Groenwolds's relaxed conservatism

See algorithm in appended Python code. The objective and the constraint function is given verbatim below. (Notice the naive choice of (a quadratic approximation to) a function linearised in terms of reciprocal intervening variables, `dd`.... The reader is encouraged to, for example, uncomment the analytic second order information.)

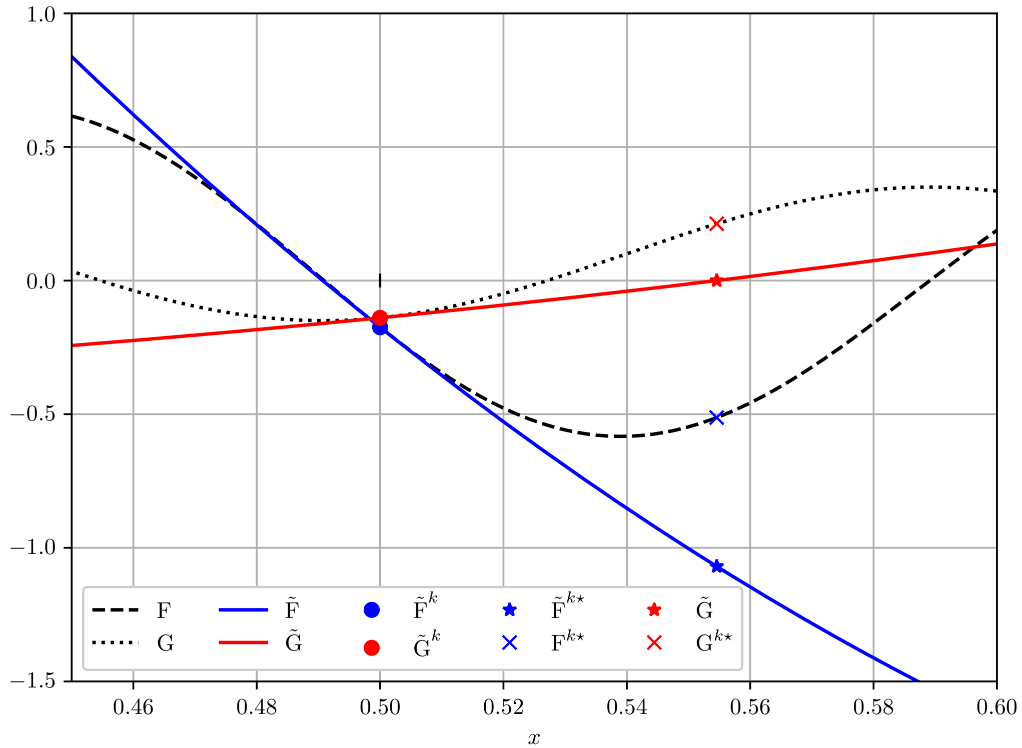
```
#
import numpy as np
#
def obj(x):
#
    a = 1.; b = 32.; c = -1.
    f = a*np.sin(b*x)*np.exp(c*x)
    df = a*c*np.sin(b*x)*np.exp(c*x) + a*b*np.exp(c*x)*np.cos(b*x)
#   ddf = 2*a*c*b*np.cos(b*x)*np.exp(c*x)+a*c*c*np.sin(b*x)*np.exp(c*x)-a*b*b*np.exp(c*x)*np.sin(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddf = -2./x*df
    ddf = abs(-2./x*df ) # nonconvex variant
#
    return [f, df, ddf]
#
def con(x):
#
    a = 1./4.; b = 32.
    g = a*np.cos(b*x)+0.1
    dg = -a*b*np.sin(b*x)
#   ddg = -a*b*b*np.cos(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddg = -2./x*dg
    ddg = abs(-2./x*dg)  # nonconvex variant
#
    return [g, dg, ddg]
```

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 1.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 5.130$
at the solution
$x^{k\star} = 0.555$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -1.069746e+00$
$\tilde{\mathrm{G}}^{k\star} = -8.274515e-06$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.126921e-01$
$\mathrm{G}^{k\star} = 2.126648e-01$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

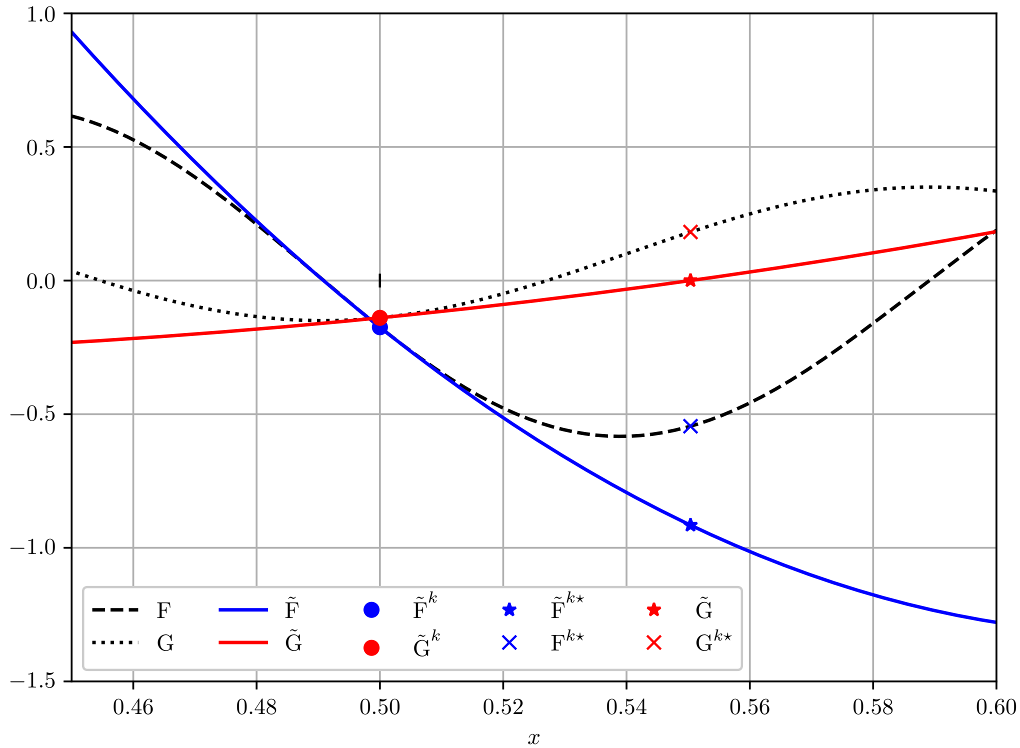
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 2.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 2.000$

with Lagrange multiplier
$\lambda^{k\star} = 3.402$
at the solution
$x^{k\star} = 0.550$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -9.152876e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.946269e-06$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.449878e-01$
$\mathrm{G}^{k\star} = 1.817974e-01$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

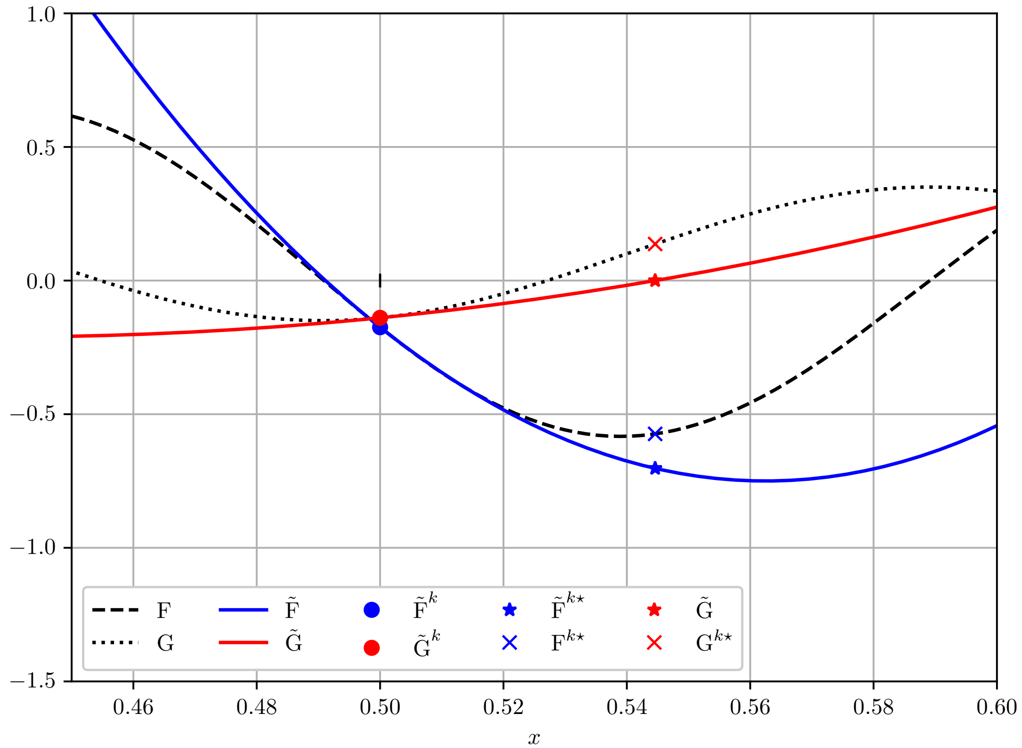
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 4.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 4.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.337$
at the solution
$x^{k\star} = 0.545$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -7.027671e-01$
$\tilde{\mathrm{G}}^{k\star} = -7.757163e-05$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.737270e-01$
$\mathrm{G}^{k\star} = 1.368968e-01$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

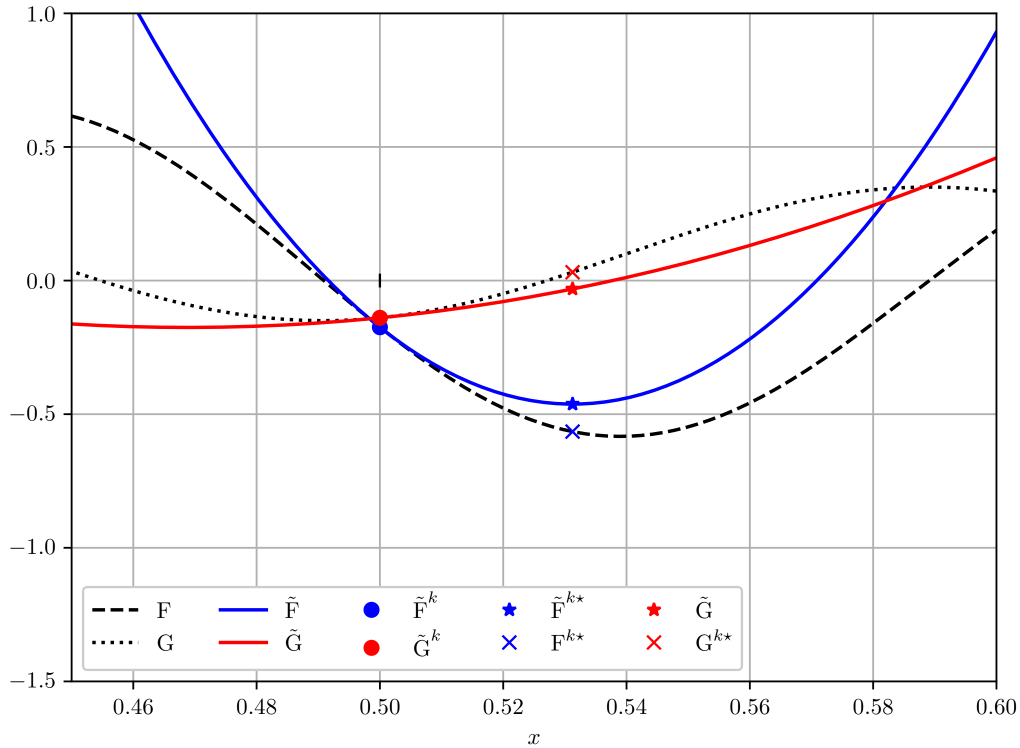
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.000$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.623186e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.145113e-02$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.651764e-01$
$\mathrm{G}^{k\star} = 3.120917e-02$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

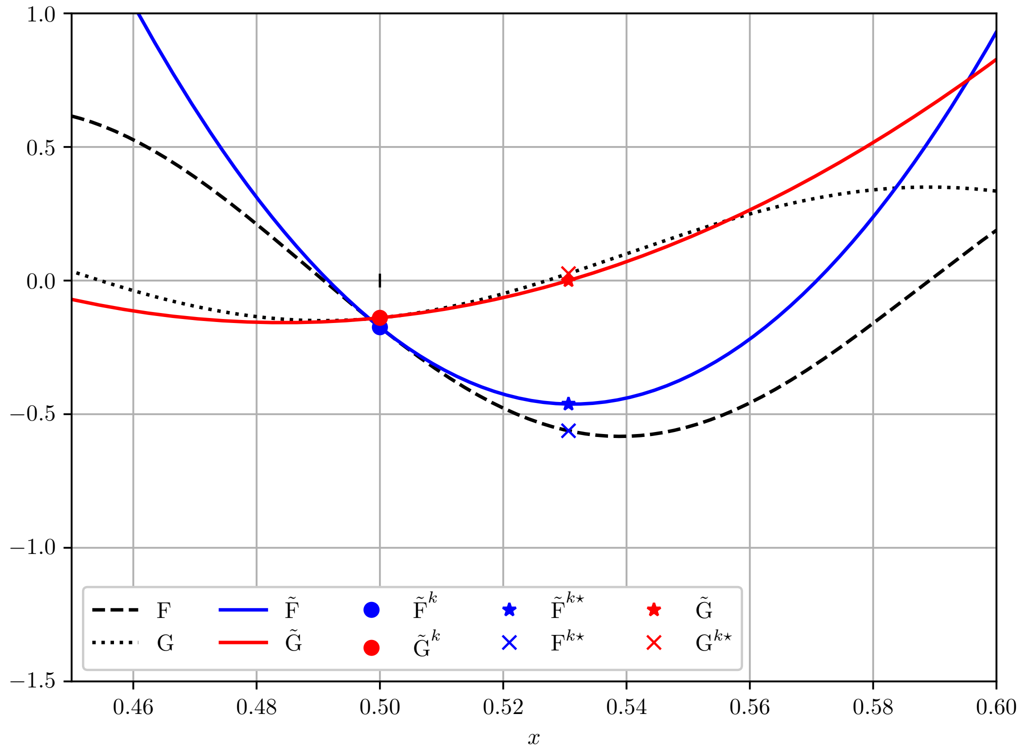
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 16.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.057$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.621899e-01$
$\tilde{\mathrm{G}}^{k\star} = 6.450742e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.619993e-01$
$\mathrm{G}^{k\star} = 2.613989e-02$

However, the solution (step) is not a feasible descent step, and it is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

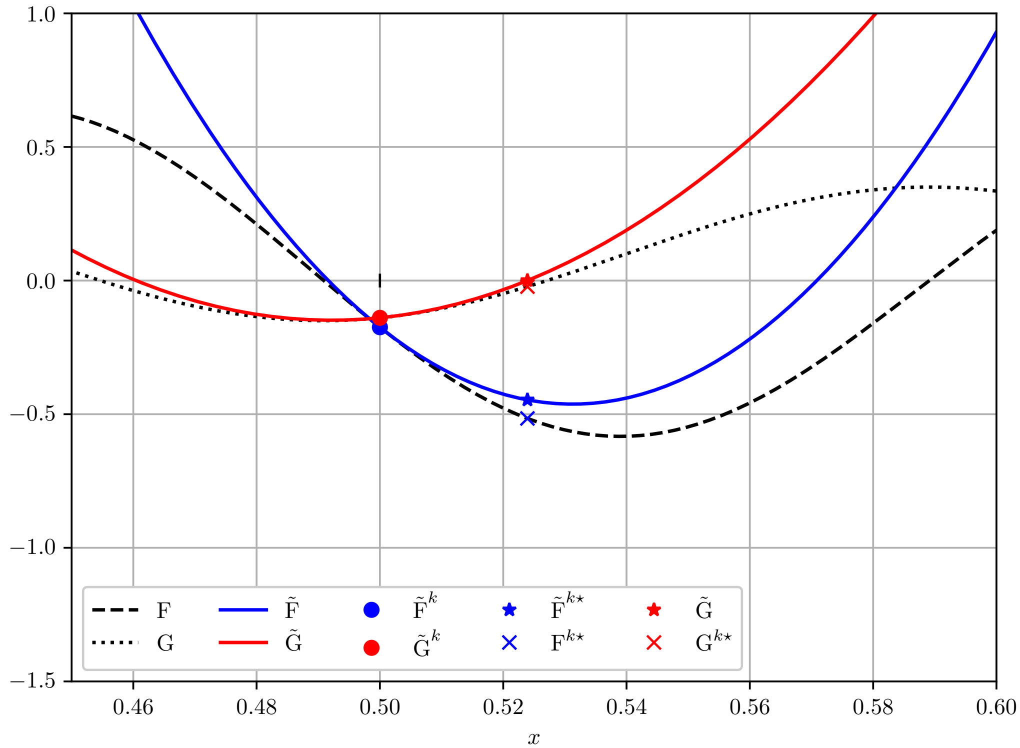
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 32.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.461$
at the solution
$x^{k\star} = 0.524$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.465202e-01$
$\tilde{\mathrm{G}}^{k\star} = 8.467353e-05$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.159334e-01$
$\mathrm{G}^{k\star} = -2.271927e-02$

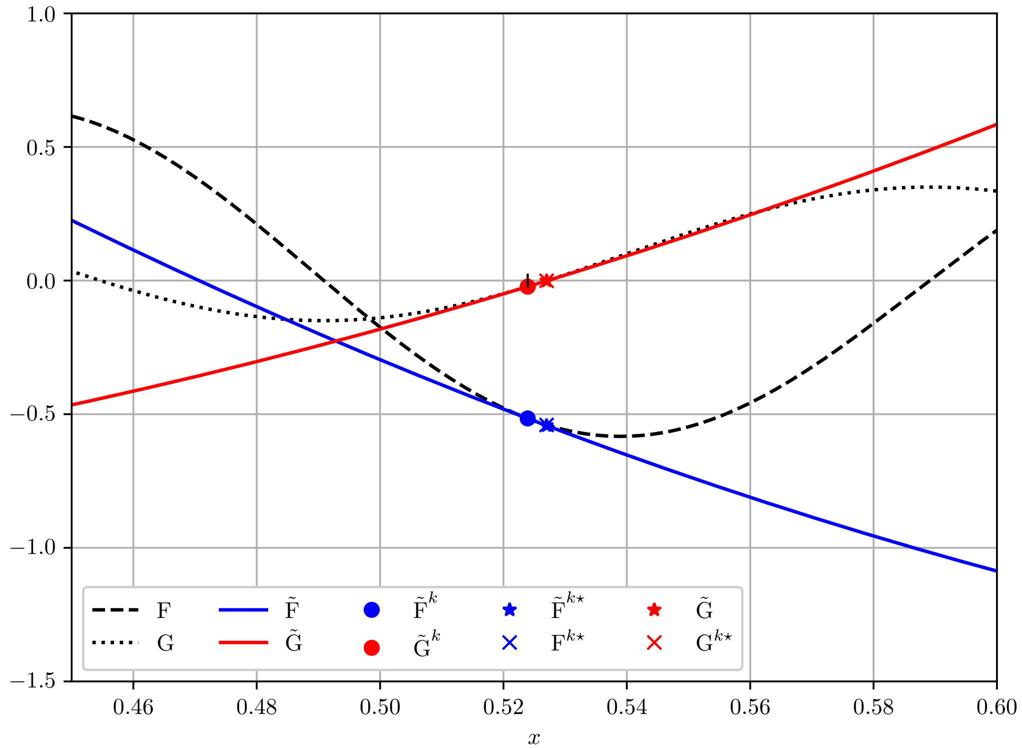
The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(In this case both approximations are indeed conservative, nevertheless.)

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.786$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.540$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.970$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.606$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.231$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.427021e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.234656e-03$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.402579e-01$
$\mathrm{G}^{k\star} = -8.041847e-04$

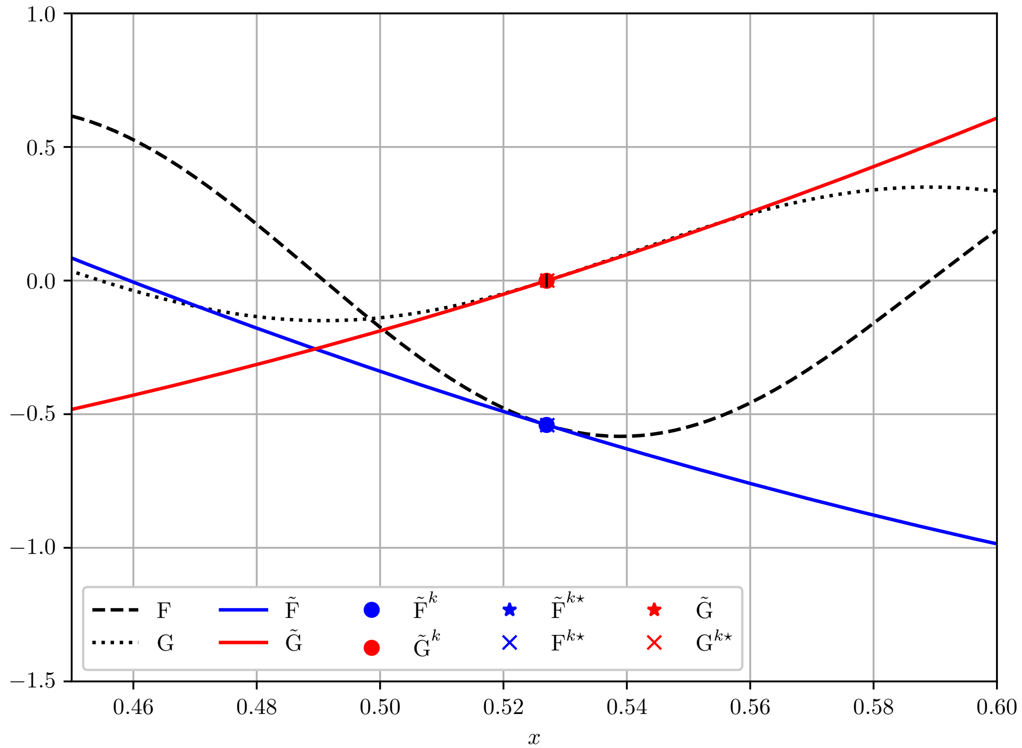
The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(Note that the objective and/or constraint function approximation is not conservative, in this case.)

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.540$
$\mathrm{d}_x^k\mathrm{F} = -7.077$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.859$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.001$
$\mathrm{d}_x^k\mathrm{G} = 7.321$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.784$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.966$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.410859e-01$
$\tilde{\mathrm{G}}^{k\star} = 5.261601e-05$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.410822e-01$
$\mathrm{G}^{k\star} = 5.313044e-05$

The solution leads to a feasible descent step and is therefore deemed *acceptable*, as is.
(In this case both approximations are indeed conservative, nevertheless.)

Terminated on $|x^{k\star} - x^k| < 1.0\text{e-}03$

Python code