

Logical equivalence identities

(Some) logical equivalences

Can replace p and q with any compound proposition

$$\neg(\neg p) \equiv p$$

Double negation

$$p \vee q \equiv q \vee p$$

$$p \wedge q \equiv q \wedge p$$

Commutativity Ordering of terms

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

Associativity Grouping of terms

$$p \wedge F \equiv F$$

$$p \vee T \equiv T$$

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

Domination aka short circuit evaluation

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

DeMorgan's Laws

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Contrapositive

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

$$\neg(p \leftrightarrow q) \equiv p \oplus q$$

$$p \leftrightarrow q \equiv q \leftrightarrow p$$

Extra examples:

$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because _____

$p \rightarrow q$ is not logically equivalent to $q \rightarrow p$ because _____

Logical operators truth tables

Truth tables: Input-output tables where we use T for 1 and F for 0.

| Input | | Output | | |
|-------|-----|---|---|---|
| | | Conjunction | Exclusive or | Disjunction |
| p | q | $p \wedge q$ | $p \oplus q$ | $p \vee q$ |
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | F | F |
| | |  |  |  |

| Input | Output |
|---|----------|
| Negation | |
| p | $\neg p$ |
| T | F |
| F | T |
|  | |

Logical equivalence

Logical equivalence : Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.

Tautology: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T .

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F .

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Logical equivalence extra example

Extra Example: Which of the compound propositions in the table below are logically equivalent?

| Input | | Output | | | | |
|-------|-----|-------------------------|----------------------------|-------------------|------------------------|----------------|
| p | q | $\neg(p \wedge \neg q)$ | $\neg(\neg p \vee \neg q)$ | $(\neg p \vee q)$ | $(\neg q \vee \neg p)$ | $(p \wedge q)$ |
| T | T | | | | | |
| T | F | | | | | |
| F | T | | | | | |
| F | F | | | | | |