under Graduate Homework In Mathematics

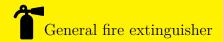
AlgebraicGeometry 12

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ROBLEM I Assume $x, y \in \mathbb{C}$ and $\mathrm{Im} x, \mathrm{Im} y > 0$. Assume $\begin{pmatrix} b & a \\ d & c \end{pmatrix} \in \mathrm{GL}(2, \mathbb{Z}),$ and $x = \frac{a + b y}{c + d y}$. Prove that $\begin{vmatrix} b & a \\ d & c \end{vmatrix} = 1$.

SOLION. Assume $y=m+ni, m, n\in\mathbb{R}, n>0$. Then we get $\mathrm{Im}x=\mathrm{Im}\frac{a+bm+bni}{c+dm+dni}=\frac{(c+dm)bn-(a+bm)dn}{(c+dm)^2+(dn)^2}>0$. So we get cbn+dmbn-adn-bmdn=n(cb-ad)>0. Since n>0 we get cb-ad>0, so $\begin{vmatrix} b & a \\ d & c \end{vmatrix}>0$.

Since
$$\begin{pmatrix} b & a \\ d & c \end{pmatrix} \in GL(2, \mathbb{Z})$$
, we get $\begin{vmatrix} b & a \\ d & c \end{vmatrix} = \pm 1$, so finally we get $\begin{vmatrix} b & a \\ d & c \end{vmatrix} = 1$.

 $\mathbb{R}^{O} \to \mathbb{E} M \text{ II Assume } \tau, \rho \in \mathbb{C}, \tau^2 = \rho^3 = 1. \text{ Prove that } \exists \sigma \in \mathbb{C} \text{ such that } \rho = \sigma^4, \tau = \sigma^6.$

SOUTHOW. Let
$$\sigma \in \sqrt{\tau \rho^2}$$
, i.e., $\sigma^2 = \tau \rho^2$. Then $\sigma^4 = \tau^2 \rho^4 = \rho$ and $\sigma^6 = \tau^3 \rho^6 = \tau$.