## In Mathematics

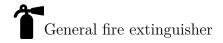
**GroupRepresentation 3** 

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ROBEM I Let  $\phi$  is representation of  $GL_n(K)$  over  $K^n$ . And  $\phi(A)\alpha := A\alpha$ . Prove: $\phi$  is faithful and irreducible and n-dimensional.

SOLITION. Obviously it's n-dimentional. If  $A \neq B$ , then exists  $\alpha \in K^n$  s.t.  $(A - B)\alpha \neq 0$ . So  $\phi(A)\alpha \neq \phi(B)\alpha$ . So  $\phi(A) \neq \phi(B)$ , so  $\phi$  is faithful. To prove  $\phi$  is irreducible, we only need to prove there is no invariant subspace of  $K^n$ . Obviously for  $\alpha, \beta \in K^n \setminus \{0\}$ , obviously there exists  $A \in GL_n(K)$  such that  $A\alpha = \beta$ . So there is no nontrival invariant subspace of  $K^n$ . So it's irreducible.

ROBEM II For  $A \in GL_n(K)$ , let  $\psi(A)X = AX, \forall X \in M_n(K)$ . Then:

- 1.  $\psi$  is  $n^2$ -dimentional representation of  $GL_n(K)$  over K.
- 2. For  $j: 1 \leq j \leq n$ , let  $M_n^{(j)}(K) := \{(a_{ik})_{n \times n} : a_{ik} \neq 0 \to k = j\}$ . Prove  $M_n^{(j)}$  is invariant subspace of  $GL_n(K)$ . Let  $\psi$  is subrepresentation of  $\psi$  in  $M_n^{(j)}$ , prove  $\psi_j$  is irreducible and  $\psi = \bigoplus_{j=1}^n \psi_j$ .
- 3. Prove  $\psi_i \cong \phi$ , where  $\phi = (??).\phi$

SOUTION. 1.

ROBEM III Let  $K = \mathbb{C}$  and n = 2 in (Group representation second homework).(Problem 3), prove the subrepresentation of  $\phi$  over  $M_2^n(\mathbb{C})$  is irreducible.

ROBEM IV Assume  $n \geq 3$  and  $n \nmid \text{char } K$ , proof: then then n- dimentional permutate representation of  $S_n$  can be decomposed as the direct sum of a main representation and a n-1- dimentional irreducible subrepresentation

 $\mathbb{R}^{OBEM}$  V Caculate the 1- dimentional  $\mathbb{C}$  representation:

- 1. (2,4)—type of 8— order elementary Abel group.
- 2. the addition group of  $\mathbb{Z}_p^n$