Graduate Homework In Mathematics

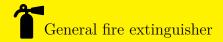
GroupRepresentation 4

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ROBEM I Find all of 1-dimentional complex representation of the alternating group A_4 . ROBEM II Consider $N \subseteq S_4$ and $N = \{(1), (12)(34), (13)(24), (14)(23)\}$.

- 1. Prove: $S_4/N \cong S_3$.
- 2. Find a 2-dimentional irreducible complex matrix representation of S_4 .

ROBEM III Assuem K is a field and $m \in \mathbb{N}^*$. Let $\phi_m(t) := t^m, \forall t \in K^*$, then ϕ_m is a 1-dimentional K-representation of (K^*, \cdot) . Use ϕ_m to find a 1-dimentional K- representation of $\mathrm{GL}_n(K)$. ROBEM IV Prove that if ϕ is 1-dimentional complex representation of finite group G, then $G/\ker\phi$ is a cyclic group.

 \mathbb{R}^{O} BEM V Prove: If G is a non-cyclic finite group, then there is no faithful 1—dimentional complex representation of G.

ROBEM VI Assume (ϕ, V) and (ψ, W) are two K-representation of group G. Prove: $(\phi \dot{+} \psi) \approx \phi^* \dot{+} \psi^*$.