Division algorithm

-20 mod 3

Integer division and remainders (aka The Division Algorithm) Let n be an integer and d a positive integer. There are unique integers q and r, with $0 \le r < d$, such that n = dq + r. In this case, d is called the divisor, n is called the dividend, q is called the quotient, and r is called the remainder.

Because these numbers are guaranteed to exist, the following functions are well-defined:

- **div** : $\mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$ given by **div** ((n, d)) is the quotient when n is the dividend and d is the divisor.
- $\mathbf{mod}: \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$ given by \mathbf{mod} ((n,d)) is the remainder when n is the dividend and d is the divisor.

Because these functions are so important, we sometimes use the notation n **div** d =**div** ((n, d)) and n **mod** d =**mod** ((n, d)).

Pro-tip: The functions **div** and **mod** are similar to (but not exactly the same as) the operators / and % in Java and python.

Example calculations:
20 div 4
20 mod 4
20 div 3
20 1 2
20 mod 3
$-20 \ \mathbf{div} \ 3$