Alpha Decay

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Abstract

One method used in nuclear spectroscopy to probe the decay properties of isotopes is to measure the energy of emitted α-particles with a charged particle detector. The rate of detection over time informs regarding the lifetime of the isotope, and the energy spectrum allows for characterization of the subprocesses. In this experiment, we aim at determining the half-lives of ^{212}Pb and ^{212}Bi, as well as the branching ratios and energies of emitted α-particles in the decay of ^{212}Bi, by recording the energies of α-particles emitted during the decay of ^{212}Pb to ^{208}Pb over 24-hour periods. So far, we have calibrated the detection apparatus by using a known decay peak energy of ^{241}Am [1], and found the parameters in the relation between the α-particle's energy E (MeV) and the detected MCA channel number n_{ch} to be $E(n_{ch}) = (4.306\times 10^{-3} \pm 7\times 10^{-6})\cdot n_{ch} + (0.074 \pm 0.008)$.

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1 Methods

1.1 Experiment Setup

The setup to detect the α-particles emitted during decay and measure their energy is presented in Fig. 1. It consists of a silicon surface barrier detector placed at a small distance (of the order of centimeters, which will be precisely measured later) from a radioactive source, both located inside a chamber. A rotary vacuum pump evacuates the air from the chamber to minimize collisions of α-particles with air molecules between decay and detection. A pre-amplifier then produces a voltage (clean signal) proportional to the integrated current received from the detector, and this voltage is amplified through a linear spectroscopy amplifier. The amplified voltage is recorded on a computer through a PCI multi-channel analyzer (MCA). For calibration purposes, a precision pulse generator was also connected to the pre-amplifier, along with an oscilloscope showing the output of the amplifier. The oscilloscope is not only used for debugging, but also to have more precise measurements of the voltage peaks produced by the pulse generator and amplified by the amplifier.

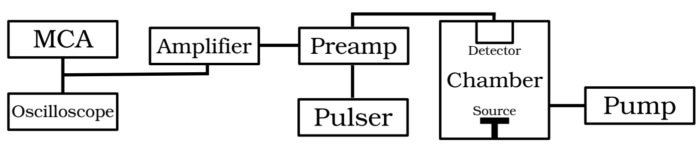


Figure 1: Block diagram of the setup needed for calibration. The pump evacuates the air from the chamber containing the radioactive source and the detector. The preamplifier integrates the current received from the detector and outputs a voltage peak proportional to that integrated value. That peak is then amplified and recorded. The pulse generator and oscilloscope are used for calibration.

2 Results

2.1 Calibration

The magnitude of an amplified voltage peak is recorded through the MCA as a channel number ranging between 0 and 2048, and that peak size is proportional to the energy of the α-particle. Thus, a calibration must be made to get an energy value from the channel number, using a relation of the form

$$E(n_{ch}) = m \cdot n_{ch} + b \tag{1}$$

where E is the energy in MeV, n_{ch} is the recorded channel number, and m and b are, respectively, the slope and the intercept of the linear relation.

To obtain these conversion parameters m and b, one characterization of the MCA is made. Namely, pulses of amplified magnitudes ranging from (1.00 ± 0.04) V to (8.00 ± 0.04) V are sent by the pulser, and the corresponding channel numbers are recorded. The corresponding voltage values are measured off the oscilloscope. Upon linearly fitting the voltage against the channel number, the acquired intercept determines the conversion parameters.

Additionally a reference decay energy is required of the well-studied alpha decay of ^{241}Am, the peak of which is known to be (5.48556 ± 0.00012) MeV [1]. The Americium is placed in the chamber, which is brought to a near-vacuum pressure of (60 ± 2.5) mTorr (the lowest stable pressure attainable with the working apparatus). Its decay spectrum is recorded as described in Sec. 1.1 during five minutes, as to have enough points for a fit, extracting the channel number associated with this decay energy.

Upon combining this data with the assumption that a voltage peak of zero from the pulser yields zero energy, the conversion parameters are obtained via

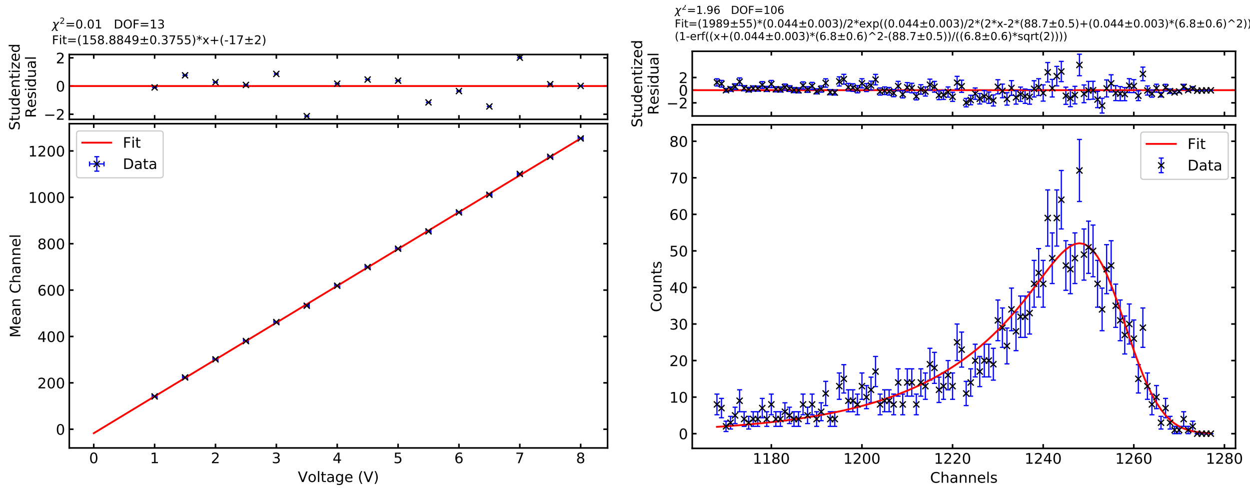
$$m = \frac{E_{Am}}{n_{Am} - \beta} \qquad b = \frac{-E_{Am}\beta}{n_{Am} - \beta} \tag{2}$$

where E_{Am} and n_{Am} are the energy and fitted channel number of the fitted americium spectrum respectively, and β is the intercept on the voltage versus channel number linear fit. This relation is derived in Appendix A.2.

For a singly peaked emission spectrum of a radioactive element, we model this stochastic process with the probability distribution

$$p(x; \lambda, \mu, \sigma) = \frac{\lambda}{2} \exp\left(\lambda\left(x - \mu + \frac{\lambda\sigma^2}{2}\right)\right)\left(1 - \mathrm{erf}\left(\frac{x - \mu + \sigma^2\lambda}{\sqrt{2}\sigma}\right)\right), \tag{3}$$

where x is the energy of the α-particle, λ parameterizes the energy losses in the chamber, μ is the energy of the transition and σ is the standard deviation of the (loss-less) Gaussian energy distribution. See Sec. A.3 for a derivation of this distribution from probability theory. A scaling factor, A, is applied to the distribution above when fitting the observed ^{241}Am spectrum, along with an interchange of energy with channel number.



(1) **(2)**

Figure 2: **(1)** For the calibration of MCA, it shows the mean channel of the pulse peak as a function of the pulse voltage. It is fitted a linear function with slope $\gamma = 158.9 \pm 0.03$, y-intercept $\beta = 17 \pm 2$, and with $\chi^2_{red} = 0.01$ (degree of freedom $dof = 13$). This very small χ^2_{red} indicates an overfit. The voltage error comes from the voltage measurement of the oscilloscope and the mean channel error comes from the Gaussian fit of the voltage pulse. **(2)** The detected signal from the α-spectrum of a ^{241}Am source in the vacuum chamber with pressure at $60 \pm 2.5)$ mTorr. The fit is based on equation Eq. 3 with $\mu = 1256.7 \pm 0.5$, the scaling factor $A = 199 \times 10 \pm 6 \times 10$, $\sigma = 6.8 \pm 0.6$, $\lambda = 0.044 \pm 0.003$, and with $\chi^2_{red} = 1.96$ ($dof = 106$). This indicates an underfit. The value of μ is interpreted as the value of the channel number associated to the decay energy. The error bar is the square root of the count. The upper plots show the studentized residuals.

The required conversion parameters are calculated and summarized in Table 1. We thus have the channel number to energy conversion formula, with values substituted in Eq. 1, as

$$E(n_{ch}) = (4.306 \times 10^{-3} \pm 0.007 \times 10^{-3}) \cdot n_{ch} + (0.074 \pm 0.008).$$

Table 1: Intermediate and final MCA calibration parameters required to relate binned channel number to energy of α-particles.

Quantity	**Value**
Intercept of Channel Number vs. Voltage Linear Fit, β	-17 ± 2
^{241}Am Reference Energy [1], E_{Am} (MeV)	$5.48556 \pm 1.2 \times 10^{-4}$
^{241}Am Peak (Fitted) Channel Number, n_{Am}	1256.8 ± 0.4
Slope of Channel Number to Energy Conversion, m (MeV)	$4.306 \times 10^{-3} \pm 0.007 \times 10^{-3}$
Intercept of Channel Number to Energy Conversion, b (MeV)	0.074 ± 0.008

A Appendix

A.1 Calibration Data and Fits

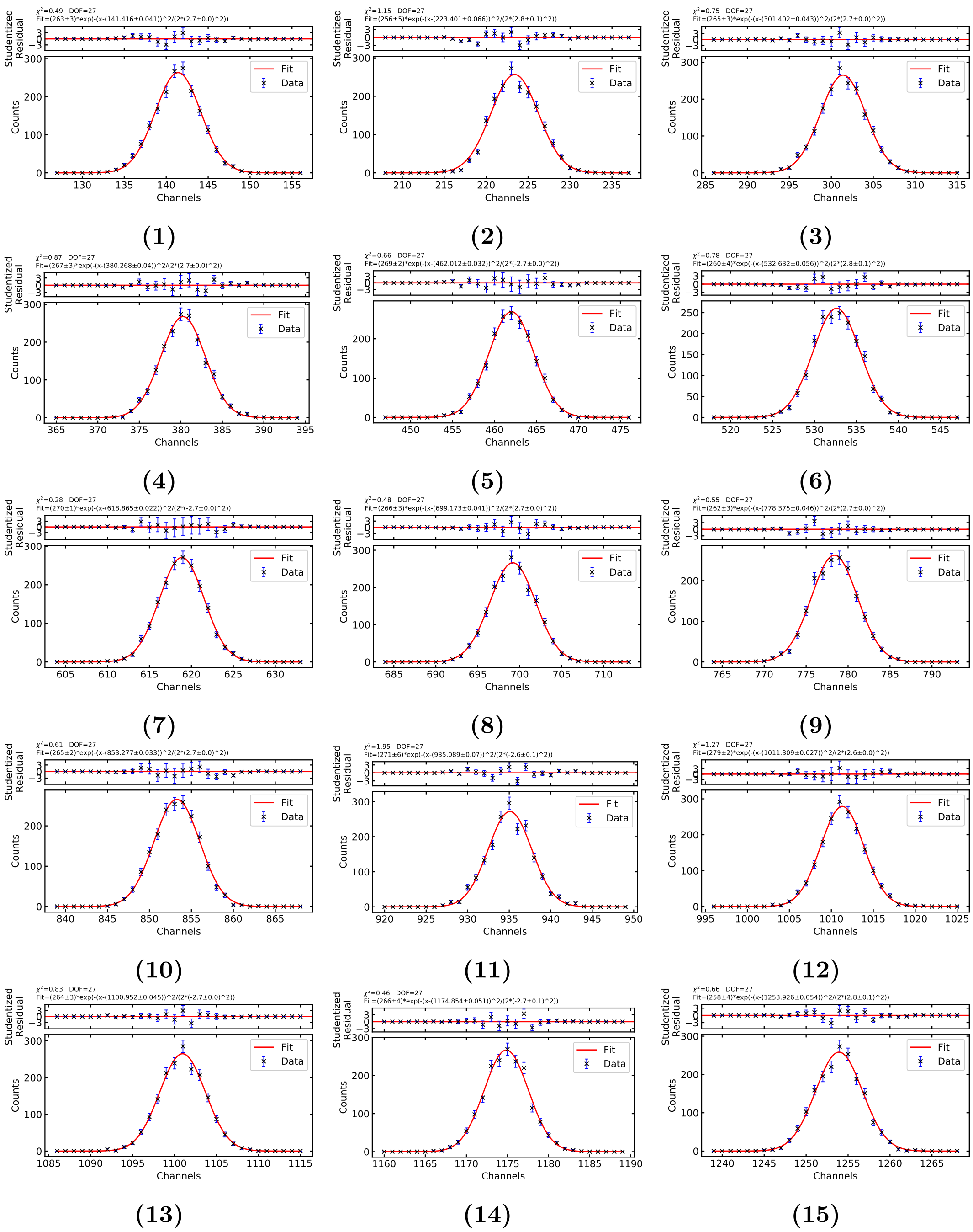


Figure 3: The plots of all voltage peaks used for the calibration of voltage and channel number relation. Each voltage peak is fitted with a Gaussian distribution, with fitted parameters shown in Table 2. All voltage signals (peaks) were recorded for 30 s duration.

Table 2: The corresponding mean channel value obtained from fitting the voltage signals, voltage measurement from the oscilloscope and the reduced χ^2 assesment of the fit in Fig.3. All voltage signals (peaks) were recorded for 30 s duration.

Pulse Index	Mean Channel of the Pulse Peak (channels)	Voltage Measured by the Oscilloscope (V)	χ^2_{red} of Fit
1	141.42 ± 0.04	1.00 ± 0.04	0.49
2	223.40 ± 0.07	1.50 ± 0.04	1.2
3	301.40 ± 0.04	2.00 ± 0.04	0.75
4	380.27 ± 0.04	2.50 ± 0.04	0.87
5	462.01 ± 0.03	3.00 ± 0.04	0.66
6	532.63 ± 0.06	3.50 ± 0.04	0.78
7	618.86 ± 0.02	4.00 ± 0.04	0.28
8	699.17 ± 0.04	4.50 ± 0.04	0.48
9	778.38 ± 0.05	5.00 ± 0.04	0.55
10	853.28 ± 0.03	5.50 ± 0.04	0.61
11	935.09 ± 0.07	6.00 ± 0.04	2.0
12	1011.31 ± 0.03	6.50 ± 0.04	1.3
13	1100.95 ± 0.04	7.00 ± 0.04	0.83
14	1174.85 ± 0.05	7.50 ± 0.04	0.46
15	1253.93 ± 0.05	8.00 ± 0.04	0.66

A.2 Derivation of Conversion Parameters

As described in Sec. 2.1, a linear fit between voltage and channel is extracted from varying the voltages of amplified pulses and measuring their associated channel numbers. Succintly,

$$n_{ch} = \gamma \cdot V + \beta \tag{4}$$

where n is the channel number, V is the voltage of the pulse, γ and β are the parameters determined in the fitting procedure. Recalling the ansatz, Eq. 1, we can substitute Eq. 4 to obtain a relation between energy and pulse voltage of the form

$$E = m \cdot \gamma \cdot V + \beta \cdot m + b.$$

Assuming that zero voltage corresponds to zero energy enforces the constraint,

$$0 = \beta \cdot m + b. \tag{5}$$

Lastly, the reference americium energy and its fitted channel number give

$$E_{Am} = m \cdot n_{Am} + b. \tag{6}$$

Equations 5 and 6 are readily inverted to yield Equations 2.

A.3 Derivation of Spectrum Fitting Function

One might recall from probability theory that if a variable H is the sum of two independent random variables F and G sampled from probability distributions f and g respectively, then the probability distribution corresponding to H is

$$h = f * g,$$

where $*$ signifies convolution over the underlying probability space [2].

In the case of this experiment, we model the probability of measuring an α-particle with a particular energy as the sum of two independent random processes. First, the α-emmission process which we model by a guassian distribution with mean energy μ and standard deviatiion σ. Second, we model the probabilty of the α-particle of a particular energy being deflected along its path to the detector by an exponential decaying distribution with decay constant λ. Letting χ_S represent the characteristic function of a set S and upon normalizing both of these distributions, their convolution can be computed;

$$\begin{aligned} p(x;\lambda,\mu,\sigma) &= \left\{ \left(\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}\left(y-\mu\right)^2 \right) \right) * \left(\frac{1}{\lambda} \exp\left(-\lambda y\right) \chi_{(0,\infty)}(y) \right) \right\}(x) \\ &= \int_{(0,\infty)} dy \frac{1}{\lambda\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}\left(y-x-\mu\right)^2 \right) \exp\left(-\lambda y\right) \\ p(x;\lambda,\mu,\sigma) &= \frac{\lambda}{2} \exp\left(\lambda \left(x - \mu + \frac{\lambda\sigma^2}{2} \right) \right) \left(1 - \operatorname{erf}\left(\frac{x-\mu+\sigma^2\lambda}{\sqrt{2}\sigma} \right) \right) \end{aligned}$$

where erf denotes the error function defined as

$$\operatorname{erf}(z) := \frac{2}{\sqrt{\pi}} \int_{(0,z)} dt \exp\left(-t^2\right).$$

References

[1] M.S. Basunia. Nuclear Data Sheets for A = 237. *Nuclear Data Sheets*, 107:2323–2422, August 2006. 1, 3, 4

[2] R.V. Hogg, J.W. McKean, and A.T. Craig. *Introduction to Mathematical Statistics*. Pearson, 2013. 7