underGraduate Homework In Mathematics

AlgebraicGeometry 4

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General fire extinguisher

PROBLEM I Prove that $\mathbb{C}$ and $\mathbb{C}\setminus\{0\}$ are Homeomorphic by Zariski topology.

SOLUTION. Since $\mathrm{card}\mathbb{C} = \mathrm{card}\mathbb{C}\setminus\{0\}$, there is a bijection $f : \mathbb{C} \to \mathbb{C}\setminus\{0\}$. Then we prove f is homeomorphism.

First, we prove a set is closed iff it's finite or it's Universe. For a finite set A, obviously $V((\prod_{t\in A}(x-t))) = A$ is closed. For closed set $A = V(I), I \neq (0)$, consider $f \in I\setminus\{0\}$, we know $\forall a \in A, f(a) = 0$, so A is finite.

Since f is bijection, we know f preserve the cardinality, so preserve finite set. And $f(\mathbb{C}) = \mathbb{C}\setminus\{0\}$, so f preserve closed set. For the same reason f^{-1} preserve closed set. So f is homeomorphism.

$\square$

PROBLEM II Let $D_f := \{h \in \mathcal{O}_v(V)\}$