

Sum of Reciprocals Theorem

Saksham MANISH Sethi

1 Statement

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}$ is true if and only if some two of a, b , and c sum to 0.

2 Proof

Simplify the statement to $(ab+bc+ca)(a+b+c) = abc$. Expanding this, we get $a^2b+ab^2+abc+bca+b^2c+bc^2+ca^2+cab+c^2a$. Aha! We can factor this to get $(a+b)(b+c)(c+a) = 0$. Expanding this, we see it is the correct factorization. If the product of three integers is 0, one of them must be 0. Therefore, the original statement is true if and only if some two of a, b , and c sum to 0.