## Graduate Homework In Mathematics

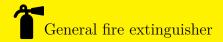
AlgebraicGeometry 5

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 $\mathbb{R}^{\mathrm{OBEM}} \text{ I If } f: V \to W, g: W \to U \text{ is two poly maps, then } (g \circ f)^* = f^* \circ g^*.$ 

SOLION. For  $u \in k[U]$ , we have  $(g \circ f)^*u = u \circ g \circ f = g^*(u) \circ f = f^*(g^*(u)) = (f^* \circ g^*)(u)$ , so  $(g \circ f)^* = (f^* \circ g^*)$ .

## $\mathbb{R}^{OB}\mathbb{E}M$ II $\mathcal{O}_{V,p}$ is local ring.

SOLION. To prove  $\mathcal{O}_{V,p}$  is local ring, we only need to prove  $\forall f \in \mathcal{O}_{V,p}$ , one of f and 1-f is unit. First we prove f is unit iff  $f(p) \neq 0$ . If f is unit then exists  $g \in \mathcal{O}_{V,p}$  s.t. fg = 1, so f(p)g(p) = 1. Then we get  $f(p) \neq 0$ . If  $f(p) \neq 0$ , then we assume  $f = \frac{g}{h}, h(p) \neq 0$ . Since  $f(p) \neq 0$  we get  $g(p) \neq 0$ , so  $\frac{h}{g} \in \mathcal{O}_{V,p}$ , then f is a unit.

Now we prove f or 1-f is a unit. Obviously  $f(p) \neq 0$  or  $1-f(p) \neq 0$ , so one of them is unit.