Percolation

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1 Permeability modification Model

The polymers passing through pores are dragged by the flow. There are two forces competeing when a polymer adheres to the surface of a pore (1) The drag force on the particle coming from flow, and (2) ionic/VanderWaals or whatever force that tries to adhere the particles to the surface. A particle attaches to the surface if the drag force exerted by the fluid flow around it is smaller than the VanderWaals/ionic force.

For simplicity, I assume that the force is ionic with a force distance profile of $F = K/X^2$, where K is a constant and X is the distance. I assume that polymers have an effective radius of d_p and fluid viscosity is μ and the flow rate is Q. This model states that the particle attaches to the surface when

$$F_{\text{drag}} \leq F_{\text{ionic}} \tag{1}$$

$$6\pi\mu d_p \frac{2Q}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right) \leq \frac{K}{(R-r)^2} \tag{2}$$

$$1 - \frac{r^2}{R^2} \leq \left(\frac{K}{12\mu d_p Q}\right)\frac{1}{(1-r/R)^2} \tag{3}$$

$$1 \leq \frac{r^2}{R^2} + \alpha\frac{1}{(1-r/R)^2} \tag{4}$$

where $\alpha = K/\left(12\mu d_p Q\right)$ is a factor that compares the ionic force and viscous force. Note that it depends on the flux rate. If we define $x = 1 - r/R$ we obtain

$$1 \leq (1-x)^2 + \frac{\alpha}{x^2} \tag{5}$$

$$x^2 - x^2(1-x)^2 \leq \alpha \tag{6}$$

$$x^2(1-(1-x)^2) \leq \alpha \tag{7}$$

$$x^3(2-x) \leq \alpha \tag{8}$$

$$\text{if } x \ll 1 \rightarrow x \leq \left(\frac{\alpha}{2}\right)^{1/3} \tag{9}$$

As a result the thickness of the adsoption layer is proportional to $\alpha^{1/3}$. Note that α depends

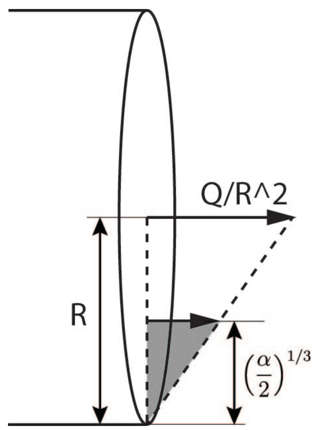

Figure 1: Flow in the pipe

on $1/Q$. Now, lets find the flux passing through this adsorption layer (any polymer in this layer would attach to the surface and therefore I call this layer adsoption layer)

$$\text{volume/time} = \frac{1}{2}R\alpha^{1/3} \cdot \frac{Q}{R^3}R\alpha^{1/3} \cdot 2\pi R = \pi Q\alpha^{2/3} = \pi \left(\frac{K}{12\mu d_p}\right)^{2/3} Q^{1/3} \tag{10}$$

$$\text{volume/time} \propto Q^{1/3} \tag{11}$$

Now, if we assume that the number of polymers passing through this area causes clogging, then the change in the total area of the pipe will be proportional to the flux through adsorption layer

$$d(\pi R^2) \propto Q^{1/3} \quad \rightarrow \quad RdR \propto Q^{1/3} \tag{12}$$

$$dR \propto \left(\frac{Q}{R^3}\right)^{1/3} \tag{13}$$

which means that the rate of change of the diameter is proportional to the shear rate to the power of 1/3. Note that we also know that $k = \pi R^4/8l^2$, as a result

$$dk \propto R^3 dR \tag{14}$$

$$dk \propto R^2 Q^{1/3} \tag{15}$$

$$dk \propto Q^{1/3} \tag{16}$$

which means the changes in the permeability is propotional to the $Q^{1/3}$ passing through the porous media.

Experimental Results

In Shima's paper, we report a figure that shows the evolution of permeability k per volume of polymer V_{pol} that passes through the porous media. Basically we are plotting Δk versus Q passing through the porous media. The figure and best polynomial fit is shown blow

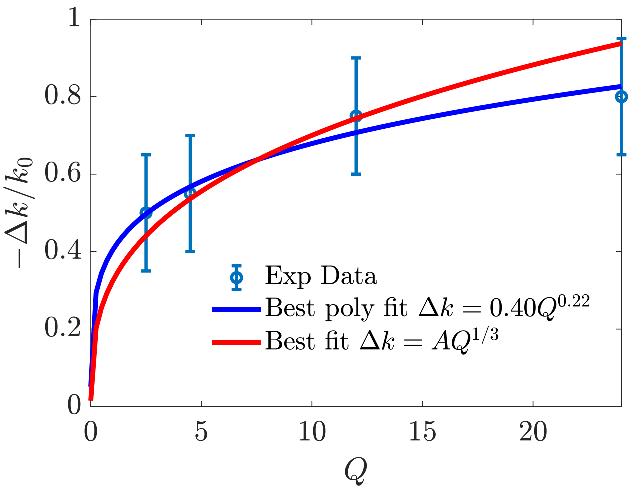

Figure 2: k versus volume of fluid passing through

1.1 Numerical Result

I also ran a numerical simulation in which, I change the diameters according to 12. The result is as follows

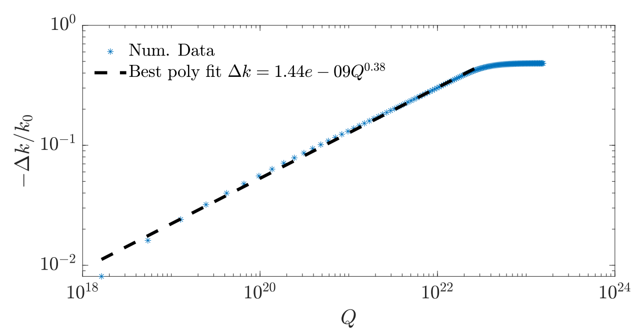

Figure 3: k versus volume of fluid passing through in numerical simulation. Note that the plateau is due to the max limit imposed for max reduction in diameter

2 General Force term

In this section we assume a general forcing term of type $F_g = K/X^n$, and as a result find the coefficient function.

$$F_{\text{drag}} \leq F_g \tag{17}$$

$$6\pi\mu d_p \frac{2Q}{\pi R^2}\left(1-\frac{r^2}{R^2}\right) \leq \frac{K}{(R-r)^n} \tag{18}$$

$$1-\frac{r^2}{R^2} \leq \frac{K}{12\mu d_p Q R^{n-2}} \frac{1}{(1-r/R)^n} \tag{19}$$

$$1-\frac{r^2}{R^2} \leq \beta \frac{1}{(1-r/R)^n} \tag{20}$$

where $\beta = K/12\mu d_p Q R^{n-2}$. If we define $x = 1 - r/R$ we obtain

$$1-(1-x)^2 \leq \beta \frac{1}{x^n} \tag{21}$$

$$x^{n+1}(2-x) \leq \beta \tag{22}$$

$$x \ll 1, \qquad x \leq \left(\frac{\beta}{2}\right)^{\frac{1}{n+1}} \tag{23}$$

$$\text{volume/time} = \frac{1}{2} R \left(\frac{\beta}{2}\right)^{\frac{1}{n+1}} \cdot \frac{Q}{R^3}\left(R\frac{\beta}{2}\right)^{\frac{1}{n+1}} \cdot 2\pi R \tag{24}$$

$$\text{volume/time} \propto Q^{1-\frac{2}{n+1}} R^{-2\frac{n-2}{n+1}} \tag{25}$$

$$\text{volume/time} \propto Q^{\frac{n-1}{n+1}} R^{-2\frac{n-2}{n+1}-1} R \tag{26}$$

$$\text{volume/time} \propto R\left(\frac{Q}{R^3}\right)^{\frac{n-1}{n+1}} \tag{27}$$

$$d(\pi R^2) \propto R\left(\frac{Q}{R^3}\right)^{\frac{n-1}{n+1}} \quad \rightarrow \quad \boxed{dR \propto \left(\frac{Q}{R^3}\right)^{\frac{n-1}{n+1}}} \tag{28}$$

If $k = \pi R^4/8l^2$, as a result

$$dk \propto R^3 dR \tag{29}$$

$$dk \propto R^3\left(\frac{Q}{R^3}\right)^{\frac{n-1}{n+1}} \tag{30}$$

$$\boxed{dk \propto Q^{\frac{n-1}{n+1}}} \tag{31}$$

3 Force Consideration

In Montgomery *et al.* (2000), the authors have summarized adhesion studies. They show that the interaction between a spherical molecule and an infinite cylinder, results in the adhesion force of

$$F \propto \frac{1}{X^2} \tag{32}$$

where X is the distance between the molecule and the surface.

4 Modelling Force

Assuming a tube shape for the pores, the following figure appears There are 3 different cases:

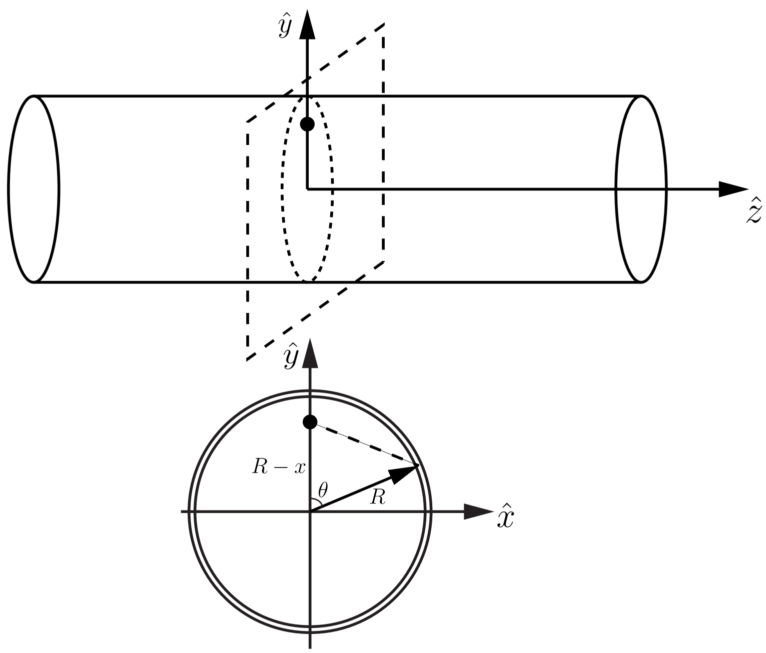

Figure 4: Polymer attachment to the surface. The polymer is assumed to be at distance x to the boundary. We pick the coordinate system such that $\hat{z}$ is along the direction of the tube and the polymer is on the y-axis.

- **Constant attraction force**: The polymer is very small compared to the pore radius and is very close to the surface such that it does not feel the curvature. For particle, the pore is like a flat infintie plate that is atttacting the particle. If the attraction force is due to columb, then the attracting force is independen of distance and has a constant value.
- If the particle feels the attraction force between an infinite cylinder with radius R and

a point particle with charge q at distance x from the edge can be calculated as

$$\text{symmetry } F_x = F_z = 0, \qquad F_y = \int_{-\infty}^{\infty}\int_0^{2\pi} \frac{R\sin\theta - R + x}{(R^2 - 2R(R-x)\sin\theta + z^2)^{3/2}} q\lambda R d\theta \tag{33}$$

Using mathematica, so far, I was unable to find the leading power of force in terms of x.

5 Lennard-Jones Force

$$F_{\text{drag}} \leq F_{LJ} \tag{34}$$

$$6\pi\mu d_p \frac{2Q}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right) \leq 48\epsilon\left[\frac{\sigma^{12}}{(R-r)^{13}} - \frac{\sigma^6}{(R-r)^7}\right] \tag{35}$$

$$6\pi\mu d_p \frac{2Q}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right) \leq 48\epsilon\left[\frac{(\sigma/R)^{12}}{(1-r/R)^{13}} - \frac{(\sigma/R)^6}{(1-r/R)^7}\right] \tag{36}$$

$$6\pi\mu d_p \frac{2Q}{\pi R^2}\left(1 - (1-x)^2\right) \leq 48\epsilon\left[\frac{(\sigma/R)^{12}}{x^{13}} - \frac{(\sigma/R)^6}{x^7}\right] \tag{37}$$

$$6\pi\mu d_p \frac{2Q}{\pi R^2}\left(2x - x^2\right)x^{13} \leq 48\epsilon\left[(\sigma/R)^{12} - (\sigma/R)^6 x^5\right] \tag{38}$$

$$x \ll 11 \to x \leq (\sigma/R)^{6/5}, \text{ or } x \leq c \tag{39}$$

which is a constant value.

$$\text{volume/time} = \frac{1}{2}Rc \cdot \frac{Q}{R^3}Rc \cdot 2\pi R = \pi Q c^2 \tag{40}$$

$$\text{volume/time} \propto Q \tag{41}$$

$$d(\pi R^2) \propto Q \quad \to \quad RdR \propto Q \tag{42}$$

$$dR \propto \frac{Q}{R} \tag{43}$$

If $k = \pi R^4/8l^2$, as a result

$$dk \propto R^3 dR \tag{44}$$

$$dk \propto R^2 Q \tag{45}$$

$$dk \propto Q \tag{46}$$

References

Montgomery, Stephen W, Franchek, Matthew A & Goldschmidt, Victor W 2000 Analytical dispersion force calculations for nontraditional geometries. *Journal of colloid and interface science* **227** (2), 567–584.