Fixed-width Addition:

- Note: excluding cases when an input is 0.

Problem formulation:

inputs: $X = X_3 X_2 X_1 X_0$

$y = y_3 y_2 y_1 y_0$

Define: $Z = Z_4 Z_3 Z_2 Z_1 Z_0 = X + y$

Output: $Out = Z_3 Z_2 Z_1 Z_0$

Goal: Compare

$$
\begin{cases}
(Out)_{2,4} \overset{?}{=} (X)_{2,4} + (y)_{2,4} \\
[Out]_{S,4} \overset{?}{=} [X]_{S,4} + [y]_{S,4} \\
[Out]_{2C,4} \overset{?}{=} [X]_{2C,4} + [y]_{2C,4}
\end{cases}
$$

① Binary Representation:

$$\begin{array}{cccccc} & & 8 & 4 & 2 & 1 \\ & & x_3 & x_2 & x_1 & x_0 \\ + & & y_3 & y_2 & y_1 & y_0 \\ \hline & z_4 & z_3 & z_2 & z_1 & z_0 \\ & 16 & 8 & 4 & 2 & 1 \end{array}$$

$$(z_4 z_3 z_2 z_1 z_0)_2 \overset{\text{always}}{=\!=} (x_3 x_2 x_1 x_0)_2 + (y_3 y_2 y_1 y_0)_2$$

But we are throwing away z_4

$$\begin{array}{cccccc} & & 8 & 4 & 2 & 1 \\ & & x_3 & x_2 & x_1 & x_0 \\ + & & y_3 & y_2 & y_1 & y_0 \\ \hline & z_4 & z_3 & z_2 & z_1 & z_0 \\ & 0 & 8 & 4 & 2 & 1 \end{array}$$

$\therefore z_4 = 1 \Rightarrow 8+8=0 \Longrightarrow$ incorrect

$z_4 = 0 \Rightarrow$ correct

② Sign-magnitude:

$$
\begin{array}{cccccc}
 & & +/- & 4 & 2 & 1 \\
 & & x_3 & x_2 & x_1 & x_0 \\
+ & & y_3 & y_2 & y_1 & y_0 \\
\hline
 & z_4 & z_3 & z_2 & z_1 & z_0
\end{array}
$$

Simple case: $x_3 = y_3 = z_3 = 0$:

$$
\begin{array}{cccccc}
 & & + & 4 & 2 & 1 \\
 & & 0 & x_2 & x_1 & x_0 \\
+ & & 0 & y_2 & y_1 & y_0 \\
\hline
 & 0 & 0 & z_2 & z_1 & z_0
\end{array}
$$

Always correct

Other cases: Mostly incorrect

Counter-example:

$$
\begin{array}{ccccccc}
 & & 1 & 1 & 0 & 0 & -4 \\
+ & & 0 & 1 & 1 & 1 & +7 \\
\hline
 & 1 & 0 & 0 & 1 & 1 & =3
\end{array}
$$

2's Complement:

-16

$? \quad -8 \quad 4 \quad 2 \quad 1$

$$\begin{array}{rcccc} & x_3 & x_2 & x_1 & x_0 \\ + & y_3 & y_2 & y_1 & y_0 \\ \hline z_4 & z_3 & z_2 & z_1 & z_0 \\ 0 & & & & \end{array}$$

Carry-over:

$-16 \rightarrow 0$

$-8+(-8)=0?$ $\quad +16$

$4+4=-8!$ $\quad -16$

$2+2=4$ $\quad 8 \rightarrow -8$

$1+1=2$

$0 \quad -8 \quad 4 \quad 2 \quad 1$

$$\begin{array}{rcccc} & x_3 & x_2 & x_1 & x_0 \\ + & y_3 & y_2 & y_1 & y_0 \\ \hline z_4 & z_3 & z_2 & z_1 & z_0 \\ 0 & -8 & 4 & 2 & 1 \end{array}$$

$$
\begin{array}{rl}
 & [0\ 0\ 1\ 0]_{2c,4} = 2 \\
+ & [1\ 1\ 0\ 1]_{2c,4} = -8+4+1 = -3 \\
\hline
0 & [1\ 1\ 1\ 1]_{2c,4} = -1
\end{array}
$$

$$= -1$$

$$
\begin{array}{rl}
 & [0\ 1\ 1\ 0]_{2c,4} = 4+2 = 6 \\
+ & [0\ 1\ 0\ 1]_{2c,4} = 4+1 = 5 \\
\hline
0 & [1\ 0\ 1\ 1]_{2c,4} = 11
\end{array}
$$

$$= -8+2+1 = -5$$

$$
\begin{array}{rl}
 & [1\ 0\ 1\ 0]_{2c,4} = -8+2 = -6 \\
+ & [1\ 0\ 0\ 1]_{2c,4} = -8+1 = -7 \\
\hline
1 & [0\ 0\ 1\ 1]_{2c,4} = -13
\end{array}
$$

$$= 2+1 = 3$$

$$
\begin{array}{r}
[1\ 1\ 1\ 1]_{2c,4} = -1 \\
+[1\ 0\ 0\ 1]_{2c,4} = -8+1 = -7 \\
\hline
1[1\ 0\ 0\ 0]_{2c,4} = -8
\end{array}
$$

Generalized result:

$$
\begin{array}{rccccc}
 & & x_n & x_{n-1} & \cdots & x_1 \\
+ & & y_n & y_{n-1} & \cdots & y_1 \\
\hline
 & z_{n+1} & z_n & z_{n-1} & \cdots & z_1
\end{array}
$$

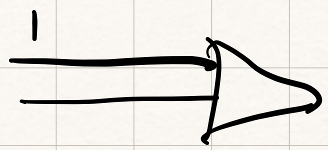
- A carry-over from col. $n-1$ to col. n will decrease the result (by 2^n)
- A carry-over from col. n to col. $n+1$ will increase the result (by 2^n)
- No carry-over in both cols. $\Rightarrow$ correct

☆

- carry-over in both cols.

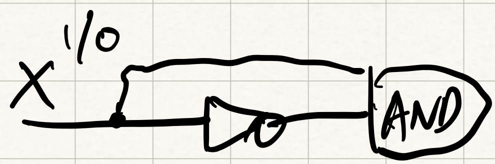
$\Rightarrow$ cancel each other ($+2^n - 2^n$)

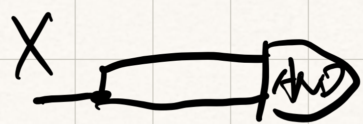
$\Rightarrow$ correct


1

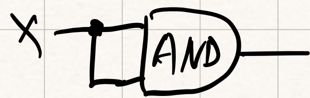

x


0


X
1/0
AND


X
AND

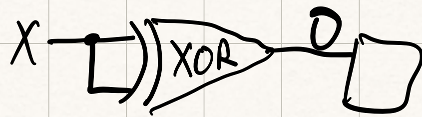
$X \wedge X$:



x	$x \wedge x$
0	0
1	1

0/1

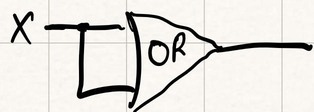
$X \otimes X$:



x	$x \otimes x$
0	0
1	0

0

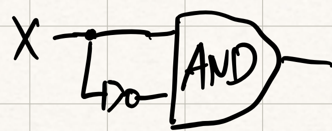
$X \vee X$:



x	$x \vee x$
0	0
1	1

0/1

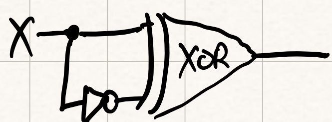
$X \wedge \neg X$



x	$\neg x$	$x \wedge \neg x$
1	0	0
0	1	0

0

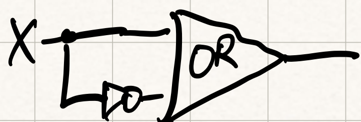
$X \otimes \neg X$



x	$\neg x$	$x \otimes \neg x$
1	0	1
0	1	1

1

$X \vee \neg X$



x	$\neg x$	$x \vee \neg x$
1	0	1
0	1	1

1