Logical equivalence identities

(Some) logical equivalences

Can replace p and q with any compound proposition

$\neg(\neg p) \equiv p$ **Double negation**

$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$ **Commutativity** Ordering of terms

$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$ **Associativity** Grouping of terms

$p \land F \equiv F$ $p \lor T \equiv T$ $p \land T \equiv p$ $p \lor F \equiv p$ **Domination** aka short circuit evaluation

$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$ **DeMorgan's Laws**

$p \to q \equiv \neg p \lor q$

$p \to q \equiv \neg q \to \neg p$ **Contrapositive**

$\neg(p \to q) \equiv p \land \neg q$

$\neg(p \leftrightarrow q) \equiv p \oplus q$

$p \leftrightarrow q \equiv q \leftrightarrow p$

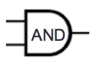
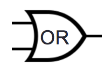
Extra examples:

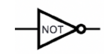
$p \leftrightarrow q$ is not logically equivalent to $p \land q$ because ______________________________

$p \to q$ is not logically equivalent to $q \to p$ because ______________________________

Logical operators truth tables

Truth tables: Input-output tables where we use T for 1 and F for 0.

Input		Output		
		Conjunction	**Exclusive or**	**Disjunction**
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F
				

Input	Output
	Negation
p	$\neg p$
T	F
F	T
	

Logical equivalence

Logical equivalence : Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.

Tautology: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Logical equivalence extra example

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					