PROBLEM I Solve this equation: $1215x \equiv 560 \mod 2755$.

SOLUTION . Easily $1215x \equiv 560 \mod 2755 \iff 243x \equiv 112 \mod 551$. Obviously $x = 200$ is a solution, and $\gcd(243, 551) = 1$, so all the solutions are $x = 200 + 551t, t \in \mathbb{Z}$. $\square$

PROBLEM II Find the solution of $\begin{cases} x + 4y - 29 \equiv 0 \mod 143 \\ 2x - 9y + 84 \equiv 0 \mod 143 \end{cases}$

SOLUTION . Double the first equation then minus the second, we get $17y - 142 \equiv 0 \mod 143$. Then $y \equiv 42 \mod 143$. Subsititute it in the first equation, we get $x \equiv 4 \mod 143$. $\square$

PROBLEM III

1. Assume $m \in \mathbb{N}^+, \gcd(a, m) = 1$, prove that $x \equiv ba^{\phi(n)-1} \mod m$ is the solution of $ax \equiv b \mod m$.
2. Assume $p$ is prime and $0 < a < p$. Prove that $x = b(-1)^{a-1}\frac{\binom{p}{a}}{p} \mod p$ is solution of $ax \equiv b \mod p$.

SOLUTION .

1. Only need to check $aba^{\phi(m)-1} \equiv b \mod m$. Since $\gcd(a, m) = 1$, easily $a^{\phi(m)} \equiv 1 \mod m$, so it's obvious.
2. We multiply $a!$ to the equation, we get $a!x \equiv b(-1)^{a-1}\prod_{k=1}^{a-1}(-k) \equiv b(a-1)! \mod p$. Since $0 < a < p$, we get $\gcd((a-1)!, p) = 1$, so $ax \equiv b \mod p$.

$\square$

PROBLEM IV Solve the equation:

$$\begin{cases} x \equiv 1 \mod 2 \\ x \equiv 2 \mod 5 \\ x \equiv 3 \mod 7 \\ x \equiv 4 \mod 9 \end{cases}$$

SOLUTION . $\square$