

25/11/2025 - Matemáticas Discretas (Ude@)

1. Deudas:

a. **Parcial 2** Tener noticias

b. **Parcial 3** (Bonus de +0.6 si alguien no usó IA)

c. Parcial 4 : Jueves 4 de diciembre (Parcial)

d. **Taller Final** : code.org (Viernes : 8:00 AM)
Ultimo tema 20%

3. Parcial 3. (Conjuntos, Relaciones y Relaciones de orden)

Talleres Opciones

Taller 6 (Conjuntos)

Taller 7 (Relaciones)

Taller 8 (Relaciones de orden)

4. Algebra Booleana

George Boole

Claude Shannon

Logica Proposicional

Algebra Booleana

(Binarias)

Circuitos Logicos

1. Proposiciones



1. Variables booleanas

Bit (0/1)

2. Operadores

- Negación: \neg - 7
- Conjunción: \wedge - ^
- Disyunción: \vee - v
- O Exclusivo: \oplus - +
- Condicional: \rightarrow
- Equivalencia: \leftrightarrow

2. Operadores:

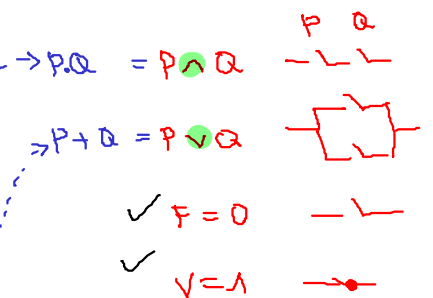
• (Producto)

+ (Suma)

\oplus

\rightarrow

\leftrightarrow



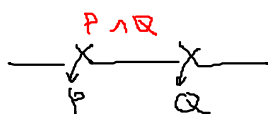
Operación

Switch

Computertas

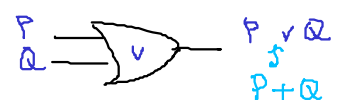
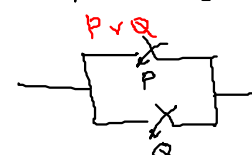
AND ^

$P \wedge Q$



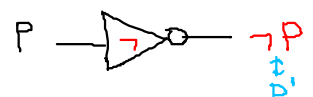
OR v

$P \vee Q$



NOT

$\neg P$



3. Identidades (Reglas de validez)

| Identidades Booleanas | | |
|---------------------------|---|---|
| Nombre | Identidad | |
| 1. Ley del doble negación | $x'' = x$ | |
| 2. Ley de idempotencia | $x \cdot x = x$ | $x + x = x$ |
| 3. Ley de identidad | $x \cdot 1 = x$ | $x + 0 = x$ |
| 4. Ley de dominación | $x \cdot 0 = 0$ | $x + 1 = 1$ |
| 5. Leyes conmutativa | $x \cdot y = y \cdot x$ | $x + y = y + x$ |
| 6. Ley asociativa | $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ | $x + (y + z) = (x + y) + z$ |
| 7. Ley distributiva | $x \cdot (y + z) = x \cdot y + x \cdot z$ | $x + y \cdot z = (x + y) \cdot (x + z)$ |
| 8. Leyes de De Morgan | $(x \cdot y)' = x' + y'$ | $(x + y)' = x' \cdot y'$ |
| 9. Ley de absorción | $x \cdot (x + y) = x$ | $x + x \cdot y = x$ |
| 10. Ley del complemento | $x \cdot x' = 0$ | $x + x' = 1$ |

5. Circuitos lógicos

Compuerta NOT

Simbols

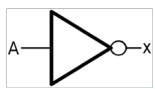


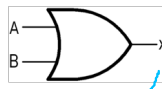
Tabla de verdad

| A | $X = A'$ |
|---|----------|
| 0 | 1 |
| 1 | 0 |

Expresión Algebraica

$$X = A'$$

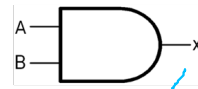
Compuerta OR



| A | B | $X = A + B$ |
|---|---|-------------|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

$$X = A + B$$

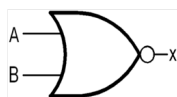
Compuerta AND



| A | B | $X = A \cdot B$ |
|---|---|-----------------|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

$$X = A \cdot B = AB$$

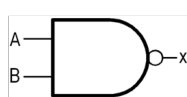
Compuerta NOR



| A | B | $X = (A + B)'$ |
|---|---|----------------|
| 0 | 0 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

$$X = (A + B)' = A \downarrow B$$

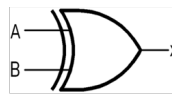
Compuerta NAND



| A | B | $X = (A \cdot B)'$ |
|---|---|--------------------|
| 0 | 0 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

$$X = (A \cdot B)' = A \uparrow B$$

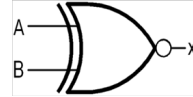
Compuerta XOR



| A | B | $X = A \oplus B$ |
|---|---|------------------|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

$$X = A \oplus B$$

Compuerta XNOR



| A | B | $X = A \odot B$ |
|---|---|-----------------|
| 0 | 0 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

$$X = A \odot B$$

6. Función Booleanas

Representación:

1. Algebraica ✓
2. Tabla de verdad ✓
3. Diagrama de flechas ✓
4. Circuito lógico

Formas → Estándar (canónica)
→ No estándar

Electronica digital

(Shannon)

Representación Algebraica

Ejemplo: $Q = (RST)' \cdot (R + S + T)' \rightarrow Q(R, S, T)$
 Variables: R, S, T

Tabla de verdad: $n = 3 \rightarrow \text{Filas} = 2^n = 2^3 = 8$

• \wedge
 + \vee

$$Q = (RST)' \cdot (R + S + T)'$$

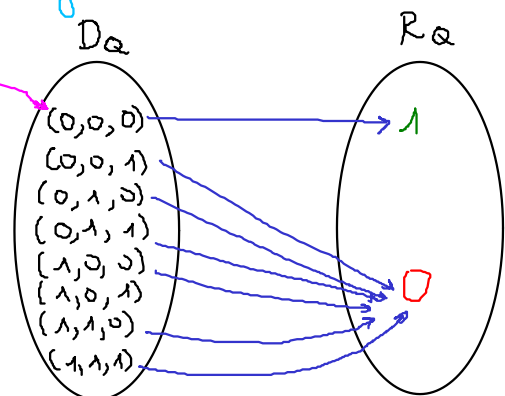
| R | S | T | ① RST | ② $R+S+T$ | ③ $(RST)'$ | ④ $(R+S+T)'$ | ⑤ Q |
|---|---|---|------------|--------------|---------------|-----------------|----------|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

Rta: Tabla de verdad

| R | S | T | Q |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

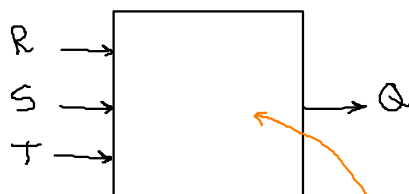
terna = 3-tupla
 (R, S, T)

Diagrama de flechas



Circuito Logico

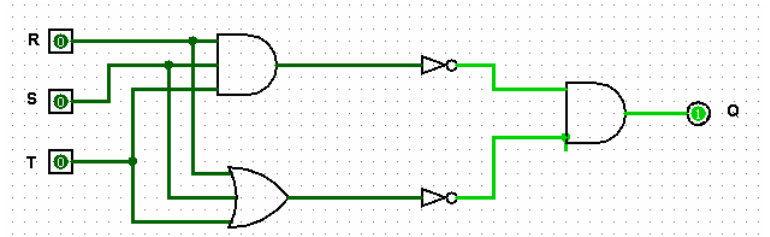
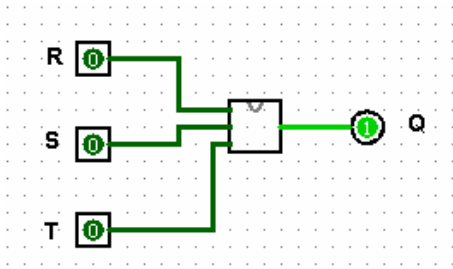
Caja Negra



$Q(R, S, T)$

Circuito Logico

$$Q = (RST)' \cdot (R + S + T)'$$



7. Simplificación

$$Q = (RST)' \cdot (R + S + T)'$$

Simplifique la expresión usando las identidades de Álgebra Booleana

Forma no simplificada

$$Q = (RST)' \cdot (R + S + T)'$$



Forma simplificada

$$Q = R'S'T'$$

| Identidades Booleanas | | |
|---------------------------|---|---|
| Nombre | Identidad | |
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| 8. Leyes de De Morgan | $(x \cdot y)' = x' + y'$ | $(x + y)' = x' \cdot y'$ |
| 9. Ley de absorción | $x \cdot (x + y) = x$ | $x + x \cdot y = x$ |
| 10. Ley del complemento | $x \cdot x' = 0$ | $x + x' = 1$ |

Multiplicación
(·)

Suma
(+)

Pasos

Razon

1. $(RST)' (R + S + T)'$

Forma original sin simplificar

2. $(R' + S' + T')(R'S'T')$

Ley de Morgan para (·) y (+) en 1

3. $R'R'S'T' + S'R'S'T' + T'R'S'T'$

Distributividad para (+) en 2

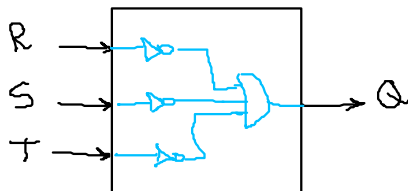
4. $R'S'T' + R'S'T' + R'T'S'$

Idempotencia para (·) en 3

5. $R'S'T'$

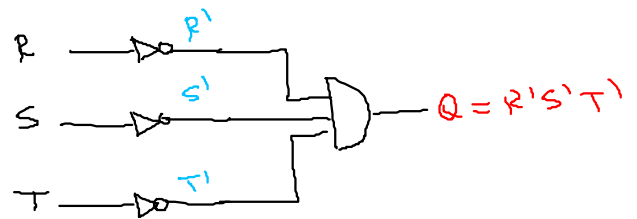
Idempotencia para la (+) en 4

Caja negra



$$Q = R'T'S'$$

Circuito Logico



\$ 25000

7. Formas de expresar una función

$$F = x + yz$$

a. Conceptos importantes

Forma No estandar

Libre

$$F_1 = xy + z(x' + y')x$$

$$F_2 = x + (y + (zx)')')$$

$$F_2 = x + yz$$

Productos de Sumas (POS)

$$F_1 = (x + y)(x' + y' + z)$$

$$F_2 = (a' + b)c'$$

$$F_3 = (a + b)(b' + c')$$

Termino Suma

Sumas de productos (SOP)

$$F_1 = xyz + x'y' + z$$

$$F_2 = ab' + ab$$

Termino producto

Forma Estandar (canonica) Unica

SOP

$$\left\{ \begin{array}{l} ab: F_1 = ab + a'b' \\ xyz: F_2 = xy'z + \underbrace{xy'z}_{\text{minitermino}} + x'zy' \end{array} \right\} F_3 = a'b \quad \{ F_4 = a'b + b'a$$

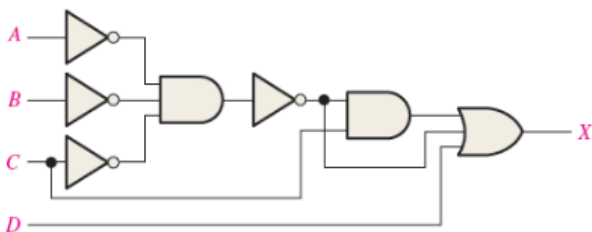
POS

$$\left\{ \begin{array}{l} ab: F_1 = (a + b)(a' + b) \\ xyz: F_2 = (x + y + z)(\underbrace{x' + y' + z'}_{\text{Maxtermino}}) \end{array} \right\} F_3 = (x' + y' + z')$$

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Ejemplo 1:

Dado el siguiente circuito logico:

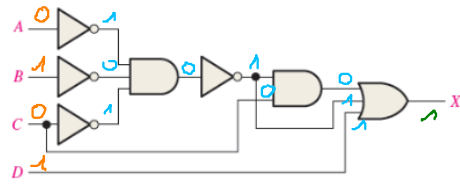
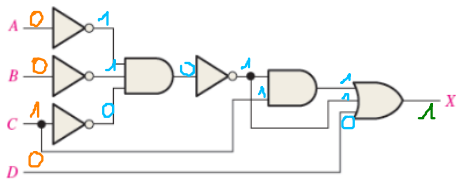


1. ¿Cual seria la salida? si:

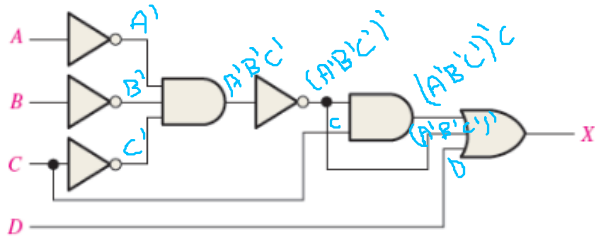
a. $A=0, B=0, C=1, D=0 \rightarrow X=1$

b. $A=0, B=1, C=0, D=1 \rightarrow X=1$

| i | A | B | C | D | X |
|----|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 0 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 1 |



2. Obtenga la función lógica asociada al circuito



$$X = f(A, B, C, D)$$

$$X = (A'B'C')'C + (A'B'C')' + D$$

4. Obtenga la tabla de verdad asociada al circuito.

| i | A | B | C | D | A' | B' | C' | (A'B'C')' | (A'B'C')'C | (A'B'C')'C + (A'B'C')' + D |
|----|---|---|---|---|----|----|----|-----------|------------|----------------------------|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 4 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 8 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 10 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 12 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 14 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |

3. Empleando las identidades Booleanas, realice la simplificación.

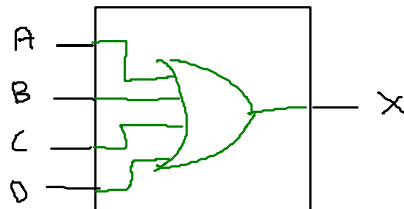
$$X = (A'B'C')'C + (A'B'C')' + D$$

| Identidades Booleanas | | | |
|---------------------------|---|-----------|---|
| Nombre | (.) | Identidad | (+) |
| 1. Ley del doble negación | | $x'' = x$ | |
| 2. Ley de idempotencia | $x \cdot x = x$ | | $x + x = x$ |
| 3. Ley de identidad | $x \cdot 1 = x$ | | $x + 0 = x$ |
| 4. Ley de dominación | $x \cdot 0 = 0$ | | $x + 1 = 1$ |
| 5. Leyes conmutativa | $x \cdot y = y \cdot x$ | | $x + y = y + x$ |
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| 7. Ley distributiva | $x \cdot (y + z) = x \cdot y + x \cdot z$ | | $x + y \cdot z = (x + y) \cdot (x + z)$ |
| 8. Leyes de De Morgan | $(x \cdot y)' = x' + y'$ | | $(x + y)' = x' \cdot y'$ |
| 9. Ley de absorción | $x \cdot (x + y) = x$ | | $x + x \cdot y = x$ |
| 10. Ley del complemento | $x \cdot x' = 0$ | | $x + x' = 1$ |

| Pasos | Razon |
|---------------------------------|-----------------------------|
| 1. $(A'B'C')'C + (A'B'C')' + D$ | Expresión inicial |
| 2. $(A'B'C')'(C+1) + D$ | Factor común para (.) en 1 |
| 3. $(A'B'C')'1 + D$ | Dominación para (+) en 2 |
| 4. $(A'B'C')' + D$ | Identidad para (.) en 3 |
| 5. $A'' + B'' + C'' + D$ | Ley de Morgan para (.) en 4 |
| 6. $A + B + C + D$ | Ley de doble negación en 5. |

$$X = (A'B'C')'C + (A'B'C')' + D \rightarrow X = A + B + C + D$$

$$X = f(A, B, C, D) :$$



5. Obtenga las formas FND (POS) y FNC (SOP) para este ejemplo.

| i | A | B | C | D | X | m_i | M_i |
|----|---|---|---|---|---|------------|---------------|
| 0 | 0 | 0 | 0 | 0 | 0 | $A'B'C'D'$ | $A+B+C+D$ |
| 1 | 0 | 0 | 0 | 1 | 1 | $A'B'C'D$ | $A+B+C+D'$ |
| 2 | 0 | 0 | 1 | 0 | 1 | $A'B'CD'$ | $A+B+C'+D$ |
| 3 | 0 | 0 | 1 | 1 | 1 | $A'B'CD$ | $A+B+C'+D'$ |
| 4 | 0 | 1 | 0 | 0 | 1 | $A'BC'D'$ | $A+B'+C+D$ |
| 5 | 0 | 1 | 0 | 1 | 1 | $A'BC'D$ | $A+B'+C+D'$ |
| 6 | 0 | 1 | 1 | 0 | 1 | $A'BCD'$ | $A+B'+C'+D$ |
| 7 | 0 | 1 | 1 | 1 | 1 | $A'BCD$ | $A+B'+C'+D'$ |
| 8 | 1 | 0 | 0 | 0 | 1 | $AB'C'D'$ | $A'+B+C+D$ |
| 9 | 1 | 0 | 0 | 1 | 1 | $AB'C'D$ | $A'+B+C+D'$ |
| 10 | 1 | 0 | 1 | 0 | 1 | $AB'CD'$ | $A'+B+C'+D$ |
| 11 | 1 | 0 | 1 | 1 | 1 | $AB'CD$ | $A'+B+C'+D'$ |
| 12 | 1 | 1 | 0 | 0 | 1 | $ABCD'$ | $A'+B'+C+D$ |
| 13 | 1 | 1 | 0 | 1 | 1 | $ABCD$ | $A'+B'+C+D'$ |
| 14 | 1 | 1 | 1 | 0 | 1 | $ABCD'$ | $A'+B'+C'+D$ |
| 15 | 1 | 1 | 1 | 1 | 1 | $ABCD$ | $A'+B'+C'+D'$ |

Representación como POS

$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}D' + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}D' + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}\bar{D}' + \bar{A}\bar{B}\bar{C}\bar{D} \\ \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}' + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}' + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$F = m_1 + m_2 + \dots + m_{15}$$

$$F = \sum m(1, 2, \dots, 15)$$

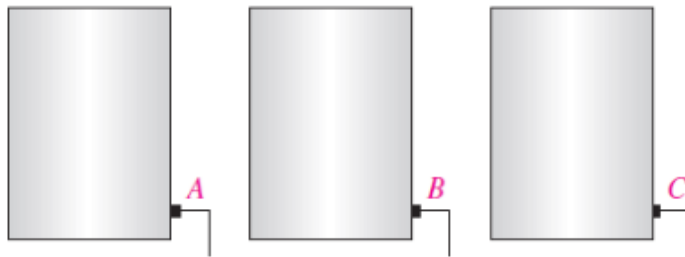
Representación como SOP

$$G = A + B + C + D$$

$$G = M_0$$

Ejemplo 2:

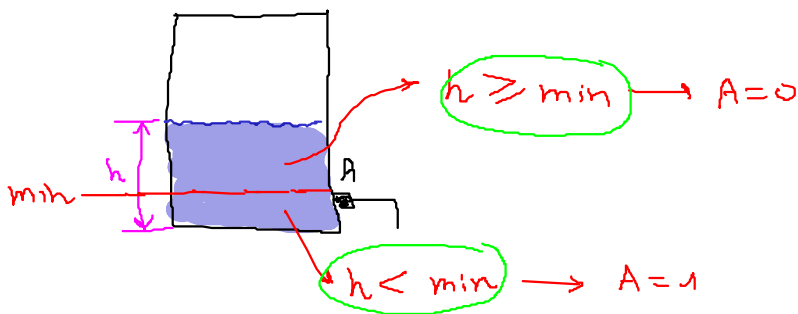
En una determinada planta de procesamiento químico se emplea un elemento químico líquido en un proceso de fabricación. Dicho elemento químico se almacena en tres tanques diferentes. Un sensor de nivel en cada tanque genera una tensión a nivel ALTO cuando el nivel de líquido en el tanque cae por debajo de un punto especificado.



Aplicando cada uno de los pasos de diseño, diseñar un circuito para supervisar el nivel del elemento químico en cada tanque, que indique cuándo el nivel de dos tanques cualesquiera cae por debajo del punto especificado. No olvide adjuntar el archivo de simulación de Logisim.

2 o mas

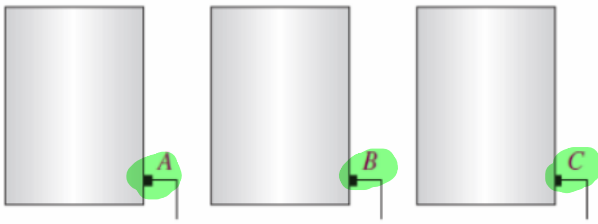
i. Entender el problema



ii. Tabla de verdad

Entradas: A, B, C $\rightarrow n=3 \rightarrow \text{filas} = 2^n = 2^3 = 8$

Salida: S (alarma)



| A | B | C | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

iii. Función lógica:

\rightarrow POS (minterminos):

| i | A | B | C | S |
|-----|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 | 0 |
| ✓ 3 | 0 | 1 | 1 | 1 |
| ✓ 4 | 1 | 0 | 0 | 0 |
| ✓ 5 | 1 | 0 | 1 | 1 |
| ✓ 6 | 1 | 1 | 0 | 1 |
| ✓ 7 | 1 | 1 | 1 | 1 |

$A'BC$
 $AB'C$
 ABC'
 ABC

$$S = A'BC + AB'C + ABC' + ABC = \sum m(3, 5, 6, 7)$$

$$S = (A'BC + ABC) + (AB'C + ABC')$$

$$S = BC(A' + A) + A(B'C + BC')$$

$$S = BC + A(B'C + BC')$$

\rightarrow SOP (Maxterminos)

| i | A | B | C | S |
|-----|---|---|---|---|
| ✓ 0 | 0 | 0 | 0 | 0 |
| ✓ 1 | 0 | 0 | 1 | 0 |
| ✓ 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| ✓ 4 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 1 |

$A + B + C$
 $A + B + C'$
 $A + B' + C$
 $A' + B + C$

$$S_2 = (A + B + C)(A + B + C')(A + B' + C)(A' + B + C)$$

$$S_2 = \prod M(0, 1, 2, 4)$$

iii. Circuito lógico: Expresión simplificada

$$S = BC + A(B'C + BC')$$

$$S = BC + A(B'c + Bc')$$

$A \oplus B = x$
 $x = A\bar{B} + \bar{A}B$

Entradas: A, B, C

Salida: S (alarma)

