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1 distribution

name	symbol	range	distribution	mean	variance	generating	characteristic
Bernoulli	$B(1,p), 0<p<1$	$\{0,1\}$	$\mu(0)=1-p, \mu(1)=p$	p	$p(1-p)$	$(1-p+pz)$	$1-p+p\mathrm{e}^{\mathrm{i}t}$
Binomial	$B(n,p), 0<p<1$	$\mathbb{N}\cap[0,n]$	$\mu(k)=\binom{n}{k}p^k(1-p)^{n-k}$	np	$np(1-p)$	$(1-p+pz)^n$	$(1-p+p\mathrm{e}^{\mathrm{i}t})^n$
Geometric	$G(p), 0<p<1$	$\mathbb{N}^+$	$\mu(k)=(1-p)^{k-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	fuck	$\frac{p\mathrm{e}^{\mathrm{i}t}}{1-(1-p)\mathrm{e}^{\mathrm{i}t}}$
Hypergeometric	$H(n,K,N)$ $N\in\mathbb{N}$ $K\in[0,N]\cap\mathbb{N}$ $n\in[0,N]\cap\mathbb{N}$	$[\max\{0,n+K-N\},$ $\min\{n,K\}]\cap\mathbb{N}$	$\mu(k)=\frac{\binom{K}{k}\binom{N-k}{n-k}}{\binom{N}{n}}$	$n\frac{K}{N}$	$n\frac{K}{N}\frac{N-k}{N}\frac{N-n}{N-1}$	fuck	fuck
Poisson	$P(\lambda), \lambda>0$	$\mathbb{N}$	$\mu(k)=\frac{\mathrm{e}^{-\lambda}\lambda^k}{k!}$	λ	λ	$\mathrm{e}^{\lambda(z-1)}$	$\mathrm{e}^{\lambda(\mathrm{e}^{it}-1)}$
Uniform	$U[a,b], a<b$	$[a,b]$	$f(x)=\frac{1}{b-a}$ $F(x)=\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	-	$\frac{\mathrm{e}^{\mathrm{i}tb}-\mathrm{e}^{\mathrm{i}ta}}{\mathrm{i}t(b-a)}$
Exponential	$Exp(\lambda), \lambda>0$	$[0,\infty)$	$f(x)=\lambda\mathrm{e}^{\lambda x}\mathbb{1}_{\{x\geq 0\}}$ $F(x)=(1-\mathrm{e}^{\lambda x})\mathbb{1}_{\{x\geq 0\}}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	-	$(1-\frac{\mathrm{i}t}{\lambda})^{-1}$
Gamma	$\Gamma(\alpha,\beta), \alpha,\beta>0$	$(0,\infty)$	$f(x)=\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\mathrm{e}^{-\beta x}\mathbb{1}_{\{x>0\}}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	-	$(1-\frac{\mathrm{i}t}{\beta})^{-\alpha}$
Normal	$N(\mu,\sigma^2), \mu,\sigma\in\mathbb{R}$	$\mathbb{R}$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ	-	$\mathrm{e}^{\mu\mathrm{i}t-\frac{\sigma^2t^2}{2}}$

2 important lemma

Lemma 1 (Borel-Cantelli). *$(\Omega,\mathcal{F},\mathbb{P})$ is a probability space. $\{A_n : n\in\mathbb{N}\}\subset\mathcal{F}$, then*

1. *if $\sum_{n\in\mathbb{N}}\mathbb{P}(A_n)<\infty$, then $\mathbb{P}(\bigcup_{k=0}^{n}\bigcap_{m=n}^{\infty}A_m)=0$;*
2. *if $\{A_n : n\in\mathbb{N}\}$ are independent, $\sum_{n\in\mathbb{N}}\mathbb{P}(A_n)=\infty$, then $\mathbb{P}(\bigcup_{k=0}^{n}\bigcap_{m=n}^{\infty}A_m)=1$.*