## Graduate Homework In Mathematics

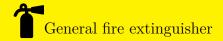
AlgebraicGeometry 4

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## $\mathbb{R}^{OBEM}$ I Prove that $\mathbb{C}$ and $\mathbb{C} \setminus \{0\}$ are Homeomorphic by Zariski topology.

SPINON. Since  $\operatorname{card}\mathbb{C} \setminus \{0\}$ , there is a bijection  $f : \mathbb{C} \to \mathbb{C} \setminus \{0\}$ . Then we prove f is homeomorphism.

First, we prove a set is closed iff it's finite or it's Universe. For a finite set A, obviously  $\mathbb{V}((\prod_{t\in A}(x-t)))=A$  is closed. For closed set  $A=\mathbb{V}(I), I\neq (0)$ , consider  $f\in I\setminus\{0\}$ , we know  $\forall a\in A, f(a)=0$ , so A is finite.

Since f is bijection, we know f preserve the cardinality, so preserve finite set. And  $f(\mathbb{C}) = \mathbb{C} \setminus \{0\}$ , so f preserve closed set. For the same reason  $f^{-1}$  preserve closed set. So f is homeomorphism.

ROBEM II Assume V is irreducible algebric set and f is a fraction function on V. Let  $D_f := \{h \in \mathcal{O}_v(V) : \exists g \in \mathcal{O}_v(V) \ s.t. \ f = \frac{g}{h}\} \cup \{0\}$ . Prove that  $D_f$  is ideal of  $\mathcal{O}_v(V)$ .

SOUTION. For  $h_1, h_2 \in D_f$ , if  $h_1 = h_2$  then  $h_1 - h_2 = 0 \in D_f$ . Else, assume  $f = \frac{g_1}{h_1} = \frac{g_2}{h_2}$ , so  $f = \frac{g_1 - g_2}{h_1 - h_2}$ . So  $h_1 - h_2 \in D_f$ .

For  $h \in D_f$  and  $j \in \mathcal{O}_v(V)$ , if hj = 0 then  $hj \in D_f$ . Else we get  $hj \neq 0$ , then  $f = \frac{g}{h} = \frac{gj}{hj}$ . So  $hj \in D_f$ .

Above all, we get  $D_f$  is a ideal.