Svanberg's (strict) conservatism

See algorithm in appended Python code. The objective and the constraint function is given verbatim below. (Notice the naive choice of (a quadratic approximation to) a function linearised in terms of reciprocal intervening variables, `dd`.... The reader is encouraged to, for example, uncomment the analytic second order information.)

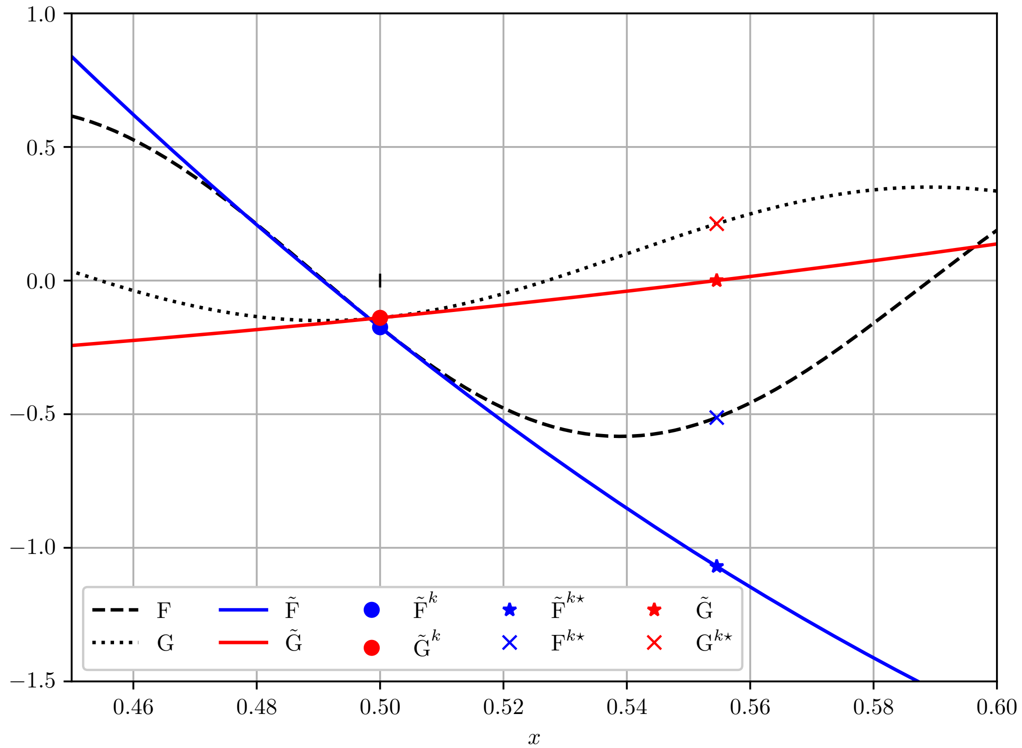
```
#
import numpy as np
#
def obj(x):
#
    a = 1.; b = 32.; c = -1.
    f = a*np.sin(b*x)*np.exp(c*x)
    df = a*c*np.sin(b*x)*np.exp(c*x) + a*b*np.exp(c*x)*np.cos(b*x)
#   ddf = 2*a*c*b*np.cos(b*x)*np.exp(c*x)+a*c*c*np.sin(b*x)*np.exp(c*x)-a*b*b*np.exp(c*x)*np.sin(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddf = -2./x*df
    ddf = abs(-2./x*df ) # nonconvex variant
#
    return [f, df, ddf]
#
def con(x):
#
    a = 1./4.; b = 32.
    g = a*np.cos(b*x)+0.1
    dg = -a*b*np.sin(b*x)
#   ddg = -a*b*b*np.cos(b*x)
#
#   quad. approx. to function linearised in terms of reciprocal intervening variables
#   ddg = -2./x*dg
    ddg = abs(-2./x*dg)  # nonconvex variant
#
    return [g, dg, ddg]
```

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 5.130$
at the solution
$x^{k\star} = 0.555$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -1.069746e+00$
$\tilde{\mathrm{G}}^{k\star} = -8.274515e-06$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.126921e-01$
$\mathrm{G}^{k\star} = 2.126648e-01$

However, the solution (step) is deemed *unacceptable*, because
→ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
→ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

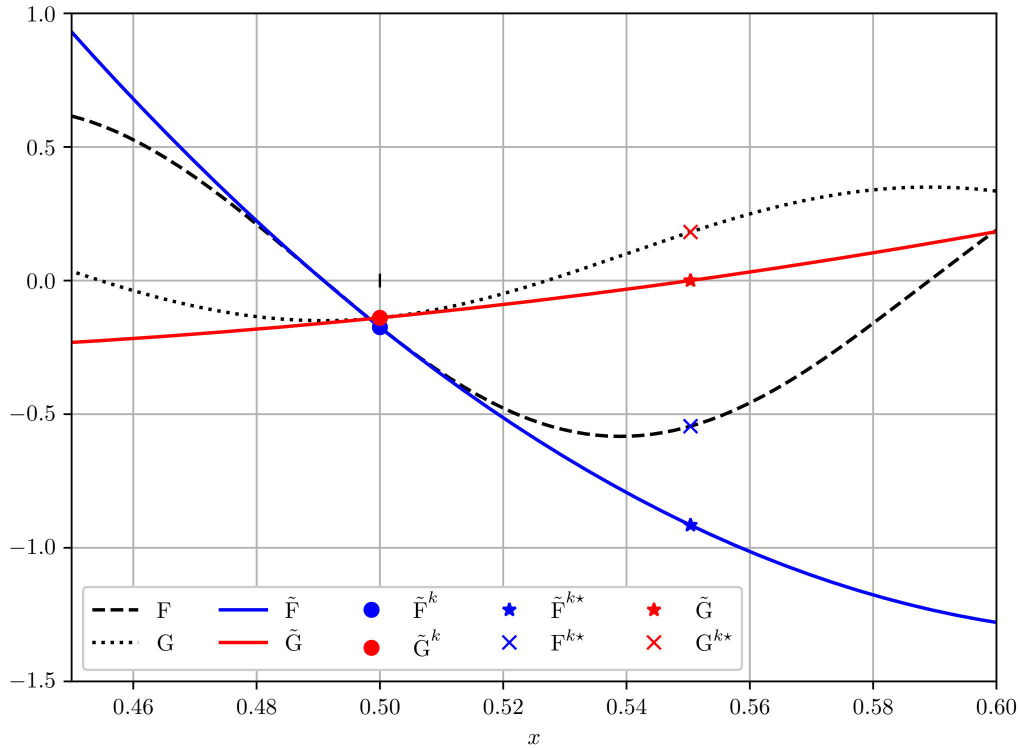
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 2.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 2.000$

with Lagrange multiplier
$\lambda^{k\star} = 3.402$
at the solution
$x^{k\star} = 0.550$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -9.152876e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.946269e-06$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.449878e-01$
$\mathrm{G}^{k\star} = 1.817974e-01$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

That is, at least one approximation function is not *conservative*.

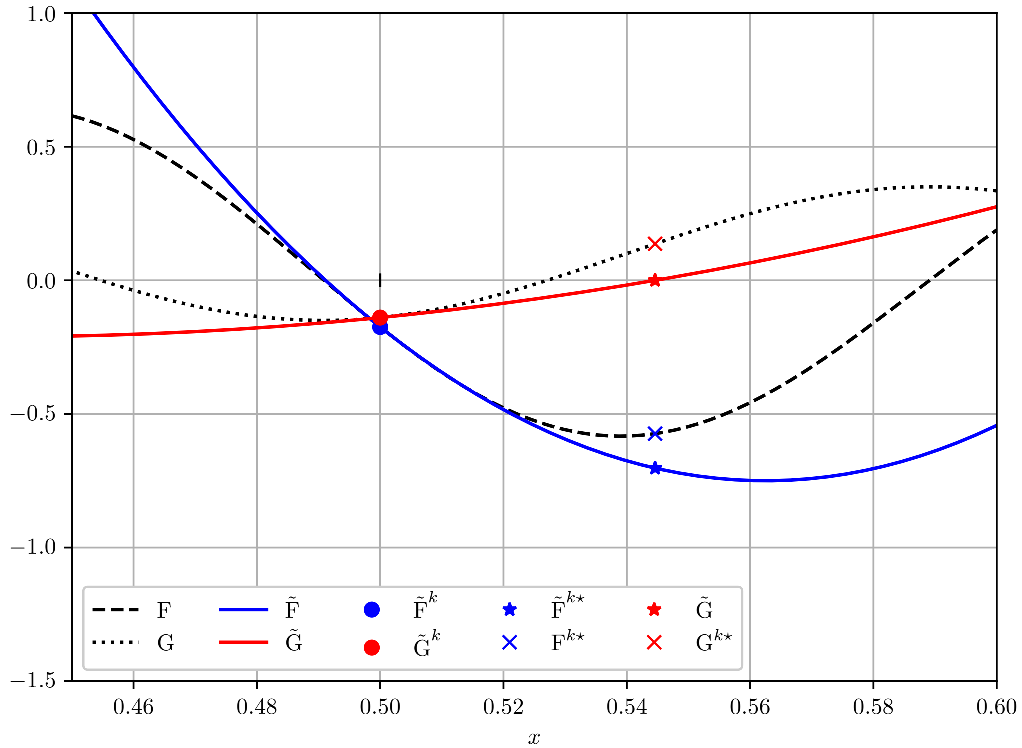
Iteration 2

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 4.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 4.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.337$
at the solution
$x^{k\star} = 0.545$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -7.027671e-01$
$\tilde{\mathrm{G}}^{k\star} = -7.757163e-05$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.737270e-01$
$\mathrm{G}^{k\star} = 1.368968e-01$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

That is, at least one approximation function is not *conservative*.

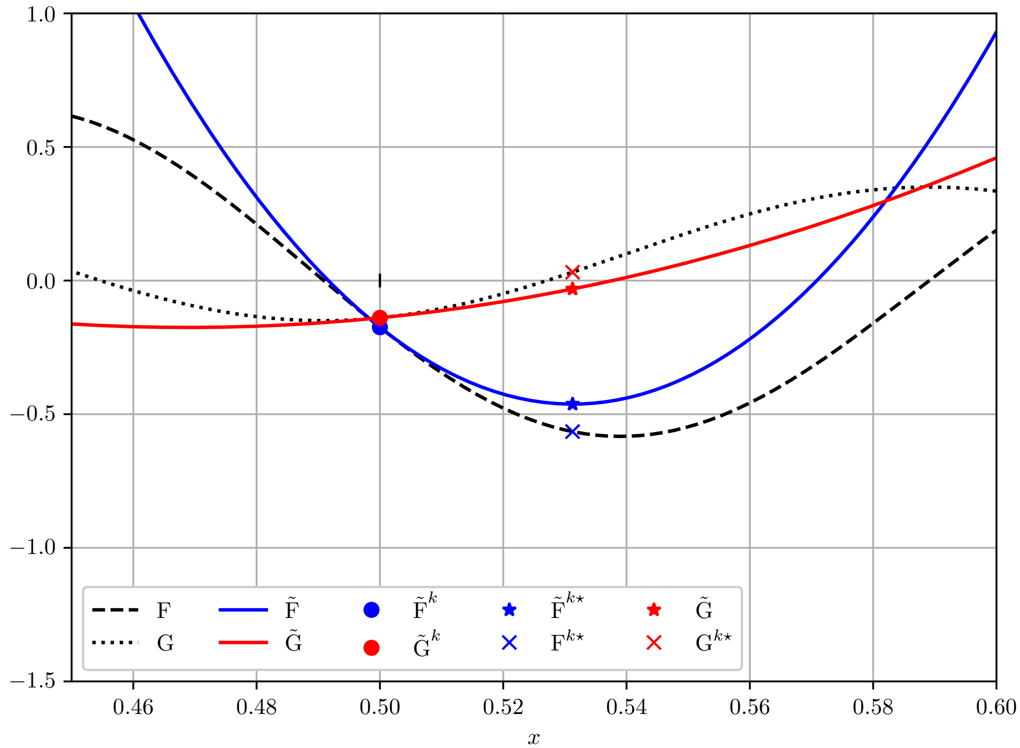
Iteration 3

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 8.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.000$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.623186e-01$
$\tilde{\mathrm{G}}^{k\star} = -3.145113e-02$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.651764e-01$
$\mathrm{G}^{k\star} = 3.120917e-02$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

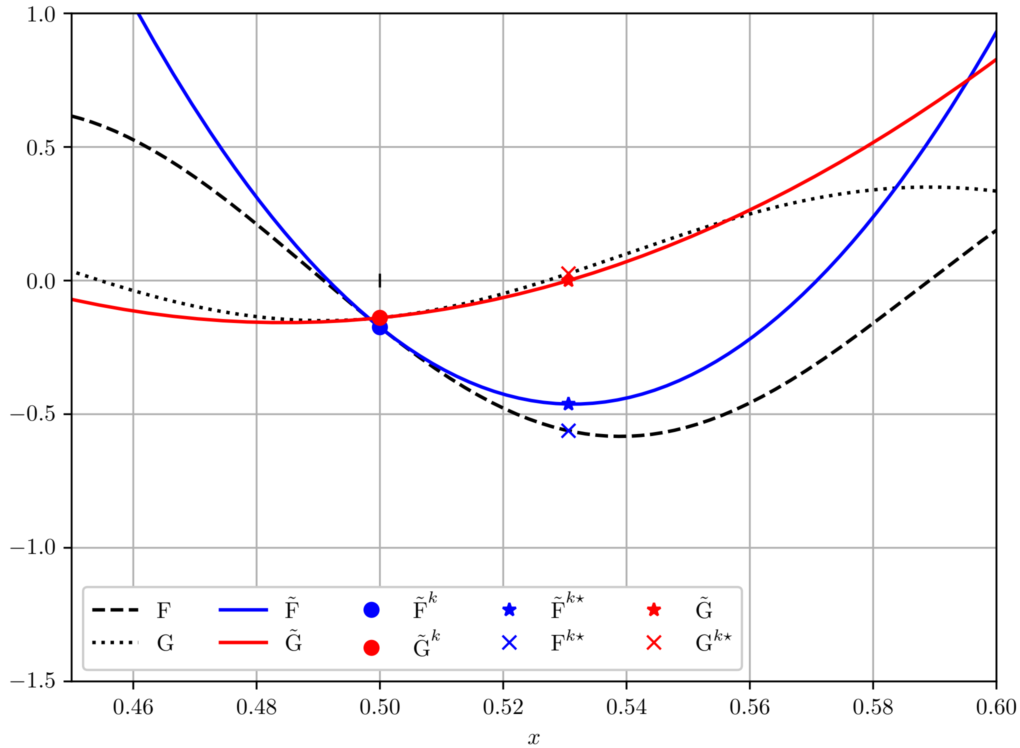
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_\mathrm{G} = 16.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.057$
at the solution
$x^{k\star} = 0.531$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.621899e-01$
$\tilde{\mathrm{G}}^{k\star} = 6.450742e-07$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.619993e-01$
$\mathrm{G}^{k\star} = 2.613989e-02$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the constraint function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{G}}^{k\star} < \mathrm{G}^{k\star}$.

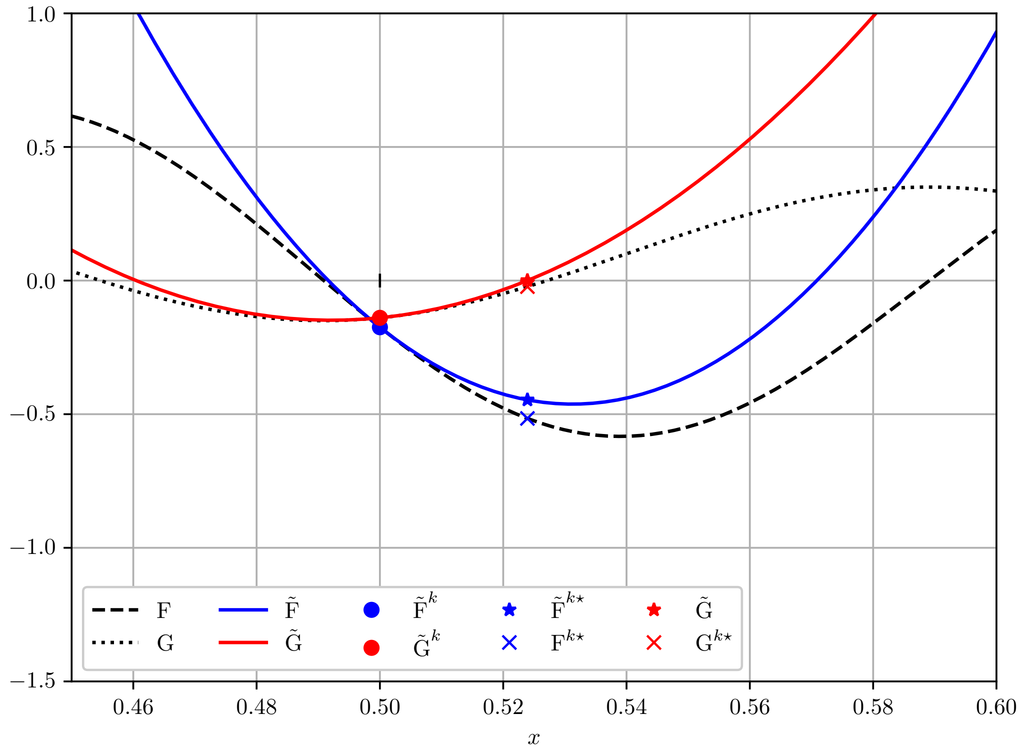
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.500$
$\mathrm{F}^k = -0.175$
$\mathrm{d}_x^k\mathrm{F} = -18.413$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 73.650$
$\alpha_{\mathrm{F}} = 8.000$

$\mathrm{G}^k = -0.139$
$\mathrm{d}_x^k\mathrm{G} = 2.303$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 9.213$
$\alpha_{\mathrm{G}} = 32.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.461$
at the solution
$x^{k\star} = 0.524$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -4.465202e-01$
$\tilde{\mathrm{G}}^{k\star} = 8.467353e-05$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.159334e-01$
$\mathrm{G}^{k\star} = -2.271927e-02$

Both the value objective function approximation and the constraint function approximation, at the new design point $x^{k\star}$, is *conservative* with respect to the actual function values, $\tilde{\mathrm{F}}^{k\star} > \mathrm{F}^{k\star}$, $\tilde{\mathrm{G}}^{k\star} > \mathrm{G}^{k\star}$

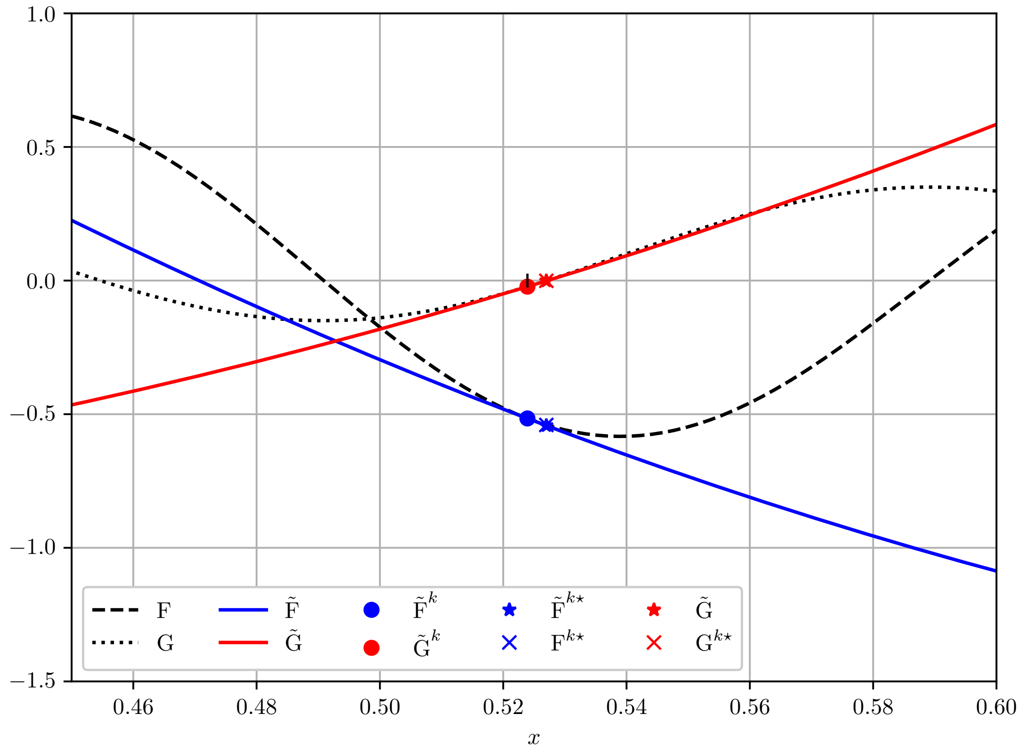
Iteration 6

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.786$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.540$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.970$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.606$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.231$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.427021e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.234656e-03$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.402579e-01$
$\mathrm{G}^{k\star} = -8.041847e-04$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

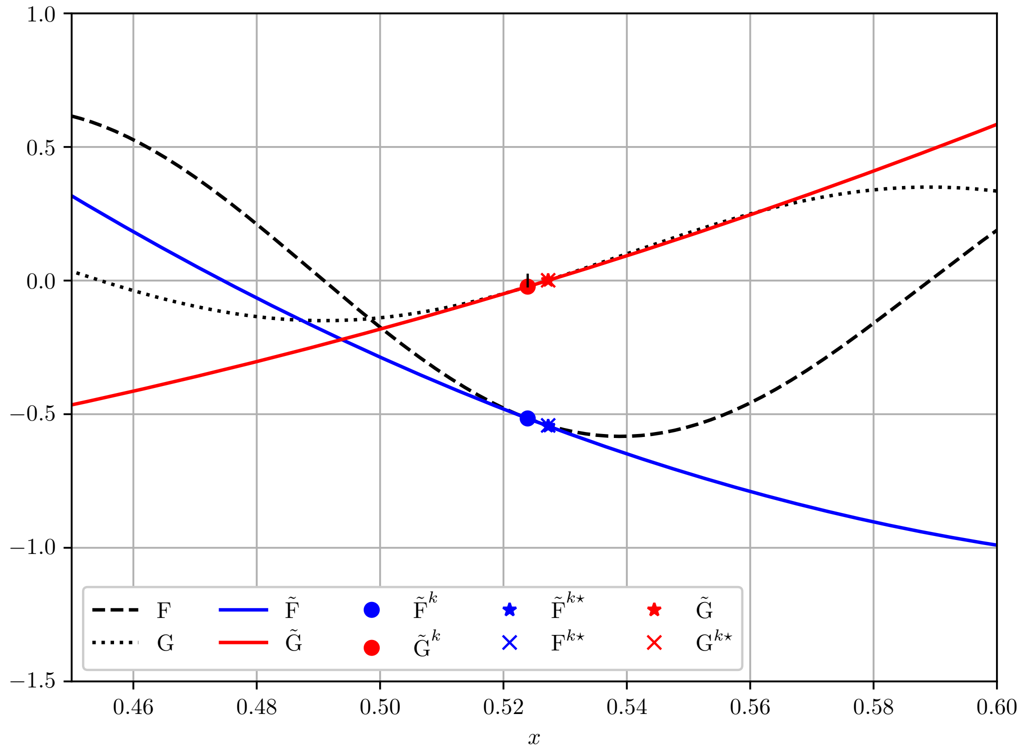
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.786$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.540$
$\alpha_{\mathrm{F}} = 2.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.970$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.606$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.213$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.447426e-01$
$\tilde{\mathrm{G}}^{k\star} = 5.743015e-04$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.420540e-01$
$\mathrm{G}^{k\star} = 1.076367e-03$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

That is, at least one approximation function is not *conservative*.

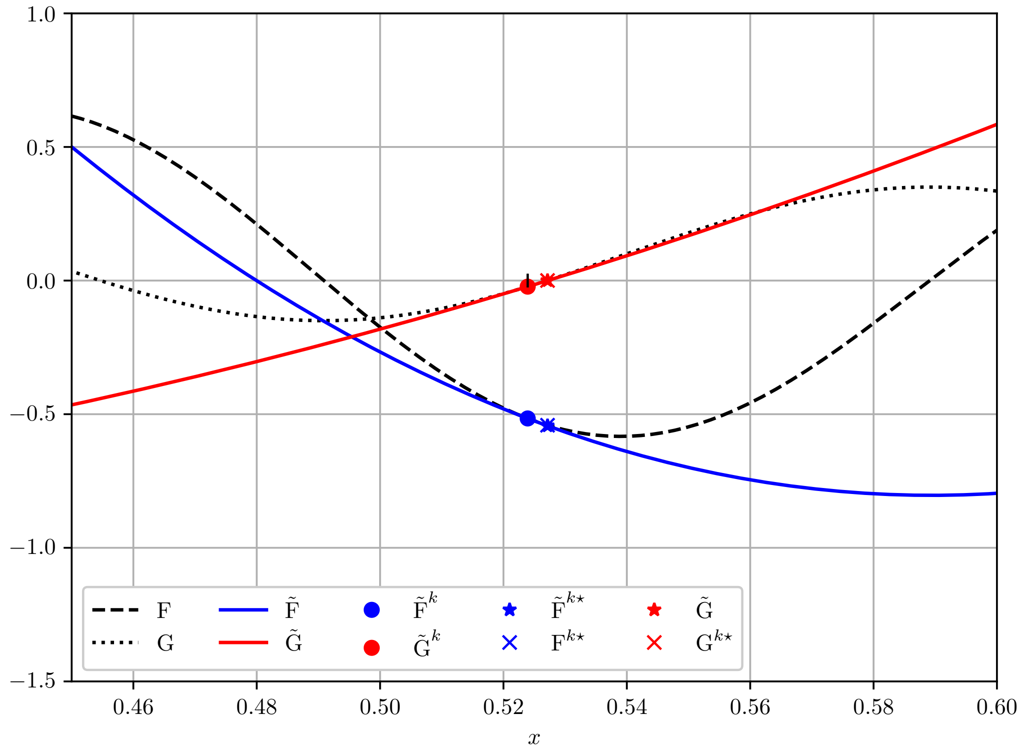
Iteration 8

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.786$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.540$
$\alpha_{\mathrm{F}} = 4.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.970$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.606$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.184$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.434984e-01$
$\tilde{\mathrm{G}}^{k\star} = -1.648436e-04$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.413249e-01$
$\mathrm{G}^{k\star} = 3.073761e-04$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

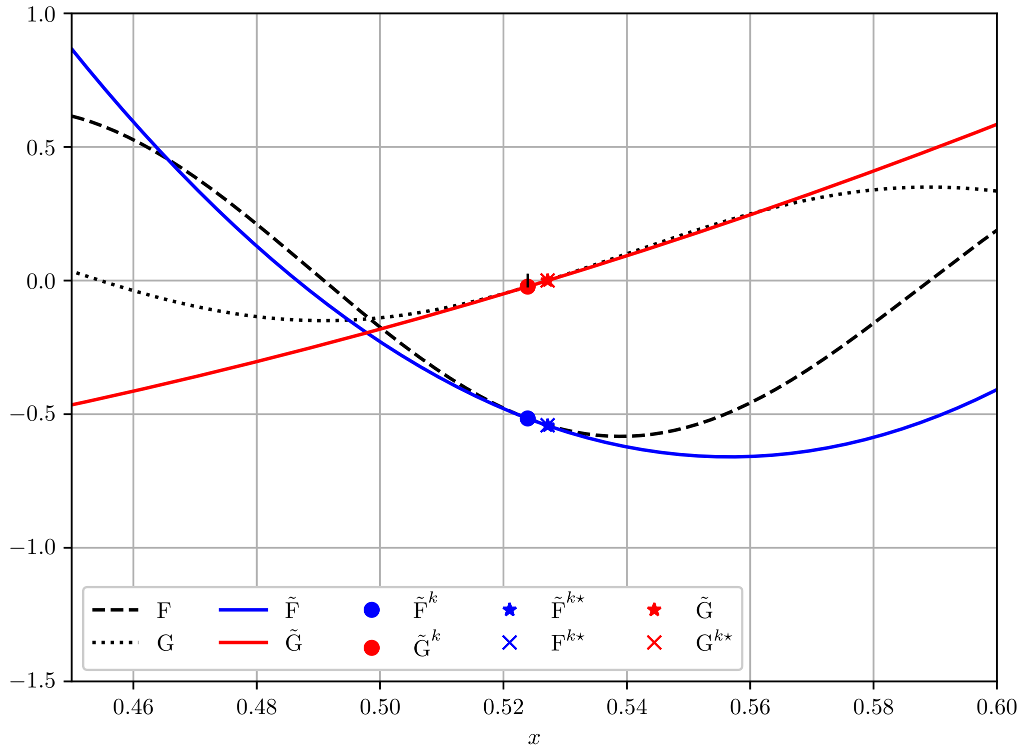
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.786$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.540$
$\alpha_\mathrm{F} = 8.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.970$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.606$
$\alpha_\mathrm{G} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 1.122$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.429334e-01$
$\tilde{\mathrm{G}}^{k\star} = -5.001063e-05$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.414386e-01$
$\mathrm{G}^{k\star} = 4.267925e-04$

However, the solution (step) is deemed *unacceptable*, because
$\rightarrow$ the value of the objective function approximation is less than the actual function value, at the new design point, $\tilde{\mathrm{F}}^{k\star} < \mathrm{F}^{k\star}$.

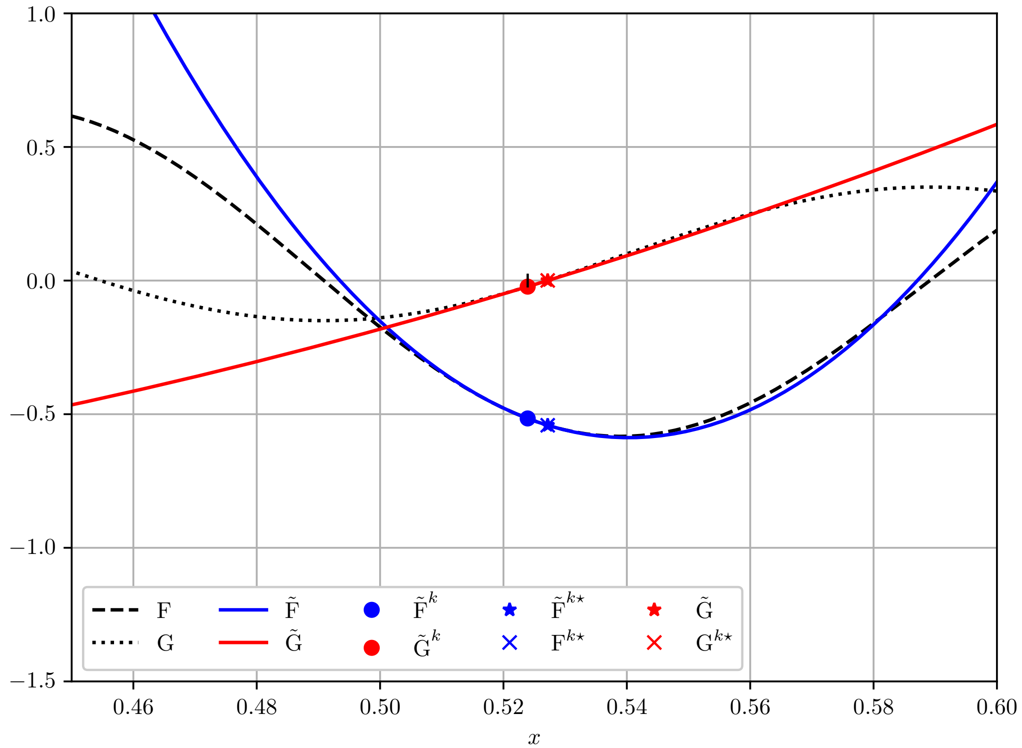
That is, at least one approximation function is not *conservative*.

QP subproblem at x^k:

$x^k = 0.524$
$\mathrm{F}^k = -0.516$
$\mathrm{d}_x^k\mathrm{F} = -8.786$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 33.540$
$\alpha_{\mathrm{F}} = 16.000$

$\mathrm{G}^k = -0.023$
$\mathrm{d}_x^k\mathrm{G} = 6.970$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 26.606$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.999$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.415990e-01$
$\tilde{\mathrm{G}}^{k\star} = 1.754802e-05$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.415054e-01$
$\mathrm{G}^{k\star} = 4.970568e-04$

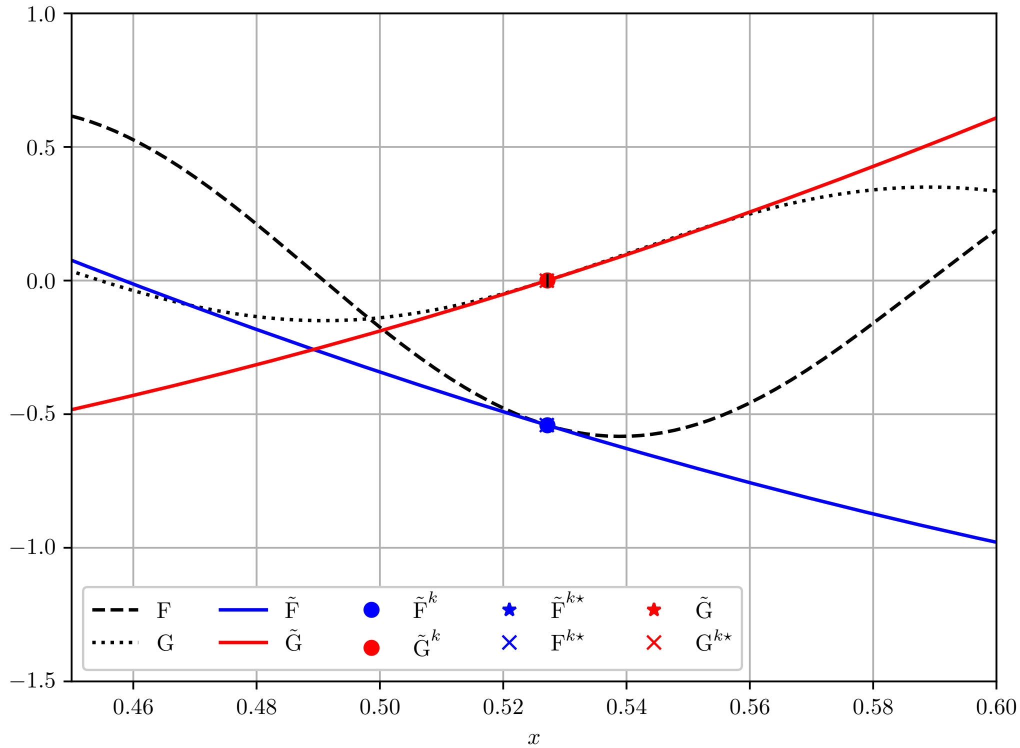
Both the value objective function approximation and the constraint function approximation, at the new design point $x^{k\star}$, is *conservative* with respect to the actual function values, $\tilde{\mathrm{F}}^{k\star} > \mathrm{F}^{k\star}$, $\tilde{\mathrm{G}}^{k\star} > \mathrm{G}^{k\star}$

QP subproblem at x^k:

$x^k = 0.527$
$\mathrm{F}^k = -0.542$
$\mathrm{d}_x^k\mathrm{F} = -6.976$
$\mathrm{dd}_x^k\tilde{\mathrm{F}} = 26.468$
$\alpha_{\mathrm{F}} = 1.000$

$\mathrm{G}^k = 0.000$
$\mathrm{d}_x^k\mathrm{G} = 7.339$
$\mathrm{dd}_x^k\tilde{\mathrm{G}} = 27.843$
$\alpha_{\mathrm{G}} = 1.000$

with Lagrange multiplier
$\lambda^{k\star} = 0.951$
at the solution
$x^{k\star} = 0.527$



At the solution of the QP the function approximations have the values
$\tilde{\mathrm{F}}^{k\star} = -5.407912e-01$
$\tilde{\mathrm{G}}^{k\star} = -2.539784e-04$

while the actual functions are evaluated to be
$\mathrm{F}^{k\star} = -5.407883e-01$
$\mathrm{G}^{k\star} = -2.535892e-04$

Both the value objective function approximation and the constraint function approximation, at the new design point $x^{k\star}$, is *conservative* with respect to the actual function values, $\tilde{\mathrm{F}}^{k\star} > \mathrm{F}^{k\star}$, $\tilde{\mathrm{G}}^{k\star} > \mathrm{G}^{k\star}$

Terminated on $|x^{k\star} - x^k| < $ 1.0e-03

Python code