Logical equivalence identities

(Some) logical equivalences

Can replace p and q with any compound proposition

$\neg(\neg p) \equiv p$		**Double negation**
$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$	**Commutativity** Ordering of terms
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	**Associativity** Grouping of terms
$p \wedge F \equiv F$ $\quad p \vee T \equiv T$	$p \wedge T \equiv p$ $\quad p \vee F \equiv p$	**Domination** aka short circuit evaluation
$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	**DeMorgan's Laws**

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$ **Contrapositive**

$\neg(p \to q) \equiv p \wedge \neg q$

$\neg(p \leftrightarrow q) \equiv p \oplus q$

$p \leftrightarrow q \equiv q \leftrightarrow p$

Extra examples:

$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because ______________________________

$p \to q$ is not logically equivalent to $q \to p$ because ______________________________

Logical operators example truth table

Input			Output		
p	q	r		$(p \wedge q) \oplus ((p \oplus q) \wedge r)$	$(p \wedge q) \vee ((p \oplus q) \wedge r)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Logical equivalence

Logical equivalence : Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.

Tautology: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Logical equivalence extra example

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					