

Groenwolds's relaxed conservatism

See algorithm in appended Python code. The objective and the constraint function is given verbatim below. (Notice the naive choice of (a quadratic approximation to) a function linearised in terms of reciprocal intervening variables, dd The reader is encouraged to, for example, uncomment the analytic second order information.)

```
#
import numpy as np
#
def obj(x):
#
    a = 1.; b = 32.; c = -1.
    f = a*np.sin(b*x)*np.exp(c*x)
    df = a*c*np.sin(b*x)*np.exp(c*x) + a*b*np.exp(c*x)*np.cos(b*x)
#   ddf = 2*a*c*b*np.cos(b*x)*np.exp(c*x)+a*c*c*np.sin(b*x)*np.exp(c*x)-a*b*b*np.exp(c*x)*np.sin(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddf = -2./x*df
#   ddf = abs(-2./x*df ) # nonconvex variant
#
    return [f, df, ddf]
#
def con(x):
#
    a = 1./4.; b = 32.
    g = a*np.cos(b*x)+0.1
    dg = -a*b*np.sin(b*x)
#   ddg = -a*b*b*np.cos(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddg = -2./x*dg
#   ddg = abs(-2./x*dg) # nonconvex variant
#
    return [g, dg, ddg]
```

QP subproblem at x^k :

$$x^k = 0.500$$

$$F^k = -0.175$$

$$d_x^k F = -18.413$$

$$dd_x^k \tilde{F} = 73.650$$

$$\alpha_F = 1.000$$

$$G^k = -0.139$$

$$d_x^k G = 2.303$$

$$dd_x^k \tilde{G} = 9.213$$

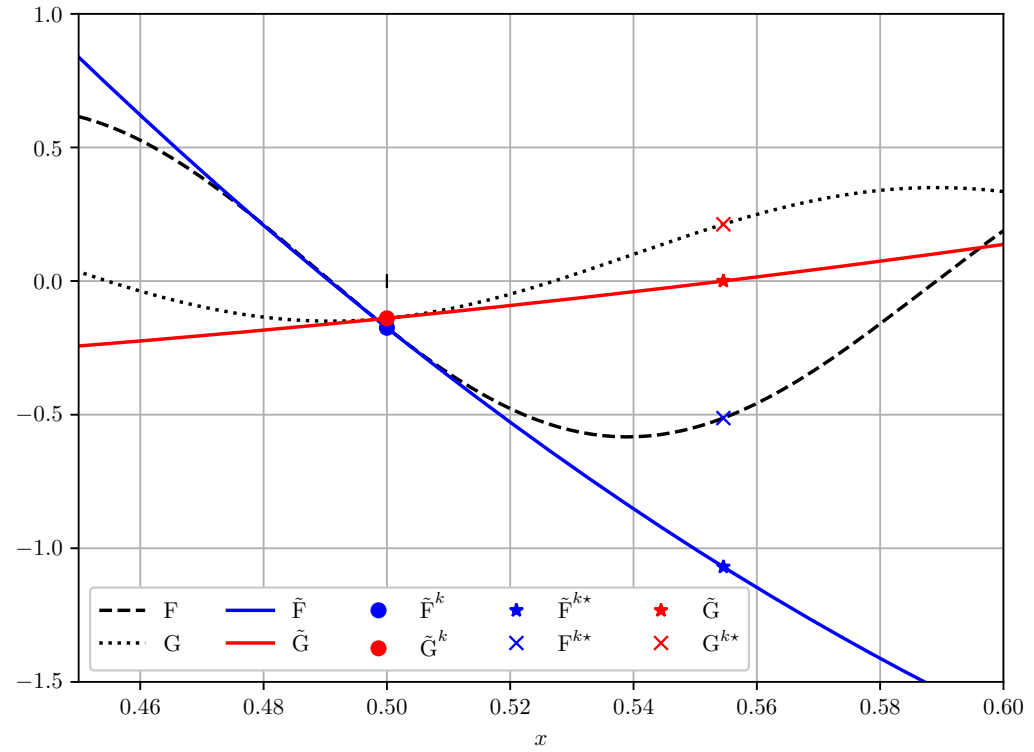
$$\alpha_G = 1.000$$

with Lagrange multiplier

$$\lambda^{k*} = 5.129$$

at the solution

$$x^{k*} = 0.555$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -1.069786e + 00$$

$$\tilde{G}^{k*} = -4.084337e - 07$$

while the actual functions are evaluated to be

$$F^{k*} = -5.126674e - 01$$

$$G^{k*} = 2.126848e - 01$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{F}^{k*} < F^{k*}$.

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.500$$

$$F^k = -0.175$$

$$d_x^k F = -18.413$$

$$dd_x^k \tilde{F} = 73.650$$

$$\alpha_F = 2.000$$

$$G^k = -0.139$$

$$d_x^k G = 2.303$$

$$dd_x^k \tilde{G} = 9.213$$

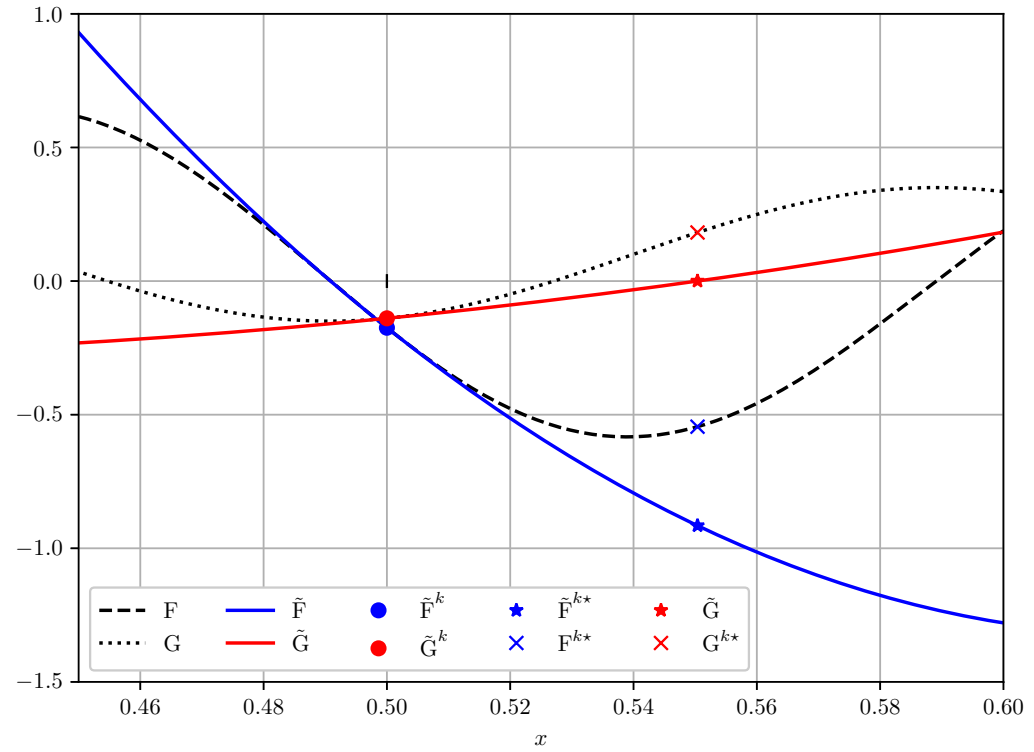
$$\alpha_G = 2.000$$

with Lagrange multiplier

$$\lambda^{k*} = 3.401$$

at the solution

$$x^{k*} = 0.550$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -9.152943e - 01$$

$$\tilde{G}^{k*} = -2.756164e - 09$$

while the actual functions are evaluated to be

$$F^{k*} = -5.449839e - 01$$

$$G^{k*} = 1.818020e - 01$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{F}^{k*} < F^{k*}$.

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.500$$

$$F^k = -0.175$$

$$d_x^k F = -18.413$$

$$dd_x^k \tilde{F} = 73.650$$

$$\alpha_F = 4.000$$

$$G^k = -0.139$$

$$d_x^k G = 2.303$$

$$dd_x^k \tilde{G} = 9.213$$

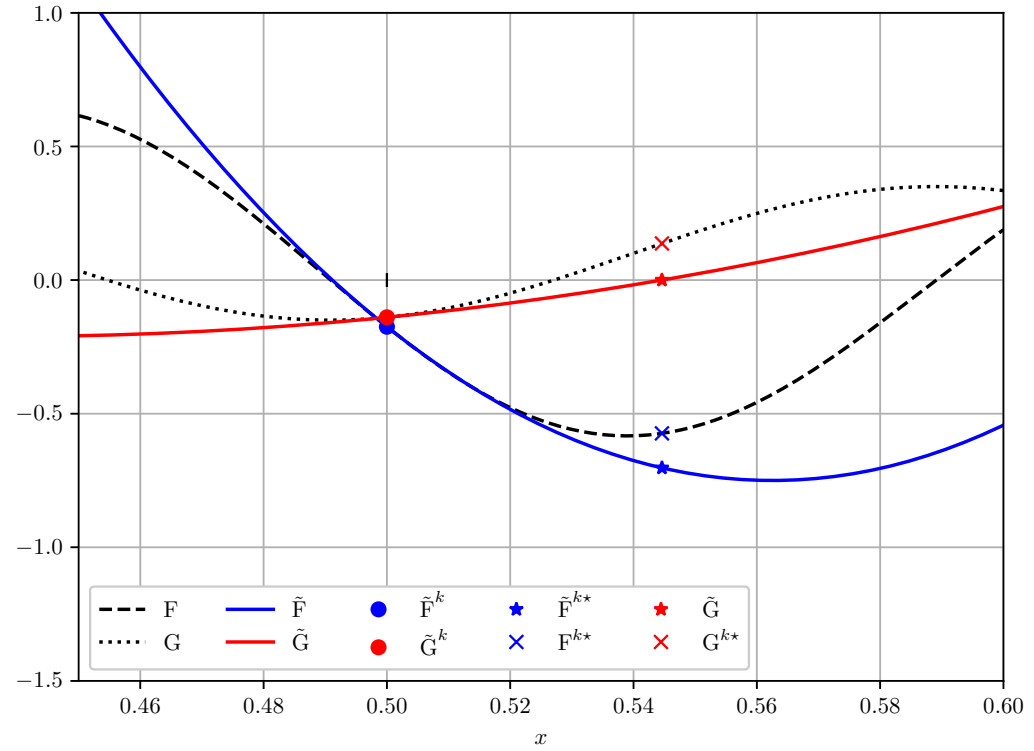
$$\alpha_G = 4.000$$

with Lagrange multiplier

$$\lambda^{k*} = 1.335$$

at the solution

$$x^{k*} = 0.545$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -7.028706e - 01$$

$$\tilde{G}^{k*} = -7.291763e - 08$$

while the actual functions are evaluated to be

$$F^{k*} = -5.736618e - 01$$

$$G^{k*} = 1.370522e - 01$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{F}^{k*} < F^{k*}$.

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.500$$

$$F^k = -0.175$$

$$d_x^k F = -18.413$$

$$dd_x^k \tilde{F} = 73.650$$

$$\alpha_F = 8.000$$

$$G^k = -0.139$$

$$d_x^k G = 2.303$$

$$dd_x^k \tilde{G} = 9.213$$

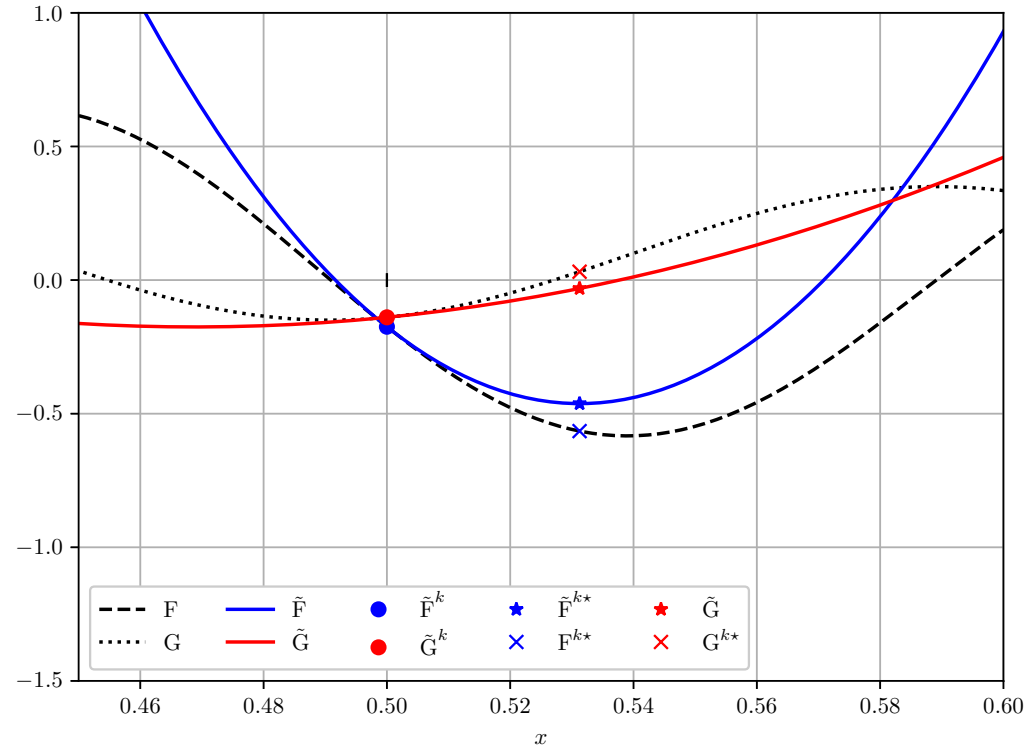
$$\alpha_G = 8.000$$

with Lagrange multiplier

$$\lambda^{k*} = 0.000$$

at the solution

$$x^{k*} = 0.531$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -4.623186e - 01$$

$$\tilde{G}^{k*} = -3.145113e - 02$$

while the actual functions are evaluated to be

$$F^{k*} = -5.651764e - 01$$

$$G^{k*} = 3.120917e - 02$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.500$$

$$F^k = -0.175$$

$$d_x^k F = -18.413$$

$$dd_x^k \tilde{F} = 73.650$$

$$\alpha_F = 8.000$$

$$G^k = -0.139$$

$$d_x^k G = 2.303$$

$$dd_x^k \tilde{G} = 9.213$$

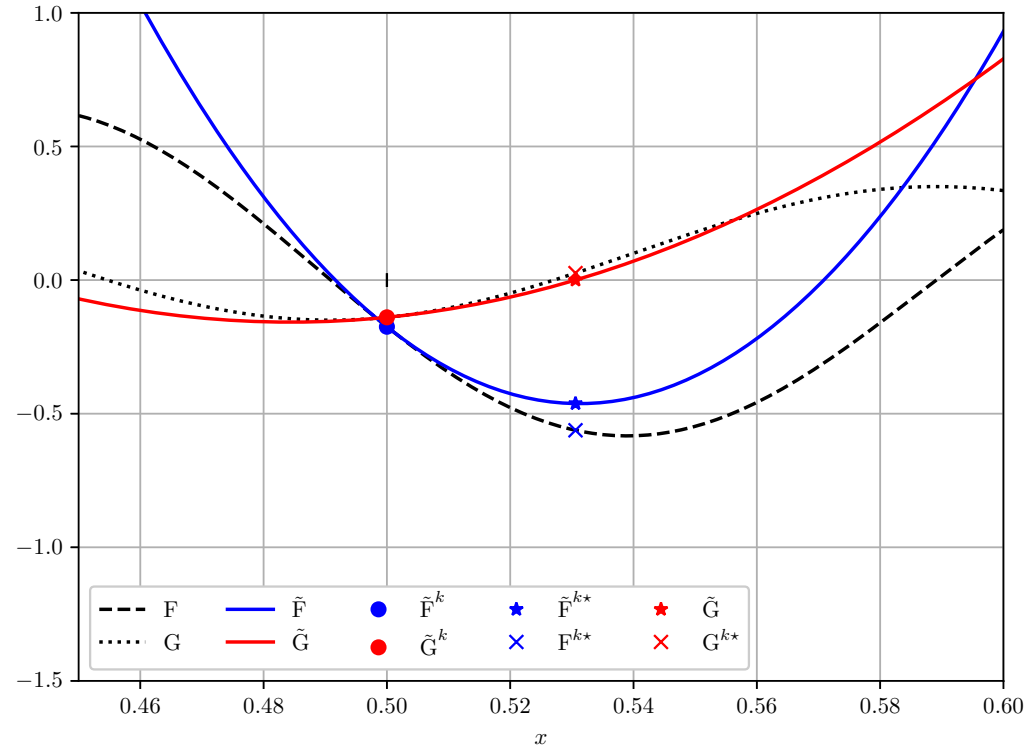
$$\alpha_G = 16.000$$

with Lagrange multiplier

$$\lambda^{k*} = 0.057$$

at the solution

$$x^{k*} = 0.531$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -4.621898e - 01$$

$$\tilde{G}^{k*} = -1.493519e - 09$$

while the actual functions are evaluated to be

$$F^{k*} = -5.619988e - 01$$

$$G^{k*} = 2.613916e - 02$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.500$$

$$F^k = -0.175$$

$$d_x^k F = -18.413$$

$$dd_x^k \tilde{F} = 73.650$$

$$\alpha_F = 8.000$$

$$G^k = -0.139$$

$$d_x^k G = 2.303$$

$$dd_x^k \tilde{G} = 9.213$$

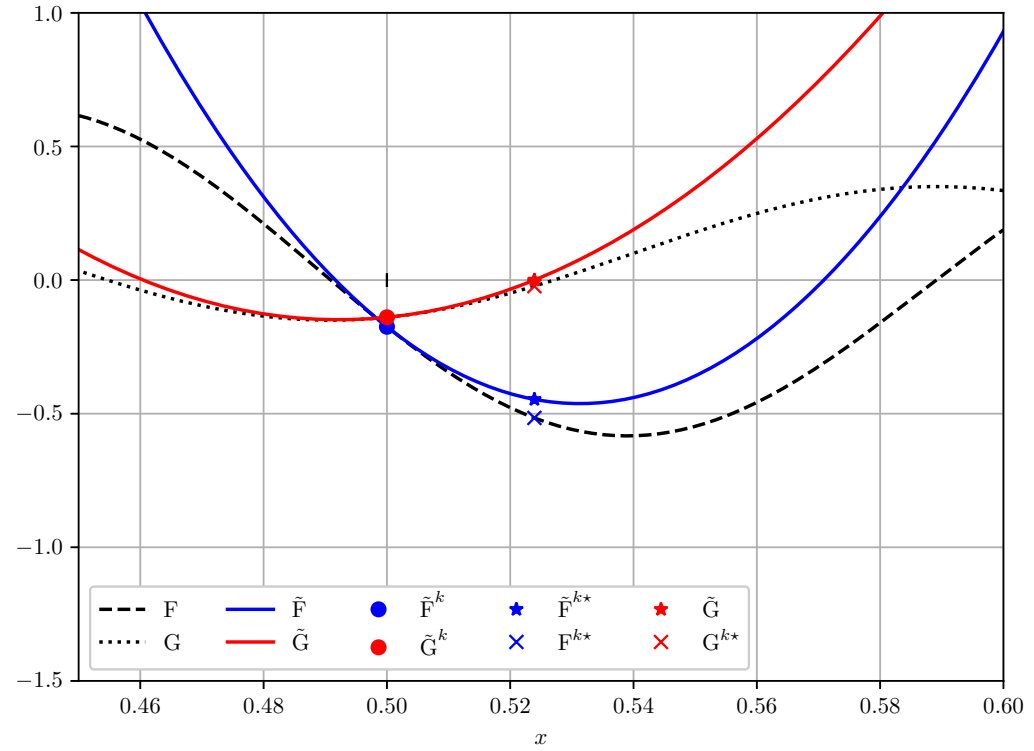
$$\alpha_G = 32.000$$

with Lagrange multiplier

$$\lambda^{k*} = 0.462$$

at the solution

$$x^{k*} = 0.524$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -4.464811e - 01$$

$$\tilde{G}^{k*} = 9.505003e - 08$$

while the actual functions are evaluated to be

$$F^{k*} = -5.158540e - 01$$

$$G^{k*} = -2.278227e - 02$$

The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(In this case both approximations are indeed conservative, nevertheless.)

QP subproblem at x^k :

$$x^k = 0.524$$

$$F^k = -0.516$$

$$d_x^k F = -8.791$$

$$dd_x^k \tilde{F} = 33.559$$

$$\alpha_F = 1.000$$

$$G^k = -0.023$$

$$d_x^k G = 6.969$$

$$dd_x^k \tilde{G} = 26.602$$

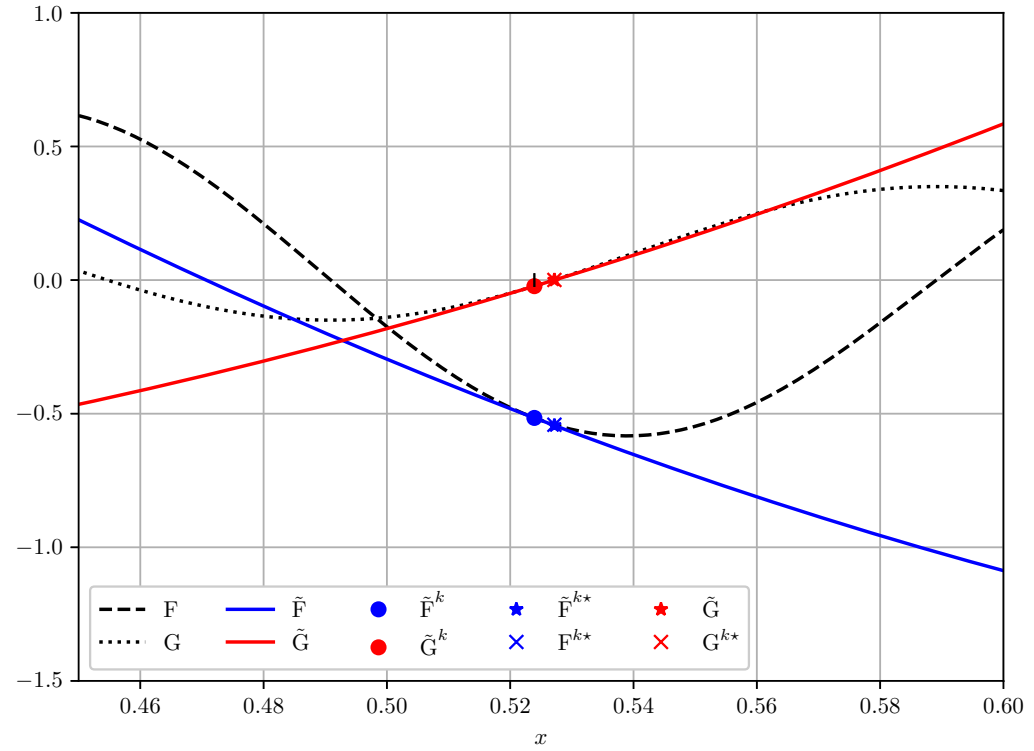
$$\alpha_G = 1.000$$

with Lagrange multiplier

$$\lambda^{k*} = 1.231$$

at the solution

$$x^{k*} = 0.527$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -5.442415e - 01$$

$$\tilde{G}^{k*} = 1.200414e - 06$$

while the actual functions are evaluated to be

$$F^{k*} = -5.414921e - 01$$

$$G^{k*} = 4.830983e - 04$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{F}^{k*} < F^{k*}$.

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.524$$

$$F^k = -0.516$$

$$d_x^k F = -8.791$$

$$dd_x^k \tilde{F} = 33.559$$

$$\alpha_F = 2.000$$

$$G^k = -0.023$$

$$d_x^k G = 6.969$$

$$dd_x^k \tilde{G} = 26.602$$

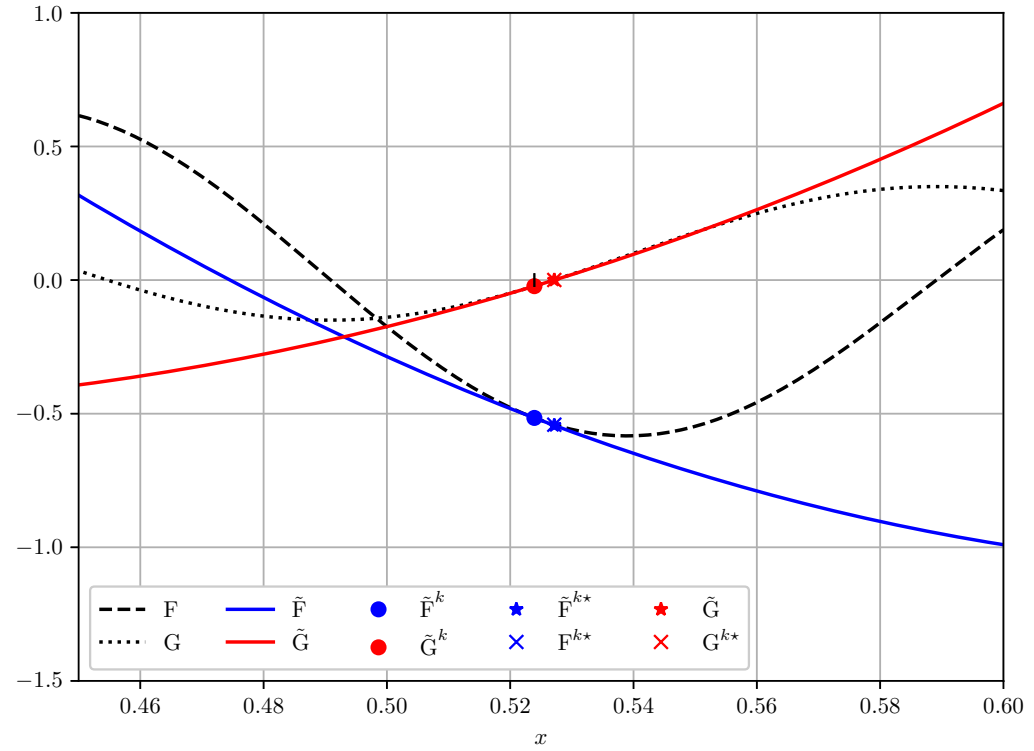
$$\alpha_G = 2.000$$

with Lagrange multiplier

$$\lambda^{k*} = 1.201$$

at the solution

$$x^{k*} = 0.527$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -5.438946e - 01$$

$$\tilde{G}^{k*} = 2.464175e - 07$$

while the actual functions are evaluated to be

$$F^{k*} = -5.413539e - 01$$

$$G^{k*} = 3.378196e - 04$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{F}^{k*} < F^{k*}$.

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.524$$

$$F^k = -0.516$$

$$d_x^k F = -8.791$$

$$dd_x^k \tilde{F} = 33.559$$

$$\alpha_F = 4.000$$

$$G^k = -0.023$$

$$d_x^k G = 6.969$$

$$dd_x^k \tilde{G} = 26.602$$

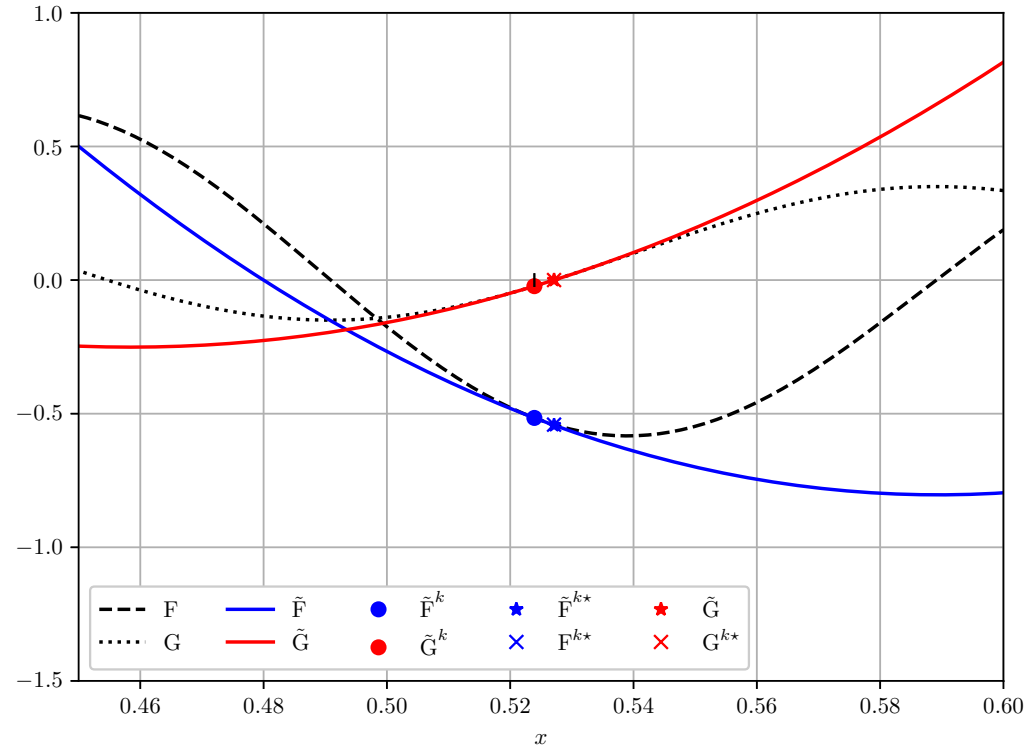
$$\alpha_G = 4.000$$

with Lagrange multiplier

$$\lambda^{k*} = 1.144$$

at the solution

$$x^{k*} = 0.527$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -5.432267e - 01$$

$$\tilde{G}^{k*} = -3.228880e - 07$$

while the actual functions are evaluated to be

$$F^{k*} = -5.410877e - 01$$

$$G^{k*} = 5.887112e - 05$$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because

→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{F}^{k*} < F^{k*}$.

→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{G}^{k*} < G^{k*}$.

That is, at least one approximation function is not *conservative*.

QP subproblem at x^k :

$$x^k = 0.524$$

$$F^k = -0.516$$

$$d_x^k F = -8.791$$

$$dd_x^k \tilde{F} = 33.559$$

$$\alpha_F = 8.000$$

$$G^k = -0.023$$

$$d_x^k G = 6.969$$

$$dd_x^k \tilde{G} = 26.602$$

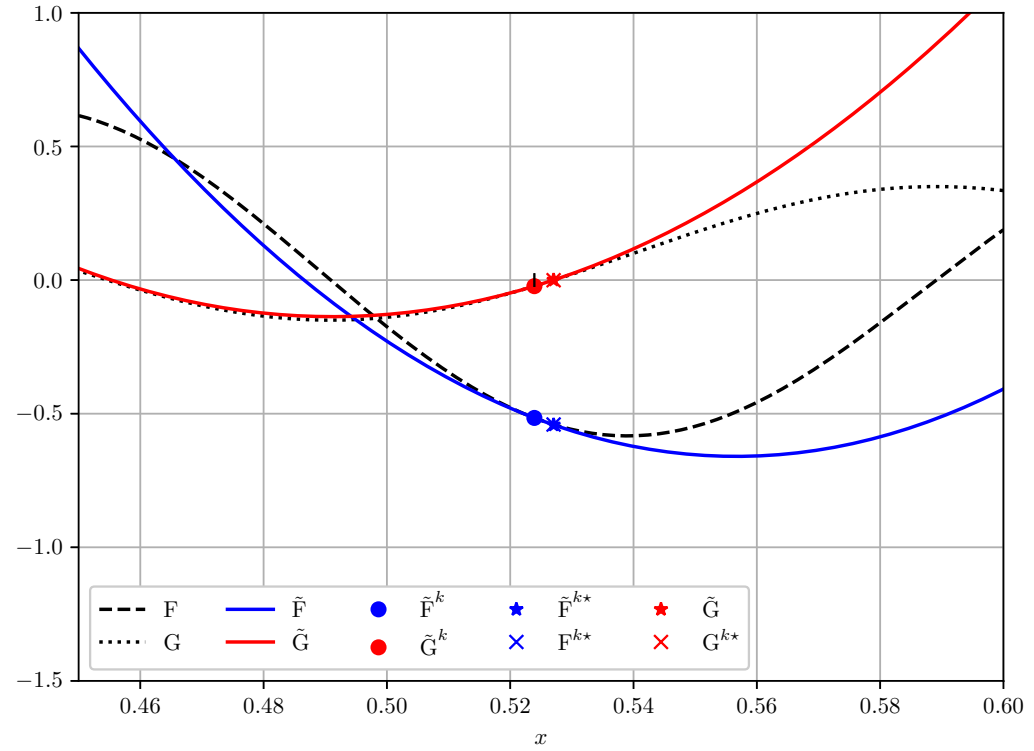
$$\alpha_G = 8.000$$

with Lagrange multiplier

$$\lambda^{k*} = 1.042$$

at the solution

$$x^{k*} = 0.527$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -5.419799e - 01$$

$$\tilde{G}^{k*} = -1.188763e - 07$$

while the actual functions are evaluated to be

$$F^{k*} = -5.405892e - 01$$

$$G^{k*} = -4.607702e - 04$$

The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.

(Note that the objective and/or constraint function approximation is not conservative, in this case.)

QP subproblem at x^k :

$$x^k = 0.527$$

$$F^k = -0.541$$

$$d_x^k F = -7.051$$

$$dd_x^k \tilde{F} = 26.756$$

$$\alpha_F = 1.000$$

$$G^k = -0.000$$

$$d_x^k G = 7.326$$

$$dd_x^k \tilde{G} = 27.799$$

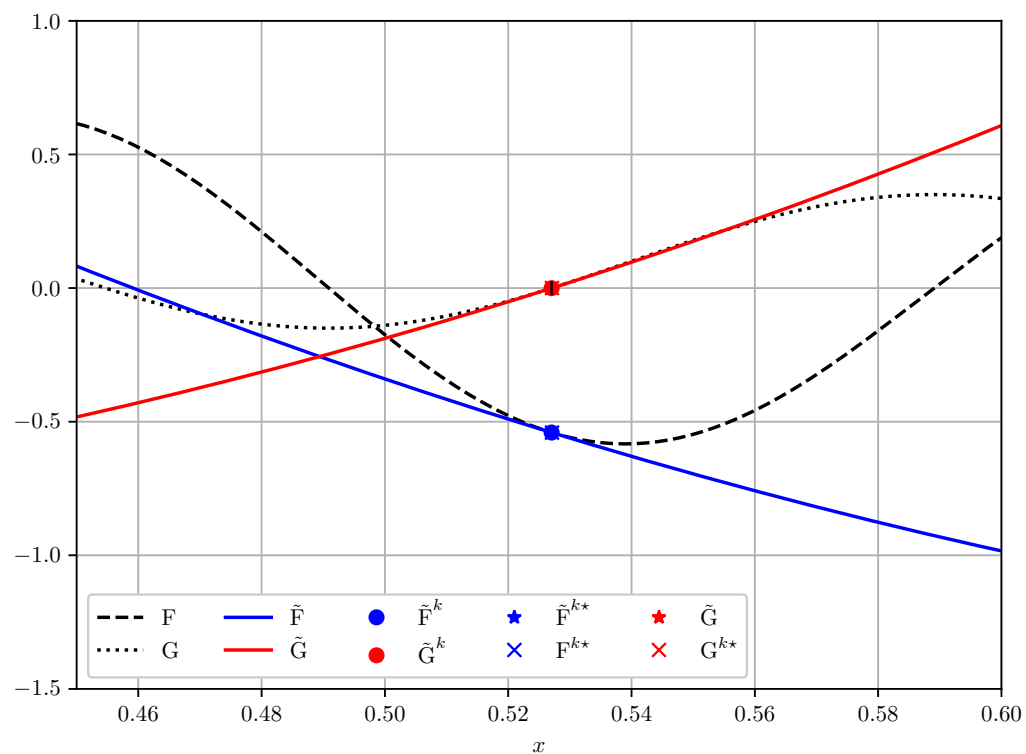
$$\alpha_G = 1.000$$

with Lagrange multiplier

$$\lambda^{k*} = 0.962$$

at the solution

$$x^{k*} = 0.527$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -5.410317e - 01$$

$$\tilde{G}^{k*} = -8.705594e - 07$$

while the actual functions are evaluated to be

$$F^{k*} = -5.410307e - 01$$

$$G^{k*} = -7.229650e - 07$$

The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.

(Note that the objective and/or constraint function approximation is not conservative, in this case.)

QP subproblem at x^k :

$$x^k = 0.527$$

$$F^k = -0.541$$

$$d_x^k F = -7.015$$

$$dd_x^k \tilde{F} = 26.617$$

$$\alpha_F = 1.000$$

$$G^k = -0.000$$

$$d_x^k G = 7.332$$

$$dd_x^k \tilde{G} = 27.821$$

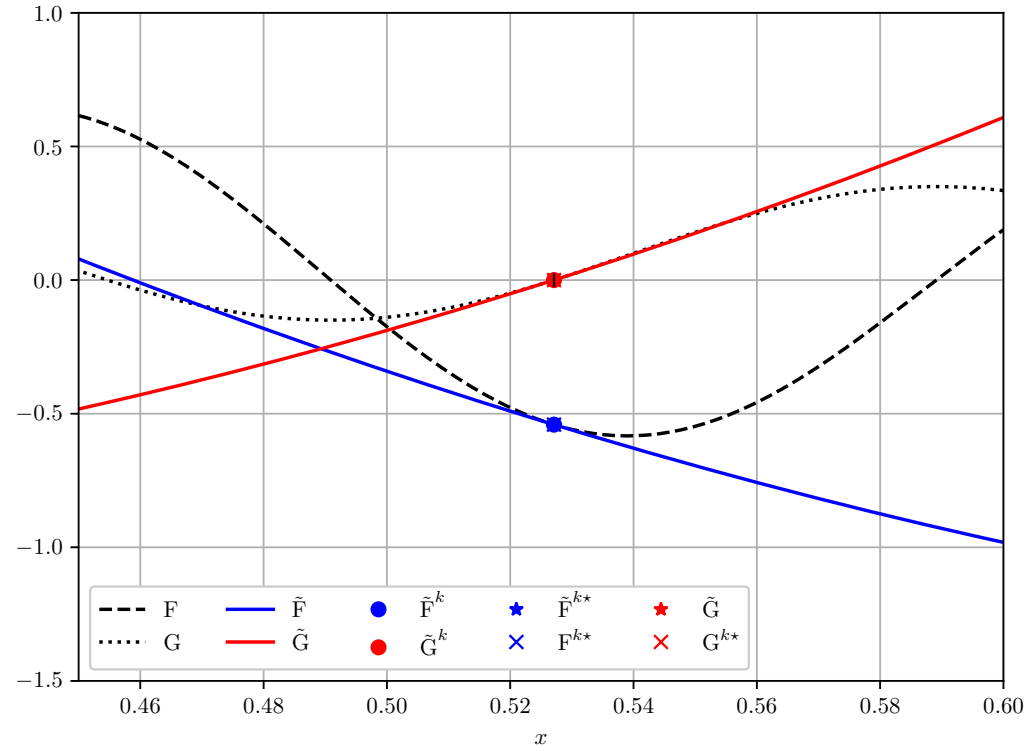
$$\alpha_G = 1.000$$

with Lagrange multiplier

$$\lambda^{k*} = 0.957$$

at the solution

$$x^{k*} = 0.527$$



At the solution of the QP the function approximations have the values

$$\tilde{F}^{k*} = -5.410317e - 01$$

$$\tilde{G}^{k*} = 3.024409e - 07$$

while the actual functions are evaluated to be

$$F^{k*} = -5.410317e - 01$$

$$G^{k*} = 3.024416e - 07$$

Both the value objective function approximation and the constraint function approximation, at the new design point x^{k*} , is *conservative* with respect to the actual function values, $\tilde{F}^{k*} > F^{k*}$, $\tilde{G}^{k*} > G^{k*}$

Terminated on $|x^{k*} - x^k| < 1.0e-06$

Python code