# The Prime Square Theorem

*Saksham Sethi*

## 1 Statement

Let $p_n$ denote the $n^{th}$ prime number. $(p_1 p_2 p_3 ... p_k) + 1$ can never be a perfect square.

## 2 Proof

Let $q = (p_1 p_2 p_3 ... p_k) + 1$.

We use contradiction. Assume $q$ IS a perfect square.

Then, $(p_1 p_2 p_3 ... p_k) + 1 = m^2$, which simplifies to $p_1 p_2 p_3 ... p_k = (m+1)(m-1)$. Note $p_1 = 2$, which is even. This means $p_1 p_2 p_3 ... p_k$ is also even, which indirectly means $q$ is odd. Since $q$ is odd, $m$ must be odd too since an even number squared can't be odd. Since $m$ is odd, $(m+1)$ and $(m-1)$ are both even, making $(m+1)(m-1) = p_1 p_2 p_3 ... p_k$ a multiple of 4. Now if we recall, 2 is the only prime and $p_1 p_2 p_3 ... p_k$ only has one factor of 2. This contradicts!

And we're done.