1. $\binom{7}{5}$, assuming each u is distinguishable.

2. Ways to choose 2 unique values for pairs:

$$\binom{13}{2}$$

Ways to choose 2 unique shapes for a particular pair:

$$\binom{4}{2}$$

Ways to choose the lone card after choosing the pairs:

$$(11)(4)$$

Total number of ways:

$$\binom{13}{2}\binom{4}{2}^2(11)(4) = 123552$$

3. Case 0: Fighting couple gets 0 songs. Ways to distribute remaining 16 songs among 6 couples:

$$\binom{6-1+16}{16}$$

Case 1: Fighting couple gets 1 song. Ways to distribute remaining 15 songs among 6 couples:

$$\binom{6-1+15}{15}$$

Total ways, assuming the same couple fights each time:

$$\binom{6-1+16}{16} + \binom{6-1+15}{15} = 35853$$

4. Let f(n) = number of trees out of n distinct nodes.

$$f(0) = 1$$

$$f(1) = 1$$

$$f(2) = f(0)f(1) + f(1)f(0) = 2$$

$$f(3) = f(1) + 2(f2) = 5$$

$$f(4) = f(0)f(3) + f(1)f(2) + f(2)f(1) + f(3)f(0) = 5 + 2 + 2 + 5 = 14$$

$$f(5) = f(0)f(4) + f(1)f(3) + f(2)f(2) + f(3)f(1) + f(4)f(0) = 14 + 5 + 4 + 5 + 14 = 42$$

In general,

$$f(n) = \sum_{k=0}^{n-1} f(k)f(n-1-k)$$

This can be represented with the following recursive code:

```
int f(int n) {
    if (n == 0) return 1;
    if (n == 1) return 1;
    int sum = 0;
    for (int i = 0; i <= n-1; i ++) {
        sum += f(i) * f(n - i - 1);
    }
    return sum;
}\]
```

The total number of 12-node sub-trees with 3 at the top is:

$$f(2)f(9) = (2)(4862) = 9274$$

5. Since nurses are identical, arrange them by non-increasing number of patients served. Case 0 (all four nurses working, 0 on break): Give each nurse 1 patient and count the number of additional patients served, of the remaining 6. This yields 8 ways:

$$\begin{aligned}&(6,0,0,0),\\&(5,1,0,0),\\&(4,2,0,0),(4,1,1,0),\\&(3,3,0,0),(3,2,1,0),(3,1,1,1),\\&(2,2,2,1)\end{aligned}$$

Case 1 (3 nurses working, 1 on break): Give each nurse 1 patient and count the number of additional patients served, of the remaining 7: This yields 8 ways:

$$\begin{aligned}&(7,0,0),\\&(6,1,0),\\&(5,2,0),(5,1,1),\\&(4,3,0),(4,2,1),\\&(3,3,1),(3,2,2)\end{aligned}$$

Adding up the cases, there are 16 ways.