

PROBLEM I Find the solution of $6x^3 + 27x^2 + 17x + 20 \equiv 0 \pmod{30}$.

SOLUTION.

□

PROBLEM II Find the solution of $31x^4 + 57x^3 + 96x + 191 \equiv 0 \pmod{225}$.

SOLUTION.

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PROBLEM III Prove: $5x^2 + 11y^2 \equiv 1 \pmod{m}$. **PROBLEM IV** If $n \mid p-1, n > 1, (a, p) = 1$, prove :

1. $x^n \equiv a \pmod{p}$ has solution $\iff a^{\frac{p-1}{n}} \equiv 1 \pmod{p}$.
2. If $x^n \equiv a \pmod{p}$ has solution, then it has n solution.

PROBLEM V $n \in \mathbb{N}^+, \gcd(a, m) = 1, x^n \equiv a \pmod{m}$ has one solution $x \equiv x_0 \pmod{m}$. Prove all the solution of $x^n \equiv a \pmod{m}$ have the form of $x \equiv yx_0 \pmod{m}$, where y is the solution of $y^n \equiv 1 \pmod{m}$.