Optimiser

Objective:

A good plant layout must achieve Cost Minimization, Safety Maximization (Risk Minimization). Total Cost of plant depends on two factors Cost factor and risk factor. We have given a function T (Total Cost Function) which depends on two variables C (Cost variable) and R (Risk variable). Our goal is to optimize the Total Cost Function (T) and minimize T in the range C [250,300] and R [50,60]. We can do this using Response Surface Model.

Model:

Using standard equation of second order response surface model, equation of Total Cost Function is given as:

$$T = \beta o+\beta 1*x1+\beta 2*x2+\beta 3*x1^2+\beta 4*x2^2+\beta 5*x1*x2$$

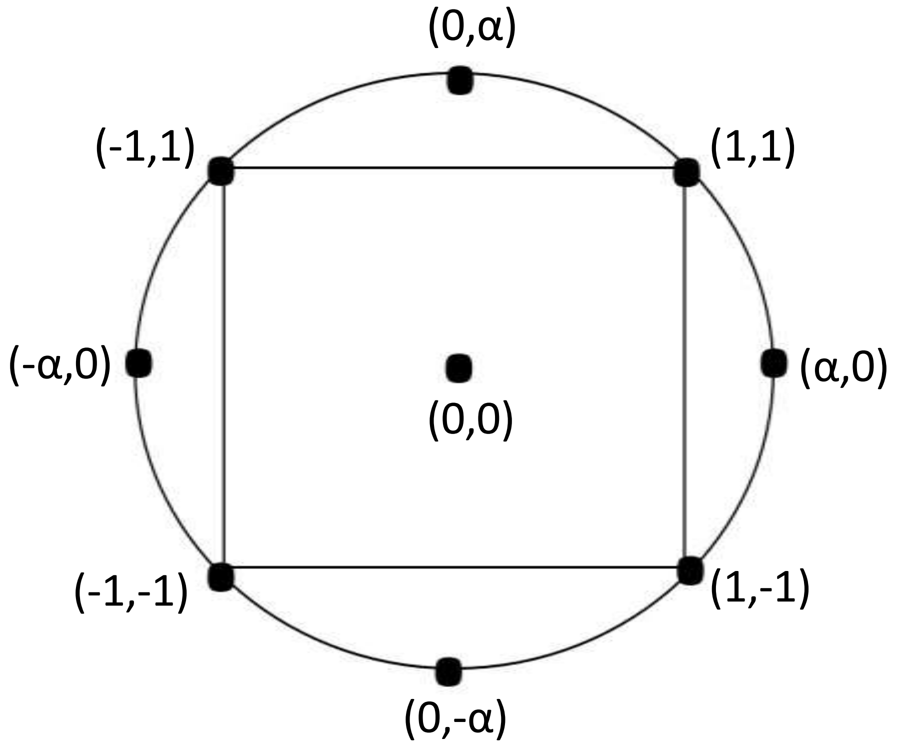
Here x1 and x2 are the coded values for C and R which we can easily find with the formula

$$\text{Coded Value} = \frac{\text{Uncoded Value} - \text{Central Value}}{\text{Step Size}}$$

Distance of each point from center is α (alpha). For two factor spherical central composite design the value of α is √2 i.e., 1.414.

We will perform 4 factorial runs, 5 central runs and 4 axial runs i.e., total 13 experimental runs. We have taken more central points to keep the prediction error in the middle of the design low.

Factorial Design for this looks as follows:



Values of T for respective values of C and R is shown in the table.

Natural Variables		Coded Variables		Output
Cost Variable (C)	Risk Variable (R)	x1	x2	Total Cost Function (T)
250	50	-1	-1	48.3009
250	60	-1	1	35.0665
300	50	1	-1	48.803
300	60	1	1	63.1786
275	55	0	0	35.1601
275	55	0	0	35.084
275	55	0	0	34.7753
275	55	0	0	35.2531
275	55	0	0	34.6404
239.645	55	-1.414	0	41.8214
310.355	55	1.414	0	62.2548
275	47.929	0	-1.414	45.2469
275	62.071	0	1.414	45.5007

Values of variables are shown in the table.

x1	x2	x1^2	x2^2	x1x2	y
-1.000	-1.000	1	1	1	48.3009
-1.000	1.000	1	1	-1	35.0665
1.000	-1.000	1	1	-1	48.8030
1.000	1.000	1	1	1	63.1786
0.000	0.000	0	0	0	35.1601
0.000	0.000	0	0	0	35.0840
0.000	0.000	0	0	0	34.7753
0.000	0.000	0	0	0	35.2531
0.000	0.000	0	0	0	34.6404
-1.414	0.000	2	0	0	41.8214
1.414	0.000	2	0	0	62.2548
0.000	-1.414	0	2	0	45.2469
0.000	1.414	0	2	0	45.5007

We can find the coefficients βo, β1, β2, β3, β4, β5 using Regression Modelling, which is fitting of curve to find the equation of curve. The steps are as follows:

Step 1: Determine Matrix X

It contains 6 columns I (all 1 values), x1 (coded values of C), x2 (coded values of R), x1^2, x2^2, x1*x2. This is a 13x6 matrix.

1	-1.000	-1.000	1	1	1
1	-1.000	1.000	1	1	-1
1	1.000	-1.000	1	1	-1
1	1.000	1.000	1	1	1
1	0.000	0.000	0	0	0
1	0.000	0.000	0	0	0
1	0.000	0.000	0	0	0
1	0.000	0.000	0	0	0
1	0.000	0.000	0	0	0
1	-1.414	0.000	2	0	0
1	1.414	0.000	2	0	0
1	0.000	-1.414	0	2	0
1	0.000	1.414	0	2	0

```
[[ 1.    -1.    -1.     1.     1.     1.   ]
 [ 1.    -1.     1.     1.     1.    -1.   ]
 [ 1.     1.    -1.     1.     1.    -1.   ]
 [ 1.     1.     1.     1.     1.     1.   ]
 [ 1.     0.     0.     0.     0.     0.   ]
 [ 1.     0.     0.     0.     0.     0.   ]
 [ 1.     0.     0.     0.     0.     0.   ]
 [ 1.     0.     0.     0.     0.     0.   ]
 [ 1.     0.     0.     0.     0.     0.   ]
 [ 1.    -1.414  0.     2.     0.     0.   ]
 [ 1.     1.414  0.     2.     0.     0.   ]
 [ 1.     0.    -1.414  0.     2.     0.   ]
 [ 1.     0.     1.414  0.     2.     0.   ]]
```

Step 2: Determination of matrix Y

It contains the value of Total cost function. This is a 13x1 matrix.

48.3009
35.0665
48.8030
63.1786
35.1601
35.0840
34.7753
35.2531
34.6404
41.8214
62.2548
45.2469
45.5007

```
[[48.3009]
 [35.0665]
 [48.803 ]
 [63.1786]
 [35.1601]
 [35.084 ]
 [34.7753]
 [35.2531]
 [34.6404]
 [41.8214]
 [62.2548]
 [45.2469]
 [45.5007]]
```

Step 3: Take Transpose of Matrix X (Let Xt)

This is a 6x13 matrix.

```
[[ 1.     1.     1.     1.     1.     1.     1.     1.     1.     1.
   1.     1.     1.   ]
 [-1.    -1.     1.     1.     0.     0.     0.     0.     0.    -1.414
   1.414  0.     0.   ]
 [-1.     1.    -1.     1.     0.     0.     0.     0.     0.     0.
   0.    -1.414  1.414]
 [ 1.     1.     1.     1.     0.     0.     0.     0.     0.     2.
   2.     0.     0.   ]
 [ 1.     1.     1.     1.     0.     0.     0.     0.     0.     0.
   0.     2.     2.   ]
 [ 1.    -1.    -1.     1.     0.     0.     0.     0.     0.     0.
   0.     0.     0.   ]]
```

Step 4: Calculate multiplication of Xt and X (Let A)

This is a 6x6 matrix.

```
[[13.        0.        0.        8.        8.        0.      ]
 [ 0.        7.998792  0.        0.        0.        0.      ]
 [ 0.        0.        7.998792  0.        0.        0.      ]
 [ 8.        0.        0.       12.        4.        0.      ]
 [ 8.        0.        0.        4.       12.        0.      ]
 [ 0.        0.        0.        0.        0.        4.      ]]
```

Step 5: Calculate inverse of A (Let B)
This is a 6x6 matrix.

```
[[ 0.2         0.          0.         -0.1        -0.1         0.        ]
 [ 0.          0.12501888  0.          0.          0.          0.        ]
 [ 0.          0.          0.12501888  0.          0.          0.        ]
 [-0.1         0.          0.          0.14375     0.01875     0.        ]
 [-0.1         0.          0.          0.01875     0.14375     0.        ]
 [ 0.          0.          0.          0.          0.          0.25      ]]
```

Step 6: Calculate multiplication of Xt and Y (Let C)
This is a 6x1 matrix.

```
[[565.0857   ]
 [ 57.5070276]
 [  1.5000732]
 [403.5014   ]
 [376.8442   ]
 [ 27.61     ]]
```

Step 7: Calculate multiplication of B and C (Let D)
This is a 6x1 matrix and these are the values of the parameters

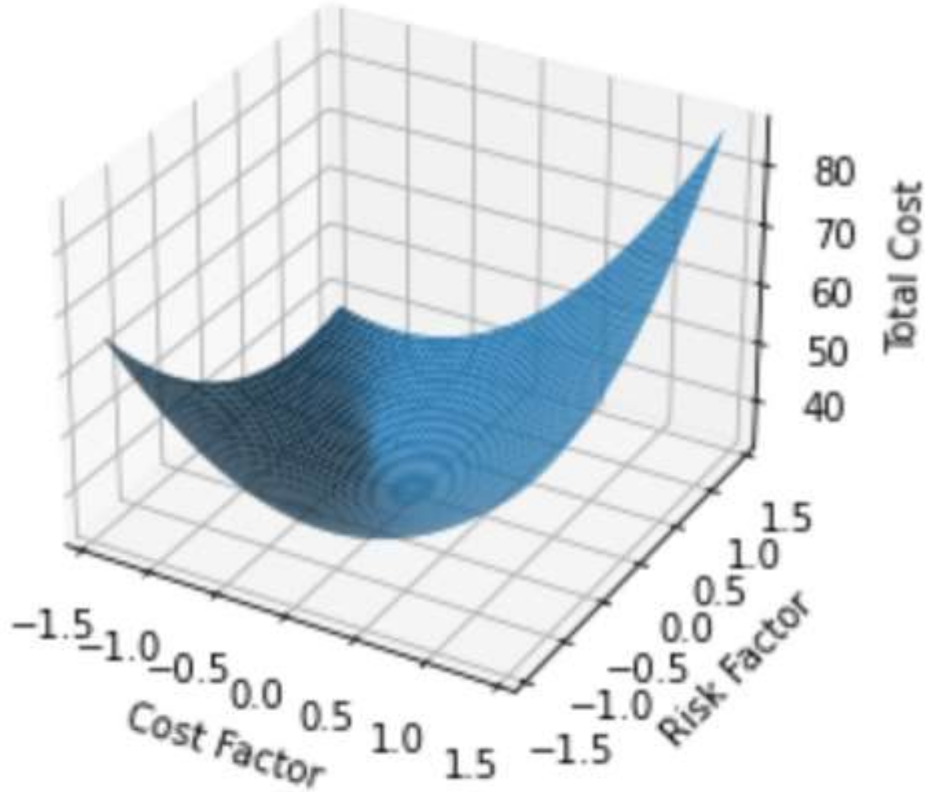
```
[[34.98258   ]
 [ 7.18946406]
 [ 0.18753747]
 [ 8.560585  ]
 [ 5.228435  ]
 [ 6.9025    ]]
```

β_o = 34.98258
β_1 = 7.18946406
β_2 = 0.18753747
β_3 = 8.560585
β_4 = 5.228435
β_5 = 6.9025

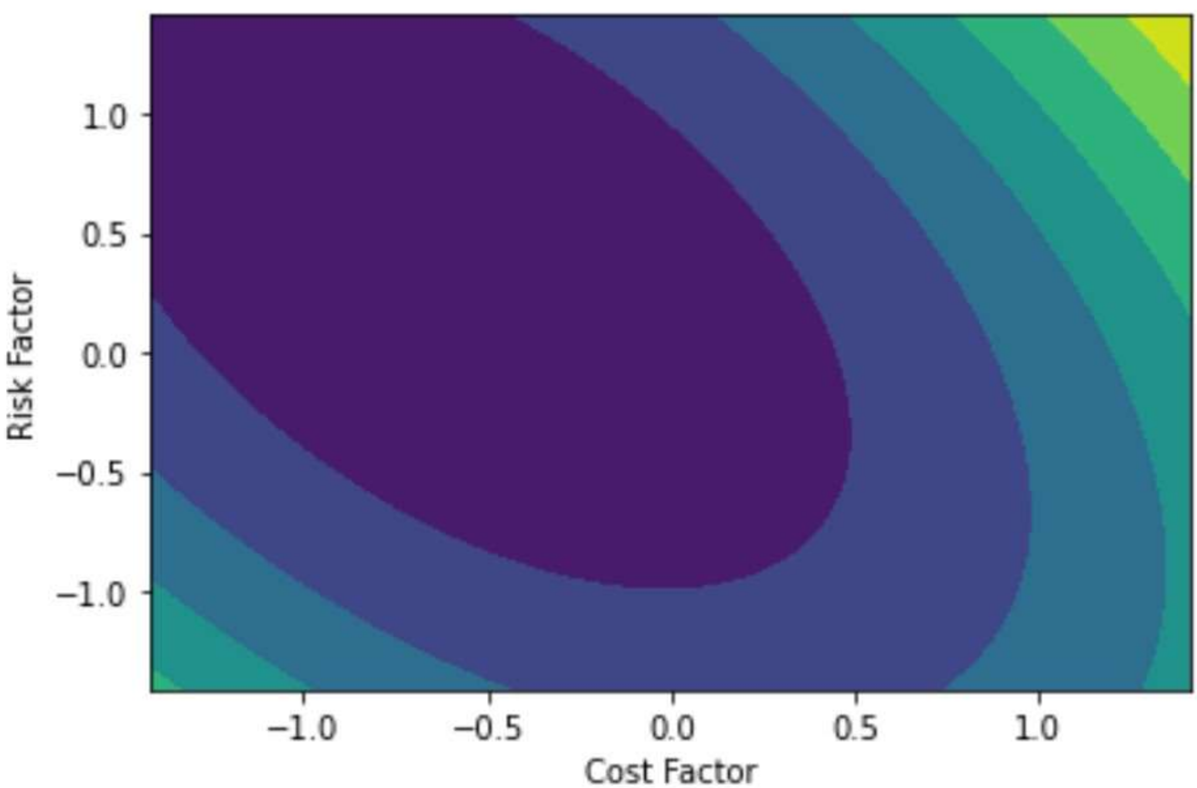
and the function,

T = 34.98258 + 7.18946406*x1 + 0.18753747*x2 + 8.560585*x1^2 +5.228435*x2^2+6.9025*x1*x2

Surface Plot of the function

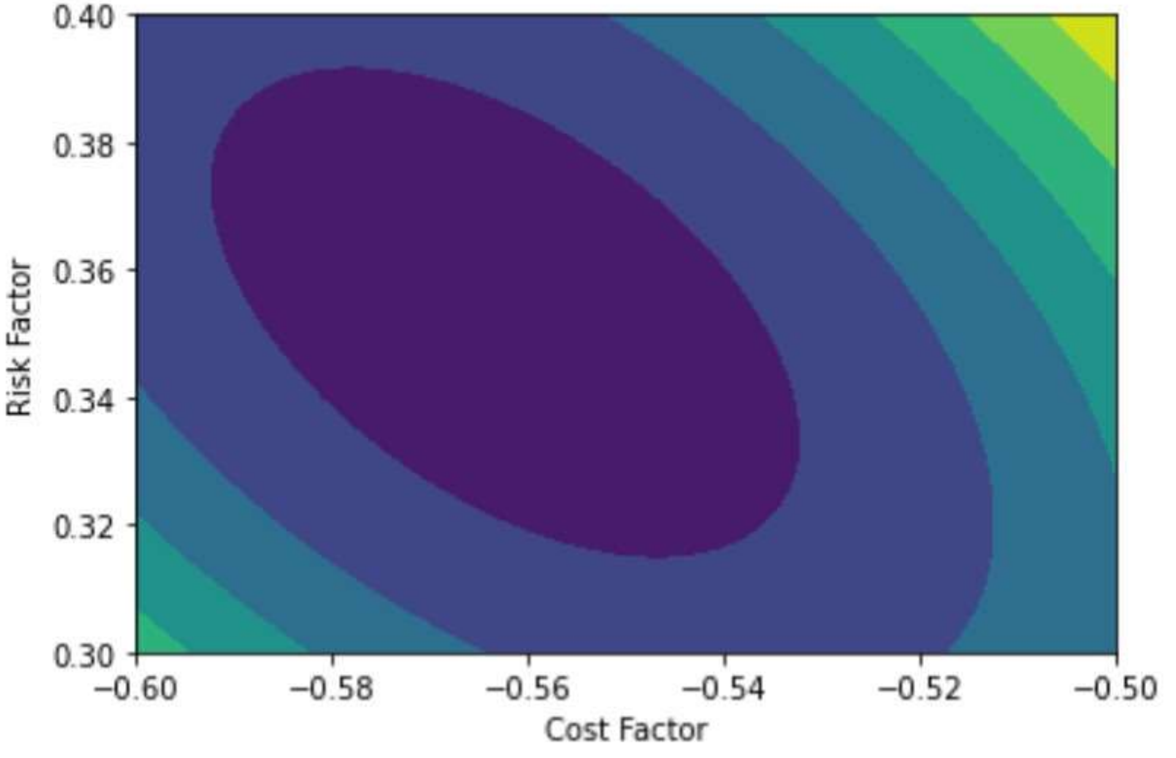


Contour Plot of the function



By looking at the plots we find that the minimum value should be between x1 [-0.6,-0.5] and x2 [0.3,0.4].

Contour plot for this range



Minimum value of function is coming 32.9943 at x1= -0.56 i.e., C=261 and x2=0.35 i.e., R=56.75

From this we can conclude that the minimum value of Total cost function is **32.9943** in the range C [250,300] and R[50,60].