

# under Graduate Homework In Mathematics

## Set Theory 5

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2023 年 11 月 25 日



General fire extinguisher

PROBLEM I Prove:  $F \subset \mathcal{N}$  is closed set  $\iff F = [T]$  for some  $T \subset {}^{<\omega}\omega$ .

SOLUTION. •  $\implies$  : Let  $T := T_F$ , now we need to prove  $F = [T]$ . From the definition of  $T_F$  and  $[T]$  easily we get  $F \subset [T]$ . Now we prove  $[T] \subset F$ . For  $f \in [T]$ , we get  $f \restriction n \in T$ . i.e.,  $\forall n \in \mathbb{N}, f \restriction n = g \restriction n$  for some  $g \in F$ . So  $d(f, F) \leq d(f, g) = \frac{1}{2^n}$ . Since  $F$  is closed, we get  $f \in F$ .

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