## Division algorithm

**Integer division and remainders** (aka The Division Algorithm) Let n be an integer and d a positive integer. There are unique integers q and r, with  $0 \le r < d$ , such that n = dq + r. In this case, d is called the divisor, n is called the dividend, q is called the quotient, and r is called the remainder.

Because these numbers are guaranteed to exist, the following functions are well-defined:

- **div** :  $\mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$  given by **div** ((n,d)) is the quotient when n is the dividend and d is the divisor.
- $\mathbf{mod}: \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$  given by  $\mathbf{mod}$  ( (n,d) ) is the remainder when n is the dividend and d is the divisor.

Because these functions are so important, we sometimes use the notation n **div** d =**div** ((n, d)) and n **mod** d =**mod** ((n, d)).

**Pro-tip**: The functions **div** and **mod** are similar to (but not exactly the same as) the operators / and % in Java and python.

Example calculations:

20 div 4

20 mod 4

20 div 3

20 mod 3

-20 mod 3

 $-20 \$ **div** $\ 3$