dumb combinatorial proof for linear squared expansion

Saksham Sethi

October 2021

We will prove $(a+b)^2 = a^2 + 2ab + b^2$ in a combinatorial way, even though this fact is very well known. Say you have a blue balls and b red balls. We will find the number of ways to pick two balls with replacement in two different ways, where order matters. For the LHS, it is straightforward: there are $a+b$ balls in total, and we have to pick two, so there are $(a+b)^2$ ways. For the RHS, we can do casework on the color of the balls. If both balls are blue, there are a^2 ways. Similarly, if both balls are red, there are b^2 ways. If one of them is red and the other is blue, we have two ways to choose which of them we pick first, a ways to choose which blue ball we pick, b ways to choose which red ball we pick, which in total gives us $2ab$ ways. We have our identity now, as desired. $\square$