under Graduate Homework In Mathematics

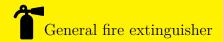
AlgebraicGeometry 13

白永乐

202011150087

202011150087@mail.bnu.edu.cn

2023年12月16日



 \mathbb{R}^{Ω} BEM I Assume $\Omega \subset \mathbb{C}$ is a domain. Prove that f is meromorphic map over Ω is equiv for Ω as opensubset of \mathbb{C} and as Remian surface.

SOUTION. First we assume $f:\Omega\to\mathbb{C}_\infty$ is holomorphic. Let $T:=\{x\in\Omega:f(x)=\infty\}$. Now we prove f is meromorphic over Ω . Since $\forall x\in T, f(x)=\infty$ and f is continous, we get $\forall x\in T, \lim_{y\to x}f(y)=\infty$. Now we only need to prove every $x\in T$ is isolated point. If not, we will prove $f\equiv\infty$. Let $V:=\{x\in\Omega:f(x)=\infty\wedge\exists x_n\in\Omega, x_n\neq x, x_n\to x, f(x_n)=\infty\}\neq\varnothing$. Easily V is closed in Ω , now we prove it's open, too. Assume $x\in V$, and $x_n\in\Omega, x_n\neq x, x_n\to x, f(x_n)=\infty$. Since f is holomorphic, we get $\exists V:\infty\in V\subset\mathbb{C}_\infty$ is open, $\exists U:x\in U\subset\Omega$ is open, such that $g:=\phi\circ f|_U\circ\mathrm{id}:U\to\mathbb{C}$ is holomorphic, where $\phi(x)=\frac{1}{x}$. Then $g(x)=g(x_n)=0$. So $g|_U\equiv0$. So we get $f|_U\equiv0$. So V is open in V. Since V is connected, we get V is V. So V is open in V. Since V is connected, we get V is V.

Second we assume $f:W\to\mathbb{C}$ is holomorphic, where $W\subset\Omega$ is open, and $\forall x\in T:=\Omega\setminus W, \lim_{y\to x}f(y)=\infty$, and x is isolated point. Now let $h:\Omega\to\mathbb{C}_\infty, h|_W=f, h|_T\equiv\infty$. We only need to prove h is holomorphic from Ω (as Remian surface) to \mathbb{C}_∞ . Only need to prove h is holomorphic on $x\in T$. Let $\phi:\mathbb{C}\setminus\{0\}\to\mathbb{C}, x\mapsto\frac{1}{x}$. Let $U:=h^{-1}(\mathbb{C}\setminus\{0\})\subset\Omega$ is a neibor of x. Now we prove $g:=\phi\circ h\circ \mathrm{id}:U\to\mathbb{C}$ is holomorphic. Easily g is well-defined since $0\notin h(U)$. And f is meromorphic on Ω , so g is holomorphic.