

under Graduate Homework In Mathematics

GroupRepresentation 4

白永乐

202011150087

202011150087@mail.bnu.edu.cn

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General fire extinguisher

PROBLEM I Find all of 1-dimensional complex representation of the alternating group A_4 .

PROBLEM II Consider $N \trianglelefteq S_4$ and $N = \{(1), (12)(34), (13)(24), (14)(23)\}$.

1. Prove: $S_4/N \cong S_3$.
2. Find a 2-dimensional irreducible complex matrix representation of S_4 .

PROBLEM III Assume K is a field and $m \in \mathbb{N}^*$. Let $\phi_m(t) := t^m, \forall t \in K^*$, then ϕ_m is a 1-dimensional K -representation of (K^*, \cdot) . Use ϕ_m to find a 1-dimensional K -representation of $\text{GL}_n(K)$.

PROBLEM IV Prove that if ϕ is 1-dimensional complex representation of finite group G , then $G/\ker \phi$ is a cyclic group.

PROBLEM V Prove: If G is a non-cyclic finite group, then there is no faithful 1-dimensional complex representation of G .

PROBLEM VI Assume (ϕ, V) and (ψ, W) are two K -representation of group G . Prove: $(\phi + \psi) \approx \phi^* + \psi^*$.