Definitions set prereqs

| Term | Notation Example(s) | We say in English |
|-----------------------|---------------------|--|
| all reals | \mathbb{R} | The (set of all) real numbers (numbers on the number |
| | | line) |
| all integers | \mathbb{Z} | The (set of all) integers (whole numbers including neg- |
| | | atives, zero, and positives) |
| all positive integers | \mathbb{Z}^+ | The (set of all) strictly positive integers |
| all natural numbers | N | The (set of all) natural numbers. Note : we use the |
| | | convention that 0 is a natural number. |

Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form "The set of all x from the universe U such that x is ..." by writing

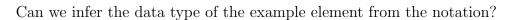
$$\{x \in U \mid ...x...\}$$

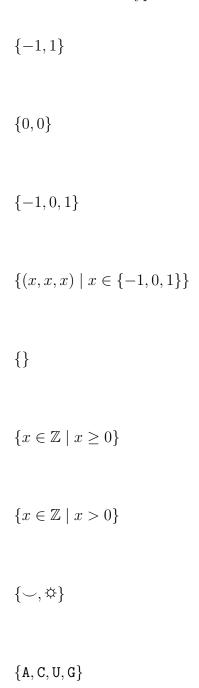
or form "the collection of all outputs of some operation when the input ranges over the universe U" by writing

$$\{...x...\mid x\in U\}$$

We use the symbol \in as "is an element of" to indicate membership in a set.

Example sets: For each of the following, identify whether it's defined using the roster method or set builder notation and give an example element.





{AUG, UAG, UGA, UAA}

Set operations

To define a set we can use the roster method, set builder notation, a recursive definition, and also we can apply a set operation to other sets.

New! Cartesian product of sets and set-wise concatenation of sets of strings

Definition: Let X and Y be sets. The **Cartesian product** of X and Y, denoted $X \times Y$, is the set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$

$$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

Conventions: (1) Cartesian products can be chained together to result in sets of n-tuples and (2) When we form the Cartesian product of a set with itself $X \times X$ we can denote that set as X^2 , or X^n for the Cartesian product of a set with itself n times for a positive integer n.

Definition: Let X and Y be sets of strings over the same alphabet. The **set-wise concatenation** of X and Y, denoted $X \circ Y$, is the set of all results of string concatenation xy where $x \in X$ and $y \in Y$

$$X \circ Y = \{xy \mid x \in X \text{ and } y \in Y\}$$

Pro-tip: the meaning of writing one element next to another like xy depends on the data-types of x and y. When x and y are strings, the convention is that xy is the result of string concatenation. When x and y are numbers, the convention is that xy is the result of multiplication. This is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.

Fill in the missing entries in the table:

| Set | Example elements in this set and their data type: |
|---|---|
| B | A C G U |
| | (A,C) (U,U) |
| $B \times \{-1, 0, 1\}$ | |
| $\{-1,0,1\} \times B$ | |
| | (0, 0, 0) |
| $\{\mathtt{A},\mathtt{C},\mathtt{G},\mathtt{U}\}\circ\{\mathtt{A},\mathtt{C},\mathtt{G},\mathtt{U}\}$ | |
| | GGGG |

Definitions functions prereqs

| Term | Notation Example(s) | We say in English |
|------------------------------|---|---|
| sequence | x_1, \ldots, x_n | A sequence x_1 to x_n |
| summation | x_1, \dots, x_n $\sum_{i=1}^n x_i \text{ or } \sum_{i=1}^n x_i$ | The sum of the terms of the sequence x_1 to x_n |
| piecewise rule definition | $f(x) = \begin{cases} \text{rule 1 for } x & \text{when COND 1} \\ \text{rule 2 for } x & \text{when COND 2} \end{cases}$ | Define f of x to be the result of applying rule 1 to x when condition COND 1 is true and the result of applying rule 2 to x when condition COND 2 is true. This can be generalized to having more than two conditions (or cases). |
| function applica- | f(7) | f of 7 or f applied to 7 or the image of 7 under f |
| 01011 | f(z) | f of z or f applied to z or the image of z under f |
| | f(g(z)) | f of g of z or f applied to the result of g applied to z |
| absolute value | -3 | The absolute value of -3 |
| square root | $\sqrt{9}$ | The non-negative square root of 9 |

Pro-tip: the meaning of two vertical lines | | depends on the data-types of what's between the lines. For example, when placed around a number, the two vertical lines represent absolute value. We've seen a single vertial line | used as part of set builder definitions to represent "such that". Again, this is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.