Graduate Homework In Mathematics

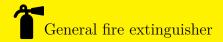
AlgebraicGeometry 4

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\mathbb{R}^{OBEM} I Prove that \mathbb{C} and $\mathbb{C} \setminus \{0\}$ are Homeomorphic by Zariski topology.

SOLION. Since $\operatorname{card}\mathbb{C} \setminus \{0\}$, there is a bijection $f : \mathbb{C} \to \mathbb{C} \setminus \{0\}$. Then we prove f is homeomorphism.

First, we prove a set is closed iff it's finite or it's Universe. For a finite set A, obviously $V((\prod_{t\in A}(x-t)))=A$ is closed. For closed set $A=V(I), I\neq (0)$, consider $f\in I\setminus \{0\}$, we know $\forall a\in A, f(a)=0$, so A is finite.

Since f is bijection, we know f preserve the cardinality, so preserve finite set. And $f(\mathbb{C}) = \mathbb{C} \setminus \{0\}$, so f preserve closed set. For the same reason f^{-1} preserve closed set. So f is homeomorphism.

ROBEM II Let $D_f := \{h \in \mathcal{O}_v(V)\}$