GroupRepresentation3

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PROBLEM I Let ϕ is representation of $\mathrm{GL}_n(K)$ over K^n. And $\phi(A)\alpha := A\alpha$. Prove:ϕ is faithful and irreducible and $n-$dimentional.

PROBLEM II For $A \in \mathrm{GL}_n(K)$, let $\psi(A)X = AX, \forall X \in M_n(K)$. Then:

1. ψ is n^2-dimentional representation of $\mathrm{GL}_n(K)$ over K.
2. For $j : 1 \leq j \leq n$, let $M_n^{(j)}(K) := \{(a_{ik})_{n\times n} : a_{ik} \neq 0 \to k = j\}$. Prove $M_n^{(j)}$ is invariant subspace of $\mathrm{GL}_n(K)$. Let ψ is subrepresentation of ψ in $M_n^{(j)}$, prove ψ_j is irreducible and $\psi = \bigoplus_{j=1}^n \psi_j$.
3. Prove $\psi_j \cong \phi$, where $\phi =$ (PROBLEM $I).\phi$

PROBLEM III Let $K = \mathbb{C}$ and $n = 2$ in (Group representation second homework).(Problem 3), prove the subrepresentation of ϕ over $M_2^n(\mathbb{C})$ is irreducible.

PROBLEM IV Assume $n \geq 3$ and $n \nmid \operatorname{char} K$