## under Graduate Homework In Mathematics

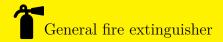
AlgebraicGeometry 5

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ROBEM I If  $f: V \to W, g: W \to U$  is two poly maps, then  $(g \circ f)^* = f^* \circ g^*$ .

SOLION. For  $u \in k[U]$ , we have  $(g \circ f)^*u = u \circ g \circ f = g^*(u) \circ f = f^*(g^*(u)) = (f^* \circ g^*)(u)$ , so  $(g \circ f)^* = (f^* \circ g^*)$ .

## $\mathbb{R}^{OBEM}$ II $\mathcal{O}_{V,p}$ is local ring.

SOLION. To prove  $\mathcal{O}_{V,p}$  is local ring, we only need to prove  $\forall f \in \mathcal{O}_{V,p}$ , one of f and 1-f is unit. First we prove f is unit iff  $f(p) \neq 0$ . If f is unit then exists  $g \in \mathcal{O}_{V,p}$  s.t. fg = 1, so f(p)g(p) = 1. Then we get  $f(p) \neq 0$ . If  $f(p) \neq 0$ , then we assume  $f = \frac{g}{h}, h(p) \neq 0$ . Since  $f(p) \neq 0$  we get  $g(p) \neq 0$ , so  $\frac{h}{g} \in \mathcal{O}_{V,p}$ , then f is a unit.

Now we prove f or 1-f is a unit. Obviously  $f(p) \neq 0$  or  $1-f(p) \neq 0$ , so one of them is unit.

## ROBEM III Prove: $\{V_h : h \in k[V]\}$ is topological basis of V.

SPETION. Only need to prove for any open set  $U \subset V$ , we can find a subclass of  $\{V_h : h \in k[V]\}$  such that U is union of the class. Obviously from ROBEM IV we get exists a finite subclass satisfy the requirement, it's even stronger!

## ROBEM IV Prove: For open set $U \subset V, \exists h_1, h_2, \cdots h_n$ s.t. $U = \bigcup_{k=1}^n V_{h_k}$ .

SOLITION. Since U is open set in V, so  $\exists I$  is ideal in  $k[x_1, \dots x_n]$  such that  $\forall I = U^c$ . Since  $k[x_1, \dots x_n]$  is Noetherian, we obtain  $\exists f_1, \dots f_k, I = (f_1, \dots f_n)$ . Let  $h_k := f_k|_V$ , then  $U^c = \{p \in V : \forall k, h_k(p) = 0\}$ , so  $U^c = \cap_{k=1}^n (V_{h_k}^c)$ , i.e.,  $U = \bigcup_{k=1}^n V_{h_k}$ .