

Prim's algorithm using this indexed priority queue

Project by:

Dipti Shrinivas Desai.

NetId:dsd140330

Problem Statement:

Implement indexed priority queues. A binary heap is used to store the elements. If an element x moves to a new index i , then you must call $x.putIndex(i)$ to inform the element of its location in the heap. The function `decreaseKey` will be called with the index of the element whose key has decreased, and the heap order needs to be restored at that index.

Implement Prim's algorithm using this indexed priority queue

Abstract:

- **Generic MST**

$T = \{ \}$

While T is not a spanning tree

Do find an edge (u,v) that is safe for set A

$T = T \cup \{(u,v)\}$

Return A ;

- **Prim's Algorithm:**

- Input: A non-empty connected weighted graph with vertices V and edges E (the weights can be negative).
- Initialize: $V_{\text{new}} = \{x\}$, where x is an arbitrary node (starting point) from V , $E_{\text{new}} = \{ \}$
- Repeat until $V_{\text{new}} = V$:
 - Choose an edge $\{u, v\}$ with minimal weight such that u is in V_{new} and v is not (if there are multiple edges with the same weight, any of them may be picked)
 - Add v to V_{new} , and $\{u, v\}$ to E_{new}
- Output: V_{new} and E_{new} describe a minimal spanning tree

Minimum Edge weight Data Structure	Time Complexity
Adjacency Matrix, searching	$O(V ^2)$
Binary Heap and adjacency List	$O(E \log V)$

Implementation:

- Indexed Priority queue represents binary Min-heap structure. Every vertex in graph is an element in a priority queue and every vertex has name, weight, Edge Adjacency list and Index in the priority queue.
- Index value gives the access to the element in the queue in $O(1)$ time.
- Main functionality includes decrease key, percolate up, percolate down, DeleteMin for priority queue
- Decrease key will be called in prim's algorithm when node key of the vertex is reduced (In this implementation key is weight that is associated with vertex).
- Decrease key will restore heap order property after the priority of x has decreased. It will call percolate up function which restores the heap property.
- DeleteMin function removes the 1st node of the heap (minimum weight node) and then call Percolate down because Heap order may be violated.

System Configuration:

Processor: Intel Core I7-4700MQ 2.40GHz

RAM: 8 GB

HDD: 1TB

IDE: Eclipse

Input:

- <http://www.utdallas.edu/~rbk/teach/now/idsa.html>
- Sample input project 2

OUTPUT

File Name	Vertices	Edges	WMST	Time Taken
Prim1	50	140	84950	0mSec
Prim2	100	284	110419	3mSec
Prim3	200	580	153534	4mSec
Prim-ck	100000	299971	3384476	220mSec

(Time taken is only for prims algorithm for calculating Minimum spanning tree, it does not consider time taken for initialization of graph)**

References:

http://www.wikiwand.com/en/Prim's_algorithm

<http://www.personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/GraphAlgor/genericMST.html>

Balaji Raghavachari's class notes