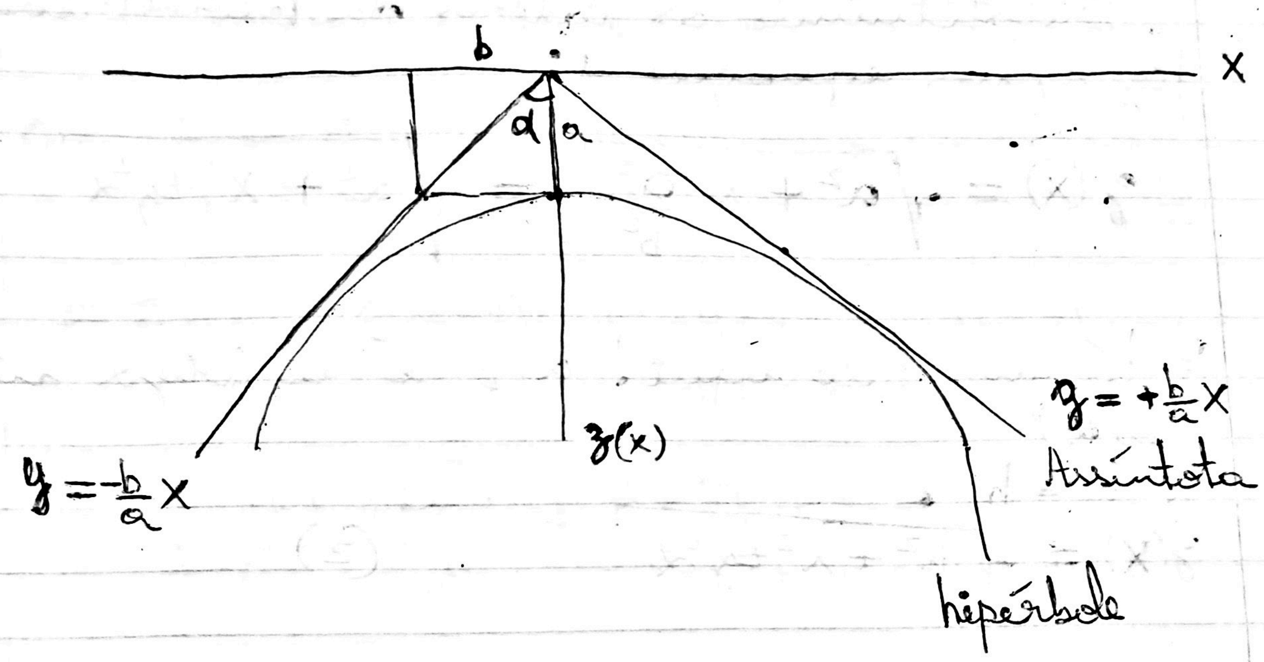
* Desenvolvimento: Refletor Hiperbólico.

1. Partindo da Equação da hipérbole.

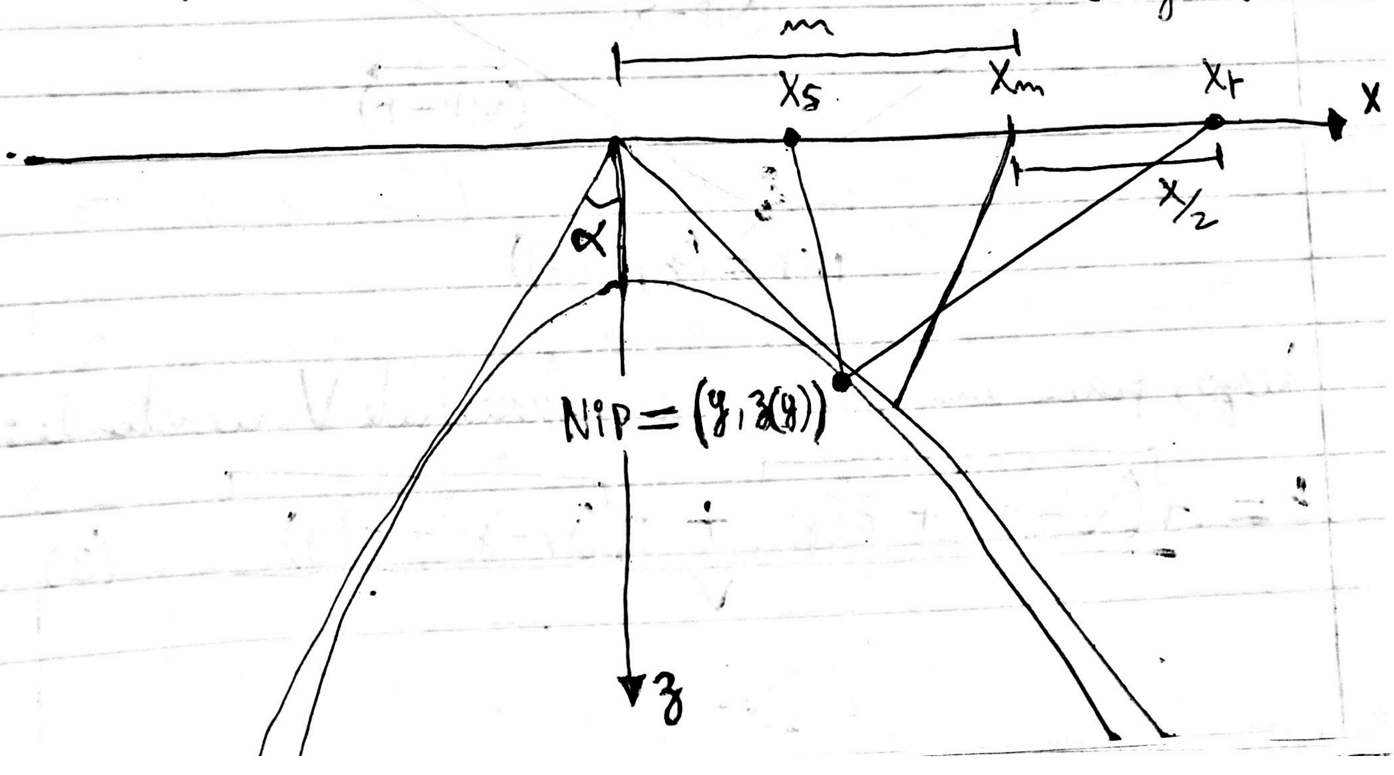
(Fig. 1)



$$\frac{z^2(x)}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow z = \sqrt{a^2 + x^2\frac{a^2}{b^2}} \qquad (I)$$

2. Definindo a geometria

(Fig. 2)



O ponto de reflexão é $NiP = (y, z(y))$; as coordenadas da fonte e do receptor são X_s e X_r respectivamente. X_m e $X/2$ são as coordenadas do cmp e o valor do half-offset $X/2 = h$.

Substituindo os valores da Geometria na fig. 2, na equação 1:

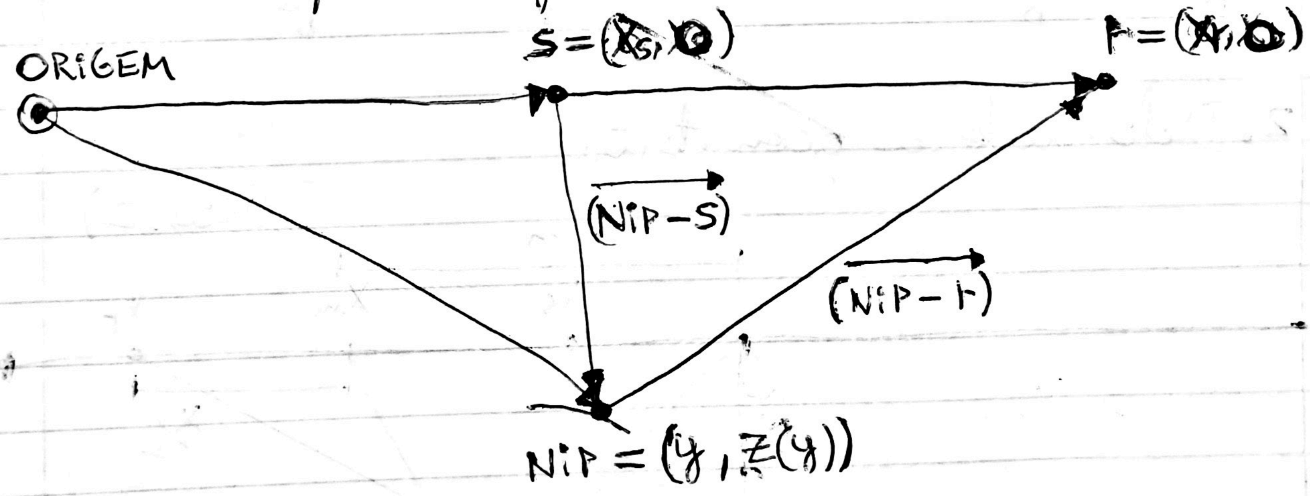
$$z(x) = \sqrt{a^2 + x^2 \frac{a^2}{b^2}} = \sqrt{a^2 + x^2 tg^2\alpha} \quad (2)$$

* há um erro aqui! a/b é a cotg α não a tg α!

se $a = h$ e

$$z(x) = \sqrt{h^2 + x^2 tg^2\alpha} \quad (2)$$

3. O tempo de Reflexão será:



Logo, para um meio de velocidade V constante:

$$t = \frac{\sqrt{(x_s - y)^2 + Z^2(y)} + \sqrt{(x_r - y)^2 + Z^2(y)}}{V} \quad (3)$$

$$t = \frac{\sqrt{(x_s - y)^2 + h^2 + y^2 tg^2\alpha} + \sqrt{(x_r - y)^2 + h^2 + y^2 tg^2\alpha}}{V} \quad (3)$$

4. De acordo com o princípio de Fermat o tempo de trânsito deve ser estacionário.

$$0 = \frac{\partial t}{\partial y} = \frac{\partial}{\partial y} \frac{\sqrt{(x_s - y)^2 + h^2 + y^2 tg^2\alpha}}{V} + \frac{\partial}{\partial y} \frac{\sqrt{(x_r - y)^2 + h^2 + y^2 tg^2\alpha}}{V}$$

$$= \frac{2y\, tg^2\alpha - 2(x_s - y)}{2V\sqrt{\ldots}} + \frac{2y\, tg^2\alpha - 2(x_r - y)}{2V\sqrt{\ldots}}$$

$$0 = \frac{y - x_s + y\, tg^2\alpha}{V\sqrt{\ldots \text{①}}} + \frac{y - x_r + y\, tg^2\alpha}{V\sqrt{\ldots \text{②}}}$$

$$0 = (y - x_s + y\, tg^2\alpha)\sqrt{②} + (y - x_r + y\, tg^2\alpha)\sqrt{①}$$

$$= (y - x_s + y\, tg^2\alpha)^2 \left((x_r - y)^2 + h^2 + y^2 tg^2\alpha\right) =$$

$$(y - x_r + y\, tg^2\alpha)^2 \left((x_s - y)^2 + h^2 + y^2 tg^2\alpha\right)$$

Usando a identidade $tg^2\theta = sec^2\theta - 1$

$$= \left(y \cdot (1 + tg^2\alpha) - x_s\right)^2 (\ldots ①) = \left(y(1 + tg^2\alpha) - x_r\right)^2 (\ldots ②)$$

$$= (y\, sec^2\alpha - x_s)^2 (\ldots ①) = (y\, sec^2\alpha - x_r)^2 (\ldots ②)$$

$$0 = \left[\frac{y}{\cos^2\alpha} - x_s\right]^2 \left[(x_r - y)^2 + h^2 + y^2 tg^2\alpha\right] =$$

$$\left[\frac{y}{\cos^2\alpha} - x_r\right]^2 \left[(x_s - y)^2 + h^2 + y^2 tg^2\alpha\right] \quad (4)$$

GAP ①

*Não sei como passar da eq. (4) para a eq. (5) abaixo...

$$y^2 (x_s + x_r)\, tg^2\alpha - 2y(x_s x_r\, sen^2\alpha - h^2) - h^2 (x_s + x_r) \cos^2\alpha = 0 \qquad (5)$$

$$\Delta = b^2 - 4a.c = (-2(x_s x_r\, sen^2\alpha - h^2))^2 + 4.(x_s + x_r)\, tg^2\alpha \,.\, h^2 (x_s + x_r) \cos^2\alpha$$

$$= 4(x_s x_r\, sen^2\alpha - h^2)^2 + 4h^2 (x_s + x_r)^2\, sen^2\alpha$$

$$y = \frac{2y(x_s x_r\, sen^2\alpha - h^2)^4 + \sqrt{4(x_s x_r\, sen^2\alpha - h^2)^2 + 4h^2 (x_s + x_r)^2\, sen^2\alpha}}{2.(x_s + x_r)\, tg^2\alpha} \qquad (6)$$

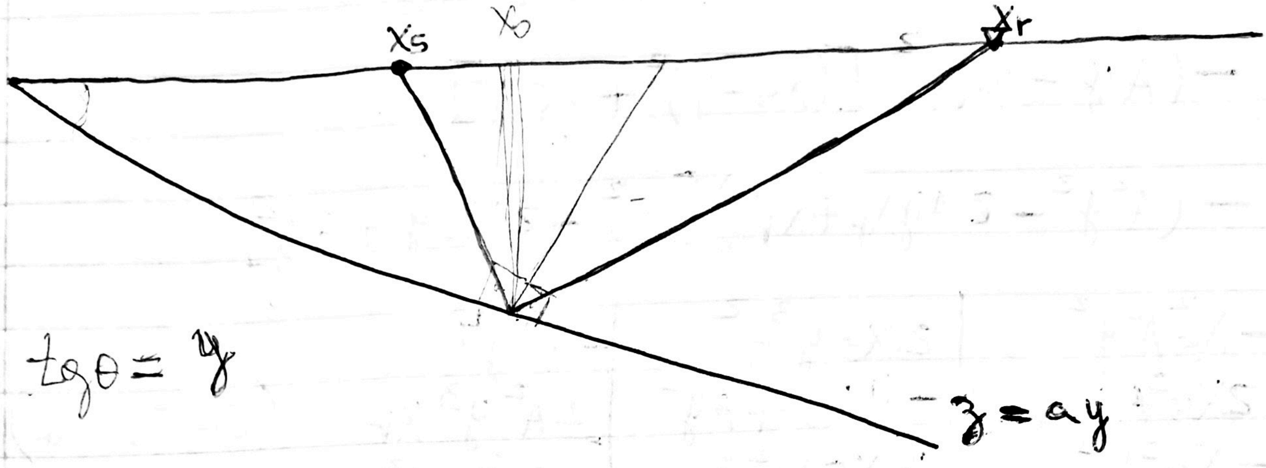
$$= \frac{(x_s x_r\, sen^2\alpha - h^2)^4 + \sqrt{(x_s x_r\, sen^2\alpha - h^2)^2 + h^2 (x_s + x_r)^2\, sen^2\alpha}}{(x_s + x_r)\, tg^2\alpha}$$

GAP ②

*Não sei como passa de (6) para (7), abaixo...

$$y = \frac{h^2 (x_s + x_r) \cos^2\alpha}{h^2 - x_s x_r\, sen^2\alpha + \sqrt{(h^2 + x_s^2\, sen^2\alpha)(h^2 + x_r^2\, sen^2\alpha)}}$$

* Desenvolvimento: Refletor Plano (Esboço)



$$t = \frac{\sqrt{(X_s - y)^2 + z^2(y)} + \sqrt{(X_r - y)^2 + z^2(y)}}{V}$$

$$\frac{\partial t}{\partial y} = 0 = \frac{-2(X_s - y) + 2a^2 y}{2V\sqrt{(X_s - y)^2 + z^2(y)}} + \frac{-2(X_r - y) + 2a^2 y}{2V\sqrt{(X_r - y)^2 + z^2(y)}}$$

$$= \frac{y - X_s + a^2 y}{\sqrt{(X_s - y)^2 + z^2(y)}} + \frac{y - X_r + a^2 y}{\sqrt{(X_r - y)^2 + z^2(y)}}$$

$$0 = (y - X_s + a^2 y)^2\left[(X_r - y)^2 + z^2(y)\right] - (y - X_r + a^2 y)^2\left[(X_s - y)^2 + z^2(y)\right]$$

$$\left(\overbrace{(1 + a^2)}^{A}\, y - X_s\right)^2\left[X_r^2 - 2X_r y + y^2 + a^2 y^2\right]$$

$$(A^2 y^2 - 2AyX_s + X_s^2)\left[X_r^2 - 2X_r y + Ay^2\right]$$

$X_r^2 A^2 y^2$	$-2A^2 y^3 X_r$	$A^3 y^4$
$-2X_r^2 A y X_s$	$+4Ay^2 X_r X_s$	$-2A^2 y^3 X_s$
$X_r^2 X_s^2$	$-2X_s^2 X_r y$	$X_s^2 A y^2$

$$-(y-X_r+a^2y)^2\left[(X_s-y)^2+z^2(y)\right]$$

$$-(Ay-X_r)^2\left[(X_s-y)^2+a^2y^2\right]$$

$$-(A^2y^2-2AyX_r+X_r^2)\left[X_s^2-2X_sy+Ay^2\right]$$

$-X_s^2A^2y^2$	$\cancel{2X_sy^3A^2}$	$\cancel{-A^3y^4}$	(2º Termo)
$\cancel{2X_s^2AyX_r}$	$\cancel{-4X_sX_rAy^2}$	$\cancel{2A^2y^3X_r}$	
$\cancel{-X_r^2X_s^2}$	$2X_sX_r^2y$	$-X_r^2Ay^2$	

$X_r^2A^2y^2$	$\cancel{-2A^2y^3X_r}$	$\cancel{A^3y^4}$	(1º Termo)
$\cancel{-2X_r^2AyX_s}$	$\cancel{+4Ay^2X_rX_s}$	$\cancel{-2A^2y^3X_s}$	
$\cancel{X_r^2X_s^2}$	$\cancel{-2X_s^2X_ry}$	$X_s^2Ay^2$	

$$X_r^2A^2y^2-X_s^2A^2y^2+X_s^2Ay^2-X_r^2Ay^2=0 \qquad \div y \text{ e } \div A$$

$$(X_r^2-X_s^2)Ay+(X_s^2-X_r^2)y=0$$

$$(X_r^2-X_s^2)(1+a^2)+(X_s^2-X_r^2)=0$$

$$(X_r^2-X_s^2)+a^2(X_r^2-X_s^2)+(X_s^2-X_r^2)=0$$

$$-(X_s^2-X_r^2)+a^2(X_s^2-X_r^2)+(X_s^2-X_r^2)=0$$

$$a^2(X_s^2-X_r^2)=0 \qquad \boxed{X_s=X_r}$$