underGraduate Homework In Mathematics

AlgebraicGeometry 4

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General fire extinguisher

PROBLEM I Prove that $\mathbb{C}$ and $\mathbb{C}\setminus\{0\}$ are Homeomorphic by Zariski topology.

SOLUTION. Since $\text{card}\mathbb{C} = \text{card}\mathbb{C}\setminus\{0\}$, there is a bijection $f:\mathbb{C}\to\mathbb{C}\setminus\{0\}$. Then we prove f is homeomorphism.

First, we prove a set is closed iff it's finite or it's Universe. For a finite set A, obviously $\mathbb{V}((\prod_{t\in A}(x-t))) = A$ is closed. For closed set $A=\mathbb{V}(I), I\neq(0)$, consider $f\in I\setminus\{0\}$, we know $\forall a\in A, f(a)=0$, so A is finite.

Since f is bijection, we know f preserve the cardinality, so preserve finite set. And $f(\mathbb{C})=\mathbb{C}\setminus\{0\}$, so f preserve closed set. For the same reason f^{-1} preserve closed set. So f is homeomorphism. $\square$

PROBLEM II Assume V is irreducible algebric set and f is a fraction function on V. Let $D_f := \{h\in\mathcal{O}_v(V) : \exists g\in\mathcal{O}_v(V)\ s.t.\ f=\frac{g}{h}\}\cup\{0\}$. Prove that D_f is ideal of $\mathcal{O}_v(V)$.

SOLUTION. For $h_1,h_2\in D_f$, if $h_1=h_2$ then $h_1-h_2=0\in D_f$. Else, assume $f=\frac{g_1}{h_1}=\frac{g_2}{h_2}$, so $f=\frac{g_1-g_2}{h_1-h_2}$. So $h_1-h_2\in D_f$.

For $h\in D_f$ and $j\in\mathcal{O}_v(V)$, if $hj=0$ then $hj\in D_f$. Else we get $hj\neq 0$, then $f=\frac{g}{h}=\frac{gj}{hj}$. So $hj\in D_f$.

Above all, we get D_f is a ideal. $\square$