligned}$$

where $x_\perp + x_\parallel = 1$ as these are momentum fractions. Further, β^f is approximated to be 1.
Now, to calculate the value of $\langle \sin\theta \rangle^s$, we give a physical argument. The initial direction of $\vec{\beta}^s$ points along the parallel direction while $\Delta\vec{p^s}$is directed along the perpendicular direction. Therefore, initial angle is $\pi/2$. However, post recombination, since the dominant momentum component is the perpendicular one, the angle is 0. Therefore, the average angle is close to $\pi/4$ and hence $\langle \sin\theta \rangle^s \approx 1/\sqrt{2}$. Since, we have characterised this one as a slow quark, instead of our initial argument where we put $\gamma/\gamma + 1 \approx 1$, we write

$$\omega^s = a\frac{\Delta p^s}{\Delta t} \tag{5.16}$$

where $a = \frac{\gamma^s}{1+\gamma^s}\beta^s\langle\sin\theta\rangle^s$ is a factor between 0 and 1.
By similar arguments for $\langle\sin\theta\rangle^f$, we have $\langle\sin\theta\rangle^f \approx 1$.
The change in energy is common to both the accelerating q^s and the decelrating q^f.

$$\Delta E = \left[\left(p_\perp^f\right)^2 + \left(p_\parallel^f\right)^2 + \left(m^f\right)^2\right]^{1/2} + \left[(p_\perp^s)^2 + \left(p_\parallel^s\right)^2 + (m^s)^2\right]^{1/2} - \left[\left(p_\perp^H\right)^2 + \left(p_\parallel^H\right)^2 + \left(m^H\right)^2\right]^{1/2} \quad (5.17)$$

Under the assumption that $p_\perp^s = p_\parallel^f = 0$ and $(p_\parallel^H)^2 << (p_\perp^H)^2$, we have

$$\begin{aligned}\Delta E &\approx \left[\left(p_\perp^f\right)^2 + \left(m^f\right)^2\right]^{1/2} + \left[\left(p_\parallel^s\right)^2 + (m^s)^2\right]^{1/2} - \left[\left(p_\perp^H\right)^2 + \left(m^H\right)^2\right]^{1/2} \\ &= \left\{p_\perp^f\left[1 + \frac{(m^f)^2}{2(p_\perp^f)^2}\right] + [(p_\parallel^s)^2 + (m^s)^2]^{1/2} - p_\perp^H\left[1 + \frac{(m^H)^2}{2(p_\perp^H)^2}\right]\right\} \quad (5.18)\end{aligned}$$

where the binomial expansion is done with the assumption that the transverse momentum is large compared to the mass for the fast quark and the hadron.
Now, for the second term, we have

$$\begin{aligned}[(p_\parallel^s)^2 + (m^s)^2]^{1/2} &= \left[\left(\frac{m^s}{m_\perp^H}\right)^2 (p_\parallel^H)^2 + (m^s)^2\right]^{1/2} \\ &= m^s \cosh y^H \quad (5.19)\end{aligned}$$

Here we introduce another factor $0 < z < 1$ which relates the hadron and q^f transverse momenta by $p_\perp^f = p_\perp^H/z$. This reduces the equation to

$$\Delta E = \left\{\left(\frac{1-z}{z}\right)p_\perp^H + \left[\frac{z(m^f)^2 - (m^H)^2}{2p_\perp^H}\right] + m^s \cosh y^H\right\} \quad (5.20)$$

Finally, we get the polarisation P^s and P^f.

$$P^s = -\frac{\frac{\gamma^s}{1+\gamma^s}\frac{\beta^s x_\perp p_\perp^H}{\sqrt{2}\Delta t}}{\left\{\left(\frac{1-z}{z}\right)p_\perp^H + \left[\frac{z(m^f)^2-(m^H)^2}{2p_\perp^H}\right] + m^s\cosh y^H\right\}} \tag{5.21}$$

$$P^f = \frac{\frac{(1-x_\parallel)p_\parallel^H}{\Delta t}}{\left\{\left(\frac{1-z}{z}\right)p_\perp^H + \left[\frac{z(m^f)^2-(m^H)^2}{2p_\perp^H}\right] + m^s\cosh y^H\right\}} \tag{5.22}$$

Given our choice of quantisation axis along the normal to the reaction plane, we can find the spin density matrices of the slow and the fast quarks. For q^s, it is given by

$$\rho^{q^s} = \frac{1}{2}\begin{pmatrix} 1+P_{q^s} & 0 \\ 0 & 1-P_{q^s}\end{pmatrix}$$

Similarly for q^f, we have

$$\rho^{q^f} = \frac{1}{2}\begin{pmatrix} 1+P_{q^f} & 0 \\ 0 & 1-P_{q^f}\end{pmatrix}$$

We assume there is no particular correlation between the two quarks that combine to give a vector meson. Now we can calculate the spin density matrix of the vector meson by taking a direct product of ρ^{q^s} and ρ^{q^f} and transforming it to the coupled basis [8].

$$\rho = \begin{pmatrix} \frac{(1+P_{q^s})(1+P_{q^f})}{3+P_{q^s}P_{q^f}} & 0 & 0 \\ 0 & \frac{1-P_{q^s}P_{q^f}}{3+P_{q^s}P_{q^f}} & 0 \\ 0 & 0 & \frac{(1-P_{q^s})(1-P_{q^f})}{3+P_{q^s}P_{q^f}} \end{pmatrix}$$

Hence, we have the $\rho_{0,0}$ element in terms of the polarisation of the two quarks.

$$\rho_{0,0} = \frac{1-P_{q^s}P_{q^f}}{3+P_{q^s}P_{q^f}} \tag{5.23}$$

Using equations 5.22 and 5.21 in equation 5.23, we can find the $\rho_{0,0}$ element for the produced hadron. In case of no polarisation in the system, we have $\rho_{0,0} = \frac{1}{3}$. It is clear from equation 5.23 that presence of polarisation would deviate the $\rho_{0,0}$ element from $\frac{1}{3}$. This will affect the angular distribution as for $\rho_{0,0} = \frac{1}{3}$, $\frac{dN}{d\cos\theta}$ is a constant, however in presence of polarisation, it will be a function of $\cos\theta$ as found in equation 2.11.

Chapter 6

Analysis

6.1 Properties of Φ meson

We have analysed the decay of the Φ vector meson in this report. Some of the important properties of Φ particle is listed below [9].

1. Rest mass = 1019.461 ± 0.019 MeV
2. Quark Content = $s\bar{s}$
3. Charge = 0
4. Width = 4.266 ± 0.031 MeV
5. Lifetime = $(1.55 \pm 0.01) \times 10^{-22}$ s
6. Strangeness = 0, Charmness = 0, Bottomness = 0
7. Total Spin = 1
8. Isospin = 0
9. Major decay channels with branching ratios in parenthesis
   - $\Phi \rightarrow K^+ + K^-$ $(48.9 \pm 0.5\%) \rightarrow$ (We have used this in our analysis)
   - $\Phi \rightarrow K_L^0 + K_S^0$ $(34.2 \pm 0.4\%)$
   - $\Phi \rightarrow \rho\pi + \pi^+\pi^-\pi^0$ $(15.32 \pm 0.32\%)$

6.2 Event generator - PYTHIA

The data for this analysis has been generated using a Monte-Carlo event generator PYTHIA(v. 8186) [5] and [6]. The Pythia program can be used to simulate high energy events, i.e. sets of outgoing particles produced in the interaction between two incoming particles. The emphasis is on multiparticle production in collisions between elementary particles, particularly e^+e^-, p^+p^+ and e^-p^+ collisions. It contains a library of hard processes for initial and final state parton showers, multiple parton-parton interaction, string fragmentation, beam remnants and particle decays.
The different hard processes in PYTHIA can be classified in different ways:[5], [6]

- **Number of final particles produced :** $2 \to 1$, $2 \to 2$, $2 \to 3$ processes. This aspect is important as higher the number of particles produced, the more complicated the phase space structure is and hence the whole generation procedure. PYTHIA is optimized for $2 \to 1$, $2 \to 2$ processes.
- **Physics scenario :** Hard QCD processes, Soft QCD processes, Heavy flavour production etc.

This data has been generated using all inelastic soft QCD processes(SoftQCD:inelastic = on) for pp collisions at $\sqrt{s} = 13$ TeV. Soft QCD involves processes where there is low momentum transfer between the interacting processes. Only the inelastic interactions have been included as they are dominated by soft QCD. As mentioned in the discussion on scattering, inelastic interactions result in formation of new particles. Elastic processes do not allow the same.
Inelastic processes can be further categorised into non-diffractive, single diffractive, double diffractive and central diffractive processes. The details of these processes are beyond the scope of this project.

6.3 Analysis techniques

6.3.1 Invariant Mass Reconstruction

Φ decays to K^+ and K^- with a branching ratio of 0.489. Since the lifetime of Φ is of the order of 10^{-22} s, we cannot directly detect Φ. However, we can use the four momenta of the daughter particles(K^+ and K^-) to reconstruct the mass of Φ using the invariant mass reconstruction method.
Suppose a particle M is moving with four momentum $P^\mu = (E, \vec{p})$ in the center of mass frame. Let it decay to particles A and B with four momenta $P_1^\mu = (E_1, \vec{p_1})$ and $P_2^\mu = (E_2, \vec{p_2})$ respectively.

Now using conservation of four momenta, we can write $(E, \vec{p}) = (E_1, \vec{p_1}) + (E_2, \vec{p_2})$.

$$\therefore E^2 - p^2 = (E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2 \quad (6.1)$$

$$\implies m_M^2 = m_A^2 + m_B^2 + 2(E_1 E_2 - \vec{p_1}.\vec{p_2}) \quad (6.2)$$

We used K^+ and K^- pairs to generate the invariant mass for reconstruction. Since we have taken all the possible pairs of K^+ and K^- , there will be a combinatorial background which we have to reduce using a statistical procedure.

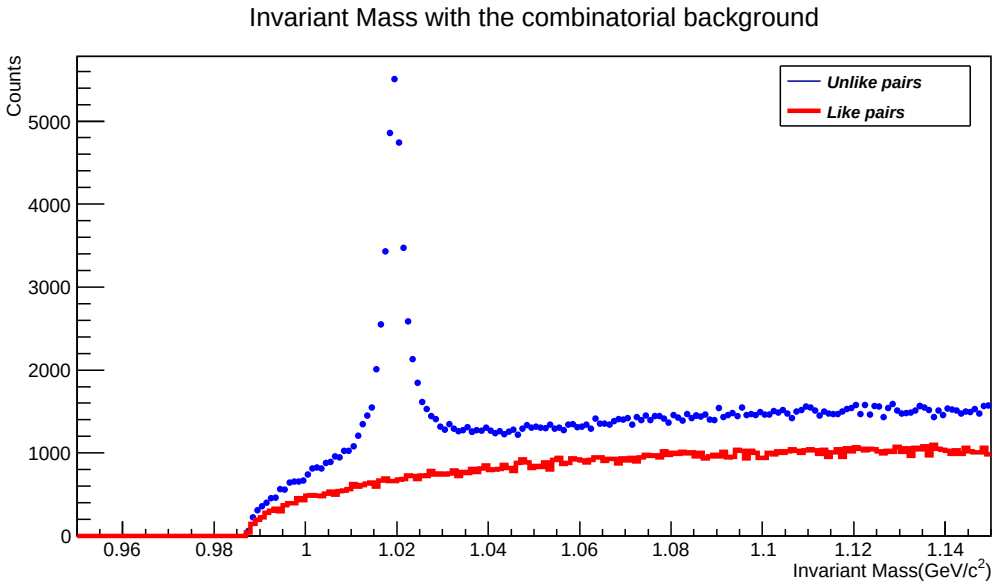


Figure 6.1: Invariant Mass reconstruction along with the combinatorial background for one $\cos(\theta)$ bin

6.4 Removal of background : Like-Sign Technique

The four momenta distribution of K^+ and K^- are statistically same. However, for a pair of K^+, there will never be a peak in the reconstructed invariant mass around the mass of Φ. The uncorrelated background in the unlike sign pair sample is estimated by the number of like-sign pairs within each event. If the numbers of like sign pairs K^+K^+ and K^-K^- are N_{++} and N_{--} respectively, the combinatorial background N is given by their geometric mean multiplied by a factor R. $N = 2R\sqrt{N_{++}N_{--}}$.

Therefore, the signal is given by $S = N^{+-} - 2R\sqrt{N_{++}N_{--}}$. The factor R accounts for possible asymmetry in the production of positively and negatively charged particle and(or) asymmetry due to a detector trigger or acceptance bias relative to the particle charge. This has no bearing on our data though, as it is generated in a simulation. This technique has the disadvantage that the statistics in the background spectrum is limited to the number of available events. On the other hand, since the number of unlike-sign pairs and like-sign pairs are calculated within the same event, the normalization of the determined background to the signal+background spectrum is straightforward, provided that one knows the R factor with good accuracy.

6.4.1 Fitting

The signal which is obtained after the background reduction is fitted to a Briet-Wigner plus a polynomial function where the Breit-Wigner function is for the resonance peak and the polynomial function is to fit the residual background. Here, we have taken a second order polynomial of the form $ax^2 + bx + c$ where a, b, c are free parameters. The Breit-Wigner function is taken as

$$f(E) = Y\frac{\Gamma/2}{(E - E_R)^2 + \Gamma^2/4} \tag{6.3}$$

where Y is the yield, Γ is the width of the resonance and E_R is the invariant mass at the resonance peak.
Therefore, the total fit function is given by

$$f(E) = Y\frac{\Gamma/2}{(E - E_R)^2 + \Gamma^2/4} + ax^2 + bx + c \tag{6.4}$$

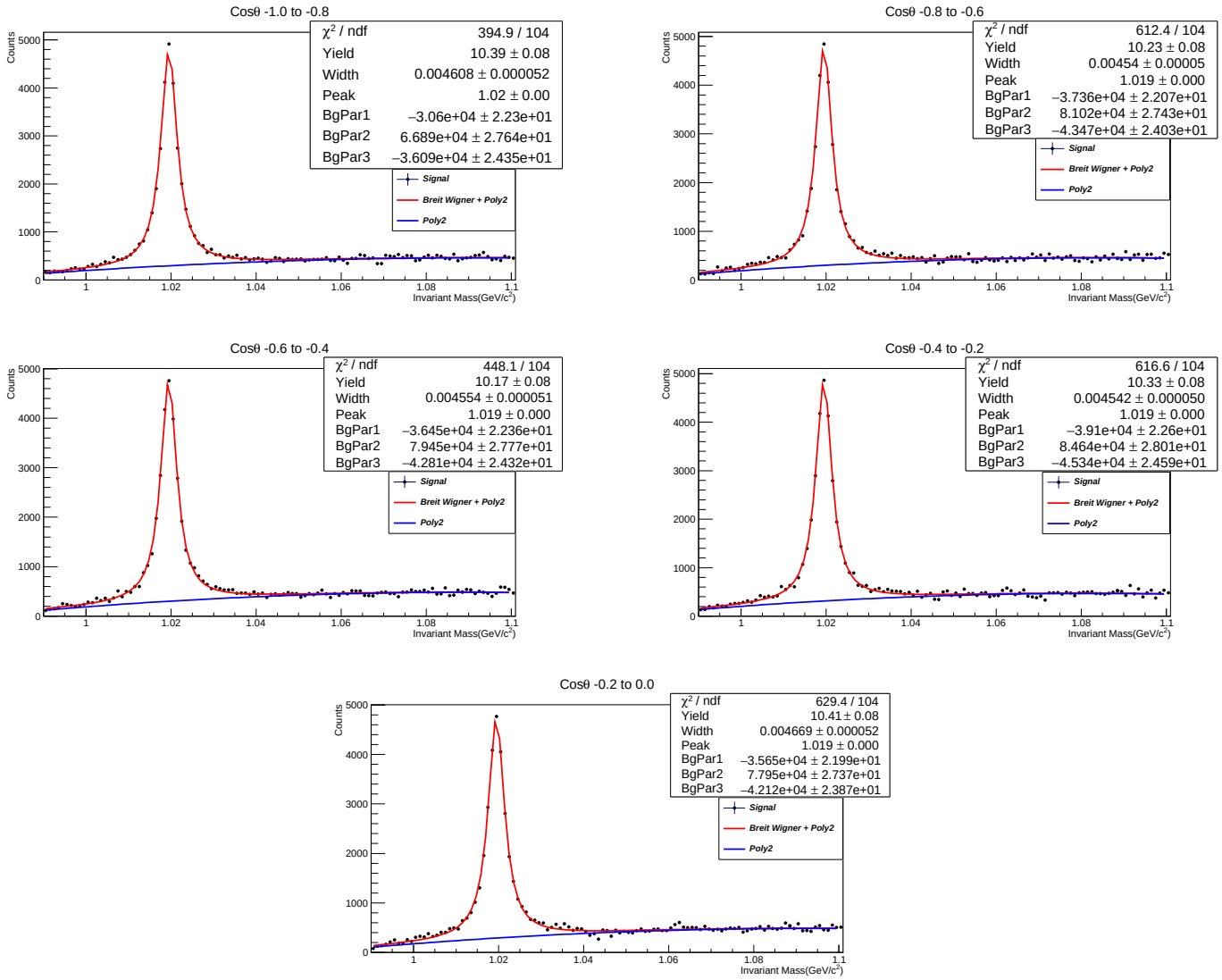


Figure 6.2: Invariant mass distribution of $m_{K^+ + K^-}$ after combinatorial background subtraction fitted with a Briet Wigner + Residual Background function (red line) in different $\cos\theta$ bins, from $\cos\theta$ = -1 to $\cos\theta = 0$, for pp collisions at $\sqrt{s} = 13$ TeV. The contribution from residual background is shown by blue line.

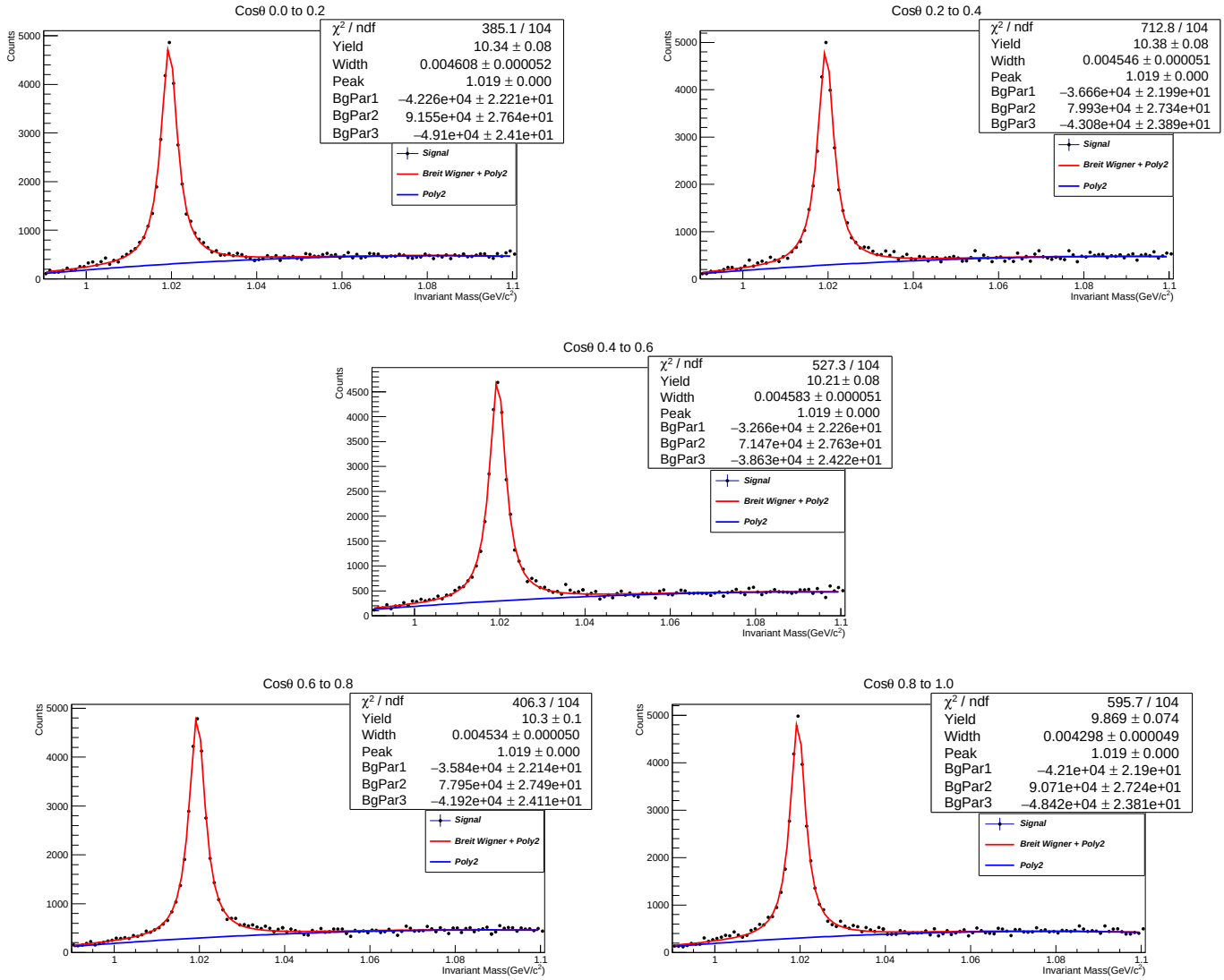
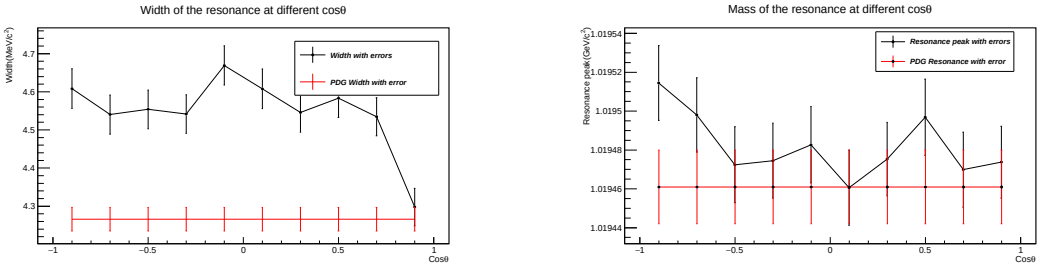


Figure 6.3: Invariant mass distribution of $m_{K^+ + K^-}$ after combinatorial background subtraction fitted with a Briet Wigner + Residual Background function (red line) in different $\cos\theta$ bins, from $\cos\theta = 0$ to $\cos\theta = 1$, for pp collisions at $\sqrt{s} = 13$ TeV. The contribution from residual background is shown by blue line.

6.5 Results

6.5.1 Mass and Width

From our analysis, we can find the the mass of the resonance peak and the width of the resonance for each $\cos\theta$ bin from the fit parameters of the Breit Wigner function. The mass peak positions of Φ are found to be consistent with the PDG value within statistical uncertainties while the width is slightly higher.



(a) (b)

Figure 6.4: (a) Width and (b) Mass of the resonance for different $\cos\theta$ bins.

6.6 Angular Distribution and $\rho_{0,0}$

Our main aim is to find the $\rho_{0,0}$ element from the $\frac{dN}{d\cos\theta}$ vs $\cos\theta$ curve. For this, we find the total number of counts in the invariant mass range $0.9 - 1.1$ GeV/c^2 for each of the 10 $\cos\theta$ bins. From this, we will get the $\frac{dN}{d\cos\theta}$ vs $\cos\theta$ curve.
After that, we fit it using the equation 2.11 given by

$$\frac{dN}{d\cos\theta} = N_0[1 - \rho_{0,0} + \cos^2\theta(3\rho_{0,0} - 1)] \tag{6.5}$$

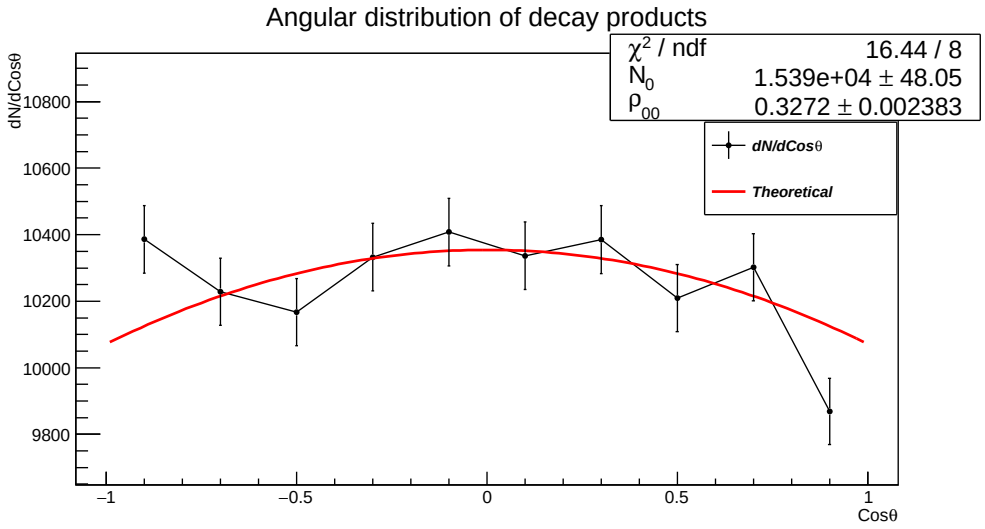


Figure 6.5: The angular distribution of the decay particles as a function of $\cos\theta$.

From the fit parameter, we get $\rho_{0,0} = 0.3272 \pm 0.0024$.

Chapter 7

Inferences and Discussion

"All of physics is either impossible or trivial. It is impossible until you understand it, and then it becomes trivial."

— Ernest Rutherford

We finally have the mass of the resonance, the width of the resonance and the value of the density matrix element $\rho_{0,0}$ from our analysis. Since our data is produced in a simulation using an event generator, there is no angular momentum of the initial beams which collide, hence there is no orbital angular momentum involved in the recombination process. If that is the case, we have already argued using equation 2.11 that $\rho_{0,0}$ should be equal to $\frac{1}{3}$. The value we obtained after analysis agrees with this expectation.

pp collisions are taken as the baseline for heavy ion collisions as many *pp* collisions mimic a heavy ion collision, sans a few properties. The spin polarisation is one such property absent in *pp* collision as it is essentially central. The analyses performed here are fairly basic and they can be improved upon by using better statistical methods, some of which include using p_T cuts, rapidity cuts etc. to filter out spurious data. However, the absence of spin polarisation is clear from the angular distribution of Φ .

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Appendices

Appendix A

The Code for generating data from PYTHIA

```
// File: tree.cc
// This is a simple test program.
// Modified by Rene Brun and Axcel Naumann to put the Pythia::event
// into a TTree.
// Copyright (C) 2014 Torbjorn Sjostrand

// Header file to access Pythia 8 program elements.
#include "Pythia8/Pythia.h"

// ROOT, for saving Pythia events as trees in a file.
#include "TTree.h"
#include "TFile.h"

using namespace Pythia8;

int main() {

// Create Pythia instance and set it up to generate hard QCD processes
// above pTHat = 20 GeV for pp collisions at 14 TeV.
Pythia pythia;

// Pick new random number seed for each run, based on clock.
pythia.readString("Random:setSeed = on");
pythia.readString("Random:seed = 0");

// setup parametes
const float Ecm = 13000.0; // CM energy in GeV

//pythia.readString("HardQCD:all = on");
//pythia.readString("PhaseSpace:pTHatMin = 20.");
//pythia.readString("Beams:eCM = 14000.");
//pythia.readString("SoftQCD:nonDiffractive = on");
//pythia.readString("SoftQCD:all = on");
```

```
//pythia.readString("SoftQCD:elastic = on");
//pythia.readString("SoftQCD:singleDiffractive = on");
//pythia.readString("SoftQCD:doubleDiffractive = on");
//pythia.readString("SoftQCD:centralDiffractive = on");
pythia.readString("SoftQCD:inelastic = on");

//pythia.readString("BeamRemnants:reconnectColours=on");
//pythia.readString("BeamRemnants:reconnectRange=5.0");
pythia.init(2212, 2212, Ecm);


//Tune, amount of CR, ... have to be included in config.cmnd
pythia.readFile("config.cmnd");
// pythia.readString("Random:seed");
//  pythia.readString("Random:seed = 1245876");
int nEvents = pythia.mode("Main:numberOfEvents");
int nAbort = pythia.mode("Main:timesAllowErrors");

// Set up the ROOT TFile and TTree.
TFile *file = TFile::Open("pytree.root","recreate");
Event *event = &pythia.event;
TTree *T = new TTree("T","ev1 Tree");
T->Branch("event",&event);

// Begin event loop. Generate event; skip if generation aborted.
for (int iEvent = 0; iEvent < 2000000; ++iEvent) {
if (!pythia.next()) continue;

// Fill the pythia event into the TTree.
// Warning: the files will rapidly become large if all events
// are saved. In some cases it may be convenient to do some
// processing of events and only save those that appear
// interesting for future analyses.

for (int i = 0; i < pythia.event.size(); ++i) {
Int_t ist = pythia.event[i].status();

//select final particles
if ( ist <= 0 )continue;

Float_t eta = pythia.event[i].eta();
Float_t rap = pythia.event[i].y();
Float_t rphi = pythia.event[i].phi();
Float_t pt = pythia.event[i].pT();
Float_t pxx = pythia.event[i].px();
Float_t pyy = pythia.event[i].py();
Float_t pzz = pythia.event[i].pz();
Float_t energy = pythia.event[i].e();
Float_t mass = pythia.event[i].m();
Int_t pidd = pythia.event[i].id();
```

```
Int_t momid1 = pythia.event[i].mother1();
Int_t momid2 = pythia.event[i].mother2();
Int_t dauid1 = pythia.event[i].daughter1();
Int_t dauid2 = pythia.event[i].daughter2();

//cout<<"I am HERE ================= "<<eta<<"\t"<<pt<<"\t"<<sqrt(pxx*pxx
    + pyy*pyy)<<"\t"<<pidd<<"\t"<<ist<<endl;
//cout<<"I am HERE ==== 2nd Step =============
    "<<pidd<<"\t"<<energy<<"\t"<<mass<<"\t"<<rap<<endl;
//cout<<"I am HERE ==== 3rd Step =============
    "<<momid1<<"\t"<<momid2<<"\t"<<dauid1<<"\t"<<dauid2<<endl;
//|eta|<1
//if (TMath::Abs(eta)>1)continue;
}
T->Fill();


// End event loop.
}

// Statistics on event generation.
pythia.stat();

//  Write tree.
T->Print();
T->Write();
T->MakeClass("plot");
delete file;

// Done.
return 0;
}
```

Appendix B

The Code (For finding the angular distribution)

```
#define TwoMillion_cxx
#include "TwoMillion.h"
#include <TH2.h>
#include <TStyle.h>
#include <TCanvas.h>

void TwoMillion::Loop()
{

if (fChain == 0) return;

TH2D *KpKm = new TH2D("K+K-","InvariantMassandCosTheta", 200,0.95,1.15,
    10, -1, 1);
TH2D *KpKp = new TH2D("K+K+","InvariantMassandCosTheta", 200,0.95,1.15,
    10, -1, 1);
TH2D *KmKm = new TH2D("K-K-","InvariantMassandCosTheta", 200,0.95,1.15,
    10, -1, 1);

Long64_t nentries = fChain->GetEntriesFast();

Long64_t nbytes = 0, nb = 0;
for (Long64_t jentry=0; jentry<nentries;jentry++) {
Long64_t ientry = LoadTree(jentry);
if (ientry < 0) break;
nb = fChain->GetEntry(jentry); nbytes += nb;
Int_t r = 0, s = 0;
Double_t Px1[300], Py1[300], Pz1[300], MassKp[300];
Double_t Px2[300], Py2[300], Pz2[300], MassKm[300];

Int_t Multiplicity = entry_;
for (Int_t a =0; a<Multiplicity; a++) \\Saving the necessary data
{
Int_t Stat = entry_idSave[a];
```

```
if(Stat == 321|| Stat == -321)
{
if(Stat == 321)
{
Px1[r] = entry_pSave_xx[a];
Py1[r] = entry_pSave_yy[a];
Pz1[r] = entry_pSave_zz[a];
MassKp[r] = entry_mSave[a];
r+=1;
}
else
{
Px2[s] = entry_pSave_xx[a];
Py2[s] = entry_pSave_yy[a];
Pz2[s] = entry_pSave_zz[a];
MassKm[s] = entry_mSave[a];
s+=1;
}

}

}

TFile *file = TFile::Open("2mEvents.root");

TH2D* KpKm = (TH2D*)file->Get("K+K-");
TH2D* KpKp = (TH2D*)file->Get("K+K+");
TH2D* KmKm = (TH2D*)file->Get("K-K-");

TH1D* KpKmUnlike[10];
for(int i=0; i< 10; i++)
{
KpKmUnlike[i] = new TH1D(Form("KpKmUnlike%d",i), Form("KpKmUnlike%d",i),
    200, 0.95, 1.15);
}

TH1D* KpKplike[10];
for(int i=0; i<10; i++)
{
KpKplike[i] = new TH1D(Form("KpKplike%d",i), Form("KpKplike%d",i), 200,
    0.95, 1.15);
}

TH1D* KmKmlike[10];
for(int i=0; i<10; i++)
{
KmKmlike[i] = new TH1D(Form("KmKmlike%d",i), Form("KmKmlike%d",i), 200,
    0.95, 1.15);
}
```

```
TH1D* bg[10];
for(int i=0; i<10; i++)
{
bg[i] = new TH1D(Form("Bg%d",i), Form("Bg%d",i), 200, 0.95, 1.15);
}

TH1D* sg[10];
for(int i=0; i<10; i++)
{
sg[i] = new TH1D("", "", 200, 0.95, 1.15);
}

TH1D* correctedbg[10];
for(int i=0; i<10; i++)
{
correctedbg[i] = new TH1D(Form("CorrectedBg%d",i),
    Form("CorrectedBg%d",i), 200, 0.95, 1.15);

}

for(int i=0; i<10; i++)
{
KpKmUnlike[i] = KpKm->ProjectionX(Form("Unlike%d", i), i+1, i+1);
}


for(int i=0; i<10; i++)
{
KpKplike[i] = KpKp->ProjectionX(Form("pplike%d",i), i+1, i+1);
}

for(int i=0; i<10; i++)
{
KmKmlike[i] = KmKm->ProjectionX(Form("mmlike%d",i), i+1, i+1);
}

for(int i = 0; i<10; i++)
{
for(int b = 1; b<=200; b++)
{
Double_t nbkg
    =2*TMath::Sqrt(KpKplike[i]->GetBinContent(b)*KmKmlike[i]->GetBinContent(b));
bg[i]->SetBinContent(b, nbkg);
}
}

Double_t R[10];
for(int i=0; i<10; i++)
{
R[i] = KpKmUnlike[i]->Integral(150,200, "")/bg[i]->Integral(150, 200, "");
```

```
}

for(int i = 0; i<10; i++)
{
for(int b = 1; b<=200; b++)
{
Double_t backgr
    =2*TMath::Sqrt(KpKplike[i]->GetBinContent(b)*KmKmlike[i]->GetBinContent(b));
correctedbg[i]->SetBinContent(b, backgr);
}
}

TCanvas *ComBg = new TCanvas("ComBg","Total",10,10,600,600);
ComBg->cd();
KpKmUnlike[1]->SetMarkerColor(4);
KpKmUnlike[1]->Draw();
correctedbg[1]->SetTitle("Invariant Mass with combinatorial background");
correctedbg[1]->GetXaxis()->SetTitle("Invariant Mass(GeV/c^{2})");
correctedbg[1]->GetYaxis()->SetTitle("Counts");
correctedbg[1]->SetMarkerColor(1);
correctedbg[1]->Draw("same");
TLegend *legendn=new TLegend(0.6,0.65,0.88,0.85);
legendn->SetTextFont(72);
legendn->SetTextSize(0.03);
legendn->AddEntry(KpKmUnlike[1],"Signal","l");
legendn->AddEntry(correctedbg[1],"Background","l");
legendn->Draw();


for(int i=0; i<10; i++)
{
sg[i] -> Add(KpKmUnlike[i], bg[i], 1, -1);
}

TF1 *f[10];


Double_t Gamma[10];
Double_t Resonance[10];
Double_t dNdCosTheta[10];
Double_t CosThetaBinCenter[] = {-0.9, -0.7, -0.5, -0.3, -0.1, 0.1, 0.3,
    0.5, 0.7, 0.9};

TCanvas *Canvas[10];
for(int i = 0; i<10; i++)
{
Canvas[i] = new
    TCanvas(Form("Canvas%d",i),Form("Canvas%d",i),10,10,600,600);
}
```

```
Double_t YieldIn[] = {10, 10, -5, -10, 10, 10, 10, 10, 10, 9};

TF1* BgFunc[10];
for(int i=0; i< 10; i++)
{

BgFunc[i] = new TF1(Form("BGround%d",i), "[0]*x**2 + [1]*x + [2]", 0.99,
    1.1);
f[i] = new TF1(Form("Breit-Wigner%d",i),"[0]*[1]/(2*((x-[2])**2 +
    [1]**2/4.)) + [3]*x**2 + [4]*x + [5]",0.99, 1.1);

f[i]->SetParName(0,"Yield");
f[i]->SetParName(1,"Width");
f[i]->SetParName(2,"Peak");
f[i]->SetParName(3, "BgPar1");
f[i]->SetParName(4, "BgPar2");
f[i]->SetParName(5, "BgPar3");
f[i]-> SetParameter(0,YieldIn[i]);
f[i]-> SetParameter(1,0.04);
f[i]-> SetParameter(2,1.02);
f[i]-> SetParameter(3,1 );
f[i]-> SetParameter(4,1);
f[i]-> SetParameter(5,1);

}
Double_t CosBin[] = {-1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6 ,0.8,
    1};
Double_t Error[10];

Double_t Width[10];
Double_t WidthErr[10];
Double_t Res[10];
Double_t ResErr[10];

for(int i=0; i<10; i++)
{
Canvas[i]->cd();
sg[i]->SetMarkerColor(1);
sg[i]->SetMarkerStyle(20);
sg[i]->SetTitle(Form("Cos#theta %.1f to %.1f",CosBin[i],CosBin[i+1]));
sg[i]->GetXaxis()->SetRangeUser(0.99, 1.1);
sg[i]->GetXaxis()->SetTitle("Invariant Mass(GeV/c^{2})");
sg[i]->GetYaxis()->SetTitle("Counts");
sg[i]->Draw("p");
sg[i]-> Fit(f[i], "REM", "", 0.99, 1.1);
gStyle->SetOptFit(1);
gStyle->SetOptStat(0000000);

Int_t a = sg[i]->FindBin(0.99);
Int_t b = sg[i]->FindBin(1.1);
```

```
BgFunc[i]->SetParameter(0, f[i]->GetParameter(3));
BgFunc[i]->SetParameter(1, f[i]->GetParameter(4));
BgFunc[i]->SetParameter(2, f[i]->GetParameter(5));
BgFunc[i]->SetLineColor(4);
BgFunc[i]->Draw("same");

Width[i] = f[i]->GetParameter(1)*1000;
WidthErr[i] = f[i]->GetParError(1)*1000;
Res[i] = f[i]->GetParameter(2);
ResErr[i] = f[i]->GetParError(2);

TLegend *legend=new TLegend(0.6,0.65,0.88,0.85);
legend->SetTextFont(72);
legend->SetTextSize(0.03);
legend->AddEntry(sg[i],"Signal","lpe");
legend->AddEntry(f[i],"Breit Wigner + Poly2","l");
legend->AddEntry(BgFunc[i],"Poly2","l");
legend->Draw();

dNdCosTheta[i] = f[i]->GetParameter(0)*1000;
Error[i] = TMath::Sqrt(dNdCosTheta[i]);

}

TF1 *fitf= new TF1("AngularDistro","[0]*(1-[1] + x**2*(3*[1]-1))",-1,1);
fitf->SetParName(0, "N_{0}");
fitf->SetParName(1, "#rho_{00}");
fitf->SetParameter(0,100);
fitf->SetParameter(1, 0.33);

TCanvas *C2 = new TCanvas("Angular","",10,10,600,600);
TCanvas *D3 = new TCanvas("New Canvas","",10,10,600,600);
C2->cd();
//
Double_t ErrX[] = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
TGraphErrors *Gr = new TGraphErrors(10, CosThetaBinCenter, dNdCosTheta,
   ErrX, Error);
Gr->SetMarkerColor(1);
Gr->SetMarkerStyle(20);
Gr->GetXaxis()->SetTitle("Cos#theta");
Gr->GetYaxis()->SetTitle("dN/dCos#theta");
Gr->SetTitle("Angular distribution of decay products");
Gr -> Fit(fitf, "", "", -1, 1);
Gr->Draw();
TLegend *legend2=new TLegend(0.6,0.65,0.88,0.85);
legend2->SetTextFont(72);
legend2->SetTextSize(0.03);
legend2->AddEntry(Gr,"dN/dCos#theta","lpe");
legend2->AddEntry(fitf,"Theoretical","l");
```

```
legend2->Draw();

D3->cd();
Double_t PDGWid[10];
Double_t PDGWidErr[10];
for(int i = 0; i<10; i++)
{
PDGWid[i] = 0.004266*1000;
PDGWidErr[i] = 0.000031*1000;
}
TGraphErrors *Gr1 = new TGraphErrors(10, CosThetaBinCenter, Width, ErrX,
    WidthErr);
Gr1->SetMarkerColor(1);
Gr1->SetMarkerStyle(20);
Gr1->GetXaxis()->SetTitle("Cos#theta");
Gr1->GetYaxis()->SetTitle("Width(GeV/c^{2})");
Gr1->SetTitle("Width of the resonance at different cos#theta");
Gr1->Draw();
TGraphErrors *GrErr2 = new TGraphErrors(10, CosThetaBinCenter, PDGWid,
    ErrX, PDGWidErr);
GrErr2->SetLineColor(2);
GrErr2->Draw("pl same");
TLegend *legend3=new TLegend(0.6,0.65,0.88,0.85);
legend3->SetTextFont(72);
legend3->SetTextSize(0.03);
legend3->AddEntry(Gr1,"Width with errors","lpe");
legend3->AddEntry(GrErr2, "PDG Width with error", "lpe");
legend3->Draw("p");

TCanvas *D4 = new TCanvas("Resonance","",10,10,600,600);
D4->cd();

Double_t PDGRes[10];
Double_t PDGResErr[10];
for(int i = 0; i<10; i++)
{
PDGRes[i] = 1.019461;
PDGResErr[i] = 0.000019;
}

TGraphErrors *Gr2 = new TGraphErrors(10, CosThetaBinCenter, Res, ErrX,
    ResErr);
Gr2->SetMarkerColor(1);
Gr2->SetMarkerStyle(20);
Gr2->GetXaxis()->SetTitle("Cos#theta");
Gr2->GetYaxis()->SetTitle("Resonance peak(GeV/c^{2})");
Gr2->SetTitle("Value of the resonance at different cos#theta");
Gr2->Draw("apl");
TGraphErrors *GrErr = new TGraphErrors(10, CosThetaBinCenter, PDGRes,
    ErrX, PDGResErr);
```

```
GrErr->SetLineColor(2);
GrErr->SetMarkerStyle(20);
GrErr->Draw("pl same");
TLegend *legend4=new TLegend(0.6,0.65,0.88,0.85);
legend4->SetTextFont(72);
legend4->SetTextSize(0.03);
legend4->AddEntry(Gr2,"Resonance peak with errors","lpe");
legend4->AddEntry(GrErr, "PDG Resonance with error", "lpe");
legend4->Draw();

TFile *MyFile = new TFile("Plots.root","RECREATE");
MyFile -> cd();
KpKm -> Write();
KpKp -> Write();
KmKm -> Write();
Gr2->Write();
GrErr->Write();
legend4->Write();

for(Int_t i = 0; i<10; i++)
{
KpKmUnlike[i]->Write();
KpKplike[i] -> Write();
KmKmlike[i] -> Write();
correctedbg[i] -> Write();
sg[i]->SetMarkerColor(1);
sg[i]->SetMarkerSize(1);
sg[i]->SetMarkerStyle(20);
sg[i] -> Write();
BgFunc[i] ->Write();


}

}
```