under Graduate Homework In Mathematics
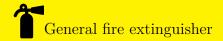
Set Theory 4

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2023 年 10 月 30 日

General fire extinguisher

PROBLEM I Consider $\mathbb{Q} = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})/\sim$, where $(a,b) \sim (c,d) \iff ad = bc$. Define $+_\mathbb{Q}, \cdot_\mathbb{Q}$ and $<_\mathbb{Q}$ and verify that your definitions doesn't depend on the choice of representatives.

SOLUTION. Let $[(a,b)] +_\mathbb{Q} [(c,d)] = [(ad+bc,bd)], [(a,b)] \cdot_\mathbb{Q} [(c,d)] = [(ac,bd)]$, and $[(a,b)] <_\mathbb{Q} [(c,d)] \iff abd^2 < cdb^2$. Now we prove they are well-defined, i.e., doesn't depend on the choice of representatives.

For $+_\mathbb{Q}$, assume $(a,b) \sim (e,f)$, we need to prove $(ad+bc,bd) \sim (ed+fc,df)$. Since $af = be$, we have $(ad+bc)bf = ad^2f + bdcf = bed^2 + bdcf = (ed+fc)bd$. So $+_\mathbb{Q}$ is well defined. $\sqrt{w}$ mod $sw \deg f$

$\square$