The Sethi Factorial Theorem

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1 Statement

$\sum_{k=1}^{n} k(k!) = (n+1)! - 1.$, for all positive integers n.

2 Interpretation

In simple terms, the statement says:

$(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + ... + (n \times n!) = (n+1)! - 1$, for all positive integers n.

We will refer to this interpretation of the statement in the proof, and not the original.

3 Proof

We use induction.

Base Case: $n = 1$**:** $(1 \times 1!) = (1+1)! - 1 = 2! - 1 = 1$, which is true.

Inductive step: As n **varies:** We have to prove that if n works, $n+1$ works as well. That is, we have to prove

$(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + ... + ((n+1) \times (n+1)!) = (n+2)! - 1,$

given the statement works for n.

Well, lets find the value of it and see if it really does equal $(n+2)! - 1$.

$(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + ... + ((n+1) \times (n+1)!) = (n+1)! - 1 + (n+1)(n+1)!$

$= (n+1)!(1 + (n+1)) - 1 = (n+1)!(n+2) - 1 = (n+2)! - 1$, as desired. $\square$

And we're done.