1.2 Eikonal and transport equations; the problem of validity of the ray series

$$u(t,x,y,z) = e^{-i\omega t}\, u(x,y,z)$$

$$\nabla^2 u + u\frac{\omega^2}{c^2} = 0 \qquad u = A(x,y,z)\, e^{i\omega \tau(x,y,z)}$$

$$\frac{\partial u}{\partial x} = \frac{\partial A}{\partial x} e^{i\omega\tau} + \frac{\partial e^{i\omega\tau}}{\partial x} A \qquad \frac{\partial y}{\partial t} = \frac{\partial y}{\partial x}\cdot\frac{\partial x}{\partial t} \quad \boxed{x(t)}$$

$$\frac{\partial u}{\partial x} = \frac{\partial A}{\partial x} e^{i\omega\tau} + e^{i\omega\tau}\frac{\partial \tau}{\partial x} A\, i\omega$$

$$\frac{\partial u}{\partial x} = e^{i\omega\tau}\left(i\omega\frac{\partial\tau}{\partial x}A + \frac{\partial A}{\partial x}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[e^{i\omega\tau}\left(i\omega\frac{\partial\tau}{\partial x}A + \frac{\partial A}{\partial x}\right)\right]$$

$$= i\omega e^{i\omega\tau}\frac{\partial\tau}{\partial x}\left(i\omega\frac{\partial\tau}{\partial x}A + \frac{\partial A}{\partial x}\right) + \left(i\omega\frac{\partial^2\tau}{\partial x^2}A + i\omega\frac{\partial\tau}{\partial x}\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial x^2}\right)e^{i\omega\tau}$$

$$= e^{i\omega\tau}\left\{i\omega\frac{\partial\tau}{\partial x}\left(i\omega\frac{\partial\tau}{\partial x}A + \frac{\partial A}{\partial x}\right) + i\omega\frac{\partial^2\tau}{\partial x^2}A + i\omega\frac{\partial\tau}{\partial x}\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial x^2}\right\}$$

$$= e^{i\omega\tau}\left\{-\omega^2\left(\frac{\partial\tau}{\partial x}\right)^2 A + i\omega\left(2\frac{\partial\tau}{\partial x}\frac{\partial A}{\partial x} + \frac{\partial^2\tau}{\partial x^2}A\right) + \frac{\partial^2 A}{\partial x^2}\right\}$$

$$\nabla \gamma = \frac{\partial \gamma}{\partial x}\hat{i} + \frac{\partial \gamma}{\partial y}\hat{j} + \frac{\partial \gamma}{\partial z}\hat{k}$$

$$\nabla \gamma \cdot \nabla \gamma = \left(\frac{\partial \gamma}{\partial x}\right)^2 + \left(\frac{\partial \gamma}{\partial y}\right)^2 + \left(\frac{\partial \gamma}{\partial z}\right)^2$$

$$\nabla \gamma \cdot \nabla A = \frac{\partial \gamma}{\partial x}\frac{\partial A}{\partial x} + \frac{\partial \gamma}{\partial y}\frac{\partial A}{\partial y} + \frac{\partial \gamma}{\partial z}\frac{\partial A}{\partial z}$$

$$\nabla^2 u + \frac{\omega^2}{c^2}u = \frac{\omega^2}{c^2}u + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\omega^2}{c^2}e^{i\omega\gamma}A + e^{i\omega\gamma}\left\{-\omega^2\left(\frac{\partial \gamma}{\partial x}\right)^2 A + i\omega\left(2\frac{\partial \gamma}{\partial x}\frac{\partial A}{\partial x} + \frac{\partial^2 \gamma}{\partial x^2}A\right) + \frac{\partial^2 A}{\partial x^2}\right\}$$

$$+ e^{i\omega\gamma}\left\{-\omega^2\left(\frac{\partial \gamma}{\partial y}\right)^2 A + i\omega\left(2\frac{\partial \gamma}{\partial y}\frac{\partial A}{\partial y} + \frac{\partial^2 \gamma}{\partial y^2}A\right) + \frac{\partial^2 A}{\partial y^2}\right\}$$

$$+ e^{i\omega\gamma}\left\{-\omega^2\left(\frac{\partial \gamma}{\partial z}\right)^2 A + i\omega\left(2\frac{\partial \gamma}{\partial z}\frac{\partial A}{\partial z} + \frac{\partial^2 \gamma}{\partial z^2}A\right) + \frac{\partial^2 A}{\partial z^2}\right\}$$

$$= e^{i\omega\gamma}\Bigg\{-\omega^2 A\left[\left(\frac{\partial \gamma}{\partial x}\right)^2 + \left(\frac{\partial \gamma}{\partial y}\right)^2 + \left(\frac{\partial \gamma}{\partial z}\right)^2\right]$$

$$+ 2i\omega\left[\left(\frac{\partial \gamma}{\partial x}\frac{\partial A}{\partial x}\right) + \left(\frac{\partial \gamma}{\partial y}\frac{\partial A}{\partial y}\right) + \left(\frac{\partial \gamma}{\partial z}\frac{\partial A}{\partial z}\right)\right]$$

$$+ i\omega A\left[\frac{\partial^2 \gamma}{\partial x^2} + \frac{\partial^2 \gamma}{\partial y^2} + \frac{\partial^2 \gamma}{\partial z^2}\right]$$

$$+ \left[\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}\right]\Bigg\} + \frac{\omega^2}{c^2}A e^{i\omega\gamma}$$

1.2

$$= e^{i\omega\tau}\left\{\frac{\omega^2}{c^2}A + (\nabla\tau)^2\omega^2 A + i\omega\left(2(\nabla\tau\cdot\nabla A) + A\nabla^2\tau\right) + \nabla^2 A\right\} = 0$$

$\div\, \omega^2$

$$\left\{\underbrace{\frac{1}{c^2}A + (\nabla\tau)^2 A}_{} + \underbrace{\frac{i}{\omega}\left(2(\nabla\tau\cdot\nabla A) + A\nabla^2\tau\right)}_{} + \frac{\nabla^2 A}{\omega^2}\right\} = 0$$

$$(\nabla\tau)^2 = \frac{1}{c^2} \qquad 2(\nabla\tau\cdot\nabla A) + A\nabla^2\tau = 0$$

(EIKONAL) (TRANSPORTE)

RAY SERIES

$$u \simeq e^{i\omega\tau}\sum_{n=0}^{\infty}\frac{A_n(x,y,z)}{(-i\omega)^n}$$

$$2(\nabla\tau\cdot\nabla A_{n+1}) + A_{n+1}\nabla^2\tau = \nabla^2 A_n \qquad n = -1, 0, 1 \ldots$$

$$A_{-1} \equiv 0$$

$\boxed{n=-1}$ $2(\nabla\tau\cdot\nabla A_0) + A_0\nabla^2\tau = 0$

Teste da razão

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L \begin{cases} \text{se } 0 \leq L < 1, & \text{converge} \\ \text{se } L > 1, & \text{diverge} \\ \text{se } L = 1, & \text{não sei} \end{cases}$$

$$\left|\frac{A_1}{\omega A_0}\right|$$