Title of project

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1 The detailed setup of the problem

Ising model is used to successfully explain the phase transition between paramagnetism and ferromagnetism in Statistical Physics. The phenomenon that a magnet disappears its magnetism after being heated over a critical temperature and obtains its magnetism through cooling down the same critical temperature, is called the phase transition between paraamagnetism and ferromagnetism.

This model assumes that magnet is made by small needles regularly arranged. Each needle only has two different directions (upwards and downwards). Generally we consider a magnet as a square matrix with period boundary, call it A. The direction of each needle in magnet is regarded as an element in A, and ± 1 represent two direction. Thus, we can use a matrix $A \in M_n(\mathbb{R})$ to represent the state of a magnet. The macroeconomic appearance of magnet (that is whether to appear magnetism) is depended on directions of all needles. We use Hamiltonian to measure a microstate of magnet, i.e. A:

$$H(A) = -\sum_{\sigma_i \in A} \sum_{i \sim j} J\sigma_i\sigma_j - \sum_{\sigma_i \in A} H\sigma_j \tag{1}$$

, where J represents the spin-spin interaction, H represents the external field, σ_i is individual spins on eacch of lattice sites. Since each needle will interact with sourrounding needles (these needles on it's left, right, up and down). The first sum represents the interactions between each small needle; latter one means the external field trying to align in one direction.

How does temperature influence magnetism? It turns out that each needle can convert its direction randomly influenced by temperature T. Assume that each needle convert in different time. Whether to accept the convert of a needle is deicided by the change of Hamiltonian before and after the convert, , and temperature at the same time, let's call the change as ΔH.

If $\Delta H > 0$, then accept the convert. If $\Delta H > 0$, then we accept the convert with probabilty of $\mathrm{e}^{\frac{-\Delta H}{k_B T}}$ where k_B is a constant. After the convert of the needle, a convert will happen afterwards. After enough long time, the process may reach a dynamic balance. Thus, the magnet will perform may different property.

Our aim is to find out how does the temperature influence the property of magnet and determine the critical temperature. In this article, we focus on these two problem:

1. Take $J = 1, k_B = 1, H = 0$, N is the size of A. Fixing T, choosing an initial microstate and initial needle randomly, let a convert, and judge whether to accept it, and then repaet the process as before. We will get a chain of microstates. Caculate internal energy u in the process:

$$u = \frac{U}{N^2} U = \mathbb{E}(H) \tag{2}$$

   and specific heat c:

$$c = \frac{C}{N^2}, C = \frac{k_B}{T^2}\mathbb{D}(H) \tag{3}$$

   where $\mathbb{D}$ is the variance. And plot $u - T, c - T$ graph.

2. Take $J = 1, k_B = 1$, $H \neq 0$. Fixing T, H, choosing an initial microstate and initial needle randomly, let a convert, and judge whether to accept it, and then repaet the process as before. We will get a chain of microstates. Caculate magnetization m in the process:

$$m = \frac{M}{N^2}, M = \mathbb{E}(\sum_{\sigma_i \in A} \sigma_i). \tag{4}$$

   And plot $m - (T, H)$ graph.

2 The procedure you take to do the computation

3 Analysis of the numerical results

4 The issues you encounter and how you overcome

5 Possible discussion about the results and further thinking

Ising model can be applied to many other situations and fields. For example, opinion dynamics. Consider a group of people, each person can support one of two different candidates (like A and B), let the