Svanberg's (strict) conservatism

See algorithm in appended Python code. The objective and the constraint function is given verbatim below. (Notice the naive choice of (a quadratic approximation to) a function linearised in terms of reciprocal intervening variables, `dd`.... The reader is encouraged to, for example, uncomment the analytic second order information.)

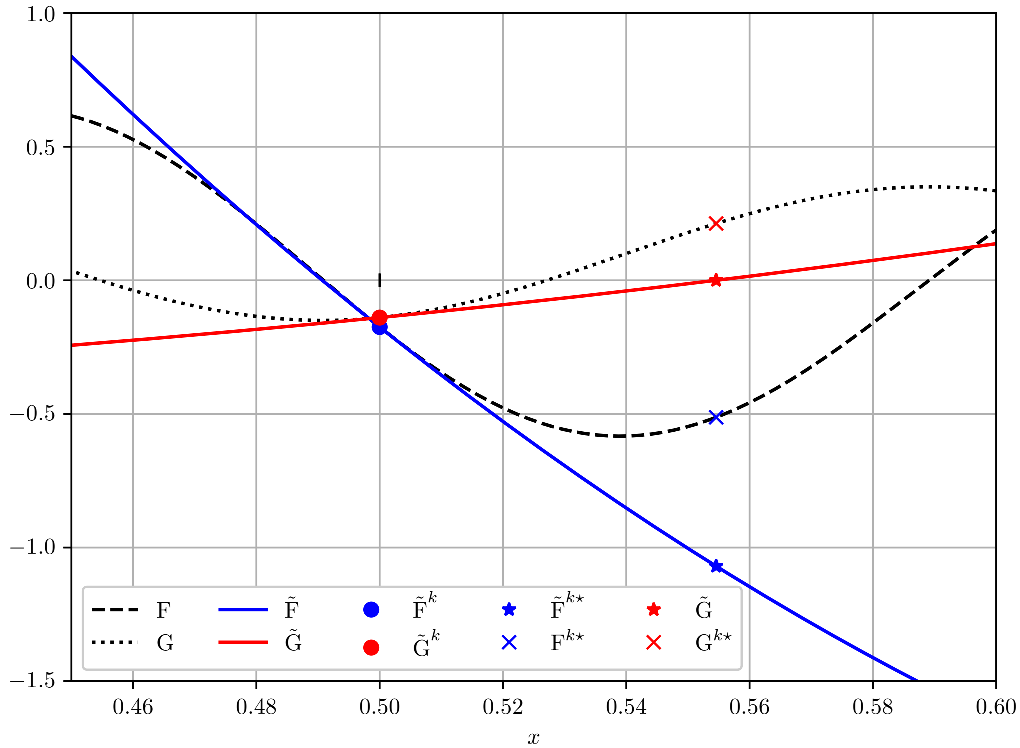
```
#
import numpy as np
#
def obj(x):
#
    a = 1.; b = 32.; c = -1.
    f = a*np.sin(b*x)*np.exp(c*x)
    df = a*c*np.sin(b*x)*np.exp(c*x) + a*b*np.exp(c*x)*np.cos(b*x)
#   ddf = 2*a*c*b*np.cos(b*x)*np.exp(c*x)+a*c*c*np.sin(b*x)*np.exp(c*x)-a*b*b*np.exp(c*x)*np.sin(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddf = -2./x*df
    ddf = abs(-2./x*df ) # nonconvex variant
#
    return [f, df, ddf]
#
def con(x):
#
    a = 1./4.; b = 32.
    g = a*np.cos(b*x)+0.1
    dg = -a*b*np.sin(b*x)
#   ddg = -a*b*b*np.cos(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddg = -2./x*dg
    ddg = abs(-2./x*dg)  # nonconvex variant
#
    return [g, dg, ddg]
```

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 1.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 5.129$
at the solution
$x^{k\star} = 0.555$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -1.069786e+00$
$\tilde{\mathrm{G}}^{k\star} = -4.084337e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.126674e-01$
$\mathrm{G}^{k\star} = 2.126848e-01$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

That is, at least one approximation function is not *conservative*.

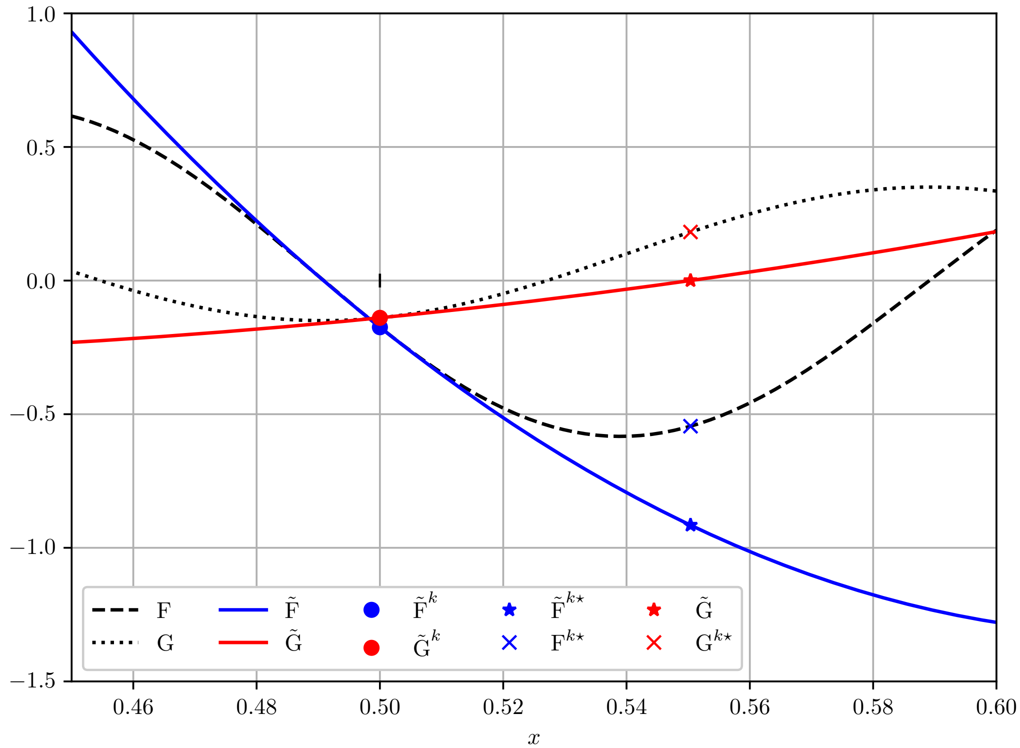
Iteration 1

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 2.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 2.000$

with Lagrange multiplier
$\lambda^{k\star} = 3.401$
at the solution
$x^{k\star} = 0.550$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -9.152943e-01$
$\tilde{\mathrm{G}}^{k\star} = -2.756164e-09$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.449839e-01$
$\mathrm{G}^{k\star} = 1.818020e-01$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

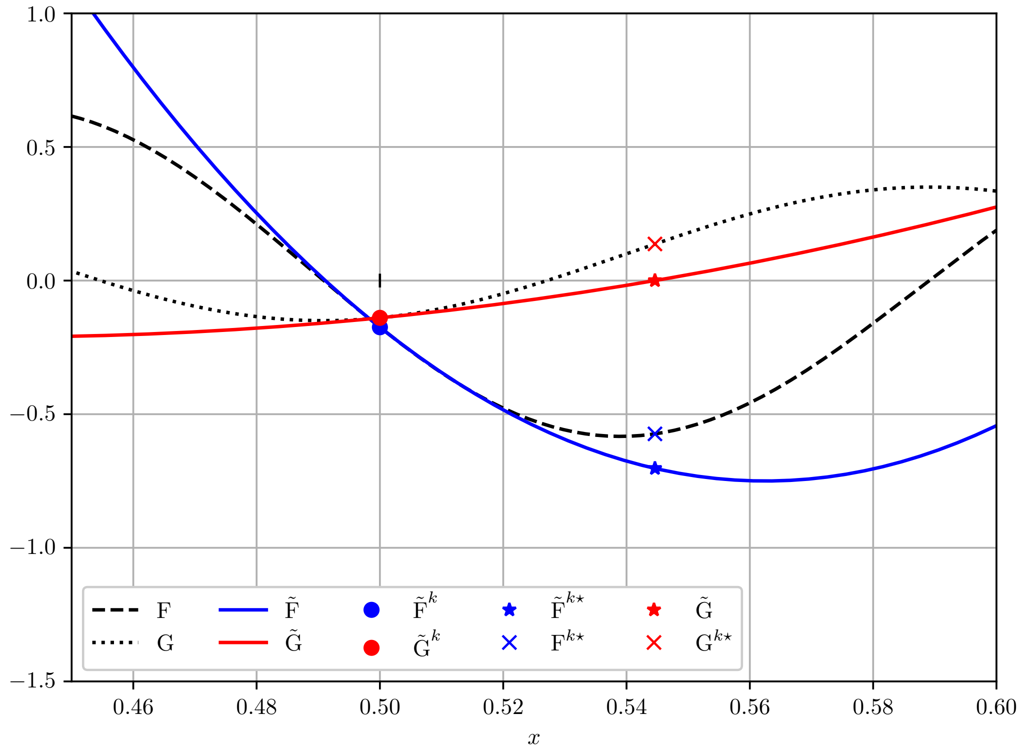
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 4.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 4.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.335$
at the solution
$x^{k\star} = 0.545$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -7.028706e-01$
$\tilde{\mathrm{G}}^{k\star} = -7.291763e-08$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.736618e-01$
$\mathrm{G}^{k\star} = 1.370522e-01$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

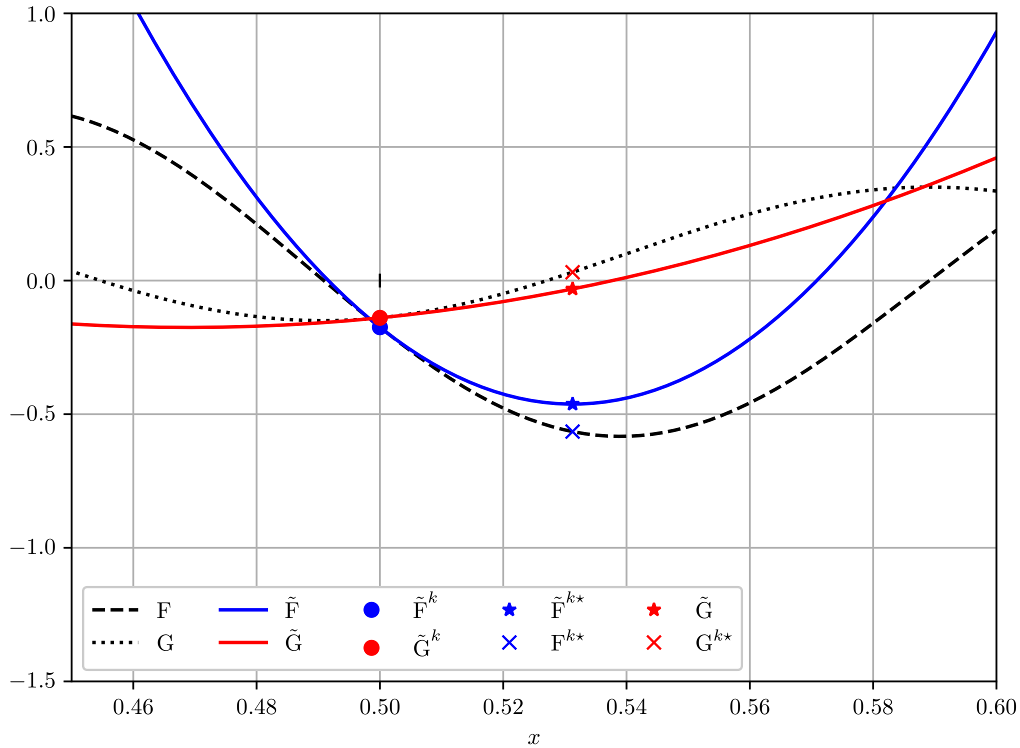
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.000$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.623186e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.145113e-02$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.651764e-01$
$\mathrm{G}^{k\star} = 3.120917e-02$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

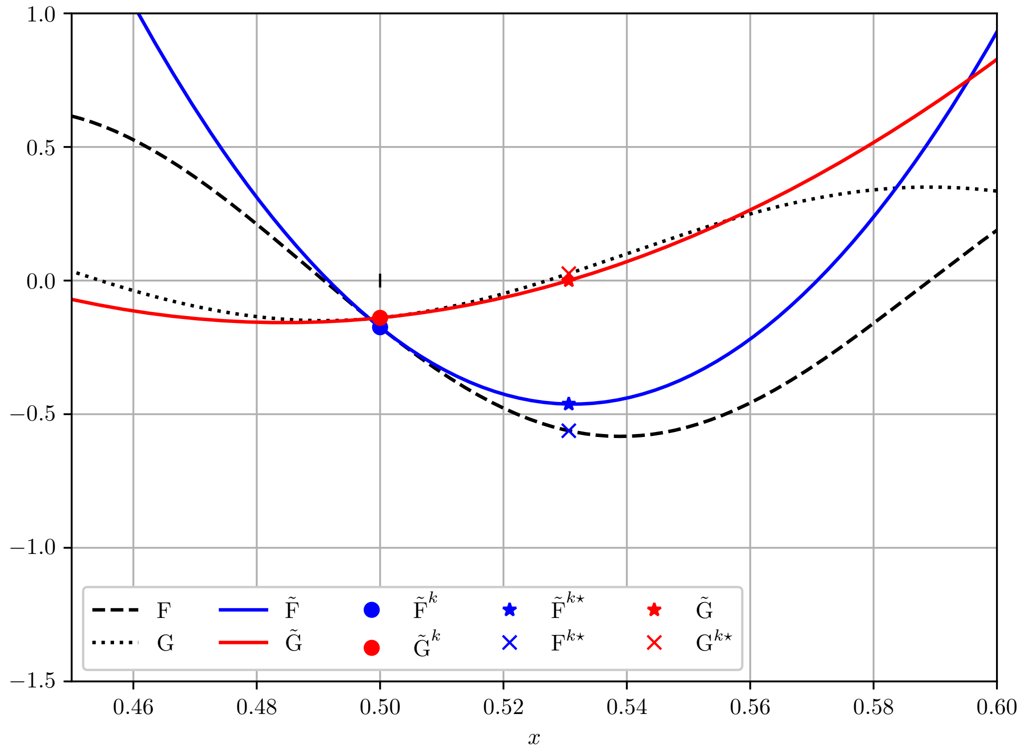
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 16.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.057$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.621898e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.493519e-09$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.619988e-01$
$\mathrm{G}^{k\star} = 2.613916e-02$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

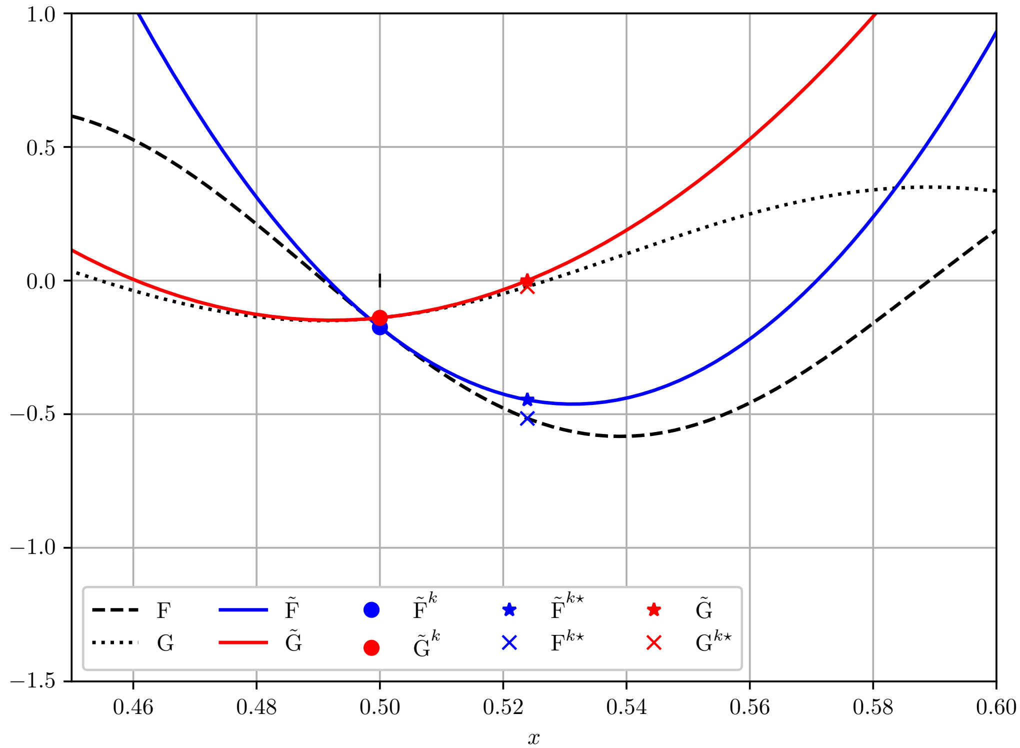
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 32.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.462$
at the solution
$x^{k\star} = 0.524$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.464811e-01$
$\tilde{\mathrm{G}}^{k\star} = 9.505003e-08$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.158540e-01$
$\mathrm{G}^{k\star} = -2.278227e-02$

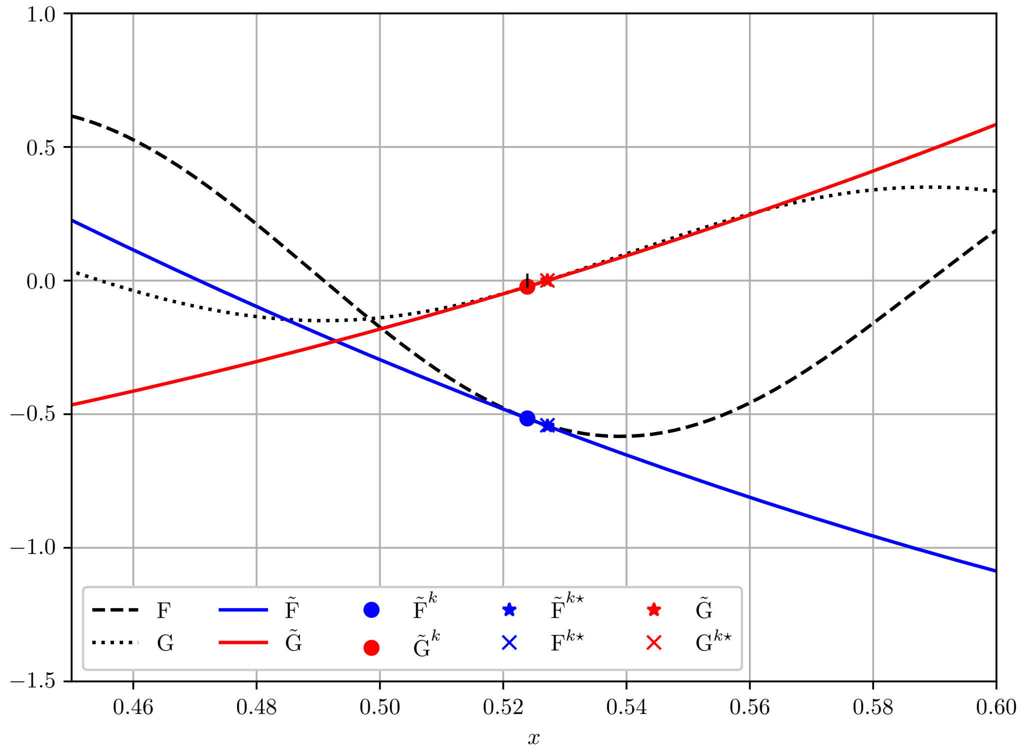
Both the value objective function approximation and the constraint function approximation, at the new design point $x^{k\star}$, is *conservative* with respect to the actual function values, $\tilde{\mathrm{F}}^{k\star} > \mathrm{F}^{k\star}$, $\tilde{\mathrm{G}}^{k\star} > \mathrm{G}^{k\star}$

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.231$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.442415e-01$
$\tilde{\mathrm{G}}^{k\star} = 1.200414e-06$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.414921e-01$
$\mathrm{G}^{k\star} = 4.830983e-04$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

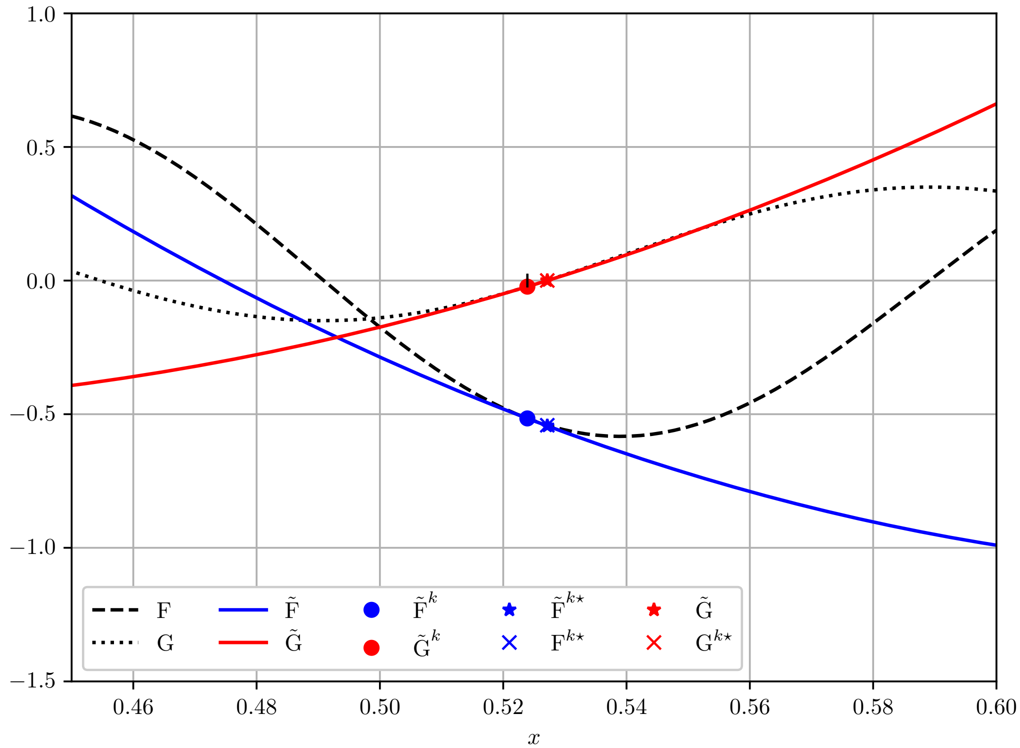
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_{\mathrm{F}} = 2.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_{\mathrm{G}} = 2.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.201$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.438946e-01$
$\tilde{\mathrm{G}}^{k\star} = 2.464175e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.413539e-01$
$\mathrm{G}^{k\star} = 3.378196e-04$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

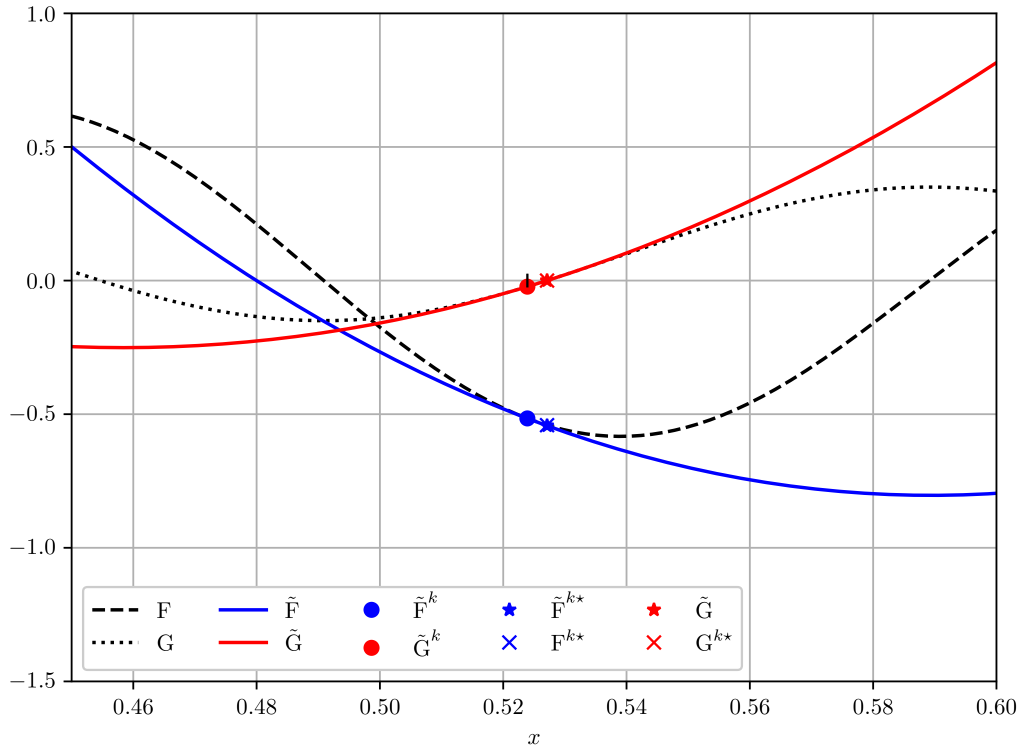
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_{\mathrm{F}} = 4.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_{\mathrm{G}} = 4.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.144$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.432267e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.228880e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410877e-01$
$\mathrm{G}^{k\star} = 5.887112e-05$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

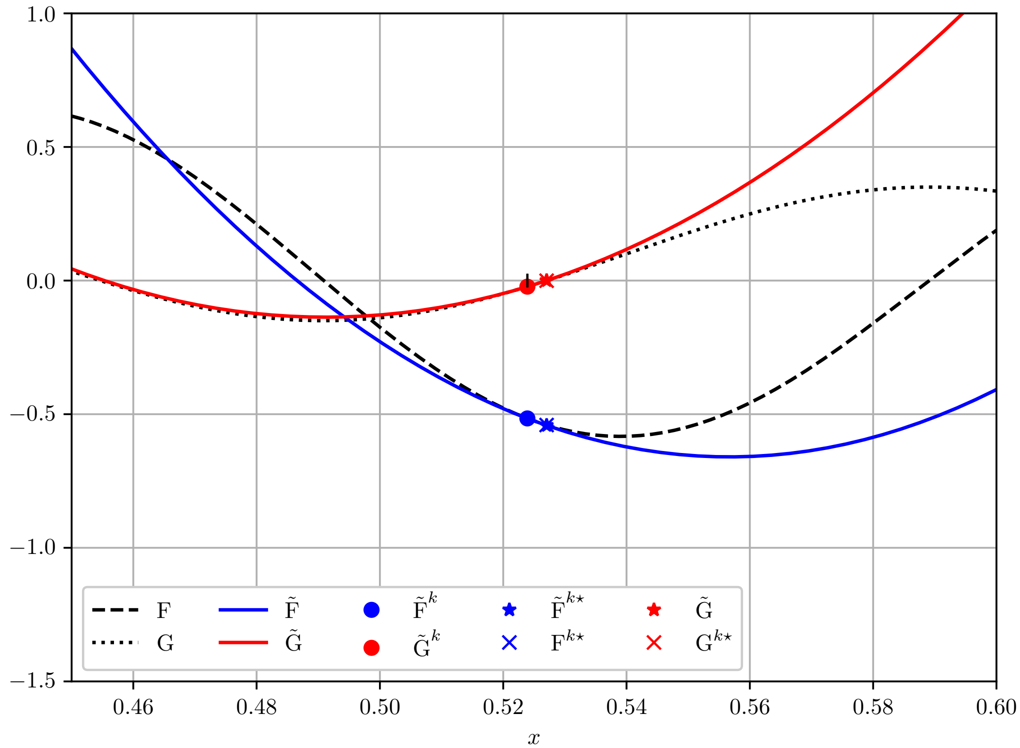
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_\mathrm{G} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.042$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.419799e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.188763e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.405892e-01$
$\mathrm{G}^{k\star} = -4.607702e-04$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

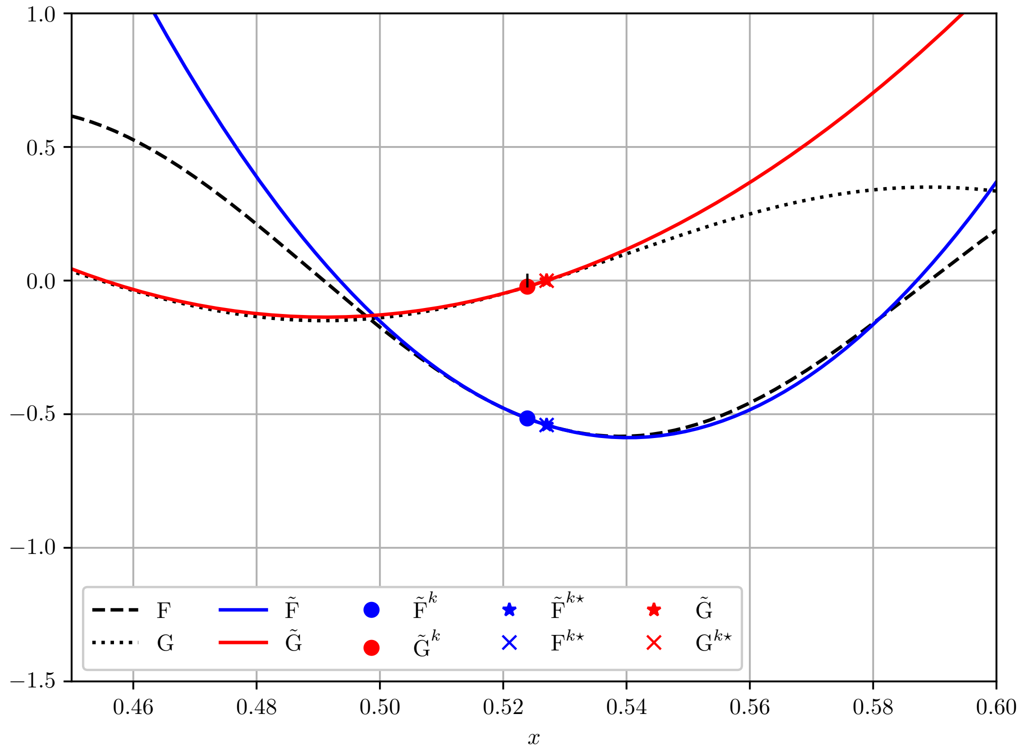
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_\mathrm{F} = 16.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_\mathrm{G} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.932$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.406727e-01$
$\tilde{\mathrm{G}}^{k\star} = -9.766279e-08$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.405892e-01$
$\mathrm{G}^{k\star} = -4.607498e-04$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

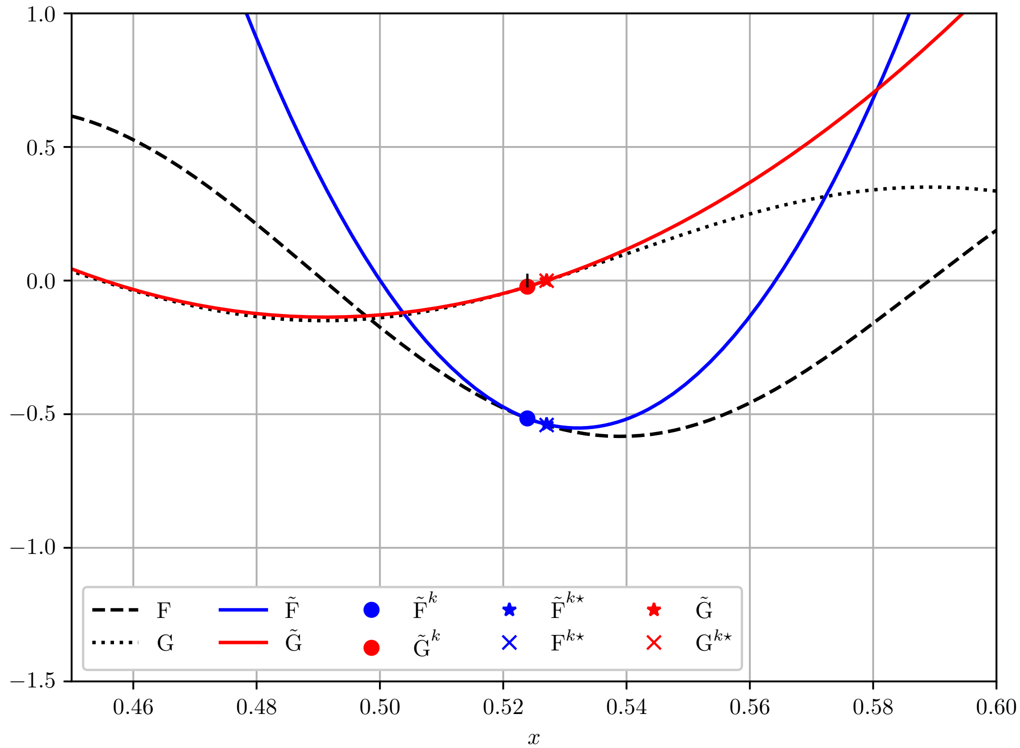
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_{\mathrm{F}} = 32.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_{\mathrm{G}} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.713$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.380584e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.847116e-08$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.405893e-01$
$\mathrm{G}^{k\star} = -4.606930e-04$

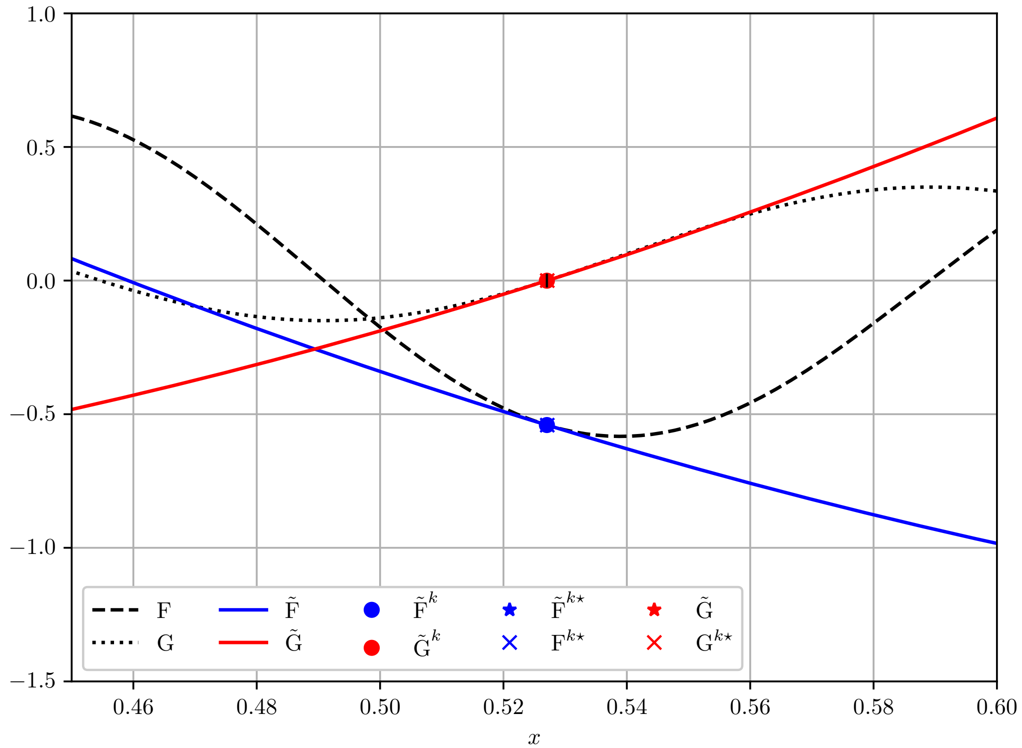
Both the value objective function approximation and the constraint function approximation, at the new design point $x^{k\star}$, is *conservative* with respect to the actual function values, $\tilde{\mathrm{F}}^{k\star} > \mathrm{F}^{k\star}$, $\tilde{\mathrm{G}}^{k\star} > \mathrm{G}^{k\star}$

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.051$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.756$
$\alpha_\mathrm{F} = 1.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.326$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.799$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.962$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410309e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.755736e-06$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410298e-01$
$\mathrm{G}^{k\star} = -1.608758e-06$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

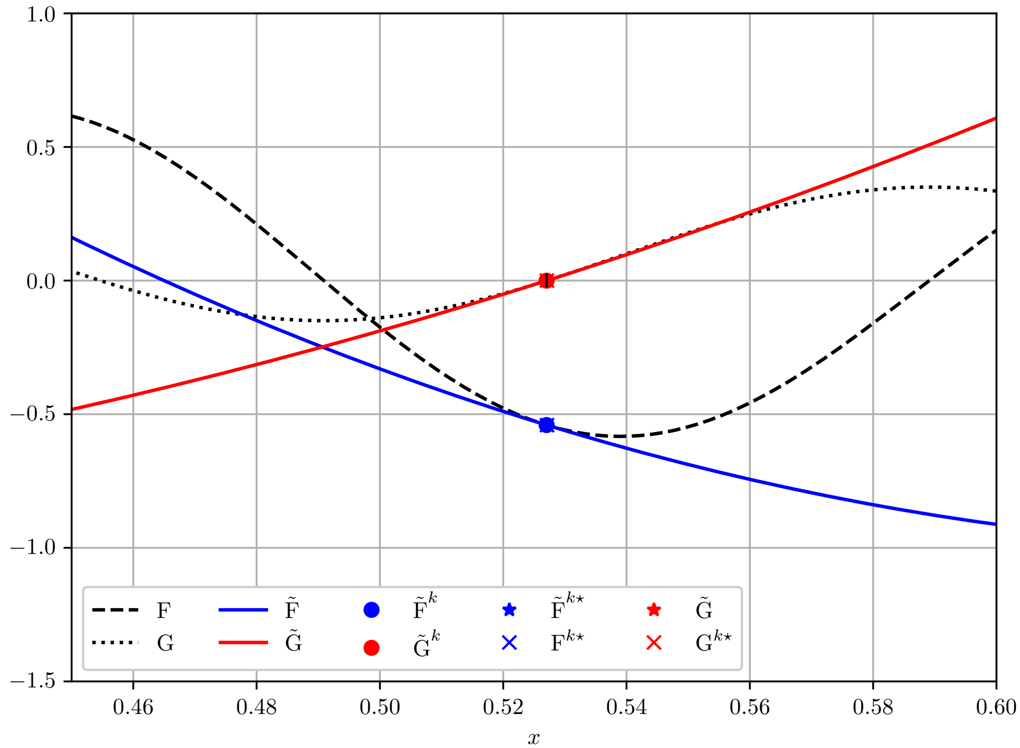
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.051$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.756$
$\alpha_\mathrm{F} = 2.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.326$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.799$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.962$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410322e-01$
$\tilde{\mathrm{G}}^{k\star} = -2.867190e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410312e-01$
$\mathrm{G}^{k\star} = -1.387999e-07$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

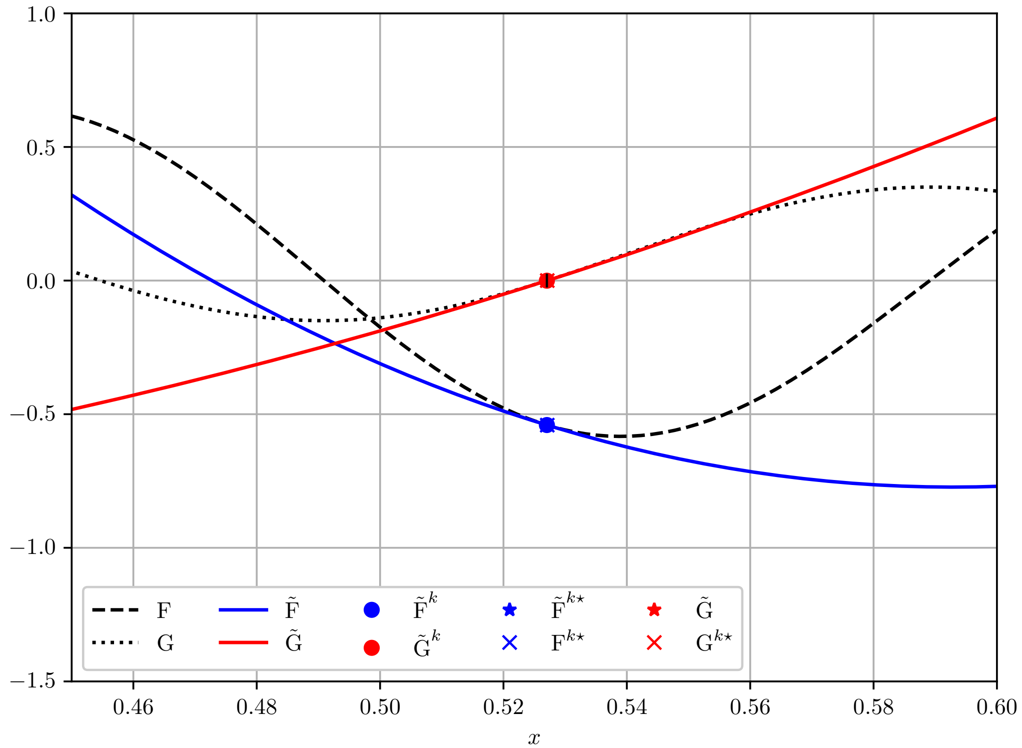
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.051$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.756$
$\alpha_\mathrm{F} = 4.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.326$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.799$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.961$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410319e-01$
$\tilde{\mathrm{G}}^{k\star} = -5.376857e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410310e-01$
$\mathrm{G}^{k\star} = -3.899276e-07$

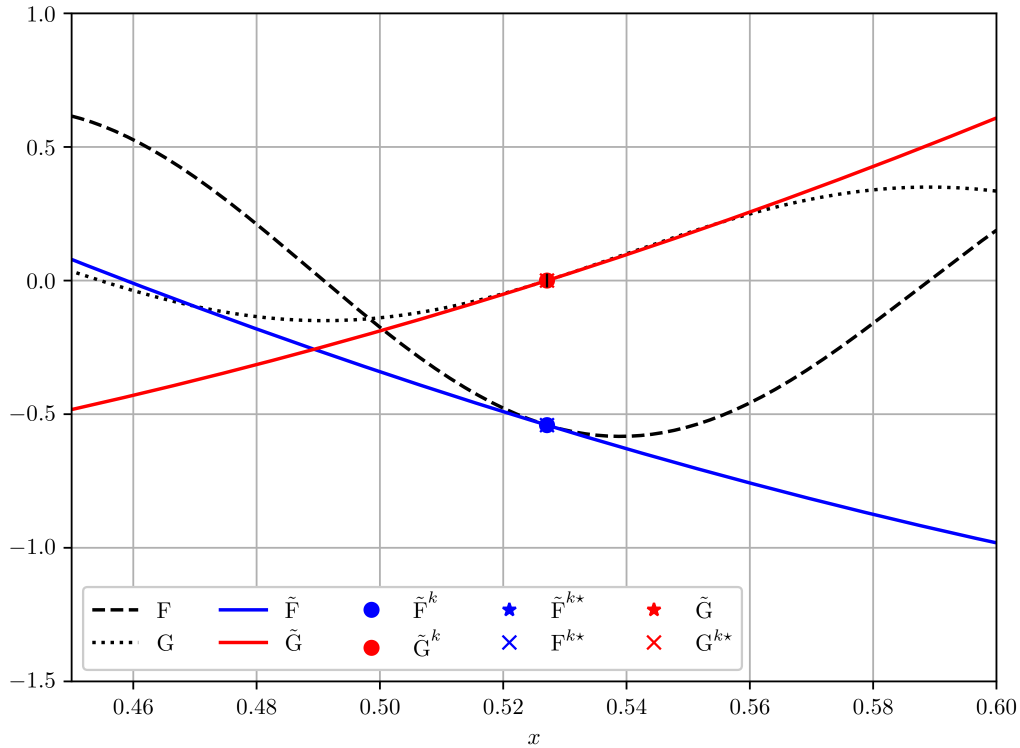
Both the value objective function approximation and the constraint function approximation, at the new design point $x^{k\star}$, is *conservative* with respect to the actual function values, $\tilde{\mathrm{F}}^{k\star} > \mathrm{F}^{k\star}$, $\tilde{\mathrm{G}}^{k\star} > \mathrm{G}^{k\star}$

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.015$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.617$
$\alpha_\mathrm{F} = 1.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.332$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.821$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.957$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410314e - 01$
$\tilde{\mathrm{G}}^{k\star} = 5.323905e - 08$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410314e - 01$
$\mathrm{G}^{k\star} = 5.323919e - 08$

Both the value objective function approximation and the constraint function approximation, at the new design point $x^{k\star}$, is *conservative* with respect to the actual function values, $\tilde{\mathrm{F}}^{k\star} > \mathrm{F}^{k\star}$, $\tilde{\mathrm{G}}^{k\star} > \mathrm{G}^{k\star}$

Terminated on $|x^{k\star} - x^k| < 1.0\text{e-}06$

Python code