Capítulo 1 Popov



$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

(operador laplaciano)

$$u = e^{i\varphi} = \cos\varphi + i\,\text{sen}\,\varphi\,, \quad i^2 = -1$$

$$\varphi = -\omega t + k_1 x + k_2 y + k_3 z$$

$$\frac{\partial u}{\partial t} = e^{i\varphi}\, i\frac{\partial \varphi}{\partial t} = -i\omega u\,; \quad \frac{\partial^2 u}{\partial t^2} = (-i\omega)^2 u\,;$$

$$\frac{\partial u}{\partial x} = i e^{i\varphi}\frac{\partial \varphi}{\partial x} = i k_1 u\,; \quad \frac{\partial^2 u}{\partial x^2} = (ik_1)^2 u\,;$$

$$-\frac{1}{c^2}(-i\omega)^2 u + \left[(ik_1)^2 u + (ik_2)^2 u + (ik_3)^2 u\right] = 0$$

$$u\left[-\frac{(-i\omega)^2}{c^2} + (ik_1)^2 + (ik_2)^2 + (ik_3)^2\right] = 0$$

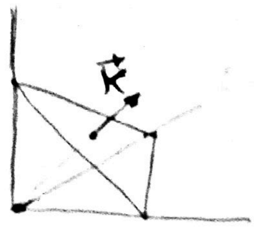
$$-\frac{\omega^2}{c^2} + \left[k_1^2 + k_2^2 + k_3^2\right] = 0$$

$$\frac{\omega^2}{c^2} = k_1^2 + k_2^2 + k_3^2 \qquad |\vec{k}| \equiv \sqrt{k_1^2 + k_2^2 + k_3^2} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{c^2} = |\vec{k}|^2 \rightarrow \frac{\omega}{c} = |\vec{k}| = \frac{2\pi}{\lambda}$$

$u = A e^{i\varphi}$ $\qquad \varphi = -\omega t + \vec{k}\cdot\vec{r} \qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\varphi = 0 \to \vec{k}\cdot\vec{r} = \omega t$ em $t = 0$ $\quad \vec{k}\cdot\vec{r} = 0$



$$|\vec{r}| = \frac{\vec{k}}{|\vec{k}| s}, \quad \vec{k}\cdot\vec{r} = \omega t$$

$$|\vec{k}| s = \omega t$$

$$|\vec{k}| \times \frac{\vec{k}}{|\vec{k}| s} = \omega . t$$

$$|\vec{r}|\,|\vec{k}| s = \vec{k} \Rightarrow \vec{k}\cdot\vec{r} = \frac{\vec{k}}{s}$$

$$|\vec{k}| \frac{ds}{dt} = \omega \to \frac{ds}{dt} = \frac{\omega}{|\vec{k}|} = c$$

$$|\vec{k}|\,|\vec{r}| = \omega t \qquad |\vec{k}| s = \omega t \to |\vec{k}| \frac{ds}{dt} = \omega$$

"Plane wave solutions play a remarkable role in mathematical physics because many types of solutions can be presented as a superposition of plane waves. Obviously, a plane wave solution does not exists if the velocity varies."

"But suppose that the velocity varies slowly. In this case it is natural to seek for a solution for the wave equation in a form of the so-called deformed plane wave $u = A(t, x, y, z)\, e^{i\varphi(t,x,y,z)}$ where the amplitude A is no longer constant, but depends on coordinates, and the phase function $\varphi$ is not a linear function".

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"We arrive at a special analytical form of an approximate solution of the wave equation which is called now the ray series, and we may assert that the ray method is an extension of the plane wave theory to slowly varying media."