Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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Margaret L. Brandeau, Samuel S. Chiu,

MANAGEMENT SCIENCE
Vol 35, No 6, June 1989
Printed in U.S.A.

AN OVERVIEW OF REPRESENTATIVE PROBLEMS IN LOCATION RESEARCH*

MARGARET L. BRANDEAU AND SAMUEL S. CHIU
Department of Industrial Engineering and Engineering Management, Stanford University, Stanford, California 94305
Engineering-Economic Systems Department, Stanford University, Stanford, California 94305

We present a survey of over 50 representative problems in location research. Our goal is not to review all variants of different location models or to describe solution results, but rather to provide a broad overview of major location problems that have been studied, indicating briefly how they are formulated and how they relate to one another. We review standard problems such as median, center, and warehouse location problems, as well as less traditional location problems which have emerged in recent years. Our primary focus is on problems for which operations research-type models have been developed. Most of the problems we review have been formulated as optimization problems.
(LOCATION THEORY)

1. Background and Introduction

Location theory was first formally introduced in 1909 by Alfred Weber (1909), who considered the problem of locating a single warehouse to minimize the total travel distance between the warehouse and a set of spatially distributed customers. This work was reconsidered by Isard (1956) with his study of industrial location, land use, and related problems. Another early location problem was formulated by Hotelling (1929), an economist who considered the problem of locating two competing vendors along a straight line. This work was later extended by Smithies (1941) and Stevens (1961).

A number of authors in the 1950's and early 1960's considered the problem of facility layout and design (Apple 1963, Armour and Buffa 1963, Ireson 1952, Moore 1962, Muther 1955, Reed 1961). Losch (1954) and Moses (1958) considered economic factors associated with production center location. Miehle (1958) considered the problem of minimizing link length in networks.

Before the mid-1960's, however, work in the field of location theory consisted primarily of a number of separate applications, not tied together by a unified theory. These include the location of: firefighting vehicles (Valinsky 1955); a classification yard in a rail network (Mansfield and Wein 1958); solid waste disposal sites (Wersan et al. 1962); exchange locations in a telephone network (Rapp 1962); factory sites (Burstall et al. 1962); and track checking stations on a rail line (Young 1963).

More theoretical interest in location problems was sparked by a seminal paper of Hakimi (1964) who considered the general problem of locating one or more facilities on a network to minimize either the sum of distances or the maximum distance between facilities and points on a network. Since then, considerable research has been carried out in the field of location theory. A number of different classes of problems have been identified and solved, and location methodologies have been extended to a variety of practical applications.

Although location theory has been an active area of research for the last 20 years, only limited attempts to date have been made to systematically review and describe location problems. Francis et al. (1983), in an excellent survey of selected location research,

* Accepted by Alexander H. G. Rinnooy Kan; received November 12, 1984. This paper has been with the authors 12 months for 3 revisions.

0025-1909/89/3506/0645$01.25

reviewed four classes of problems (continuous planar; discrete planar; mixed planar; and discrete network problems), concentrating on optimization models for which "reliable algorithms" have been developed. In another comprehensive survey, Tansel et al. (1983a, b) provided a review of p-center and p-median problems, as well as location problems on tree networks, describing algorithms and solution results for each problem. Krarup and Pruzan (1983) reviewed planar and network versions of the p-median and p-center problems, and in another work surveyed capacitated and uncapacitated plant location problems (1979). Aikens (1985) presented a survey of warehouse location models. Scott (1970) reviewed a selected set of multi-server location problems, while ReVelle et al. (1970) surveyed public and private sector location models. Francis and White (1974, 1978) examined problems relating to facility layout and location, and Handler and Mirchandani (1979) considered network location problems. Domschke and Drexl (1985) provided a bibliography of location-related problems.

In this paper we present a representative survey of major location problems. Unlike the surveys mentioned above, our goal is not to focus on problem applications, specific classes of problems, or theoretical solution results; rather, our goal is to provide a broad overview of major problems in the field—to briefly describe different types of location models that have been studied, and to indicate how they relate to one another. We review more than 50 location problems, including standard problems such as the median, center, and warehouse location problems, as well as less traditional problems which have emerged in recent years. Our primary focus is on problems for which operations research-type models have been developed, and most of the problems we review have been formulated as optimization problems. (We note that when referring to a location problem in this paper, we really mean a model of a location problem.) In selecting the problems to review, we have not attempted to describe all variants of all problems, but rather have attempted to identify a representative sample of problems that have been studied, including a number which have not appeared in published surveys. Similarly, our list of references is not intended to be exhaustive; rather, we have attempted for each type of problem to cite one or two early papers which introduced the problem and (where appropriate) several more recent papers which contain key results. Our hope is that this survey will provide the reader with a general overview of the types of location problems that have been and are being studied.

In §2, we define what we mean by a location problem, present examples of typical problems, and indicate the variety of applications of location models. In §3 we present a fairly detailed location taxonomy, which we use in §4 to concisely describe more than 50 representative location problems. We then briefly describe typical solution methods for location problems, and conclude by remarking on trends which have emerged in recent location research.

2. Examples and Applications of Location Theory

2.1. *Definition of a Location Problem*

A location problem is a spatial resource allocation problem. In the general location paradigm, one or more service facilities ("servers") serve a spatially distributed set of demands ("customers"). The spatial topology being modeled may be a general network, or a specialized network (e.g., a tree). The objective is to locate facilities (and perhaps allocate customers to servers) to optimize an explicit or implicit spatially dependent objective. Typical criteria for such decisions include: minimizing average travel time or distance between demands and servers; minimizing average response time (that is, travel time plus any queue delay); minimizing a cost function of travel or response time; minimizing maximum travel time; or maximizing minimum travel time. In general, the objective function consists of terms involving travel distances (or times) related to facility-facility and/or facility-customer interactions.

2.2. *Examples of Typical Location Problems*

Network Design. A city is faced with the problem of designing a water treatment network. Untreated water emanates from a number of different sources in the city. A central water treatment facility is to be located to minimize the total length of piping needed to conduct the untreated water to the treatment facility.

Warehouse Location. A firm wants to locate an undetermined number of warehouses in a region to serve a number of customers. Associated with each warehouse is a fixed investment cost, a distribution cost per unit of demand satisfied, and possibly a capacity constraint. The firm wishes to minimize its total warehouse cost.

Fire Box Coverage. A fire department wants to locate a fixed number of fire boxes in its service area with the objective of minimizing the maximum distance that any citizen must travel to the nearest fire box.

Competitive Facility Location. A firm wants to introduce a new store in an area already served by competing firms. Customers in the area have exhibited a certain level of demand for the product or service, but the demand could increase if a new store were added. The firm's objective is to locate its new store to yield maximum profit or market share.

2.3. *Applications of Location Theory*

Researchers have developed location models for a wide variety of public and private sector problems, as indicated in Table 1. Some of the published literature contains a description of an actual application (with real data), while other works describe a model designed for a particular type of problem, but not an actual application; we have highlighted this distinction in Table 1. In the private sector, the problem that has received the most attention is the location of warehouses or production centers, although many other types of location problems have also been considered. In the public sector, an important application area has been the location of emergency service vehicles or facilities.

3. Classification Scheme for Location Problems

Although location theory encompasses a wide range of problems, they all share certain common elements. In this section we present a natural taxonomy which we will use to distinguish different location problems.

As a preliminary, we observe that a model of any problem must consist of: an objective (optimizing or nonoptimizing); decision variable(s); and system parameters. To distinguish between different location models, we create a "menu" of choices within each of these categories. By selecting appropriate entries from each category, any location problem/model can be specified. We have designed our menu of choices to specifically account for the most common problem variations, but we include a catch-all "other" within each category to account for any additional types of problems.

Our taxonomy for location problems is shown in Table 2. The menu items in that table are self-explanatory. In §4 we use this framework to distinguish more than 50 representative location problem types with referenced articles indicated in Table 3.

4. Major Problems in Location Research

In this section we briefly review a representative sample of more than 50 different problem types in location research, and show how each type fits into our taxonomy. For

TABLE 1

*Application Areas of Different Location Models**

I *Private Sector Application Areas*	
Warehouse/Production Center Location	Drysdale and Sandiford 1969,* Efroymson and Ray 1966, Eilon et al. 1971, Maranzana 1964,* Nambiar et al. 1981,* Weber 1909
Factory Work Center Location	Armour and Buffa 1963, Francis and White 1974, Hillier and Connors 1966
Communication Network Design/Exchange Location	Miehle 1958, Rapp 1962
Electric Power Stations	Hochbaum 1982
Private Service Vehicles (e.g., Taxicab Fleets, Bloodmobiles)	Cerveny 1980,* Larson and Stevenson 1972
Private Service Equipment (e.g., Oil Spill Cleanup, Cotton Gins, Lock Boxes)	Belardo et al. 1984,* Klingman et al. 1976,* Mavrides 1979,* ReVelle and Swain 1970, Psaraftis et al. 1986*
Private Service Center Location (e.g., Tax Collection Offices)	Fitzsimmons and Allen 1983*
Transportation Centers (e.g., Shipping Ports, Railroad Classification Yards, Bus Garages)	Mansfield and Wein 1958, Osleeb et al. 1986,* Wirasinghe and Waters 1984* Young 1963
"Obnoxious Facilities" (e.g., Toxic Dumps, Nuclear Power Plants)	Church and Garfinkel 1978
Bank Accounts	Cornuejols et al. 1977, Hopmans 1986*
II. *Public Sector Application Areas*	
Emergency Service Vehicles/Facilities	Brandeau and Larson 1986,* Daskin and Stern 1981,* Eaton et al. 1985,* Fitzsimmons 1973, Larson 1974, Plane and Hendrick 1977,* Savas 1969,* Scarapi 1984,* Swoveland et al. 1973,* Toregas et al. 1971, Walker 1974*
Public Service Centers (e.g., Health Centers, Blood Banks, Waste Treatment Plants)	Dokmeci 1977, Hua 1962,* Larson and Stevenson 1972, Price and Turcotte 1986,* Patel 1979,* Smeers and Tyteca 1983, Wagner and Falkson 1975, Wersan et al. 1962
Public Network Design (e.g., Water Treatment Networks)	Katz and Cooper 1974
Residential Neighborhoods	Lundqvist 1984, Mattson 1986*
Defense Installations	Dasarathy and White 1980

* An asterisk next to a reference indicates that the reference describes an application with real data; otherwise, the authors have stated that their model is designed with the indicated application area in mind.

TABLE 2

A Taxonomy to Distinguish Location Problems

I. OBJECTIVE

- Optimizing:
  - Minimize Average Travel Time/Average Cost
  - Maximize Net Income
  - Minimize Average Response Time
  - Minimize Maximum Travel Time/Cost
  - Maximize Minimum Travel Time/Cost
  - Maximize Average Travel Time/Cost
  - Minimize Server Cost Subject to a Minimum Service Constraint
  - Optimize a Distance-Dependent Utility Function
  - Other
- Non-Optimizing
- Type of Location Dependence of Objective Function:
  - Server-Demand Point Distances
    - Weighted vs. Unweighted
    - Some vs. All Demand Points
    - Routed vs. Closest
  - Inter-Server Distances
  - Absolute Server Location*
  - Server-Distribution Facility Distances
  - Distribution Facility-Demand Point Distances
  - Other

II. DECISION VARIABLES

- Server/Facility Location
- Service Area/Dispatch Priorities
- Number of Servers and/or Service Facilities
- Server Volume/Capacity
- Type of Goods Produced by Each Server (in a Multi-Commodity Situation)
- Routing/Flows of Server or Goods to Demand Points
- Queue Capacity
- Other

III. SYSTEM PARAMETERS

- Topological Structure:
  - Link vs. Tree vs. Network vs. Plane vs. *n*-Dimensional Space*
  - Directed vs. Undirected
- Travel Metric:
  - Network-Constrained vs. Rectilinear vs. Euclidean vs. Block Norm vs. Round Norm vs. L_p vs. Other
- Travel Time/Cost:
  - Deterministic vs. Probabilistic
  - Constrained vs. Unconstrained
  - Volume-Dependent vs. Nonvolume-Dependent
- Demand:
  - Continuous vs. Discrete
  - Deterministic vs. Probabilistic
  - Cost-Independent vs. Cost-Dependent
  - Time-Invariant vs. Time-Varying
- Number of Servers
- Number of Service Facilities
- Number of Commodities

TABLE 2 (*cont'd*)

Server Location
Constrained vs. Unconstrained
Finite vs. Infinite Number of Potential Locations
Fixed vs. Dependent on System Status
Zero vs. Nonzero Relocation Cost
Deterministic vs. Probabilistic Location
Zero vs. Nonzero Fixed Cost
Server Capacity:
Capacitated vs. Uncapacitated
Reliable vs. Unreliable
Service Area and Dispatch Priorities:
Cooperating vs. Noncooperating Servers
Closest Distance vs. Nonclosest-Distance Service Area
Service Discipline:
FCFS vs. Priority Classes vs. Spatially-Oriented Rule vs. Other
Queue Capacity

* For certain product design and product positioning problems.

convenience in presentation, problems are grouped together by objective function, and within those groupings by topographical structure. We provide a brief description of each problem in the discussion below; a more complete characterization of each problem (in its typical formulation) is contained in Figure 1 (coded by problem numbers introduced below), while Table 3 provides a cross-index between problems and references.

4.1. *Minimization of Average Travel Time/Average Cost or Maximization of Net Income*

The objective of the *p-median problem* on a general network (P1) is to locate p servers to minimize average weighted or unweighted travel distance between servers and demand points (Goldman 1971, Hakimi 1964, 1965, Kariv and Hakimi 1979b, Minieka 1977). Most formulations assume demands at a finite number of points. The continuous median problem on a network allows demands to be continuously distributed over the links (Chiu 1987a, Cavalier and Sherali 1986). Another version of this problem is a *median problem with uncertainty* (P2) in which demands and/or travel times on the network are random variables (Frank 1966, Louveaux 1986, Mirchandani and Odoni 1979, Mirchandani et al. 1985, Weaver and Church 1983). Other authors (Berman and LeBlanc 1984, Berman and Odoni 1982) have considered a *mobile server location problem on a stochastic network* (P3) in which one or more servers are to be located and can be moved at a cost in response to changes in relative travel times.

The objective of the L_p *norm network location problem* (P4) is to locate a single server on a network to minimize average travel cost, but unlike the 1-median problem, the sum of (weighted or unweighted) distances is measured by an L_p metric (Shier and Dearing 1983, Brandeau and Chiu 1988a). Hooker (1986) has considered the general class of such problems with convex cost functions.

For the *m-facility minisum problem with mutual communication* (P5), the objective is to locate m servers on a tree network to minimize the weighted sums of distances between facilities and demand points plus a weighted sum of inter-facility distances subject to inter-facility and facility-demand point distance constraints (Dearing et al. 1976).

The simplest planar minisum problem is the *Weber problem* (P6), which assumes that a single server is to be located to respond to a set of discrete demands (Weber 1909) to minimize travel cost. The *generalized Weber problem* (P7), also referred to as the *ware-*

TABLE 3
Index of Problems and References

Problem Number	Problem Name	References
P1	p-median	Goldman 1971, Hakimi 1964, 1965, Hung 1973, Jarvinen et al. 1972, Kariv and Hakimi 1979b, Matula and Kolde 1976, Minieka 1977.
P2	median problem with uncertainty	Frank 1966, Louveaux 1986, Mirchandani and Odoni 1979, Mirchandani et al. 1985, Weaver and Church 1983.
P3	mobile server on a stochastic network	Berman and LeBlanc 1984, Berman and Odoni 1982.
P4	L_p norm network location problem	Brandeau and Chiu 1988a, Shier and Dearing 1983.
P5	m-facility minisum with mutual communication	Dearing et al. 1976.
P6	Weber problem	Weber 1909.
P7	generalized Weber (or warehouse location or location-allocation problem)	Baker 1974, Cerveny 1980, Chen 1983, Cooper 1963, 1964, 1967, Corneujols et al. 1977, Drezner and Wesolowsky 1978b, 1981, Drysdale and Sandiford 1969, Efroymson and Ray 1966, Eilon et al. 1971, Garfinkel et al. 1974, Hochbaum 1982. Juel 1981, Juel and Love 1985, Khumawala 1972, Kuehn and Hamburger 1963, Kuenne and Soland 1972, Love and Juel 1982, Love and Morris 1975a, b, Morris 1975, Picard and Ratliff 1978, Sherali and Adams 1984, Sherali and Shetty 1977, Vergin and Rogers 1967, Wesolowsky and Love 1971, 1972.
P8	simple plant location	Efroymson and Ray 1966, Erlenkotter 1978, Feldman et al. 1966, Krarup and Pruzan 1983, Manne 1964, Spielberg 1969.
P9	capacitated plant location	Akinc and Khumawala 1977, Guignard and Spielberg 1979, Sá 1969.
P10	dynamic warehouse	Geoffrion 1975, Van Roy and Erlenkotter 1982, Wesolowsky and Truscott 1975.
P11	multi-commodity service facility location	Karzakis and Boffey 1981, Laundy 1985, Neebe and Khumawala 1981, Warszawski 1973.
P12	multi-commodity distribution facility location	Geoffrion and Graves 1974.
P13	unreliable p-median	Drezner 1985.
P14	multi-stage median and hub median	Hakimi and Maheshwari 1972, O'Kelly 1986, Wendell and Hurter 1973b.
P15	facility layout	Apple 1963. Armour and Buffa 1963, Francis and White 1974, Hillier and Connors 1966, Ireson 1952, Mallette and Francis 1972, Moore 1962, Muther 1955, Reed 1961.
P16	linear assignment	Mallette and Francis 1972, Ross and Soland 1978.
P17	quadratic assignment	Gilmore 1962, Hillier and Connors 1966, Lawler 1963, Pierce and Crowston 1971, Ross and Soland 1978.
P18	location-routing	Drezner et al. 1985, Laporte and Nobert 1981, Laporte et al 1983.

TABLE 3 (*cont'd*)

Problem Number	Problem Name	References
P19	transportation-location	Cooper 1972.
P20	travelling salesman location	Berman and Simchi-Levi 1986, Burness and White 1976.
P21	production-location	Hurter and Martinich 1984, Hurter and Wendell 1972.
P22	stochastic loss median	Berman et al. 1985, Chiu and Larson 1985.
P23	stochastic queue	Batta et al. 1988a, b, Batta 1988a, b, Berman 1985, Berman et al. 1985, Brandeau and Chiu 1988e, Chiu 1987b, 1986, Chiu et al. 1985a, Jung and Cavalier 1988.
P24	absolute *p*-center	Chandrasekaran and Daughety 1981, Garfinkel et al. 1977, Hakimi 1964, Hakimi 1965, Handler and Rozman 1985, Kariv and Hakimi 1979a, Minieka 1970, 1977, Tansel et al. 1982a, b.
P25	vertex-constrained *p*-center	Farley 1982. Halfin 1974, Hedetniemi et al. 1981.
P26	*p*-facility minimax (or planar minimax)	Chen 1983, Chen and Handler 1983, Drezner and Wesolowsky 1978a, Elzinga and Hearn 1972a, Elzinga et al. 1976, Francis 1967, Megiddo 1983, Tansel et al. 1983a, Ward and Wendell 1985, Wesolowsky 1972.
P27	unreliable *p*-center	Drezner 1985.
P28	minimax assignment (or bottleneck assignment)	Garfinkel 1971, Hsu and Nemhauser 1979.
P29	*k*-centrum	Andreatta and Mason 1985.
P30	*m*-facility minimax with mutual communication	Dearing et al. 1976.
P31	covering	Elzinga and Hearn 1972b, Minieka 1970, Moon and Chaudry 1984, Tansel et al. 1982b.
P32	maximal covering	Church and ReVelle 1974, Francis and White 1974, Megiddo et al. 1983.
P33	minimum cost partial covering	Kolen 1983.
P34	*p*-cover	Elzinga and Hearn 1972a.
P35	hierarchical covering	Moore and ReVelle 1982.
P36	stochastic queue center	Brandeau and Chiu 1988c.
P37	obnoxious facility (or antimedian)	Church and Garfinkel 1978, Drezner and Wesolowsky 1985, Minieka 1983.
P38	distant point (or anticenter)	Kulshrestha 1984, Minieka 1983.
P39	*p*-dispersion (or anticovering)	Chandrasekaran and Daughety 1981, Moon and Chaudry 1984.
P40	Lorentz measure	Maimon and Halpern 1985.
P41	variance measure	Maimon and Halpern 1985.
P42	Vororoi partitioning problem	Hakimi and Labbe 1988.

TABLE 3 (*cont'd*)

Problem Number	Problem Name	References
P43	Condorcet point (or voting measure)	Hansen and Thisse 1981, Labbe 1985.
P44	cent-dian (or medi-center)	Halpern 1976, 1978, 1979, Handler 1976, McGinnis and White 1978.
P45	bi-objective multi-facility minimax	Tansel et al. 1982a.
P46	multi-criteria facility location	Ross and Soland 1980.
P47	distance constraints	Dearing et al. 1976.
P48	general competitive location	Dobson and Karmarkar 1987, Drezner 1982, Ghosh and Craig 1984, Hakimi 1983, Hotelling 1929, Tobin and Freisz 1986, Wendell and McKelvey 1981.
P49	Hotelling's problem	Hotelling 1929.
P50	duopoly location	Drezner 1982.
P51	competitive location with market externalities	Brandeau and Chiu 1988d.
P52	simple plant location under uncertainty	Hodder 1984, Jucker and Carlson 1976.
P53	price-sensitive demand problem	Erlenkotter 1977, Jurion 1983, Wagner and Falkson 1975.
P54	Hypercube problem	Larson 1974.

house location problem or the *location-allocation problem*, is to locate a set of m servers in a plane with discrete demands, and to simultaneously determine service areas for those servers (Cooper 1963, 1964, 1967, Sherali and Shetty 1977) in order to minimize travel or service cost. Distances may be measured by a rectilinear norm (Love and Morris 1975a, Picard and Ratliff 1978, Sherali and Shetty 1977, Wesolowsky and Love 1971, 1972), a Euclidean norm (Cerveny 1980, Chen 1983, Cooper 1963, 1964, Francis and Cabot 1972), an L_p norm (Drezner and Wesolowsky 1978b, 1981, Love and Morris 1975b), or a hyper-rectilinear norm (Juel and Love 1985). Thisse et al. (1984) have considered general block and round norms. Some versions of the problem include weighted distance between servers in the objective function (Drezner and Wesolowsky 1978b, Love and Morris 1975b, Vergin and Rogers 1967). Other formulations assume that the number of servers is also a decision variable, and assume fixed costs associated with each potential server location (Efroymson and Ray 1966); Soland (1974) also incorporated concave production and distribution costs. Other versions assume that servers may have fixed capacities, and some assume that servers can be located only at a finite number of possible locations. Similarly, server capacity may also be a decision variable. Other versions include location-specific production costs in the objective function (Sherali and Adams 1984).

The *simple plant location problem* (P8) is a particular version of the warehouse location problem which seeks to select an undetermined number of facility locations from a finite number of potential locations to minimize fixed setup cost plus variable service cost; facilities are assumed to have no capacity constraint (Efroymson and Ray 1966, Erlenkotter 1978, Spielberg 1969). In some cases, cost functions with economies of scale have been included (Feldman et al. 1966, Manne 1964). The *capacitated plant location problem* (P9) is a version of P8 which incorporates capacity restrictions on the servers (Akinc and Khumawala 1977, Sá 1969); when only some servers have capacity restrictions, the problem is sometimes referred to as a *mixed plant location problem* (Guignard and Spielberg 1979). There is also a class of *dynamic warehouse location problems* (P10) which pose the question of how to locate warehouses over time in response to changing demand patterns so as to minimize total long-run cost (Geoffrion 1975, Van Roy and Erlenkotter 1982, Wesolowsky and Truscott 1975). For such problems, potential improvements in service are traded off against relocation costs.

The above versions of the warehouse location problem assume that a single commodity is being handled. The *multi-commodity service facility location problem* (P11) is an extension of the simple plant location problem in which a number of different commodities must be handled, and each service facility can handle only one type (Karzakis and Boffey 1981, Laundy 1985, Neebe and Khumawala 1981, Warszawski 1973). A related problem is the *multi-commodity distribution facility location problem* (P12), in which a set of intermediate distribution facilities between service facilities and demand points are to be located (Geoffrion and Graves 1974). For this problem, the objective function costs depend not only on distribution center and demand point locations, but also on service facility locations. Many other variants of the simple plant location problem have been studied; a detailed review is given by Krarup and Pruzan (1983).

The *unreliable p-median problem* (P13) generalizes the planar minisum problem to allow for the possibility that facilities may become inactive; p facilities are to be located to minimize expected travel cost, given that up to q facilities may fail at any one time (Drezner 1985).

Some median problems involve goods or services which require several processing stages and therefore involve travel to several servers (Hakimi and Maheshwari 1972, Wendell and Hurter 1973b) or from a demand point to a server to another demand point (O'Kelly 1986). For example, the *hub location problem* (P14) seeks to locate a central or "hub" facility to minimize the sum of weighted distances from each demand point through the hub facility to every other demand point (O'Kelly 1986). Two interacting hubs have also been considered (O'Kelly 1986).

Another important type of planar minisum problem is the *facility layout* (or *plant*

KEY TO FIGURE 1

1 Key to Objective Type:
- 1 = minimize average travel time/cost
- 2 = maximize net income
- 3 = minimize average response time
- 4 = minimize maximum travel time/cost
- 5 = maximize minimum travel time/cost
- 6 = maximize average travel time/cost
- 7 = minimize server cost s.t. minimum service constraint
- 8 = optimize distance-dependent utility function
- 9 = other optimizing objective
- 10 = nonoptimizing objective

2 "✓" indicates applicable, "✓/X" indicates optional

3 Key to Decision Variables:
- 1 = server/facility location
- 2 = service area/dispatch priorities
- 3 = number of servers and/or service facilities
- 4 = server volume/capacity
- 5 = type of good produced by each server
- 6 = routing/flows to demand points
- 7 = queue capacity
- 8 = other

Note ( ) indicates an optional decision variable

4 L = link
- T = tree
- N = general network
- P = plane

5 N-C = network constrained
- R = rectilinear
- E = Euclidean
- Any = any planar norm

6 u = uncapacitated, c = capacitated
- R = reliable, N = unreliable

7 C = cooperating servers, N = noncooperating servers; CD = closest distance service area, NCD = nonclosest distance service area; "—" indicates not applicable

8 "—" indicates not applicable, FCFS = first come, first served, SOR = spatially oriented rule

FIGURE 1

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18
I OBJECTIVE[1]	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Type of Location Dependence[2]: Server-Demand Point Distances	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓
—Weighted vs Unweighted	W/U	W/U	W	W	W	W	W	W	W	W	W		W	W		W	W	W
—Some vs. All Demand Points	A	A	A	A	A	A	A	A	A	A	A		A	A		A	A	A
—Routed vs. Closest	C	C	C	C	C	C	C	C	C	C	C		C	C		C	C	R
• Inter-Server Distances															✓		✓	
• Absolute Server Location											✓	✓						✓/X
• Server-Distr. Facility Distances												✓						
• Distr. Facil-Demand Pt. Distances												✓						
• Other																		
II DECISION VARIABLE(S)[3]	1, 2	1, 2	1, 2	1	1, 2	1	1, 2, (3), (4)	1, 2, (3), (4)	1, 3	1	1, 2, 5	1, 2, 4, 6	1	1	1	1	1	1, (3), 6
III SYSTEM VARIABLES/ PARAMETERS																		
Topological Structure[4]	N	N	N	N	T	P	P	P	P	P	P	P	P	P	P	P	P	P
Travel Metric[5]	N-C	N-C	N-C	N-C	N-C	Any	Any	Any	Any	Any	Any	Any	Any	Any	Any	Any	Any	Any
Travel Time/Cost: Deterministic vs. Probabilistic	D	P	P	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Constrained vs. Unconstrained	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
Vol-Dependent vs Non-Vol-Depend.	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	VD/NVD

FIGURE 1 (*cont'd*)

Demand: Continuous vs. Discrete	C/D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Deterministic vs. Probabilistic	D	P	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Cost-Indep. vs. Cost-Dependent	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI
Time-Invariant vs. Time-Varying	TI	TI	TI	TI	TI	TI	TI	TI	TI	TV	TI	TI	TI	TI	TI	TI	TI	TI
Number of Servers:	P	P	P	1	P	1	P	P	P	P	P	P	P	1, 2	P	P	P	P
Number of Service Facilities:	P	P	P	1	P	1	P	P	P	P	P	P	P	1, 2	P	P	P	P
Number of Commodities:	1	1	1	1	1	1	1	1	1	1	r	r	1	1	1	1	1	1
Server Location: Constrained vs. Unconstrained	U	U	U	U	C	C/U	C/U	C	C	C	C	C	C	U	C	C	C	C
Finite vs. Infin. # Poten. Loc's	I	I	I	I	F	F/I	F/I	F	F	F	F	F	F	I	F	F	F	F
Fixed vs. Dependent on Sys. Status	F	F	D	F	F	F	F	F	F	D	F	F	F	F	F	F	F	F
Zero vs. NonZero Relocation Cost	Z	Z	NZ	Z	Z	Z	Z	Z	Z	NZ	Z	Z	Z	Z	Z	Z	Z	Z
Deterministic vs. Prob. Location	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Zero vs. NonZero Fixed Cost	Z	Z	Z	Z	Z	Z	Z/NZ	NZ	NZ	NZ	NZ	NZ	Z	Z	Z	Z	Z	Z/NZ
Server Capacity[6]	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	c, R	u, R	u, R	c, R	u, R	u, R	u, R	u, R	u, R	u/c, R
Service Area and Dispatch Priorities[7]	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	N, CD	—	N, CD	N, CD	N, CD/NCD
Service Discipline[8]	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Queue Capacity	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

	P19	P20	P21	P22	P23	P24	P25	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36
I. OBJECTIVE[1]	1	1	2	9	3	4	4	4	4	4	9	4	7	9	9	4	9	9
Type of Location Dependence:[2] Server-Demand Point Distances	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓/×	✓/×	✓		✓	✓
—Weighted vs. Unweighted	W	U	W	W	W	W/U	W/U	W/U	W/U	W/U	W/U	W/U	U	U	U		U	W/U
—Some vs. All Demand Points	A	A	A	A	A	A	A	A	A	A	S	A	A	A	A		A	A
—Routed vs. Closest	C	R	C	C	C	C	C	C	C	C	C	C	C	C	C		C	C
• Inter-Server Distances												✓	✓/×	✓/×		✓		
• Absolute Server Location															✓			
• Server-Distr. Facility Distances																		
• Distr. Facil-Demand Pt Distances			✓															
• Other																		
II DECISION VARIABLE(S)[3]	1, 6	1, 6	1, 4, 8	1	1	1	1	1	1	1	1	1	1, 3	1	1, 3	1	1	1
III. SYSTEM VARIABLES/ PARAMETERS																		
Topological Structure[4]	P	N	P	N	N, P	N	N	P	P	P	N	N	P	P	P	P	P	N
Travel Metric[5]	Any	N-C	Any	N-C	Any	N-C	N-C	Any	Any	Any	N-C	N-C	Any	Any	Any	Any	Any	N-C
Travel Time/Cost: Deterministic vs. Probabilistic	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Constrained vs. Unconstrained	U	U	U	U	U	U	U	U	U	U	U	U	U	C	U	U	C	U
Vol-Dependent vs. Non-Vol-Depend.	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD

FIGURE 1 (*cont'd*)

Demand. Continuous vs Discrete	D	D	D	D	C/D	D	D	D	D	D	D	D	D	D	D	—	D	D
Deterministic vs Probabilistic	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	—	D	D
Cost-Indep. vs. Cost-Dependent	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	CI	—	CI	CI
Time-Invariant vs. Time-Varying	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	—	TI	TI
Number of Servers:	P	1	P	P	1	P	P	P	P	P	1	P	P	P	P	P	P	1
Number of Service Facilities:	P	1	P	1	1	P	P	P	P	P	1	P	P	P	P	P	P	1
Number of Commodities:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	r	1
Server Location. Constrained vs. Unconstrained	U	U	C	U	U	U	C	U	U	C	U	C	U	U	U	U	U	U
Finite vs Infin. # Poten. Loc's	I	I	F	I	I	I	F	I	I	F	I	F	I	I	I	I	I	I
Fixed vs Dependent on Sys. Status	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
Zero vs. NonZero Relocation Cost	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z
Deterministic vs. Prob Location	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Zero vs NonZero Fixed Cost	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z
Server Capacity[6]	c, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, N	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R
Service Area and Dispatch Priorities[7]	N, NCD	—	N, CD	C, CD	—	N, CD	N, CD	N, CD	N, CD	N, CD	—	N, CD	N, CD	N, CD	N, CD	—	N, CD	—
Service Discipline[8]	—	—	—	FCFS	FCFS	—	—	—	—	—	—	—	—	—	—	—	—	—
Queue Capacity	—	—	—	0	∞	—	—	—	—	—	—	—	—	—	—	—	—	—

	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50	P51	P52	P53	P54
I OBJECTIVE[1]	6	5	5	9	9	9	9	9	9	9	9	2	2	9	9	9	3 or 8	10
Type of Location Dependence:[2] Server-Demand Point Distances	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
—Weighted vs. Unweighted	W/U	W/U		W/U	W/U	W	U	W/U	W	W	U	W	W	W	W	W	W	W
—Some vs. All Demand Points	A	A		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
—Routed vs Closest	C	C		C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
• Inter-Server Distances			✓						✓		✓		✓					
• Absolute Server Location										✓		✓				✓	✓	
• Server-Distr. Facility Distances																		
• Distr. Facil-Demand Pt. Distances																		
• Other												✓		✓				
II DECISION VARIABLE(S)[3]	1	1	1	1	1	1, 2	1	1	1	1	1, 3	1	1	1	1	1	1	1–4, 7
III. SYSTEM VARIABLES/ PARAMETERS																		
Topological Structure[4]	N	N	N	T	T	N	N	N	N	P	N	P	L	P	N	P	P	P
Travel Metric[5]	N-C	N-C	N-C	N-C	N-C	N-C	N-C	N-C	N-C	Any	N-C	Any	N-C	E	N-C	Any	Any	Any
Travel Time/Cost. Deterministic vs. Probabilistic	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Constrained vs. Unconstrained	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
Vol-Dependent vs. Non-Vol-Depend	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD	NVD

FIGURE 1 (*cont'd*)

Demand: Continuous vs. Discrete	D	D	—	D	D	C	D	D	D	D	D	D	D	D	D	D	D	D
Deterministic vs. Probabilistic	D	D	—	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Cost-Indep vs Cost-Dependent	CI	CI	—	CI	CI	CI	CI	CI	CI	CI	CI	CI/CD	CI	CI	CI	CI	CD	CI
Time-Invariant vs Time-Varying	TI	TI	—	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI	TI
Number of Servers:	1	1	P	1	1	P	1	1	P	P	P	P	2	2	2	1	P	P
Number of Service Facilities.	1	1	P	1	1	P	1	1	P	P	P	P	2	2	2	1	P	P
Number of Commodities.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Server Location: Constrained vs. Unconstrained	U	U	U	U	U	U	U	U	U	C	U	C	U	U	U	C	C/U	C
Finite vs Infinite, # Poten. Loc's	I	I	I	I	I	I	I	I	I	F	I	F	I	I	I	F	F/I	F
Fixed vs. Dependent on Sys. Status	F	F	F	F	F	F	F	F	F	F	F	F	F	D	F	F	F	F
Zero vs. NonZero Relocation Cost	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z
Deterministic vs. Prob. Location	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	P
Zero vs NonZero Fixed Cost	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z	Z
Server Capacity[6]	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	u, R	c, R	u, R	u, R	c, R
Service Area and Dispatch Priorities[7]	—	—	N, CD	—	—	N, NCD	—	—	N, CD	N, CD	N, CD	N, CD/ NCD	N, CD	N, CD	N, NCD	—	N, CD	N, CD
Service Discipline[8]	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	SOR
Queue Capacity	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0 or ∞

layout) problem (P15) (Armour and Buffa 1963, Francis 1973, Francis and White 1974, Hillier and Connors 1966, Mallette and Francis 1972). In the typical formulation, a fixed number of facilities are to be located to minimize the sum of interserver flow costs, and locations are chosen from a finite set of potential sites.

We note that minisum problems with discrete demands and a finite number of potential server locations are often modelled as generalized assignment problems (Ross and Soland 1978). The objective of the *linear assignment problem* (P16) is to assign new facilities to sites to minimize transportation costs between facilities and demand points (Mallette and Francis 1972), while the objective for the *quadratic assignment problem* (P17) also includes inter-facility costs (Gilmore 1962, Hillier and Connors 1966, Lawler 1963, Pierce and Crowston 1971).

For some problems, both server locations and flows of goods are to be simultaneously determined. In the *location-routing problem* (P18) an unspecified number of servers are to be located to minimize the sum of server operating costs and routing costs (Drezner et al. 1985, Laporte and Nobert 1981, Laporte et al. 1983). Servers may be capacity-constrained, and travel time/cost along any path may be volume dependent. Closely related is the *transportation location problem* (P19) which involves locating a fixed number of capacitated servers in a planar region and determining amounts of the commodity to be transported from each server to each demand point (Cooper 1972). The objective is to minimize total shipping cost. Another problem involving routing is the *travelling salesman location problem* (P20) (Burness and White 1976, Berman and Simchi-Levi 1986): a single server is to be located on a network with probabilistic calls for service to minimize the expected length of a travelling salesman tour.

For the *production-location problem* (P21), the objective is to simultaneously locate facilities and determine the optimal production input mix for each facility (Hurter and Wendell 1972), in order to maximize profit. The prices of input goods are assumed to depend on facility location, or the prices may be stochastic (Hurter and Martinich 1984).

4.2. *Minimization of Average Response Time*

Some location problems include queueing effects in the objective function. In the *stochastic loss median problem* (P22) a single facility (which may house one or many servers) is to be located on a general network to minimize a weighted function of travel cost and opportunity cost of lost customers (Berman et al. 1985, Chiu and Larson 1985). The *stochastic queue median problem* (P23) assumes that a single server with infinite queue capacity is to be located on a general network to minimize average response time (Berman et al. 1985, Chiu 1986, 1987b, Chiu et al. 1985). The problem has also been studied with two servers (Berman and Larson 1985); on a tree network (Chiu et al. 1985, Chiu 1986); on a network with probabilistic times and demands (Berman 1985); and with continuous link demands (Chiu 1986). Variations of the basic problem include rejection of calls at a cost (Batta 1988a), inclusion of service-time-dependent queueing disciplines (Batta 1988b), and priority queueing (Batta et al. 1988a). Other researchers have considered the stochastic queue median problem in a planar region (Batta et al. 1988b, Brandeau and Chiu 1988e, and Jung and Cavalier 1988).

4.3. *Minimization of Maximum Travel Time/Cost*

The objective of the *p-center problem* (or *absolute p-center problem*) (P24) is to locate p servers on a general network to minimize maximum (weighted or unweighted) distance between a set of discrete demand points and the servers (Hakimi 1964, 1965, Handler and Rozman 1985, Kariv and Hakimi 1979a, Minieka 1977). The problem has also been studied with distances measured by a nonlinear cost function (Tansel et al. 1982a, b), as well as with continuous demands. When server locations are limited to nodes of the network, the problem is the *vertex-constrained p-center problem* (P25) (Farley 1982, Halfin 1974). We note that, when $p = 1$, (P25) is referred to as the *Jordan center problem* (Hedetniemi et al. 1981).

The planar version of (P24) is the *p-facility minimax problem* or *planar minimax problem* (P26) (Francis 1967, Wesolowsky 1972). For this problem, distances may be measured by a rectilinear metric (Wesolowsky 1972), a Euclidean metric (Chen 1983, Chen and Handler 1983, Elzinga et al. 1976, Megiddo 1983), or an L_p metric (Drezner and Wesolowsky 1978a), or a block norm (Ward and Wendell 1985), and may be weighted or unweighted. Some versions of the problem also include distances between servers in the objective function, distance constraints between servers (Tansel et al. 1983a), or linear add-on's to the distance function from certain demand points (Elzinga and Hearn 1972a). The *unreliable p-center problem* (P27) is a planar minimax problem which includes the possibility that up to q servers may fail at any one time (Drezner 1985), similar to the unreliable p-median problem (P13). When m servers are to be located at m of n possible locations in order to minimize maximum cost, the problem is referred to as the *minimax assignment problem* or *bottleneck assignment problem* (P28) (Garfinkel 1971). A similar problem with a maximin objective has also been considered (Hsu and Nemhauser 1979).

Closely related to the 1-center problem is the *k-centrum problem* (P29). The objective of this problem is to locate a single server to minimize the (weighted or unweighted) sum of distances between the server and the k farthest demand points (Andreatta and Mason 1985).

The *m-facility minimax problem with mutual communication* (P30) involves locating m servers on a network to minimize the maximum of the inter-facility distances and distances between facilities and demand points subject to inter-facility and facility-demand point distance constraints (Dearing et al. 1976).

The objective of the *covering problem* (P31) is to locate a minimum number of servers on a network so that every server (or demand point) is within a specified distance of the nearest server (Elzinga and Hearn 1972b, Minieka 1970, Moon and Chaudry 1984), while the *maximal covering problem* (P32) seeks to locate a fixed number of servers to maximize the number of servers (or demand points) within a specified distance of the nearest server (Church and ReVelle 1974, Francis and White 1974, Megiddo et al. 1983). Daskin (1983) has extended this problem to allow for the case when servers are not always available to answer calls immediately. In the *minimum cost partial covering problem* (P33), the distance constraint is relaxed and the objective is to minimize the sum of facility setup costs plus a penalty function for not serving some demand points (Kolen 1983). When the number of servers is fixed and the objective is to locate the servers to minimize the maximum distance between any pair of servers, the problem is referred to as the *p-cover problem* (P34) (Elzinga and Hearn 1972a). The *hierarchical covering problem* (P35) assumes that there are N types of facilities which provide different levels of service (Moore and ReVelle 1982). A facility of type j provides a radius r_j of coverage. The problem is to locate n_j facilities of type j to maximize the total population which has access to all levels of service.

4.4. *Minimization of Maximum Response Time*

Analogous to the p-center problem is the *stochastic queue center problem* (P36), which has the objective of minimizing maximum expected response time to any demand point (that is, expected queue delay plus maximum travel time to any call). A single-server network version of the problem has been studied (Brandeau and Chiu 1988c).

4.5. *Maximization of Minimum or Average Travel Time/Cost*

The objective of the *obnoxious facility problem* (or *antimedian problem*) (P37) is to locate a single server on a network to maximize average (weighted or unweighted) travel distance between the server and a set of discrete demand points (Church and Garfinkel 1978, Minieka 1983). A version with multiple servers has also been considered (Drezner and Wesolowsky 1985). The objective of the *distant point problem* (or *anticenter problem*) (P38) is to maximize minimum (weighted or unweighted) distance between the server and demand points (Kulshrestha 1984, Minieka 1983). For the *p-dispersion problem* (or *anti-covering problem*) (P39) the objective is to locate p facilities on a network to maximize the minimum distance between any pair of facilities (Chandrasekaran and Daughety 1981, Moon and Chaudry 1984).

4.6. *Problems with Other Objectives*

Many location problems based on multiple objectives, nonoptimizing objectives, or other less traditional objectives do not fit into the above groupings.

The objective of the *Lorentz measure problem* (P40) is to locate a single server on a tree network to minimize an equity measure based on weighted distances of customers from the server (Maimon and Halpern 1985), while the *variance measure problem* (P41) is based on a variance measure of individual customer distances from the system-wide average distance (Maimon and Halpern 1985). The objective of the *Voronoi partitioning problem* (P42) is to locate p facilities so that their service areas (as defined by four different measures) are as equal as possible (Hakimi and Labbe 1988). The objective of the *Condorcet point problem* or *voting measure problem* (P43) is to find a single facility location on a network which is closest to an absolute majority of customers (Hansen and Thisse 1981, Labbe 1985).

The *cent-dian* (or medi-center *problem* (P44) is a single-server problem on a general network with an objective which is a linear combination of center and median objectives (Halpern 1976, 1978, 1979, Handler 1976). A planar version has also been studied (McGinnis and White 1978).

One class of problems has objectives based not on a single-valued function minimization but on vector minimization of different objectives (Lowe 1978, Ross and Soland 1980, Tansel et al. 1982a); rather than finding a single optimal set of server locations, the objective of these problems is to find a set of efficient (Pareto optimal) points. For the *bi-objective multifacility minimax problem* (P45) servers are to be located on a general network to vector-minimize maximum weighted distances between servers and demand points, as well as between servers (Tansel et al. 1982a). Ross and Soland (1980) studied a *multi-criteria facility location problem* (P46) in which a variable number of facilities are to be located at a subset of n planar locations in order to vector-optimize several different objectives, including: fixed facility setup cost; variable service cost; fraction of demand served; average distance travelled or average service time.

The objective of the *distance constraints problem* (P47) is based solely on satisfying a set of constraints: a fixed number of servers are to be located on a general network so that they are within a specified distance of demand points as well as one another (Dearing et al. 1976).

Another group of location problems incorporates economic and/or competitive factors. In the general *competitive location problem* (P48) as defined by Hakimi and others (Ghosh and Craig 1984, Hakimi 1983, Tobin and Freisz 1986, Wendell and McKelvey 1981), the objective is to optimally locate r new facilities to compete with p existing facilities for providing service or goods to customers at a given set of discrete demand points. Servers determine the amount of the goods they will provide and the price they will charge. Demand may be constant or may depend on the price of the good, and customers may or may not patronize the closest facility. *Hotelling's problem* (P49) assumes that two competing servers are to be located on a single link with continuous uniform demands (Hotelling 1929). Drezner (1982) has considered a more general *duopoly location problem* (P50) with two servers located in a plane with discrete demands. The objective of the first server is to find a location that will maximize its market share, given that the second server will choose the best possible location in response to the location of the first server. Customers are assumed to patronize the closest facility. A related problem (Brandeau and Chiu 1988d) is a *competitive location problem with market externalities* (P51). In the case of negative externalities, customers prefer a facility less as it serves a larger share of the market, while the reverse is true in the case of positive externalities. Dobson and Karmarkar (1987) have examined a competitive location problem in which the objective is to find a set of facilities that are economically viable while discouraging new entry.

Jucker and Carlson (1976) have formulated an economic version of the warehouse location problem which takes into account uncertainties in price, quantity supplied, quantity demanded, and risk attitude of the firm. A single firm is to be located in a planar region with discrete demands. The objective of this *simple plant location problem under uncertainty* (P52) is for the firm to maximize its total profit subject to its level of risk aversion (where the risk is a function of the variance of the firm's profit). Hodder (1984) has considered the effect on facility location of using financial market models to measure the risk associated with location investment. Another model incorporating economic factors has been considered by Hakimi and Kuo (1987).

Other researchers have studied the location of facilities when demands depend on the price of the good and/or on the distance to the server (i.e., service cost). The general *price-sensitive demand problem* (P53) is typically formulated with discrete demands occurring either on a plane or general network, and depending on the type of server (private vs. public), the objective may be maximization of profit or maximization of social welfare (Erlenkotter 1977, Wagner and Falkson 1975). A related location model, studied by Jurion (1983), considers an economy with two goods, a public good and a composite private good, which can be produced subject to a joint budget constraint. The objective is to locate n facilities to produce the public good and determine production levels in order to either maximize total consumer utility or maximize minimum utility, where consumer utility is based on the amount of each type of good consumed, as well as the distance to the facility producing the public good.

Finally, the *Hypercube model* (P54), formulated by Larson (1974), is a descriptive location model. Values of decision variables are specified in advance, and then the model evaluates system behavior given those values, and calculates a set of system performance measures. The decision maker can change the decision variables until satisfactory performance measures are obtained. Demands occur by a time-homogeneous Poisson process at discrete points in a planar region. There are n servers who respond to calls according to a dispatch preference ordering. Queue delays may occur. Because the model is descriptive, almost any system variable can become a decision variable, as indicated in Figure 1.

5. Solution Techniques for Location Problems

As shown in Table 4, a variety of different exact and heuristic solution approaches have been developed to solve location problems. We briefly describe some of them here.

5.1. *Exact Solution Approaches*

Many single-server location problems on tree networks or in a planar region have been shown to be convex (e.g., see Dearing et al. 1976), so that analytical necessary and sufficient conditions for an optimal solution can be developed (e.g., Chiu et al. 1985, Cooper 1964). Some simple two-server location problems on tree networks (which are not convex) have also been solved exactly using partial convexity properties (e.g., see Handler 1978). For single-server location problems on general networks (which are not, in general, convex), Hooker (1986) has developed a solution approach which decomposes

TABLE 4

Some Solution Techniques for Location Models

I. Exact Solution Techniques	
	A. Analytical Solution/Optimality Result*
	B. Integer Programming/Branch and Bound
	C. Dynamic Programming/Backtrack Programming
	D. Convex Programming
	E. Other
II. Heuristic Solution Techniques	
	A. Exchange Heuristics
	B. Greedy ("Add") Heuristics
	C. Drop Heuristics
	D. Sequential Location and Allocation
	E. Solution of an Approximate Problem
	F. Solution of a Relaxed Problem
	G. Solution of a Restricted Problem
	H. Other
III Techniques for Evaluation of Heuristics	
	A. Bound on Optimal Solution
	B. Worst Case Analysis
	C. Probabilistic Analysis
	D. Statistical Estimation
	E. Stopping Rule

* May be combined with numerical search methods.

the network into "treelike" segments over which the objective function is convex. These treelike segments are identical to the primary region concept introduced in Berman et al. (1985).

Most multi-server location problems of the type reviewed above are nonconvex, nonlinear optimization problems and, as such, can be difficult to solve. Many have been shown to be NP-hard (see, for example, Cornuejols et al. 1977, Kariv and Hakimi 1979a, b, Hakimi 1983).

Some location problems—such as many with discrete demand, or problems in which servers can be located only at a discrete set of points (either through an optimality result such as that for the p-median problem (Hakimi 1964) or for certain planar location problems (Wendell and Hurter 1973a), or through an explicit constraint)—have a discrete solution set. These combinatorial optimization problems have been solved using a variety of enumeration-based methods. For example, the p-center and p-median problems on a general network with discrete demands have been solved using integer and linear programming (Garfinkel et al. 1974, 1977, Geoffrion and Graves 1974, Morris 1975, ReVelle and Swain 1970). Other problems such as the warehouse location problem with rectangular distances and the p-median of a network with discrete demands have been solved by combinatorial tree search methods, including branch and bound (Efroymson and Ray 1966, El-Shaieb 1973, Gavett and Plyter 1966, Jarvinen et al. 1972, Khumawala 1972, Kuenne and Soland 1972, Matula and Kolde 1976), convex programming (Love and Morris 1975b), discrete dynamic programming (Curry and Skeith 1969, Love 1976), and backtrack programming (Hung 1973).

However, many discrete-choice location problems have too many potentially optimal solutions to be practically solved using such techniques. Furthermore, many other location problems—such as those in which servers may be located anywhere in a continuous topographic region (for example, unconstrained problems with continuous demands)—have an infinite number of potentially optimal solutions, and so cannot be solved by enumeration-based methods. This difficulty is often compounded by the fact that derivatives for many such problems do not exist, so that many differentiable optimization techniques cannot be applied. Even for problems with well-defined derivatives, the objective functions are typically highly nonconvex, with many locally optimal solutions (Baxter 1981, Cooper 1964, Eilon et al. 1971). Computational difficulties have led to the development of heuristic solution techniques.

5.2. *Heuristic Solution Techniques*

Heuristic algorithms vary greatly depending on the particular problem, but some of the general approaches are as follows:

For problems with a discrete but large set of possible solutions, an "exchange heuristic" (similar to the 2-opt, 3-opt, etc. exchange heuristics for the travelling salesman problem) works by arbitrarily selecting server locations from the allowable set, then relocating servers, each time substituting in the location point(s) which yield the greatest improvement in the objective function, until no more improvement can be made. This approach has been applied to problems such as the warehouse location problem (Cooper 1964, Cornuejols et al. 1977, Hillier and Connors 1966, Kuehn and Hamburger 1963) and the p-median of a general network with discrete demands (Teitz and Bart 1968).

Another approach, used either with large problems or problems with an infinite number of location choices, is based on successive optimization, and is referred to as a "greedy heuristic" or an "add heuristic". Servers are located one by one, each time at the location which improves the objective function by the greatest amount given the configuration of servers already located, until all servers have been located (Cooper 1964, Cornuejols et al. 1977, Kuehn and Hamburger 1963, Spielberg 1969, Vergin and Rogers 1967).

A related approach, used for problems for which m locations must be chosen from n possibilities, is a "drop heuristic": All n locations are initially chosen, and then locations are dropped one by one, each time selecting the location that has the least ill effect on the objective function (Jarvinen et al. 1972).

A fourth approach is known as "sequential location and allocation": given a fixed set of service areas, servers are located, and then given those locations, new service areas are found, etc., until no further improvement can be made. An algorithm of this type has been applied to the warehouse location and network m-median problems (Baxter 1981, Cooper 1963, 1964, 1967, Eilon et al. 1971, Love and Juel 1982, Maranzana 1964, Teitz and Bart 1968), to the 2-median problem on a tree network (Brandeau et al. 1986), and to a multi-server planar queueing model (Jarvis 1975).

Another approach is to introduce approximations so that the simplified problem can be solved using exact methods. This type of approach has been applied to the warehouse location problem with rectangular distances (Chen 1983, Wesolowsky and Love 1972), the p-center problem in a plane with discrete demands (Chen 1983), and the p-median problem on a tree with continuous demands (Cavalier and Sherali 1986).

In some cases researchers solve a relaxed problem: for example, one approach is to restrict the solution set to a manageable number of possibilities. For the warehouse location problem with Euclidean distances, one heuristic restricts the server locations to demand points (Cooper 1964), while another considers only certain types of district configurations (Hochbaum 1982). For another warehouse location problem, Baker (1974) used an exact dynamic programming algorithm, but restricted the set of solutions to be considered at any stage.

5.3. *Techniques for Evaluation of Heuristics*

Since many location problems have been solved by heuristic methods, researchers have studied ways in which the quality of solution obtained can be evaluated. For example, a variety of lower bounds have been developed for the warehouse location problem (Cornuejols et al. 1977, Efroymson and Ray 1966, Juel 1981).

Worst-case analysis has been used for some location problems (Cornuejols et al. 1977, 1980). A related approach is probabilistic analysis (Cornuejols et al. 1980), where probability distributions are assumed for problem parameters, and then the performance of the heuristic is tested over a range of simulated problems.

Golden (1978) has suggested using a statistical method for estimating the globally optimal objective function value after many locally optimal solutions have been obtained; this method has been tested on many combinatorially difficult problems including the travelling salesman problem (Golden 1978) and the planar p-median and p-center problems (Brandeau and Chiu 1988b).

6. Conclusion

As evidenced by a growing body of literature, location theory is an active field of research, with many new types of problems emerging in recent years. We remark on several trends which have appeared in recent location research. First, location models have been designed for an expanding range of applications. In addition to the location applications described in §2, other types of problems have been represented by location models. For example, in marketing and product design research, product positioning is cast as a location problem in an n-dimensional attribute space; demand points correspond to the desired attribute coordinates of representative consumer groups (e.g., see Gavish et al. 1983, Bachem et al. 1981 and Nelson 1986). Second, many location models have been integrated with other types of models, such as production models (Hurter and Martinich 1984), game theory models (Ghosh and Craig 1984, Hakimi 1983) and spatial interaction models (O'Kelly 1984). Finally, many recent models (such as Hodder 1984, Jurion 1983) have incorporated more realistic objective functions than the simple objectives used in earlier models.

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