Sum of Reverse Powers of 4

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1 Statement

$n^4 + 4^n$ is composite for all $n > 1$.

1.1 Proof

Let $m = n^4 + 4^n$. We have to prove m isn't prime.

If n is even, then n^4 and 4^n are both even, making m even as well. An even number is divisible by 2, so it isn't prime.

If n is odd, then let $n = 2k + 1$ for some positive integer k. Now, $m = n^4 + 4^n = n^4 + 4^{2k+1} = n^4 + (4 \times 4^{2k}) = n^4 + 4(2^k)^4$. That last expression $n^4 + 4(2^k)^4$ is of the form $n^4 + 4b^4$. Since $n^4 + 4b^4 = (n^2 + 2b + 2b^2)(n^2 - 2b + 2b^2)$ and n and b are both positive integers, m cannot be prime. □