underGraduate Homework In Mathematics

GroupRepresentation 3

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PROBLEM I Let ϕ is representation of $\mathrm{GL}_n(K)$ over K^n. And $\phi(A)\alpha := A\alpha$. Prove:ϕ is faithful and irreducible and $n-$dimentional.

SOLUTION. Obviously it's n-dimentional. If $A \neq B$, then exists $\alpha \in K^n$ s.t. $(A-B)\alpha \neq 0$. So $\phi(A)\alpha \neq \phi(B)\alpha$. So $\phi(A) \neq \phi(B)$, so ϕ is faithful. To prove ϕ is irreducible, we only need to prove there is no invariant subspace of K^n. Obviously for $\alpha, \beta \in K^n \setminus \{0\}$, obviously there exists $A \in \mathrm{GL}_n(K)$ such that $A\alpha = \beta$. So there is no nontrival invariant subspace of K^n. So it's irreducible. □

PROBLEM II For $A \in \mathrm{GL}_n(K)$, let $\psi(A)X = AX, \forall X \in M_n(K)$. Then:

1. ψ is n^2-dimentional representation of $\mathrm{GL}_n(K)$ over K.
2. For $j : 1 \leq j \leq n$, let $M_n^{(j)}(K) := \{(a_{ik})_{n\times n} : a_{ik} \neq 0 \to k = j\}$. Prove $M_n^{(j)}$ is invariant subspace of $\mathrm{GL}_n(K)$. Let ψ is subrepresentation of ψ in $M_n^{(j)}$, prove ψ_j is irreducible and $\psi = \bigoplus_{j=1}^n \psi_j$.
3. Prove $\psi_j \cong \phi$, where $\phi =$ (**??**).ϕ

SOLUTION. 1.

□

PROBLEM III Let $K = \mathbb{C}$ and $n = 2$ in (Group representation second homework).(Problem 3), prove the subrepresentation of ϕ over $M_2^n(\mathbb{C})$ is irreducible.

PROBLEM IV Assume $n \geq 3$ and $n \nmid \operatorname{char} K$, proof: then then $n-$ dimentional permutate representation of S_n can be decomposed as the direct sum of a main representation and a $n-1-$dimentional irreducible subrepresentation

PROBLEM V Caculate the $1-$ dimentional $\mathbb{C}$ representation:

1. $(2,4)-$type of $8-$ order elementary Abel group.
2. the addition group of $\mathbb{Z}_p^n$