1. Number of potential outcomes (order of questions matters):

$$15^8$$

Number of outcomes where no student will have to answer more than one question:

$$15!$$

Probability:

$$\frac{15!/7!}{15^8} = 0.1012369$$

2. First, find the probability of any particular generated number meeting the criteria. Number of possible integers:

$$10^5$$

even integers that start with 2 odd digits where all digits are unique:
Case 0: 0 digits between the two odds and the last even:

$$5 * 4 * 5 = 100$$

Case 1: 1 digit between the two odds and the last even:

$$5 * 4 * 5 * 7 = 700$$

Case 2: 2 digits between the two odds and the last even:

$$5 * 4 * 5 * 7 * 6 = 4200$$

Adding these cases:

$$100 + 700 + 4200 = 5000$$

Probability for particular trial:

$$\frac{5000}{10^5} = 1/20$$

Number of cases in which we get at exactly 5 of these successes in 8 trials:

$$\binom{8}{5} = 56$$

Probability of each of these cases:

$$(\frac{1}{20})^5(\frac{19}{20})^3$$

Total probability:

$$56(\frac{1}{20})^5(\frac{19}{20})^3 = 0.0000150040625$$

3. No, because given A, the values of the three dice are constrained to a smaller range. This increases the likelihood of them all being the same.

4. Total number of possible dealings (order matters): $52!/47!$
   Number of flushes of a particular suit: $13 * 12 * 11 * 10 * 9 = 13!/8!$
   Number of flushes of any suit: $4(13!/8!)$
   Probability of a flush:
   $$\frac{4(13!/8!)}{52!/47!} = 0.001980792$$
   Let X = number of hands of poker he has to play to get a flush. $X \sim \text{Binomial}(0.001980792)$
   The expected number is:
   $$\frac{1}{p} = \frac{1}{0.001980792} = 504.8484$$

5. Let E = "they won 4 of 5 games", F = "the superstar played"
   $$P(E|F) = \binom{5}{4}(0.7)^4(0.3)^1 = 0.36015$$
   $$P(E|F^c) = \binom{5}{4}(0.5)^4(0.5)^1 = 0.15625$$
   $$P(E) = P(E|F)P(F) + P(E|F^c)P(F^c) = 0.36015(0.75) + 0.15625(0.25) = 0.309175$$
   Using Bayes' Theorem
   $$P(F|E) = \frac{P(E|F)P(F)}{P(E)} = \frac{(0.36015)(0.75)}{0.309175} = 0.873655$$