under Graduate Homework In Mathematics

AlgebraicGeometry 5

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2023 年 10 月 20 日

PROBLEM I If $f : V \to W, g : W \to U$ is two poly maps, then $(g \circ f)^* = f^* \circ g^*$.

SOLUTION. For $u \in k[U]$, we have $(g \circ f)^* u = u \circ g \circ f = g^*(u) \circ f = f^*(g^*(u)) = (f^* \circ g^*)(u)$, so $(g \circ f)^* = (f^* \circ g^*)$. □

PROBLEM II $\mathcal{O}_{V,p}$ is local ring.

SOLUTION. To prove $\mathcal{O}_{V,p}$ is local ring, we only need to prove $\forall f \in \mathcal{O}_{V,p}$, one of f and $1 - f$ is unit.

First we prove f is unit iff $f(p) \neq 0$. If f is unit then exists $g \in \mathcal{O}_{V,p}$ s.t. $fg = 1$, so $f(p)g(p) = 1$. Then we get $f(p) \neq 0$. If $f(p) \neq 0$, then we assume $f = \frac{g}{h}, h(p) \neq 0$. Since $f(p) \neq 0$ we get $g(p) \neq 0$, so $\frac{h}{g} \in \mathcal{O}_{V,p}$, then f is a unit.

Now we prove f or $1 - f$ is a unit. Obviously $f(p) \neq 0$ or $1 - f(p) \neq 0$, so one of them is unit. □