DIFFERENTIAL TOPOLOGY
WINTER 2026
HOMEWORK #3

Due on Gradescope anytime before 10 p.m. EDT on Wednesday, January 28.

Problem 1. Consider the map $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (e^x \cos y, e^x \sin y)$.

(a) What is the image of f?
(b) Use the Inverse Function Theorem to show that every point in $\mathbb{R}^2$ has a neighborhood U such that $f|_U$ is injective.
(c) Show that f is not injective.
(d) Find a local inverse for f near $f(0, \pi/3)$, compute its derivative, and compare your result with what the Inverse Function Theorem tells you.

Problem 2. Consider the map $f\colon \mathbb{R}P^2 \to \mathbb{R}^3$ defined by $f([x\colon y\colon z]) = (yz, xz, xy)$. Show that f is an immersion except at six points. (Here we view $[x\colon y\colon z]$ as the equivalence class of antipodal points in S^2.)

Problem 3. Lee, Problem 4-5.

Problem 4. Lee, Problem 4-6.

Problem 5. Lee, Problem 5-10.

Problem 6. Lee, Problem 5-15.

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