Bivariate Trajectory–Undercurrent Theory (BTUT): Simplified Logic

Intuitive and Variable–Defined Edition

1. The Core Idea

BTUT is a scalable form of game theory. Instead of using heavy differential equations, it models interactions between many agents through simple local rules that spread like heat across a network.

Each agent only looks at its neighborhood, decides how to act based on what it sees, and updates smoothly over time. This makes the whole system computable in $\mathcal{O}(\underline{N})$ time — linear with population size.

2. The World: Agents and Network

We have a network of $\underline{N}$ agents (players), represented as a graph $G = (V, E)$. Each agent i has:

- a strategy $\underline{s_i} \in \{A, B\}$ - a degree $\underline{k_i}$ (number of connections) - and a connection weight $\underline{w_i}$ that measures how influential it is.

The weight depends on how "hub-like" the agent is:

$$w_i = \left(\frac{k_i}{k_{\max}}\right)^{\underline{\tau}},$$

where $\underline{\tau}$ (tau) controls how much more important big hubs are. If $\tau = 0$, all agents are equal; higher τ means hubs dominate.

3. Local Payoffs (Mini-Games)

Each connection (edge) between two agents represents one of three basic game types:

- Prisoner's Dilemma (PD)
- Hawk–Dove (HD)
- Stag Hunt (SH)

Each game type g has: - $\underline{u_A^{(g)}}, \underline{u_B^{(g)}}$: base payoffs for players A and B - $\underline{c_A^{(g)}}, \underline{c_B^{(g)}}$: cost coefficients - $\underline{f^{(g)}(p_t)}$: how cooperation strength depends on the global fraction of A-players $\underline{p_t}$

Each agent's expected payoff under game g is:

$$U_i^{(g)} = \frac{1}{2}\left[u_s^{(g)} + (p_t u_A^{(g)} + (1-p_t)u_B^{(g)})\right] f^{(g)}(p_t) - c_s^{(g)} u_s^{(g)}.$$

4. Kernel Coupling: How Influence Spreads

Each agent is connected to others through a kernel K_{ij}, which says how strongly i is influenced by j:

$$K_{ij} = e^{-\underline{\lambda} d(i,j)}.$$

Here: - $\underline{d(i,j)}$: distance between agents - $\underline{\lambda}$: decay rate (how quickly influence fades)

If two agents are close, K_{ij} is near 1 (strong influence). If far apart, K_{ij} is near 0.

5. Edge Accumulation (Information Gathering)

Each agent sums up what it feels from its neighbors:

$$\Phi_i = \sum_{j \in \mathcal{N}(i)} K_{ij} \log(1 + \underline{\eta} U_j),$$

where: - $\underline{\Phi_i}$: accumulated influence at node i - $\underline{\eta}$: scaling factor controlling how much of a neighbor's payoff is felt - $\log(1 + \eta U_j)$: smooths rapid changes so big swings don't explode

This is called the *log-intensity kernel* — it converts complex feedback into stable local information.

6. Decision and Update

Every agent compares two global utilities:

$$\tilde{U}_A = \sum_i w_i k_i U_i(A, p_t), \qquad \tilde{U}_B = \sum_i w_i k_i U_i(B, p_t).$$

Then, the fraction of A-players updates as:

$$p_{t+1} = \frac{1}{2} p_t + \frac{1}{2}\mathbb{E}[\tilde{U}_A > \tilde{U}_B].$$

This means:

- Take half of your current belief (p_t),
- Add half of the new observed tendency (how often A performs better than B).

It's a momentum-like update — slow, smooth, and stable.

7. Hidden Dynamics (Trajectory and Undercurrent)

To describe slow invisible shifts, BTUT tracks two latent variables:

$$\theta_t \text{ (trajectory)}, \qquad \psi_t \text{ (undercurrent)}.$$

They evolve as:

$$\theta_{t+1} = \theta_t + \kappa_\theta (p_{t+1} - p_t),$$
$$\psi_{t+1} = \psi_t + \kappa_\psi \log\left(\frac{\tilde{U}_A}{\tilde{U}_B}\right).$$

8. Equilibrium and Interpretation

Eventually, the system settles to a steady state:

$$p_\star = \lim_{t \to \infty} p_t.$$

We know:

$$\frac{\partial p_\star}{\partial \gamma} > 0 \quad \text{and} \quad \frac{\partial p_\star}{\partial c_A^{SH}} < 0.$$

Cooperation grows when rewards rise (γ) and falls when costs rise (c_A^{SH}).

9. Why It Scales

Because every term (K_{ij}, Φ_i, U_i) depends only on local neighbors and additive kernels, the total computation grows linearly with the number of agents:

$$\text{Complexity: } \mathcal{O}(N).$$

No exponential explosion — that's what makes BTUT a practical solution to "game theory that scales."