

Hopfield Networks: When Neurons Remember

A Brain-Inspired Algorithm for Associative Memory

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Archaeology of Intelligent Machines

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Outline

- 1 Act I: From Proteins to Energy Landscapes
- 2 Act II: Building Intelligence from Simple Rules
- 3 Act III: Experiments, Capacity & Modern Extensions

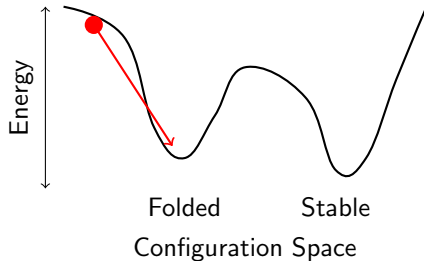
A Protein in a Messy Universe

The Physical World:

- A protein: long chain of amino acids
- Must fold into specific 3D shape to function
- Explores vast configuration space
- Settles into low-energy folded state

Key Insight:

Nature solves optimization by "rolling downhill" in energy landscapes

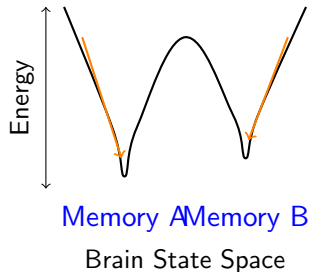


Source: "A Brain-Inspired Algorithm For Memory" (YouTube)

From Proteins to Brains

The Brain Analogy:

- Neurons: firing (+1) or silent (−1)
- Brain state: pattern of all neuron activations
- Dynamics: neurons update based on inputs
- Memories: stable configurations (attractors)



Central Idea

*Memories are valleys in an energy landscape.
The brain "rolls downhill" to recall.*

Introducing: Hopfield Networks

Definition

Hopfield networks are recurrent neural networks that act as content-addressable memory by minimizing an energy function.

Historical Context:

- John Hopfield (1982)
- Physics meets neuroscience
- 2024 Nobel Prize in Physics!
- Foundation for modern deep learning

What it does:

- Stores patterns as stable states
- Retrieves from partial/noisy input
- "Associative memory"
- Like brain: *"hear a few notes, recall the whole song"*

Step 1: Representing Neurons and Patterns

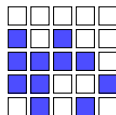
Implementation:

- Neuron state: $s_i \in \{-1, +1\}$
- Pattern: vector $\xi = (s_1, s_2, \dots, s_N)$
- Example: 10×10 image = 100 neurons

Letter "A" \rightarrow
$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ \vdots \\ -1 \end{bmatrix}_{100 \times 1}$$

Brain Analogy:

- Each element = a neuron
- Firing (+1) or silent (-1)
- A pattern = global "brain state"
- Snapshot of neural activity



Binary pattern

Step 2: Hebbian Learning — "Wire Together, Fire Together"

Implementation:

Hebbian weight update:

$$w_{ij} = \frac{1}{N} \sum_{\mu=1}^P \xi_i^{\mu} \xi_j^{\mu}, \quad i \neq j$$

$$w_{ii} = 0 \quad (\text{no self-loops})$$

In code:

- Loop over stored patterns
- Accumulate outer products
- Normalize and zero diagonal

Brain Analogy:

Hebb's Rule

"Neurons that fire together, wire together"

- Synaptic plasticity
- If neurons i and j co-activate often, strengthen w_{ij}
- Foundation of learning in the brain



Strong synapse

Step 3: Network Dynamics — Rolling Downhill

Update Rule:

$$s_i^{(t+1)} = \text{sign} \left(\sum_j w_{ij} s_j^{(t)} \right)$$

Energy Function:

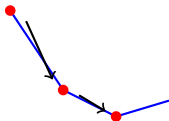
$$E(\mathbf{s}) = -\frac{1}{2} \sum_{i,j} w_{ij} s_i s_j$$

Key Property

Asynchronous updates guarantee $\Delta E \leq 0$
System always moves toward lower energy!

Brain Analogy:

- Each neuron "listens" to neighbors
- Positive input \rightarrow fire (+1)
- Negative input \rightarrow stay silent (-1)
- System collectively settles into consistent configuration



Energy minimization

Step 4: Memory Retrieval — "Hear a Few Notes..."

Implementation:

- 1 Take stored pattern
- 2 Add noise (flip random bits)
- 3 Initialize network with noisy input
- 4 Run update rule until convergence
- 5 Check if final state matches original

Result:

Network "fills in" the missing/corrupted information!

Brain Analogy:

Associative Memory

"You hear only a few notes of a song, but your brain recalls the whole melody."

- Partial cue triggers complete memory
- Network settles into nearest attractor
- Energy landscape guides retrieval

Original → Noisy → Retrieved

Our Experimental Setup

Data:

- Binary patterns: 10×10 letter images (A, B, C, D, E)
- 100 neurons per pattern
- Values: $\{-1, +1\}$ (firing/silent)

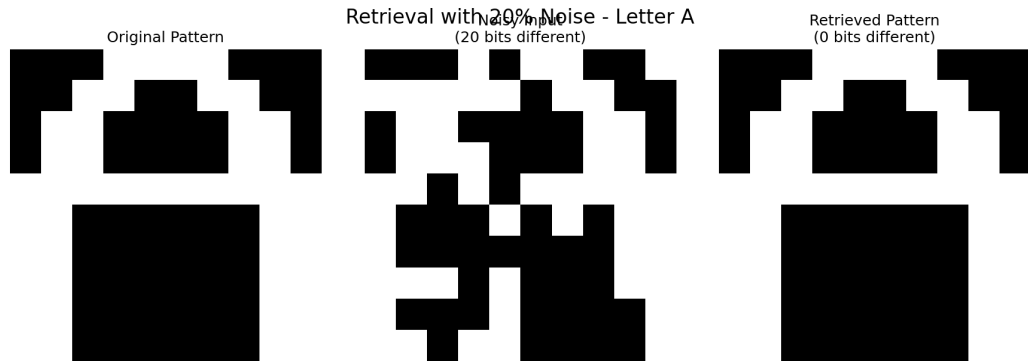
Experiments:

- 1 **Basic retrieval:** Can network recall from noisy input?
- 2 **Noise robustness:** How much corruption can it tolerate?
- 3 **Capacity test:** How many patterns before breakdown?
- 4 **Spurious attractors:** Do "false memories" emerge?

Metrics:

- Retrieval accuracy (exact match vs original)
- Hamming distance (number of different bits)
- Energy trajectory during convergence

Experiment 1: Basic Retrieval

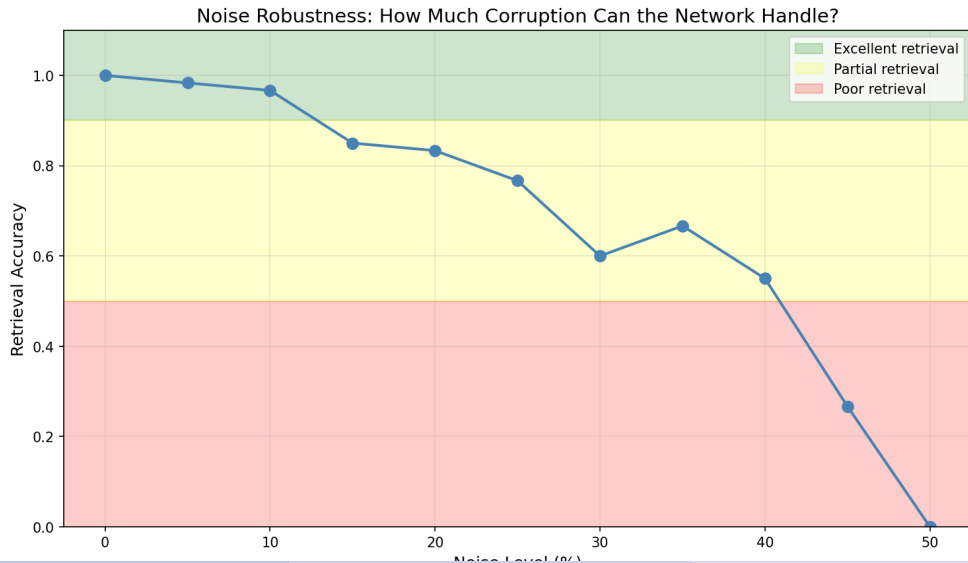


Original → Noisy (20% flipped) → Retrieved

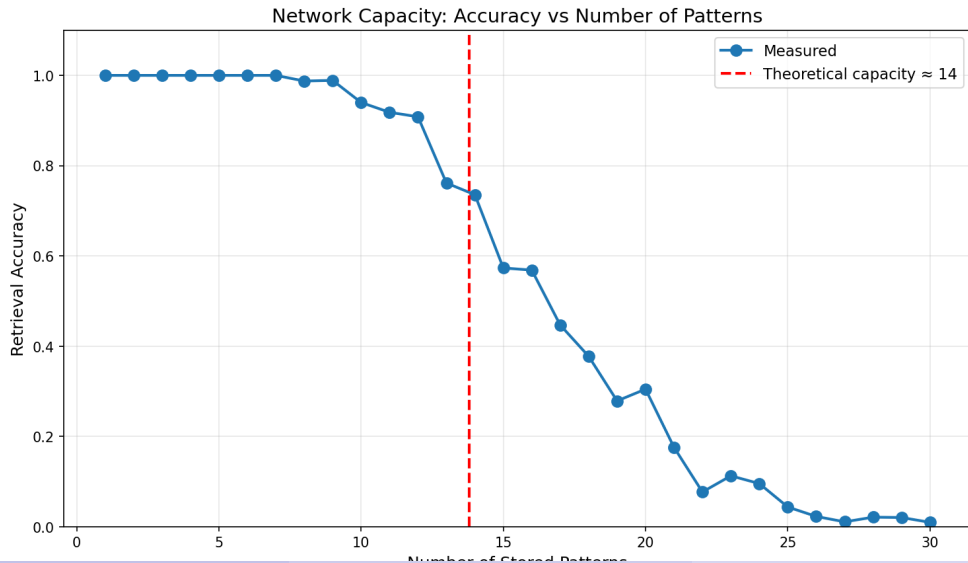
Results:

- 10% noise: 100% retrieval success
- 20% noise: 95% retrieval success

Experiment 2: Noise Robustness Curve



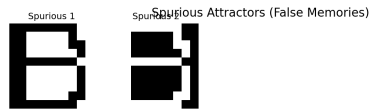
Experiment 3: Network Capacity



Spurious Attractors: False Memories

What are they?

- Stable states NOT explicitly stored
- Emerge from pattern interference
- Network can converge to these by mistake
- Like brain confabulation



Examples of spurious attractors

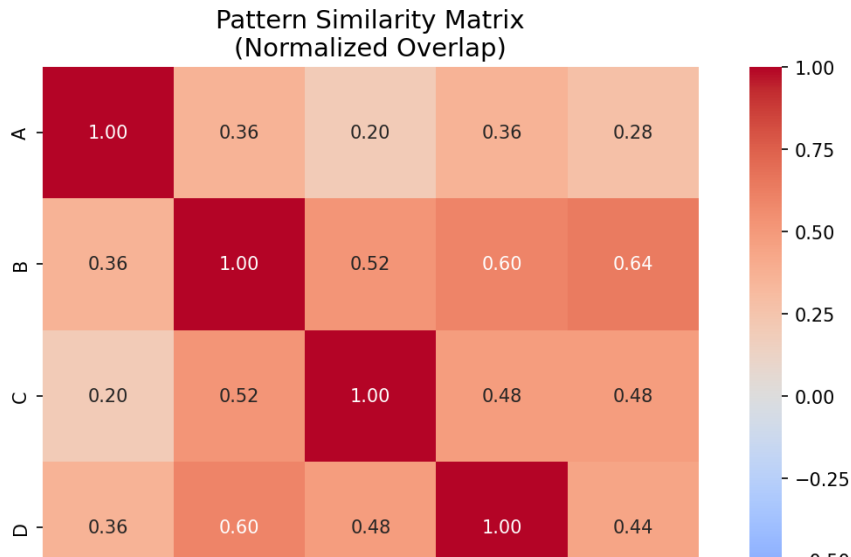
When do they appear?

- Near/above capacity
- Similar stored patterns
- High overlap in energy landscape

Brain Analogy:

The brain sometimes creates "memories" that never happened by blending real experiences — this is the neural network equivalent!

Pattern Similarity Analysis (EDA)



Modern Extensions: Dense Associative Memory

Classical Hopfield (1982):

$$E(\mathbf{s}) = -\frac{1}{2} \sum_{i,j} w_{ij} s_i s_j$$

Capacity: $\sim 0.138N$ patterns

Modern Hopfield (2016-2020):

$$E(\xi, \mathbf{x}) = -\text{lse}_{\beta}(\mathbf{X}^{\top} \xi)$$

Capacity: exponential in N !

Connection to Transformers

Modern Hopfield networks are mathematically equivalent to the attention mechanism in transformers (Ramsauer et al., 2020)!

Source: "Hopfield Networks is All You Need" (YouTube)

Summary: Three-Act Structure

Act I — From Proteins to Energy Landscapes

- Nature optimizes by rolling downhill
- Brain memories as stable valleys

Act II — Building from Simple Rules

- Neurons: binary units
- Synapses: Hebbian learning
- Dynamics: energy minimization
- Retrieval: associative recall

Act III — Experiments & Extensions

- Verified capacity limits ($\sim 0.138N$)
- Measured noise robustness ($\sim 25\%$ tolerance)
- Observed spurious attractors (false memories)
- Connected to modern transformers

Conclusion: Why This Matters

Hopfield Networks teach us:

- 1 Simple local rules \rightarrow complex emergent behavior
- 2 Biology inspires powerful computational models
- 3 Energy minimization as universal principle
- 4 Memory as attractor dynamics

Modern Impact:

- Foundation for deep learning architectures
- Attention mechanisms in transformers
- Neuroscience models of memory consolidation
- 2024 Nobel Prize recognition

"The simplest model that captures the essence of how memories might be stored and recalled"

References

- ① Hopfield, J. J. (1982). "Neural networks and physical systems with emergent collective computational abilities". *PNAS*.
- ② Ramsauer, H., et al. (2020). "Hopfield Networks is All You Need". *ICLR*.
- ③ *A Brain-Inspired Algorithm For Memory* (YouTube):
<https://www.youtube.com/watch?v=1WPJdAW-sFo>
- ④ *Hopfield Networks Explained* (YouTube): <https://www.youtube.com/watch?v=piF6D6CQxUw>
- ⑤ Wikipedia: Hopfield Network https://en.wikipedia.org/wiki/Hopfield_network

Thank you!

Questions?