Cubes and Squares Relationship

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1 Statement

$1^3 + 2^3 + ... + n^3 = (1 + 2 + 3 + ... + n)^2$

2 Proof

We will use induction.

Base Case: $n = 1$**:** Plugging $n = 1$ into the statement, we get $1^3 = (1)^2$, which is true.

Inductive step: We know it works for n, and now we have to prove that it works for $n + 1$. That is, we have to prove

$$1^3 + 2^3 + ... + (n + 1)^3 = (1 + 2 + 3 + ... + (n + 1))^2 \text{ is true.}$$

We keep simplifying the expression:

$(1 + 2 + 3 + \ldots + n + (n + 1))^2 = (1 + 2 + 3 + \ldots + n)^2 + 2(1 + 2 + 3 + \ldots + n)(n + 1) + (n + 1)^2 = 1^3 + 2^3 + \ldots + n^3 + (n + 1)(2(1 + 2 + \ldots + n) + n + 1) = 1^3 + 2^3 + \ldots + n^3 + (n + 1)(n(n + 1) + n + 1) = 1^3 + 2^3 + \ldots + n^3 + (n + 1)^3$

We see that the inductive expression does really equal what we want.

Our proof is complete by induction.