Exploiting Minimal Arrival Curves To Deal With Negative Service Curves

How MinAC Saves The Day

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Many of the results will be published in IEEE RTAS 2024.

Outline

Motivation

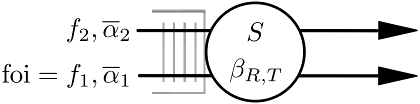
1 Motivation

2 Extension of NC Performance Bounds for (Partially) Negative Service Curves

3 Applications

4 Conclusion

What We're Up To



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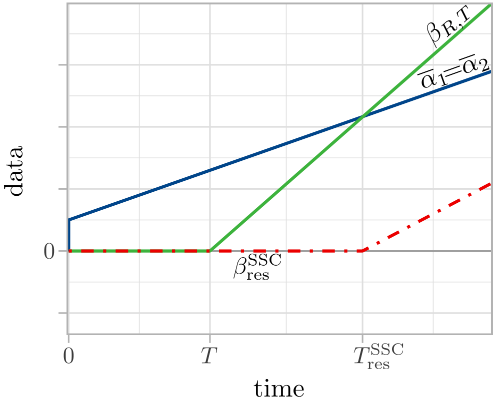

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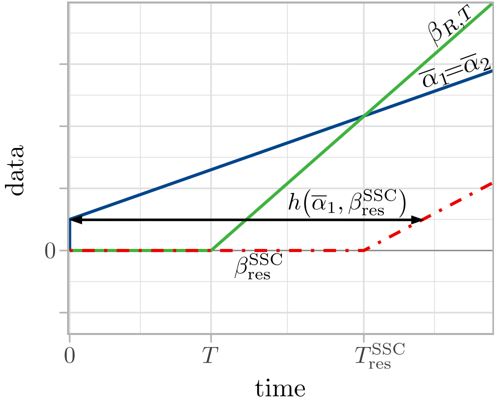

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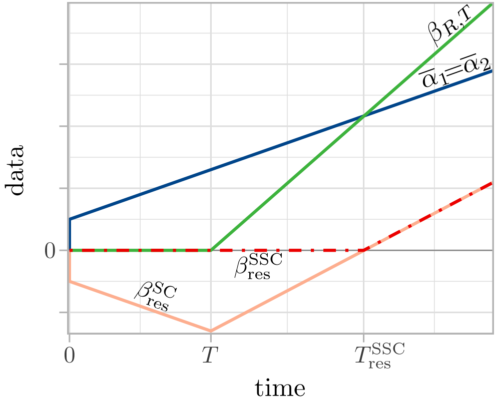

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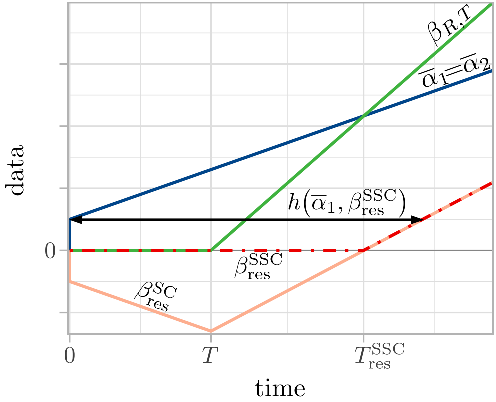

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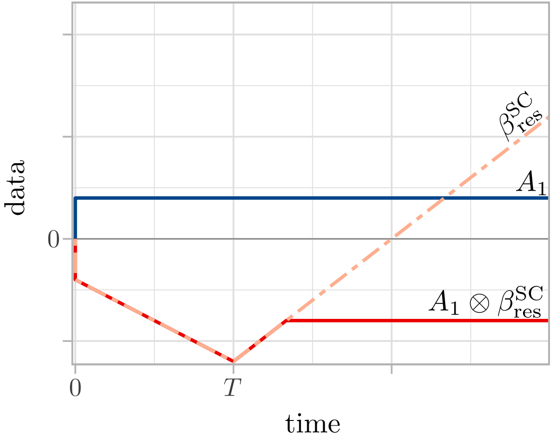
- Performance bounds are only defined for $\beta \in \mathcal{F}_0^{\uparrow}$
  $\Rightarrow$ We have no valid delay bound

What's The Issue? [Bouillard et al., 2018]



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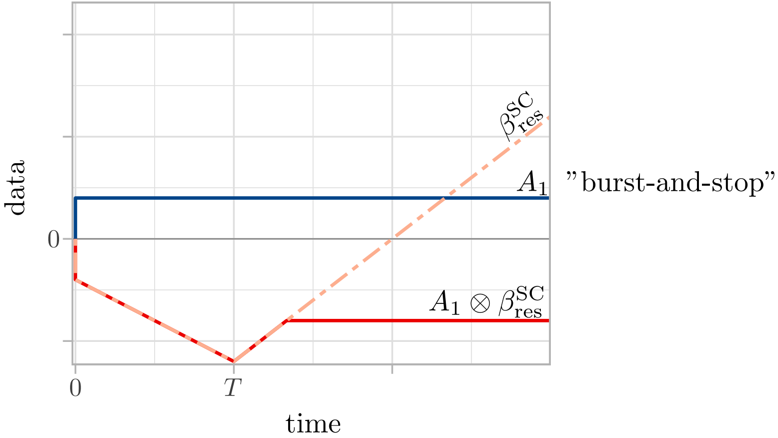

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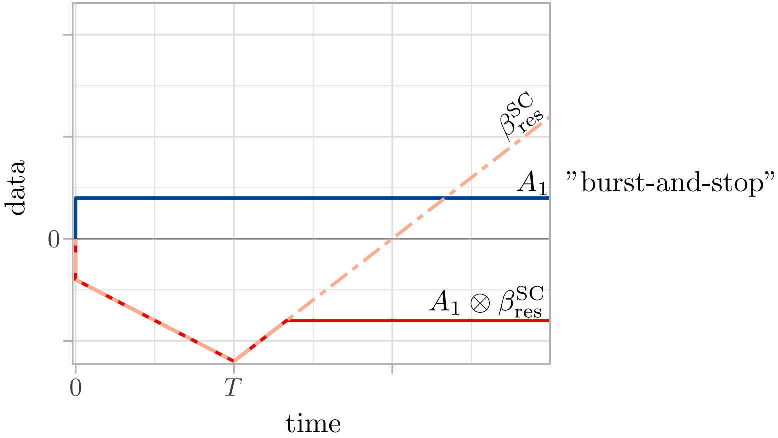

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$\Rightarrow$ $\mathcal{S}$ can delay f_1 infinitely without violating min-plus SC property

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Definition

Let $\overline{\alpha}, \underline{\alpha} \in \mathcal{F}_0^{\uparrow}$. We say that $\overline{\alpha}$ is a maximal arrival curve for arrival process A, and $\underline{\alpha}$ is a minimal arrival curve for A, if it holds for all $0 \leq s \leq t$ that

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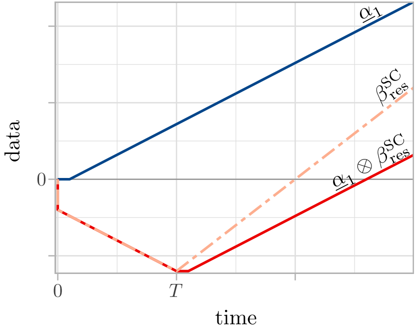

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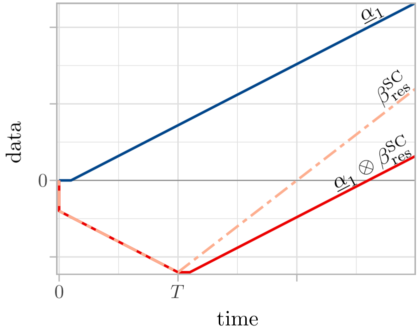
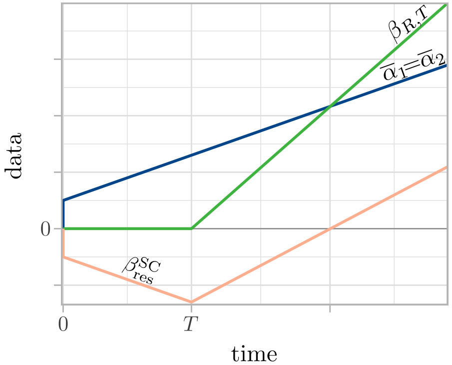


$\Rightarrow$ $\mathcal{S}$ has to eventually serve f_1

Extension of NC Performance Bounds for (Partially) Negative Service Curves

Monotony Preprocessing



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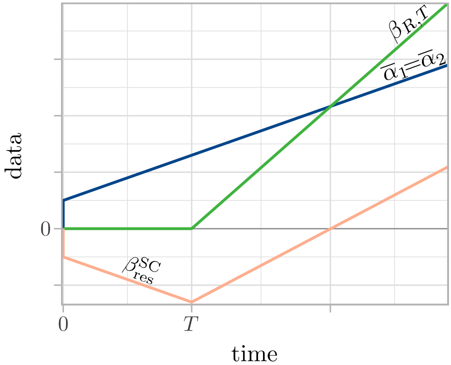

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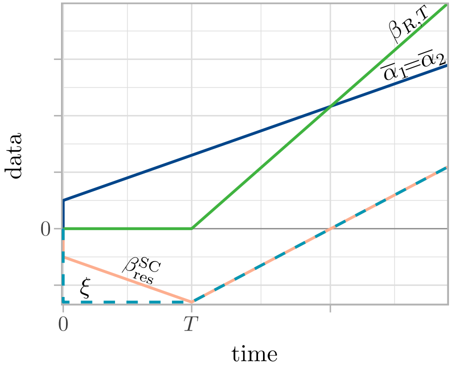

- We start with $\beta^{SC}_{res} =: \beta \in \mathcal{F}_0^{\not\uparrow}$
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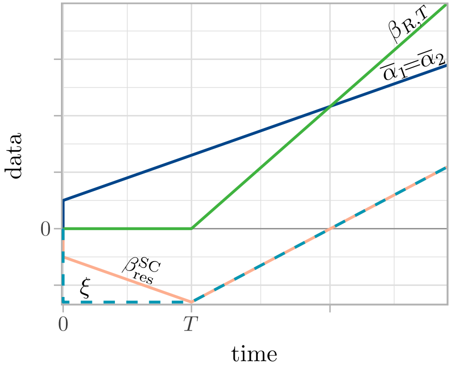

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$\Rightarrow$ $\xi \in \mathcal{F}^{\uparrow}_{\leq 0}$

Generalized Delay Bound

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Let an arrival process A traverse a system $\mathcal{S}$. Further, let the arrivals be constrained by maximal arrival curve $\overline{\alpha} \in \mathcal{F}_0^{\uparrow}$ and minimal arrival curve $\underline{\alpha} \in \mathcal{F}_0^{\uparrow}$, and let the system offer a service curve $\xi \in \mathcal{F}_{\leq 0}^{\uparrow}$. The virtual delay $d(t)$ satisfies for all $t \geq 0$

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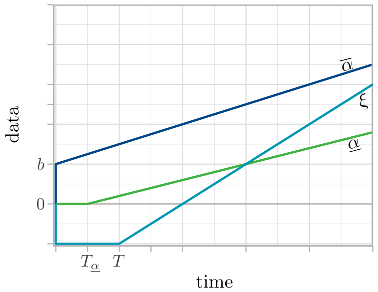

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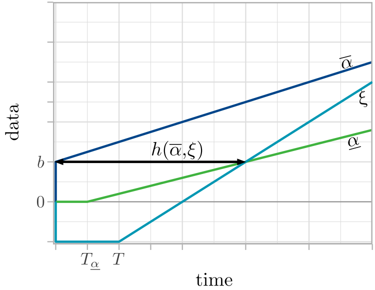

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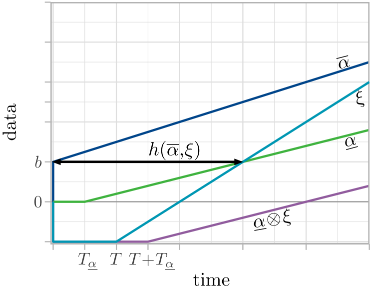

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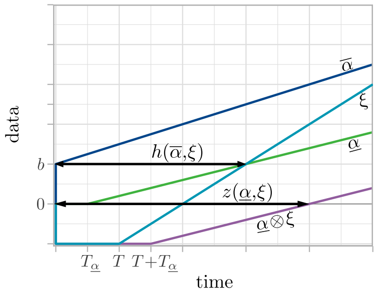

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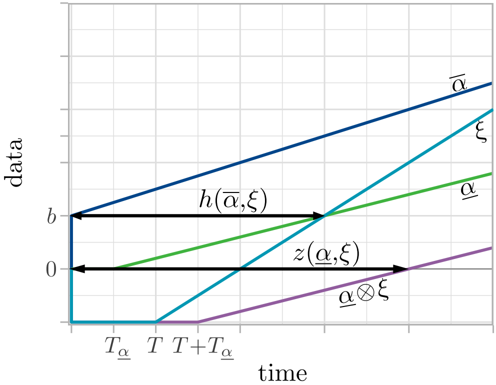

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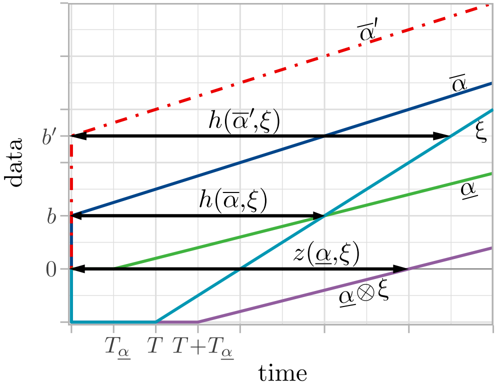

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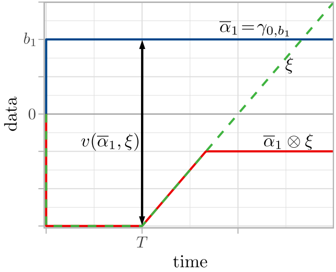

Tightness of the Backlog Bound: The Interesting Case

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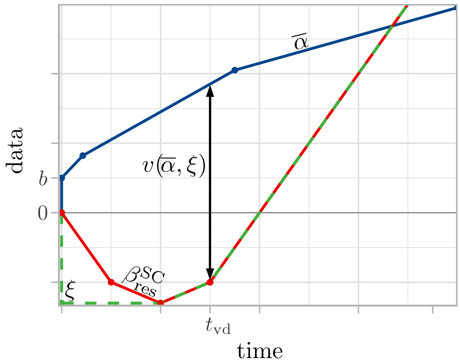

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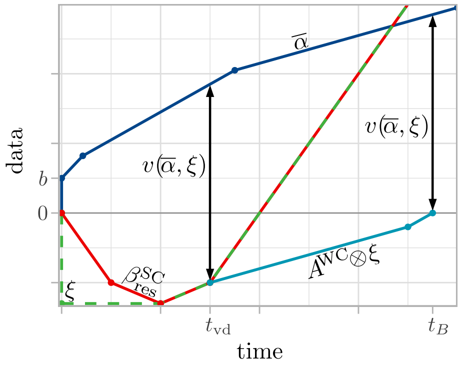

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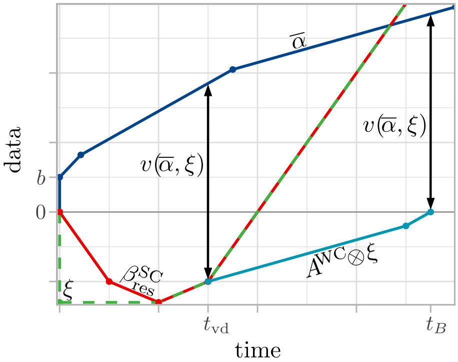

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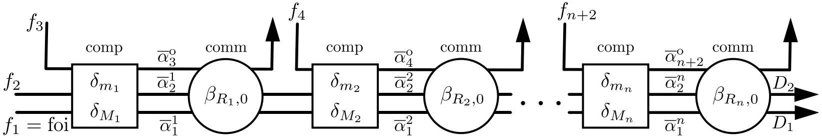
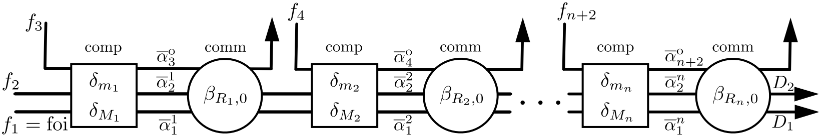


The MinAC Approach



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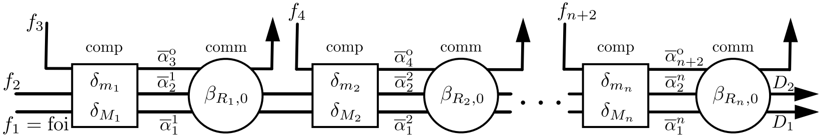

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- Determine residual service curve:

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$$= \xi_{b_2 + b_{i+2} + (r_2 + r_{i+2}) \sum_{i=1}^{n} T_i, \bigwedge_{i=1}^{n} (R_i - r_{i+2}) - r_2, \sum_{i=1}^{n} T_i}$$

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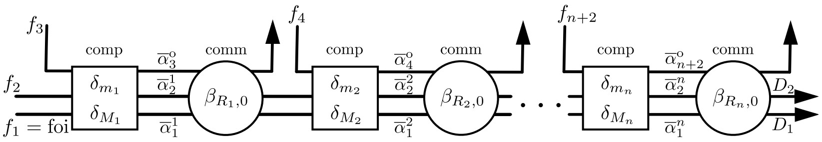

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- Calculate end-to-end delay bound:

$$d_{\text{e2e}}^{\text{mac}} = h(\overline{\alpha}_1, \beta_{\text{res}}^{\text{mac}}) \vee z(\underline{\alpha}_1, \beta_{\text{res}}^{\text{mac}})$$

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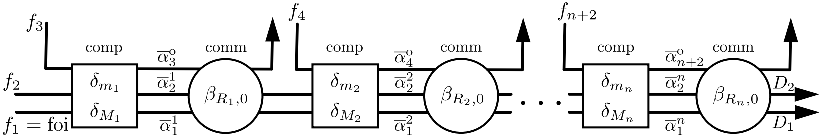

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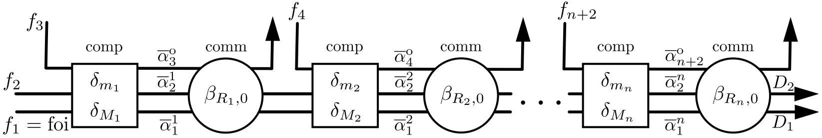

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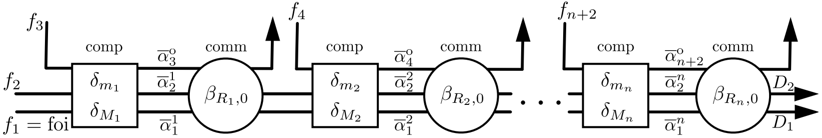

- Idea: determine input to comm by using output bound and calculate residual service curve for each component, then sum up the delay bound at each component to get end-to-end delay for f_1
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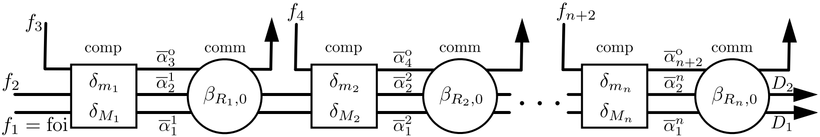

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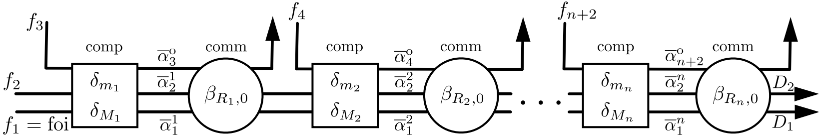

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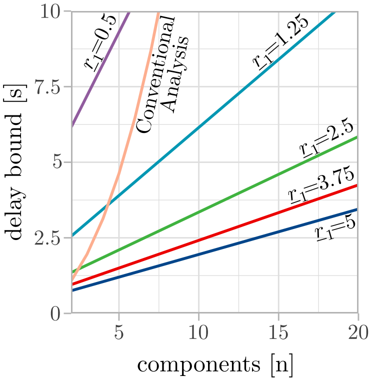
How Do They Compare?



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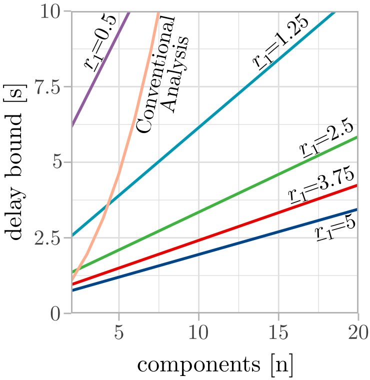

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- Different scaling of bounds
  - Min AC approach stays linear in n, while conventional analysis has super-linear scaling

Conclusion

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  - Presented C/C networks
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- What's next?
  - Investigate accuracy of LNDC preprocessing
  - Explore further applications
  - Build lower traffic shaper
  - ...

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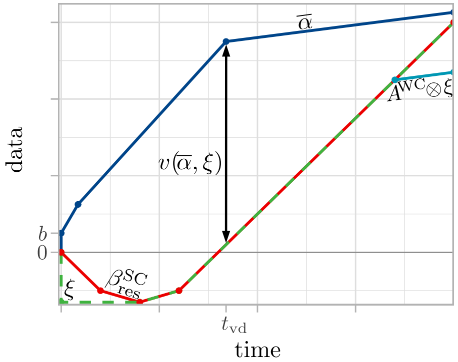
Any questions?

Tightness of the Backlog Bound: The Standard Case

- The negativity of ξ essentially plays no role



Tightness of the Backlog Bound: The Interesting Case

- Let $t_B := \overline{\alpha}^{-1}(v(\overline{\alpha}, \xi))$
- Set a worst-case sample path as:

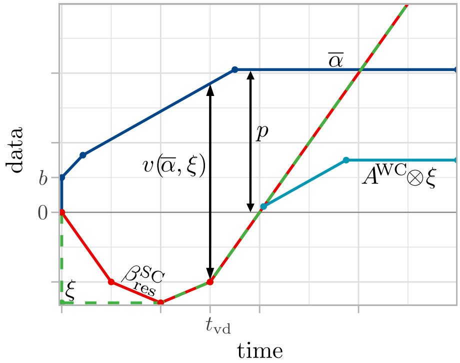
$$A^{\mathrm{WC}}(t) := \begin{cases} \overline{\alpha}(t_B) - \overline{\alpha}(t_B - t), & \text{if } t \leq t_B, \\ \overline{\alpha}(t_B), & \text{otherwise,} \end{cases}$$

and $D^{\mathrm{WC}} := \left[A^{\mathrm{WC}} \otimes \xi\right]^{+}$.

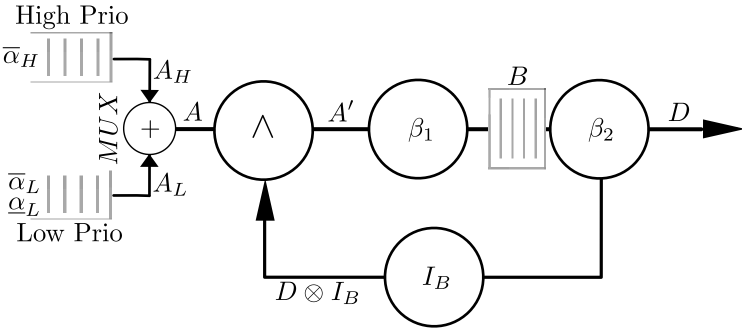
- Then $q(t_B) = v(\overline{\alpha}, \xi) \wedge \sup_{s \geq 0} \{\overline{\alpha}(s)\}$

Tightness of the Backlog Bound: The Plateau Case

- The backlog may never attain $v(\overline{\alpha}, \xi)$



Finite Buffers: Network Model

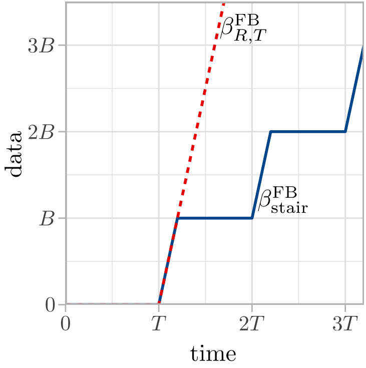


Finite Buffers: Buffer Dimensioning

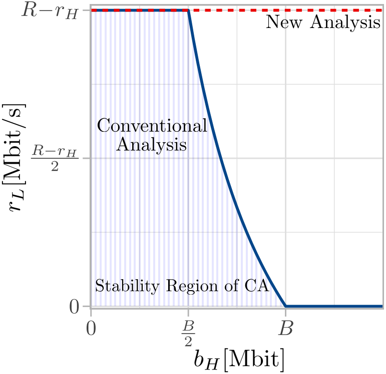
- High prio stays the same: $v(\overline{\alpha}_H, \beta^{\mathrm{FB}}) = b_H$
- Low prio changes with analysis method
  - MinAC: $v(\overline{\alpha}_L, \beta_{\mathrm{res}}^{\mathrm{mac}}) = b_H + b_L$

Finite Buffers: Stability Regions of Conventional Analysis

- Check whether $R^{\text{res}} T^{\text{res}} \leq B^{\text{res}}$
  - True? $\Rightarrow v(\overline{\alpha}, \beta^{\text{ca}}_{\text{res}}) = b_L + r_L \frac{b_H}{R - r_H}$
  - False? $\Rightarrow$ Follow shape of $\beta^{\text{FB}}_{\text{stair}}$ and watch for r_L



a) Shapes of $\beta^{\text{FB}}_{R,T}$ and $\beta^{\text{FB}}_{\text{stair}}$.



b) Stability regions of the analyses.

Finite Buffers: Delay Bounds

- High prio stays the same: $h(\overline{\alpha}_H, \beta^{\mathrm{FB}}) = \frac{b_H}{R}$
- Low prio changes with analysis method
  - MinAC uses previously shown delay bound
  - CA: Analogous to backlog; check for bandwidth delay product and r_L