1. True or False: A recursive definition of the set of integers $\mathbb{Z}$ is

$$\begin{array}{ll} \text{Basis Step:} & 20 \in \mathbb{Z} \\ \text{Recursive Step:} & \text{If } x \in \mathbb{Z}, \text{ then } x - 1 \in \mathbb{Z} \end{array}$$

Key idea: To define a set, we need to specify which elements are in and which elements are out. A description that gives some, but not all, integers is not a definition of the set of integers.

2. When $B = \{\texttt{A}, \texttt{C}, \texttt{U}\,\texttt{G}\}$ is the set of RNA bases, consider the recursively defined set X given by:

Basis Step: $\texttt{AA} \in X$, $\texttt{CC} \in X$, $\texttt{UU} \in X$, $\texttt{GG} \in X$

Recursive Step: If $x \in X$ and $b \in B$ then $bxb \in X$

where bxb is the result of string concatenations.

Give three example elements of X and an English description of the set.

Key idea: To build new example elements we can start with an element from the basis step and then apply a rule from the recursive step finitely many times.

3. Write in roster method the set given by the Cartesian product

$$\{1, 2\} \times \{a, b, c\}$$

Key idea: Cartesian product does not require the sets to have the same size as one another or to have the same types of elements as one another.

4. Consider the function d_2 : The set of ordered pairs of ratings 4-tuples $\to \mathbb{R}$ given by

$$d_2(\ ((x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4)\ )\ ) = \sqrt{\sum_{i=1}^{4} (x_i - y_i)^2}$$

Let $z = (1, 1, 1, 1)$ be a ratings 4-tuple and consider the set

$$\{x \mid d_2(\ (x, z)\ ) = 1\}$$

Rewrite this set using the roster method.

Key idea: The input to this function is an ordered pair each of whose components is a ratings 4-tuple.

5. Rewrite the set

$$\{rnalen(x) \mid x \in S \text{ and } rnalen(x) < 2\}$$

using the roster method.

Key idea: The set builder definition for this set can be read as "The collection of all outputs of the function rnalen when the input is taken from the set of RNA strands for which rnalen gives value less than 2." Informally, we consider just RNA strands whose rnalen value is less than 2 and collect their rnalen values into our set.

6. Which of the following is true? (Select all and only that apply.)

   (a) $\exists s \in S \ Mut(s, s)$
   (b) $\forall s \in S \ Mut(s, s)$
   (c) $\exists s \in S \ Ins(s, \texttt{A})$
   (d) $\exists s \in S \ Ins(\texttt{A}, s)$
   (e) $\exists s \in S \ Del(s, \texttt{A})$
   (f) $\forall s \in S \ Del(s, \texttt{A})$