Groenwolds's relaxed conservatism

See algorithm in appended Python code.
The objective and constraint function is given verbatim below.
(Notice the naive choice of (a quadratic approximation to) a reciprocal intervening variable based function, `dd`...)

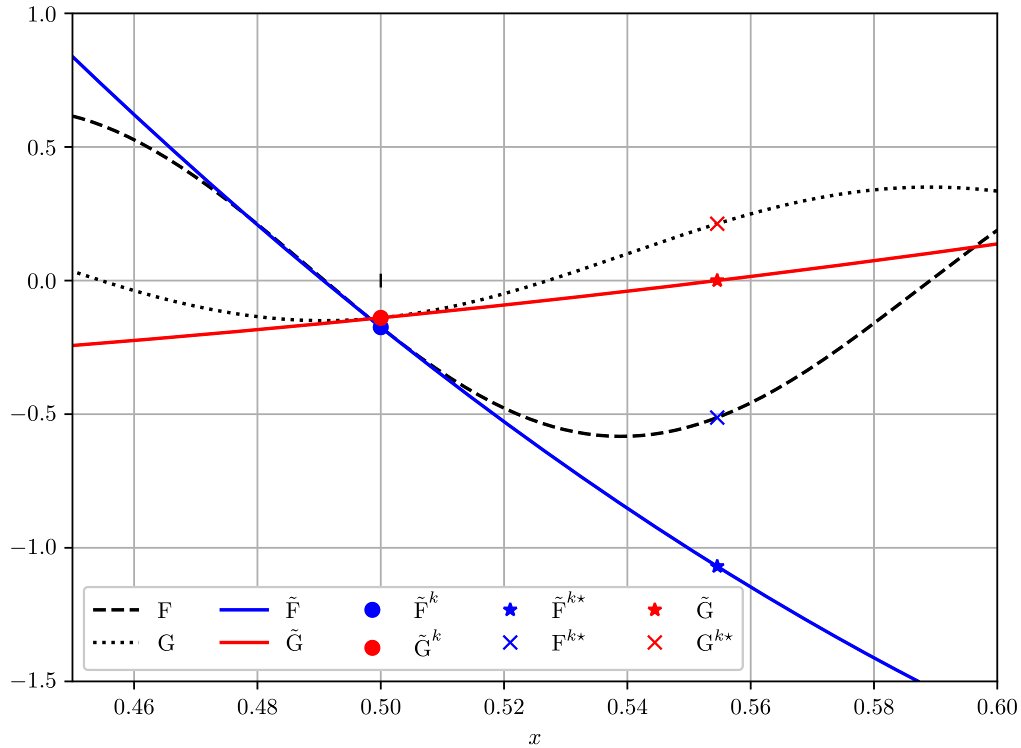
```
#
import numpy as np
#
def obj(x):
#
    a = 1.; b = 32.; c = -1.
    f = a*np.sin(b*x)*np.exp(c*x)
    df = a*c*np.sin(b*x)*np.exp(c*x) + a*b*np.exp(c*x)*np.cos(b*x)
#   ddf = a*c*b*np.cos(b*x)*np.exp(c*x) + a*c*c*np.sin(b*x)*np.exp(c*x) \
#       + a*c*b*np.exp(c*x)*np.cos(b*x) - a*b*b*np.exp(c*x)*np.sin(b*x)
    ddf = abs(-2./x*df) # quad. approx. to reciprocal intervening variables
#
    return [f, df, ddf]
#
def con(x):
#
    a = 1./4.; b = 32.
    g = a*np.cos(b*x)+0.1
    dg = -a*b*np.sin(b*x)
#   ddg = -a*b*b*np.cos(b*x)
    ddg = abs(-2./x*dg) # quad. approx. to reciprocal intervening variables
#
    return [g, dg, ddg]
#
```

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 5.129$
at the solution
$x^{k\star} = 0.555$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -1.070e+00$
$\tilde{\mathrm{G}}^{k\star} = -3.929e-10$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.127e-01$
$\mathrm{G}^{k\star} = 2.127e-01$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

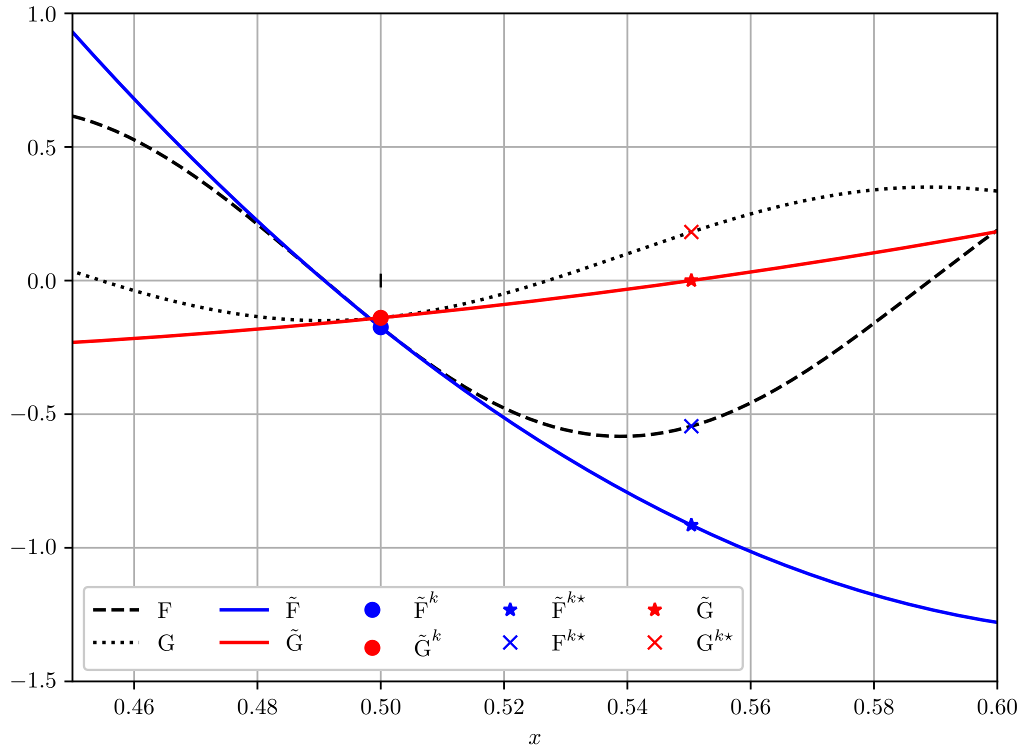
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 2.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 2.000$

with Lagrange multiplier
$\lambda^{k\star} = 3.401$
at the solution
$x^{k\star} = 0.550$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -9.153e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.680e-10$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.450e-01$
$\mathrm{G}^{k\star} = 1.818e-01$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

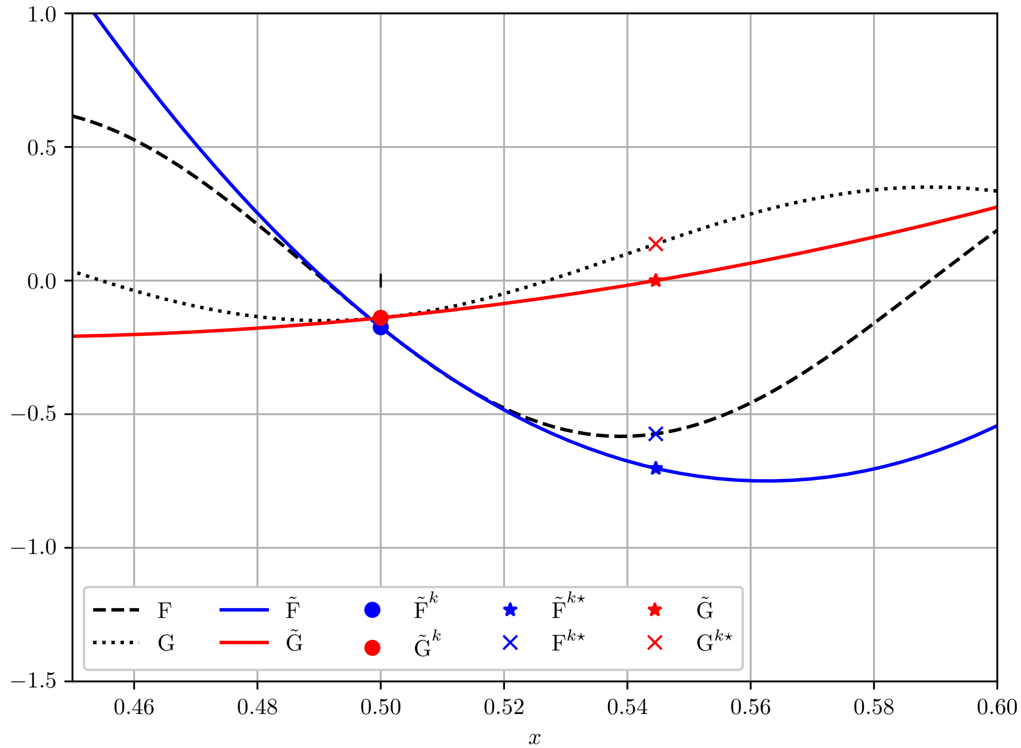
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 4.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 4.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.335$
at the solution
$x^{k\star} = 0.545$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -7.029e-01$
$\tilde{\mathrm{G}}^{k\star} = -4.449e-11$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.737e-01$
$\mathrm{G}^{k\star} = 1.371e-01$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

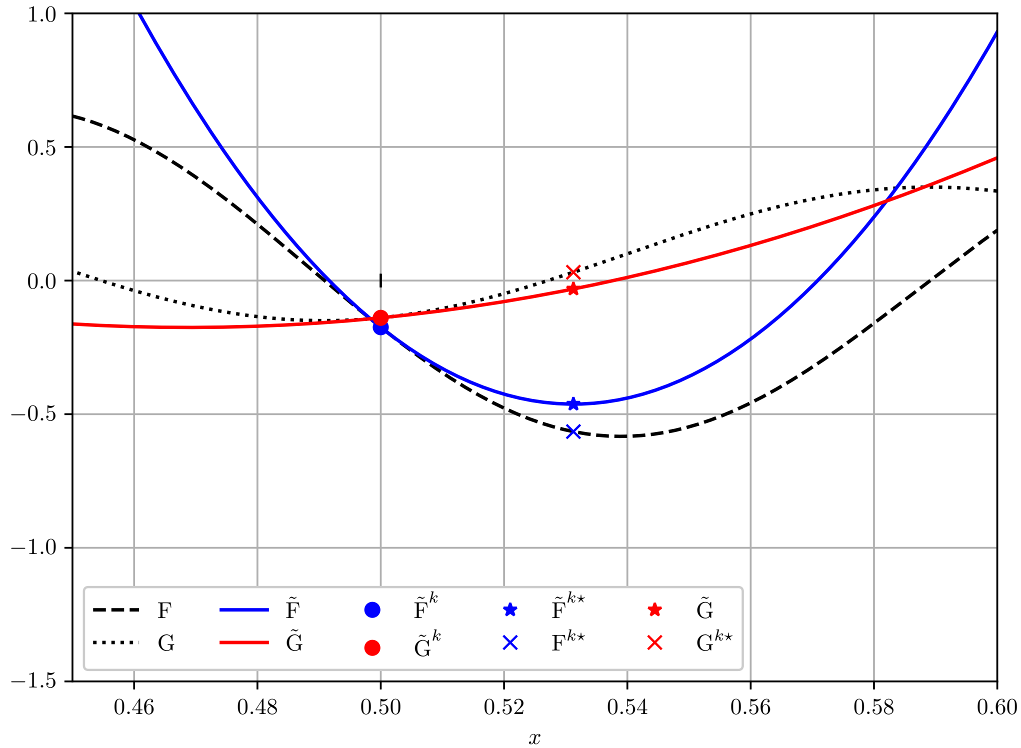
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.000$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.623e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.145e-02$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.652e-01$
$\mathrm{G}^{k\star} = 3.121e-02$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

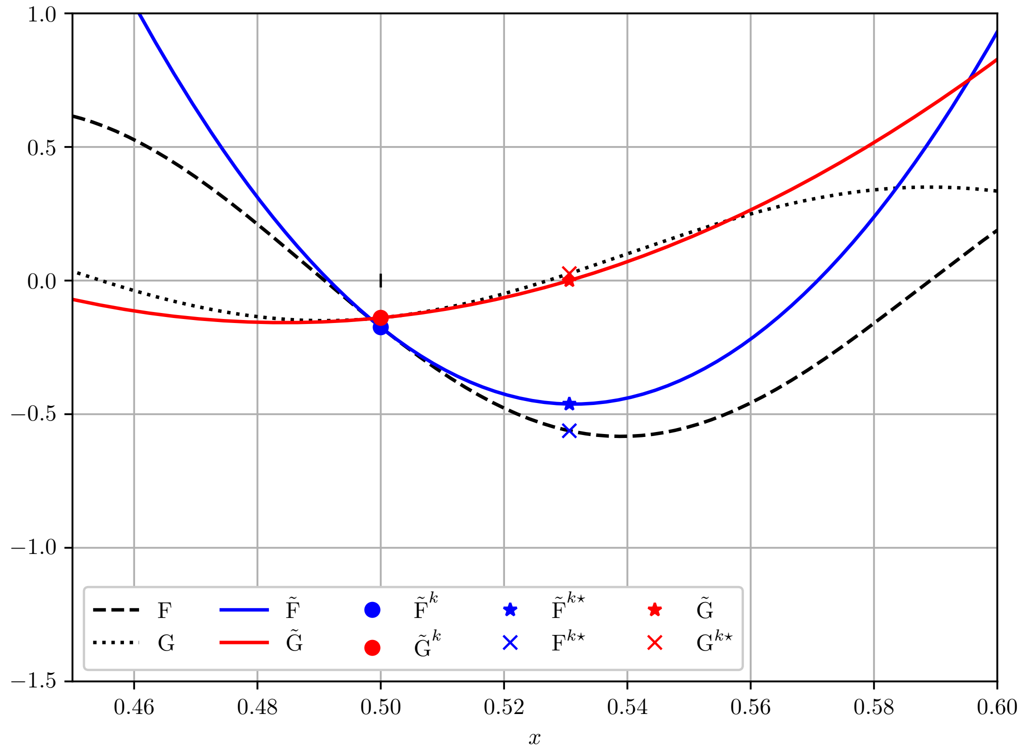
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 16.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.057$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.622e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.387e-12$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.620e-01$
$\mathrm{G}^{k\star} = 2.614e-02$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

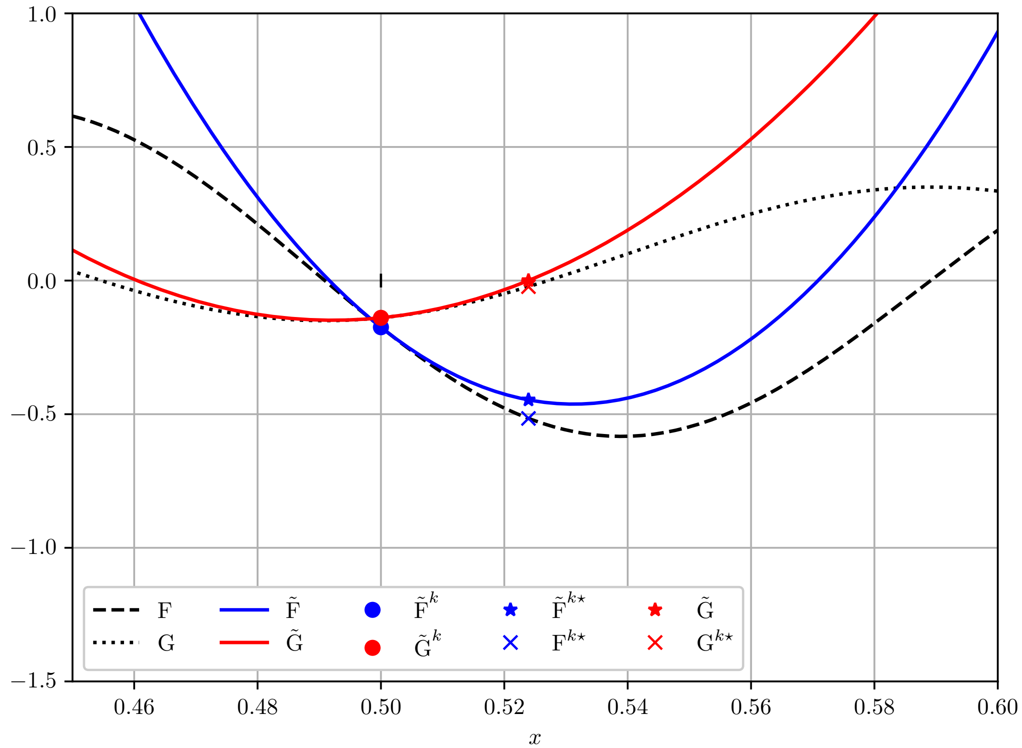
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 32.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.462$
at the solution
$x^{k\star} = 0.524$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.465e-01$
$\tilde{\mathrm{G}}^{k\star} = -6.761e-11$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.159e-01$
$\mathrm{G}^{k\star} = -2.278e-02$

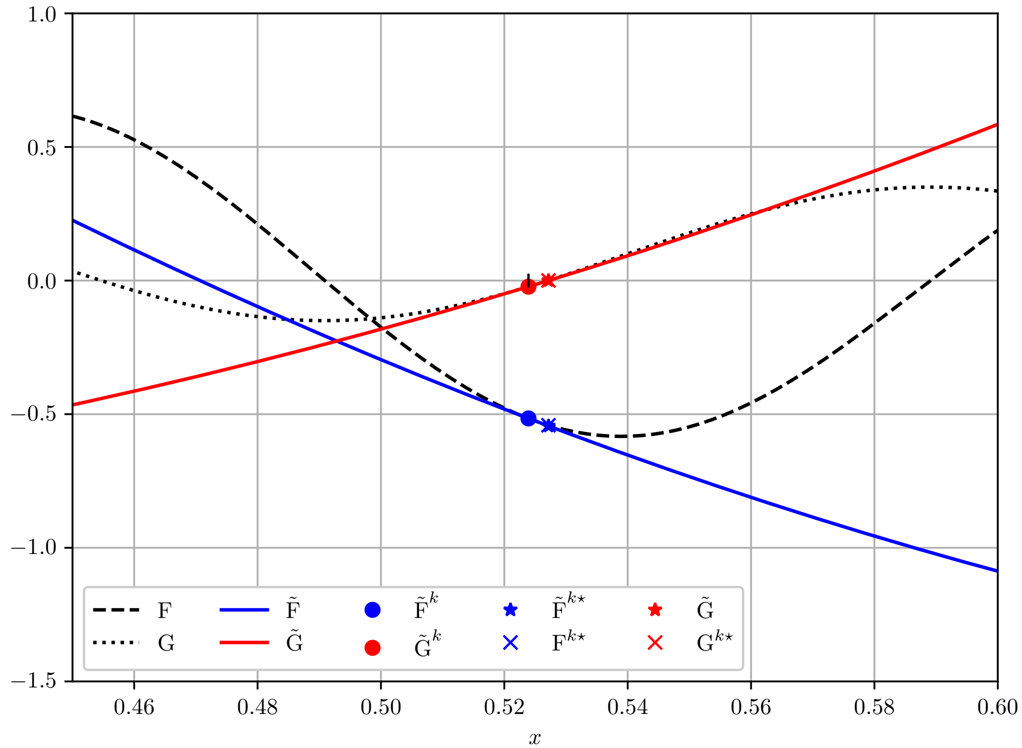
The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(In this case both approximations are indeed conservative, nevertheless.)

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.231$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.442e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.121e-09$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.415e-01$
$\mathrm{G}^{k\star} = 4.819e-04$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

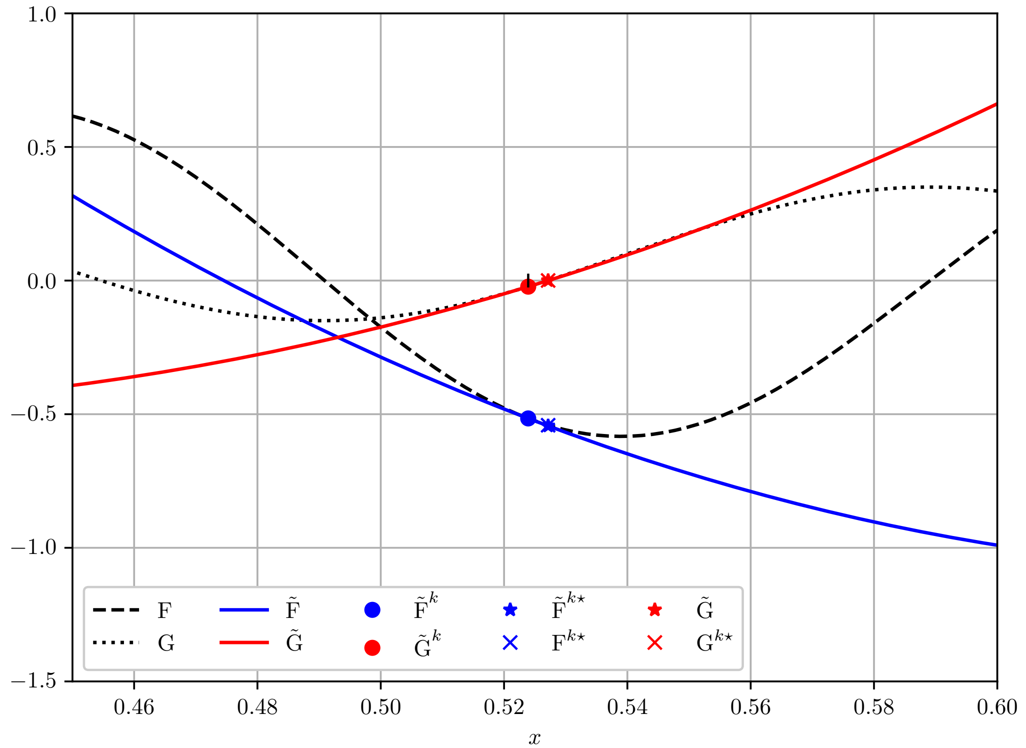
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_\mathrm{F} = 2.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_\mathrm{G} = 2.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.201$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.439e-01$
$\tilde{\mathrm{G}}^{k\star} = 4.003e-10$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.414e-01$
$\mathrm{G}^{k\star} = 3.376e-04$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

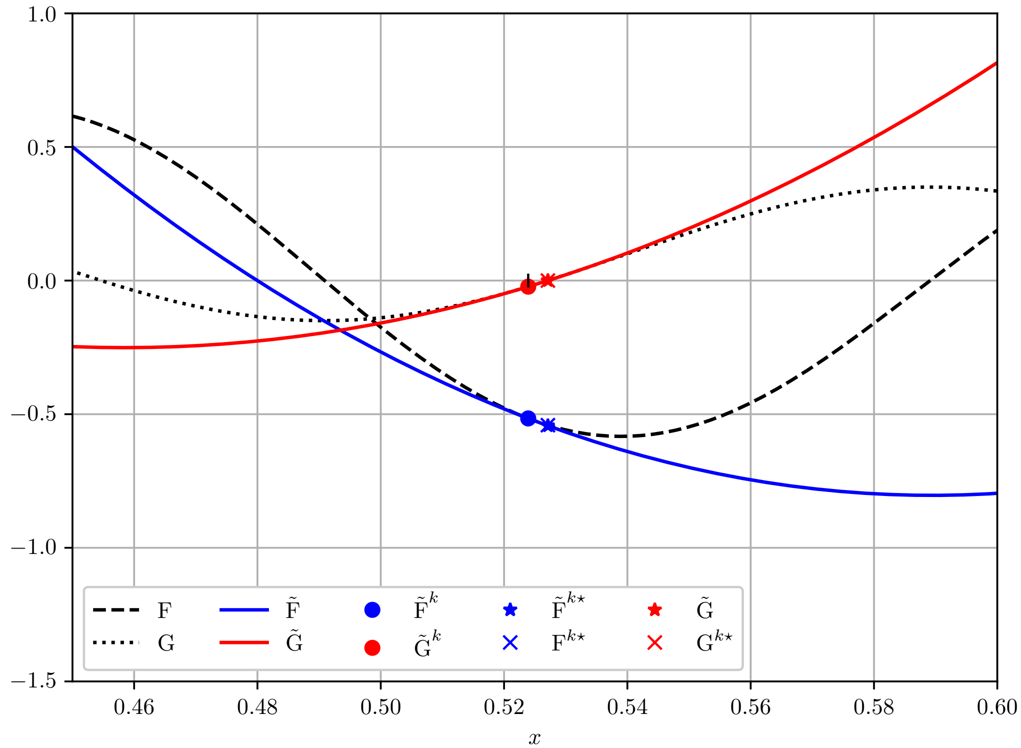
That is, at least one approximation function is not *conservative.*

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_{\mathrm{F}} = 4.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_{\mathrm{G}} = 4.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.144$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.432e-01$
$\tilde{\mathrm{G}}^{k\star} = 5.598e-11$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.411e-01$
$\mathrm{G}^{k\star} = 5.920e-05$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

That is, at least one approximation function is not *conservative*.

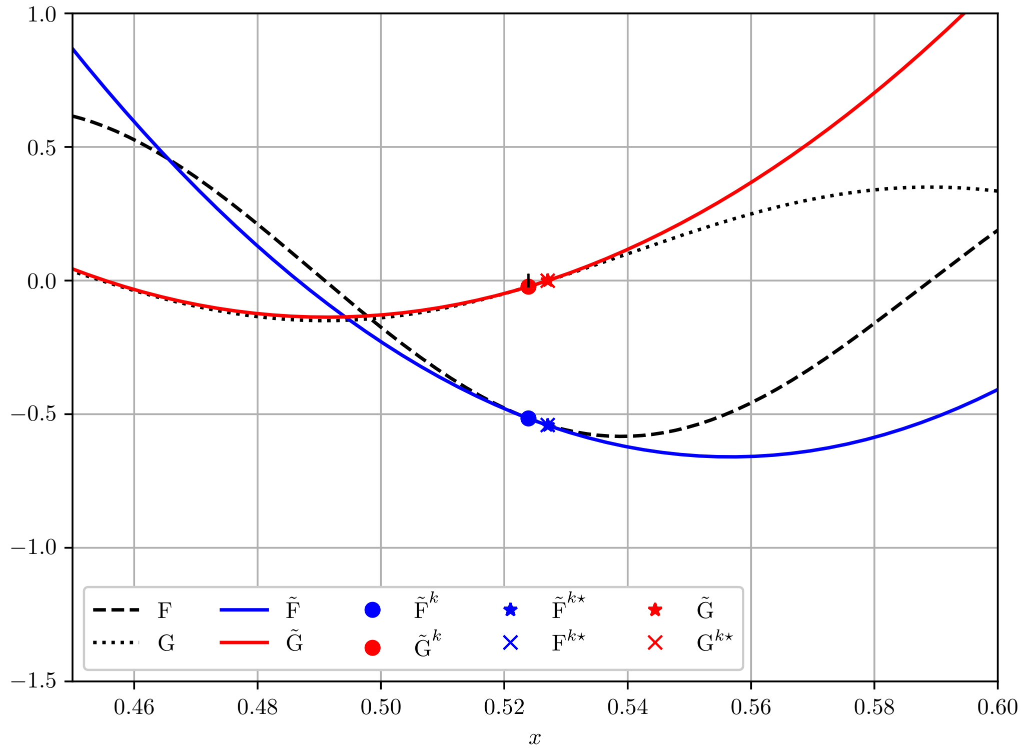
Iteration 9

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.791$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.559$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.969$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.602$
$\alpha_\mathrm{G} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.042$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.420e-01$
$\tilde{\mathrm{G}}^{k\star} = 1.244e-14$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.406e-01$
$\mathrm{G}^{k\star} = -4.607e-04$

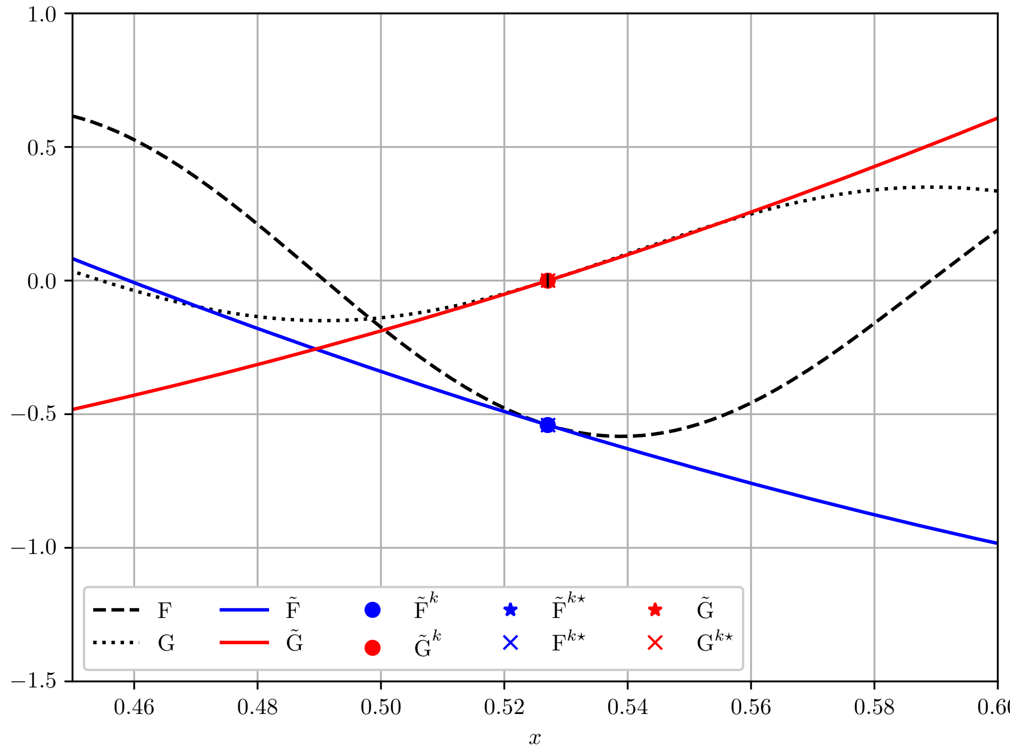
The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(Note that the objective and/or constraint function approximation is not conservative, in this case.)

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.051$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.756$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.326$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.799$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.962$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410e-01$
$\tilde{\mathrm{G}}^{k\star} = -6.488e-10$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410e-01$
$\mathrm{G}^{k\star} = 1.474e-07$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

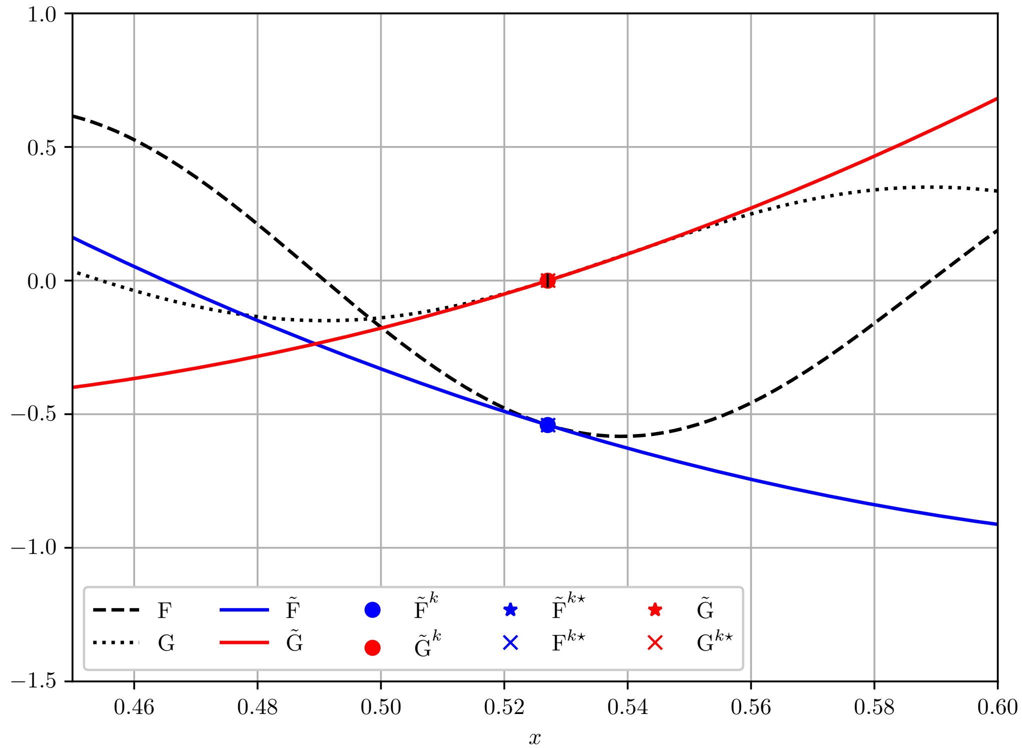
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.051$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.756$
$\alpha_\mathrm{F} = 2.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.326$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.799$
$\alpha_\mathrm{G} = 2.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.962$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.769e-10$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410e-01$
$\mathrm{G}^{k\star} = 9.293e-08$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

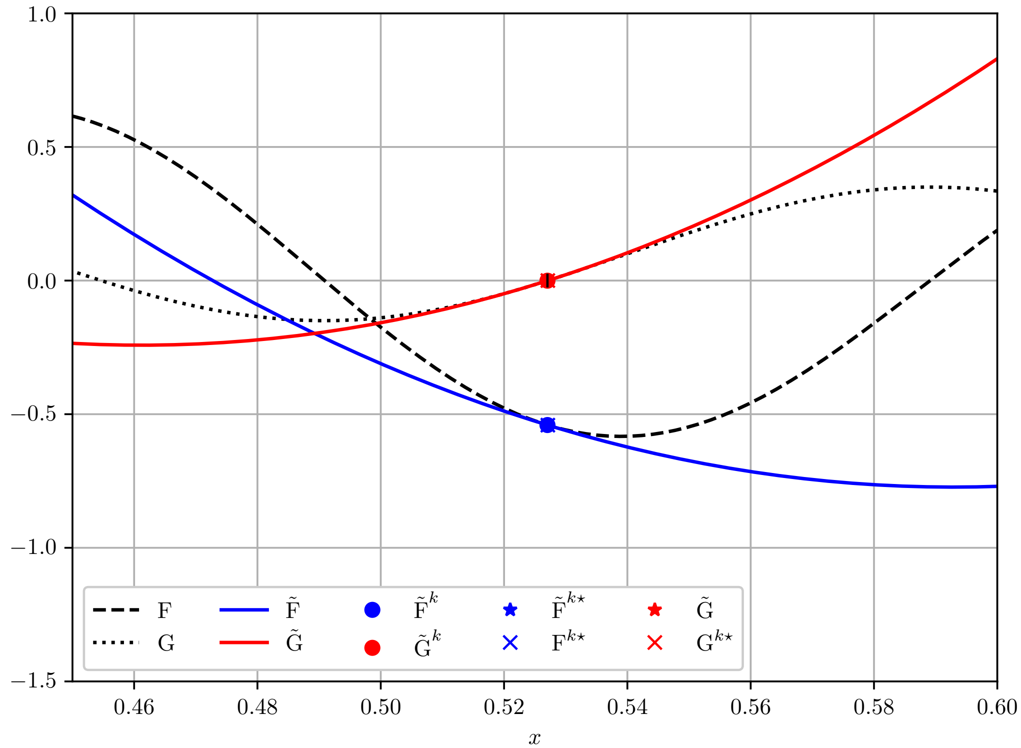
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.051$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.756$
$\alpha_{\mathrm{F}} = 4.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.326$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.799$
$\alpha_{\mathrm{G}} = 4.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.961$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.338e-10$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410e-01$
$\mathrm{G}^{k\star} = -1.709e-08$

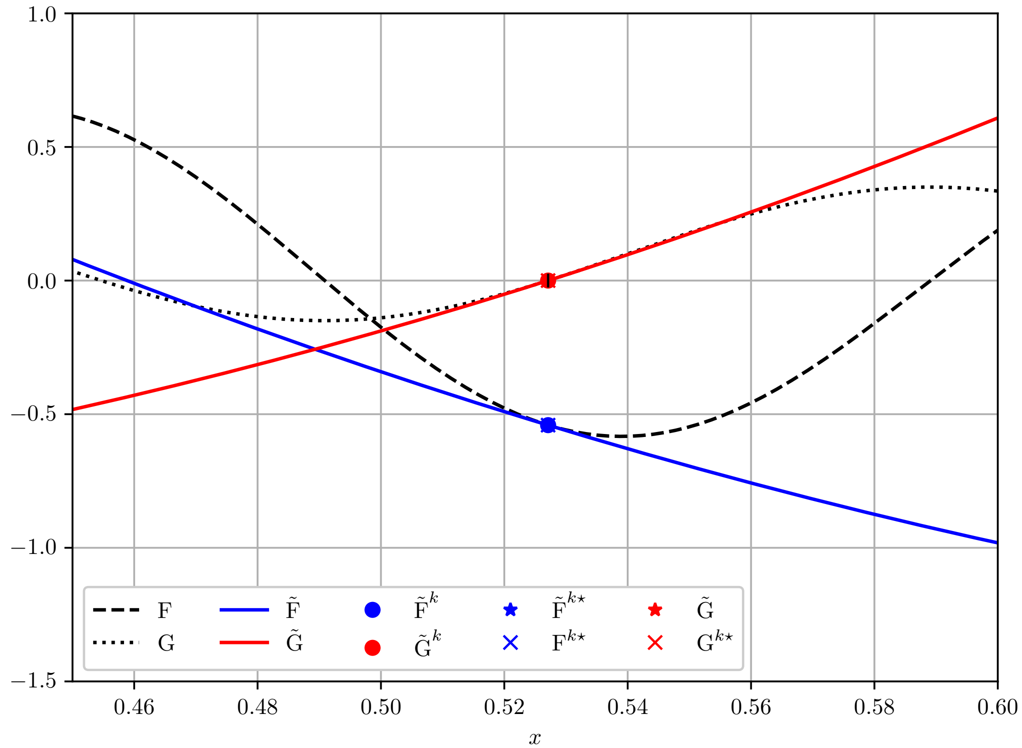
The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(Note that the objective and/or constraint function approximation is not conservative, in this case.)

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.541$
$\mathrm{d}_x^k\mathrm{F} = -7.015$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.617$
$\alpha_\mathrm{F} = 1.000$

$\mathrm{G}^k = -0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.332$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.821$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.957$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.069e-09$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410e-01$
$\mathrm{G}^{k\star} = -1.069e-09$

The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(Note that the objective and/or constraint function approximation is not conservative, in this case.)

Terminated on $|x^{k\star} - x^k| < 1\text{e-}6$

Python code