THAYER SCHOOL OF ENGINEERING • DARTMOUTH

ENGS 53

HOMEWORK #2

January 19, 2026

Contents

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1 About Homework #2

The purpose of HW#2 is to get comfortable with the concepts of quantum mechanics and to practice the mathematical tools we will use throughout the course. This assignment will primarily be on a position basis, but we will be fully embracing Dirac notation in subsequent assignments.

2 Problems - Due Monday, January 26 at 9 am

PROBLEM 1: *Macroscopic Effects of Quantum Mechanics*

A 1.00 g marble is constrained to roll inside a tube of length $L = 1.00$ cm. The tube is capped at both ends.

a Modelling this as a one-dimensional infinite square well, determine the value of the quantum number n if the marble is initially given an energy of 1.00 mJ. **[5pts]**

b Calculate the excitation energy required to promote the marble to the next available energy state. **[5pts]**

PROBLEM 2: *Quantum Tunneling*

Consider the potential

$$V(x) = \begin{cases} 0, & \text{if } |x| > a; \\ V_0, & \text{if } |x| < a. \end{cases} \quad (1)$$

and suppose that particles of fixed energy $E < V_0$ strike it coming from $x = -\infty$.

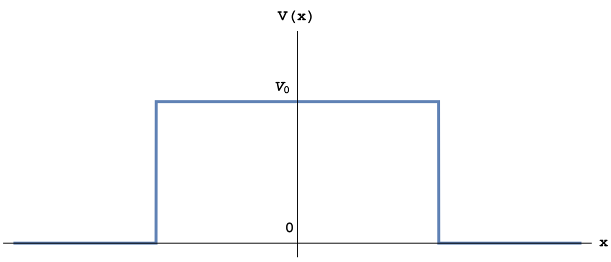

Figure 1: Potential Barrier.

a Solve the Schrödinger equation in each of the three regions $x < -a$, $-a < x < a$, and $x > a$. **[10pts]**

b What are the boundary conditions to ensure that the solution for the wave function is continuous? **[4pts]**

c What are the boundary conditions to ensure that the solution for the derivative of the wave function is continuous? **[4pts]**

d What is the transmission coefficient? **[10pts]**

PROBLEM 3 *Heisenberg Uncertainty: Particle in a Box*

Last homework, you solved the so-called 'particle in a box' problem which gives you solutions to the problem of a particle of mass m is confined to a one-dimensional box of length L. You should have found that the wave function of the particle is given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \tag{2}$$

where $n = 1, 2, 3, \ldots$.

Use the solution for the particle in a box to answer the following questions:

a Calculate $\langle x \rangle$. **[4pts]**

b Calculate $\langle p \rangle$. **[4pts]**

c Calculate $\langle x^2 \rangle$. **[4pts]**

d Calculate $\langle p^2 \rangle$. **[4pts]**

e Calculate Δx. **[4pts]**

f Calculate Δp. **[4pts]**

e In class, we showed that the Heisenberg Uncertainty Principle states that

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Calculate $\Delta x \Delta p$. How close does this get to the limit of $\frac{\hbar}{2}$. How does it scale with n? **[4pts]**

PROBLEM 4 *Heisenberg Uncertainty: Harmonic Oscillator*

The Hamiltonian for the Quantum Harmonic Oscillator is given by

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$

where $\hat{p}$ is the momentum operator, $\hat{x}$ is the position operator, m is the mass of the particle, and ω is the angular frequency of the oscillator.

The general solution to the Schrödinger equation for the quantum harmonic oscillator is given by

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left(-\frac{m\omega^2}{2\hbar} x^2\right) H_n\left(\sqrt{\frac{m\omega}{\hbar}} x\right), \tag{3}$$

where H_n are Hermite polynomials which are given by

$$H_n(z) = (-1)^n e^{z^2} \frac{d^n}{dz^n}\left(e^{-z^2}\right) \tag{4}$$

For simplicity, we will consider the ground state of the quantum harmonic oscillator that we can express as

$$\psi_0(x) = C_0 e^{-\alpha x^2} \tag{5}$$

where

$$C_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}, \quad \alpha = \frac{m\omega}{2\hbar}, \quad E = \frac{1}{2}\hbar\omega$$

$\psi_0(x)$ is the solution of the Schrödinger Eq.

Use this solution for the ground state of the quantum harmonic oscillator to answer the following questions:

a Calculate $\langle x \rangle$. **[4pts]**

b Calculate $\langle p \rangle$. **[4pts]**

c Calculate $\langle x^2 \rangle$. **[4pts]**

d Calculate $\langle p^2 \rangle$. **[4pts]**

e Calculate Δx. **[4pts]**

f Calculate Δp. **[4pts]**

e In class, we showed that the Heisenberg Uncertainty Principle states that

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Calculate $\Delta x \Delta p$. How close does this get to the limit of $\frac{\hbar}{2}$. **[4pts]**