hw4-proofs-and-sets: Sample Solutions

CSE20S24

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In this assignment, you will use propositional and predicate logic to evaluate statements and arguments. You will analyze statements and determine if they are true or false using valid proof strategies. You will also determine if candidate arguments are valid.

Relevant class material: Weeks 4,5,6.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw4-proofs-and-sets".

In your proofs and disproofs of statements below, justify each step by reference to a component of the following proof strategies we have discussed so far, and/or to relevant definitions and calculations.

- A counterexample can be used to prove that $\forall x P(x)$ is **false**.
- A witness can be used to prove that $\exists x P(x)$ is **true**.
- **Proof of universal by exhaustion**: To prove that $\forall x\, P(x)$ is true when P has a finite domain, evaluate the predicate at **each** domain element to confirm that it is always T.
- **Proof by universal generalization**: To prove that $\forall x\, P(x)$ is true, we can take an arbitrary element e from the domain and show that $P(e)$ is true, without making any assumptions about e other than that it comes from the domain.
- To prove that $\exists x P(x)$ is **false**, write the universal statement that is logically equivalent to its negation and then prove it true using universal generalization.
- **Strategies for conjunction**: To prove that $p \wedge q$ is true, have two subgoals: subgoal (1) prove p is true; and, subgoal (2) prove q is true. To prove that $p \wedge q$ is false, it's enough to prove that p is false. To prove that $p \wedge q$ is false, it's enough to prove that q is false.
- **Proof of Conditional by Direct Proof**: To prove that the implication $p \to q$ is true, we can assume p is true and use that assumption to show q is true.

- **Proof of Conditional by Contrapositive Proof**: To prove that the implication $p \to q$ is true, we can assume $\neg q$ is true and use that assumption to show $\neg p$ is true.
- **Proof by Cases**: To prove q when we know $p_1 \lor p_2$, show that $p_1 \to q$ and $p_2 \to q$.

Assigned questions

1. Evaluating predicates. Consider the following predicates, each of which has as its domain the set of all bitstrings whose leftmost bit is 1

   $E(x)$ is T exactly when $(x)_2$ is even, and is F otherwise

   $L(x)$ is T exactly when $(x)_2 < 15$, and is F otherwise

   $M(x)$ is T exactly when $(x)_2 > 2$ and is F otherwise.

   Sample response that can be used as reference for the detail expected in your answer: To prove that the statement

   $$\forall x \; L(x)$$

   is false, we can use the counterexample $x = 1111$, which is a bitstring whose leftmost bit is 1 (so is in the domain). Applying the definition of $L(x)$, since $(1111)_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 8 + 4 + 2 + 1 = 15$ which is not (strictly) less than 15, we have that $L(1111) = F$ and so the universal statement is false.

   (a) (*Graded for correctness*) [1] Use a counterexample to prove that the statement

   $$\forall x(\; L(x) \to E(x) \;)$$

   is false.

   Sample Solution:
   Idea: The counterexample needs to make $L(x)$ True (i.e. $(x)_2 < 15$), and $E(x)$ False (i.e. $(x)_2$ is not even).

   We can use the counterexample $x = 1$, which is a bitstring whose leftmost bit is 1 (so is in the domain). Applying the definition of $L(x)$, since $(1)_2 = 1 \cdot 2^0 = 1$ which is less than 15, we have $L(x) = T$. Applying the definition of $E(x)$, since $(1)_2 = 1$, and $1 \textbf{ mod } 2 = 1 \neq 0$, we have $(x)_2$ is not even, which means $E(x) = F$. Hence, for $x = 1$, $L(x) \to E(x)$ is false,

[1] This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

and so the universal statement is false.

(b) (*Graded for correctness*) Use a witness to prove that the statement

$$\exists x(\ L(x) \wedge M(x)\ )$$

is true.

Sample Solution:
Idea: the witness needs to make $L(x)$ True (i.e. $(x)_2 < 15$), and $M(x)$ True (i.e. $(x)_2 > 2$).

We can use the witness $x = 11$, which is a bitstring whose leftmost bit is 1 (so is in the domain). Applying the definition of $L(x)$, since $(11)_2 = 1 \cdot 2^1 + 1 \cdot 2^0 = 3$ which is less than 15, we have $L(x) = T$. Similarly 3 is more than 2, we have $M(x) = T$. Hence, for $x = 11$, $L(x) \wedge M(x) = T$, and so the existential statement is true.

(c) (*Graded for completeness*) [2] Translate each of the statements in the previous two parts to English.

Solution: For (a), it translates to "for all bitstrings whose leftmost bit is 1, if x in decimal is less than 15, then x in decimal is even".

For (b), it translates to "there exists bitstring whose leftmost bit is 1 such that x in decimal is less than 15 and x in decimal is greater than 2".

2. Set properties. Let $W = \mathcal{P}(\{1, 2, 3, 4, 5\})$.

Sample response that can be used as reference for the detail expected in your answer for parts (a) and (b) below:

To give an example element in the set $\{X \in W \mid 1 \in X\} \cap \{X \in W \mid 2 \in X\}$, consider $\{1, 2\}$. To prove that this is in the set, by definition of intersection, we need to show that $\{1, 2\} \in \{X \in W \mid 1 \in X\}$ and that $\{1, 2\} \in \{X \in W \mid 2 \in X\}$.

- By set builder notation, elements in $\{X \in W \mid 1 \in X\}$ have to be elements of W which have 1 as an element. By definition of power set, elements of W are subsets of $\{1, 2, 3, 4, 5\}$. Since each element in $\{1, 2\}$ is an element of $\{1, 2, 3, 4, 5\}$, $\{1, 2\}$ is a subset of $\{1, 2, 3, 4, 5\}$ and hence is an element of W. Also, by roster method, $1 \in \{1, 2\}$. Thus, $\{1, 2\}$ satisfies the conditions for membership in $\{X \in W \mid 1 \in X\}$.

[2] This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

- Similarly, by set builder notation, elements in $\{X \in W \mid 2 \in X\}$ have to be elements of W which have 2 as an element. By definition of power set, elements of W are subsets of $\{1,2,3,4,5\}$. Since each element in $\{1,2\}$ is an element of $\{1,2,3,4,5\}$, $\{1,2\}$ is a subset of $\{1,2,3,4,5\}$ and hence is an element of W. Also, by roster method, $2 \in \{1,2\}$. Thus, $\{1,2\}$ satisfies the conditions for membership in $\{X \in W \mid 2 \in X\}$.

(a) (*Graded for correctness*) Give two (different) example elements in

$$W \times \mathcal{P}(W)$$

Justify your examples by explanations that include references to the relevant definitions.

Sample Solution Example 1:
To give an example element in the set $W \times \mathcal{P}(W)$, consider $(\{1,2\}, \{\{1\},\{2\}\})$. To prove that this is in the set, by definition of Cartesian product, we need to show that $\{1,2\} \in W$ and that $\{\{1\},\{2\}\} \in \mathcal{P}(W)$.

- By definition of power set, elements of W are subsets of $\{1,2,3,4,5\}$. Since each element in $\{1,2\}$ is an element of $\{1,2,3,4,5\}$, $\{1,2\}$ is a subset of $\{1,2,3,4,5\}$ and hence is an element of W.
- By definition of power set, elements of W are subsets of $\{1,2,3,4,5\}$. Since each element in both $\{1\}$ and $\{2\}$ is an element of $\{1,2,3,4,5\}$, both $\{1\}$ and $\{2\}$ are subsets of $\{1,2,3,4,5\}$ and hence are elements of W. Also, by definition of power set, elements of $\mathcal{P}(W)$ are subsets of W. Since each element in $\{\{1\},\{2\}\}$ is an element of W, $\{\{1\},\{2\}\}$ is a subset of W and hence is an element of $\mathcal{P}(W)$.

Sample Solution Example 2:
To give an example element in the set $W \times \mathcal{P}(W)$, consider $(\emptyset, \emptyset)$. To prove that this is in the set, by definition of Cartesian product, we need to show that $\emptyset \in W$ and that $\emptyset \in \mathcal{P}(W)$.

We can prove a more general claim: $\forall$ set $S, \emptyset \in \mathcal{P}(S)$. Since there are no elements in $\emptyset$, the statement "all elements in $\emptyset$ are elements of S" will evaluate to true, and thus $\emptyset$ is a subset of S. Hence, $\emptyset \in \mathcal{P}(S)$.

We can then apply the previous result to both W (which is the power set of $\{1,2,3,4,5\}$) and $\mathcal{P}(W)$ to get $\emptyset \in W$ and $\emptyset \in \mathcal{P}(W)$. Therefore, $(\emptyset, \emptyset) \in W \times \mathcal{P}(W)$.

(b) (*Graded for correctness*) Give one example element in

$$\{X \in W \mid (1 \in X) \land (X \cap \{3,4\} = \emptyset)\}$$

Justify your example by explanations that include references to the relevant definitions.

Sample Solution:

To give an example element in the set $\{X \in W \mid (1 \in X) \wedge (X \cap \{3,4\} = \emptyset)\}$, consider $\{1,2\}$. To prove that this is in the set, by the set builder notation and the definition of conjunction, we need to show that $\{1,2\} \in W$, $1 \in \{1,2\}$, and $\{1,2\} \cap \{3,4\} = \emptyset$.

- By definition of power set, elements of W are subsets of $\{1,2,3,4,5\}$. Since each element in $\{1,2\}$ is an element of $\{1,2,3,4,5\}$, $\{1,2\}$ is a subset of $\{1,2,3,4,5\}$ and hence is an element of W.
- Since 1 is listed as one of the elements of the set in the roster notation definition $\{1,2\}$, we know $1 \in \{1,2\}$.
- By definition of set intersection, $\{1,2\} \cap \{3,4\} = \{x | x \in \{1,2\} \wedge x \in \{3,4\}\}$. If we consider the elements x where $x \in \{1,2\}$, the only elements that evaluate to true are 1 and 2. However, both elements evaluates $x \in \{3,4\}$ to false, which means there are no elements where $x \in \{1,2\}$ and $x \in \{3,4\}$ are both satisfied. Hence, we get $\{1,2\} \cap \{3,4\} = \emptyset$ as desired.

(c) (*Graded for completeness*) Consider the following statement and attempted proof:

$$\forall A \in W \, \forall B \in W \ \ (\ ((A \cap B) \subseteq A) \to (A \subseteq B) \ )$$

(1) Towards a universal generalization argument, **choose arbitrary** $A \in W, B \in W$.
(2) We need **to show** $((A \cap B) \subseteq A) \to (A \subseteq B)$.
(3) Towards a proof of the conditional by the contrapositive, **assume** $A \subseteq B$, and we need **to show** that $(A \cap B) \subseteq A$.
(4) By definition of subset inclusion, this means we need **to show** $\forall x \ (x \in A \cap B \to x \in A \ )$.
(5) Towards a universal generalization, **choose arbitrary** x; we need **to show** that
$x \in A \cap B \to x \in A$.
(6) Towards a direct proof, **assume** $x \in A \cap B$, and we need **to show** $x \in A$.
(7) By definition of set intersection and set builder notation, we have that $x \in A \wedge x \in B$.
(8) By the definition of conjunction, $x \in A$, which is what we needed to show. QED

Demonstrate that this attempted proof is invalid by providing and justifying a **counterexample** (disproving the statement). Then, explain why this attempted proof is invalid by identifying in which step a definition or proof strategy is used incorrectly, and describing how the definition or proof strategy was misused.

Sample Solution:

Counterexample:

One counterexample is $A = \{1, 2\}$ and $B = \{1\}$. As proven above, they are both subset of $\{1, 2, 3, 4, 5\}$ and hence in W. By definition of set intersection, we can show $A \cap B = \{1\}$, which is a subset of A. Hence, $(A \cap B) \subseteq A$ is true.

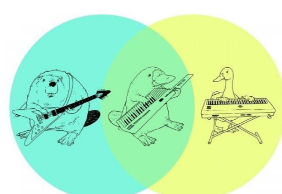
However, there exists $2 \in A$, but $2 \notin B$, which means A is not a subset of B, and $A \subseteq B$ evaluates to False.

Hence, $(A \cap B) \subseteq A) \rightarrow (A \subseteq B)$ is False for our counterexample $A = \{1, 2\}$ and $B = \{1\}$, and thus the universal statement is False.

Misuse in the proof:

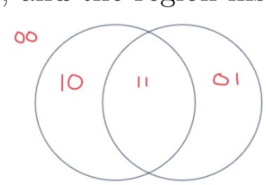
In (3), the "contrapositive" was misused. The definition of contrapositive is "to prove that the implication $p \rightarrow q$ is true, we can assume q is **false** and use that assumption to show p is also **false**" . The step (3) of the attempted proof was assuming q is **True**, and was trying to prove p is also **True**. The correct use of contrapositive in this case will be: "Towards a proof of the conditional by the contrapositive, assume A is **not** a subset of B, and we need to show that $(A \cap B)$ is **not** a subset of A."

(d) (*Graded for completeness*) A Venn diagram is a chart of overlapping regions that illustrates the similarities differences of a collection of sets. You may have seen some examples in memes, including these (from a web search):

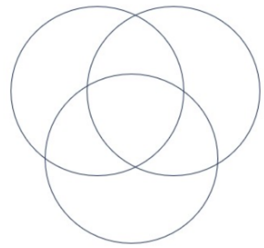


Mostly we see Venn Diagrams with 2 or 3 circles (or other shapes). In this question, we consider how to draw a Venn Diagram with 4 regions.

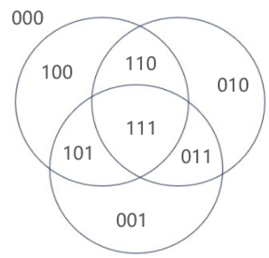
i. Here is a Venn Diagram with 2 circles. Each region is labelled (encoded) with a binary string with 2 bits. Notice that there are four regions: the region outside of the two circles, the region inside the left circle and not the right circle, the region inside the right circle and not the left circle, and the region inside both circles.



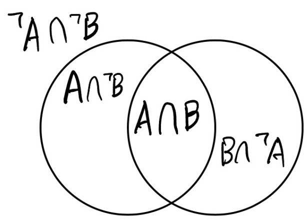
Generalize this encoding by encoding each region in this 3 circle Venn Diagram with a unique binary string with 3 bits.

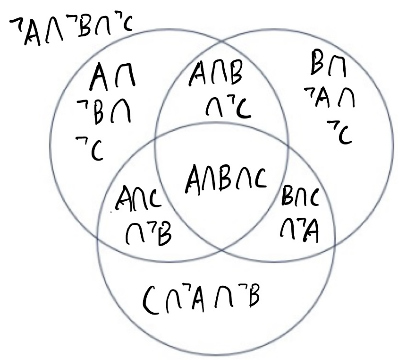


Solution:

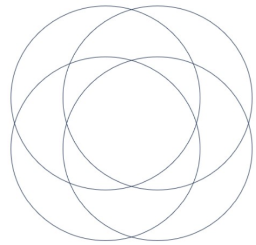


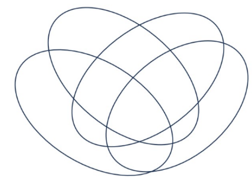
ii. Give an alternate representation of the regions in the 2 circle and 3 circle Venn Diagrams by labelling each circle with a letter (X, Y, Z, or A, B, C, for example) and then expressing each region as the result of combining these with set operations (like union, intersection, and set difference).
Solution:





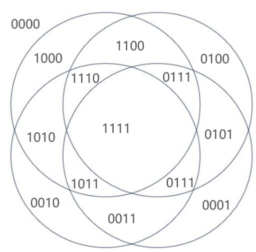
iii. Here are two attempts at a Venn Diagram with 4 shapes, one with circles and the other with ellipses.

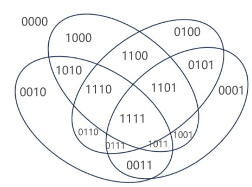




Is there anything missing from either diagram? (How many regions are there? How many regions should there be in order to use each 4 bit binary string exactly once in a generalization of the encoding we saw before?) If you had to choose one of these diagrams, which would you choose, and why?

Solution: There are 16 4 bit binary strings (from 0000 to 1111), and there should be 16 regions in the diagram.
The circles diagram has only 14 regions (0110 and 1001 missing), and the ellipse has all 16 regions:





I would choose the ellipse diagram because the circle diagram might misrepresent the data as it doesn't have enough regions.

(This question is adapted from one created by Miles Jones and is used with permission.)

3. Number properties. Consider the predicate $F(\ (a,b)\ ) =$ "a is a factor of b" over the domain $\mathbb{Z}^{\neq 0} \times \mathbb{Z}$. Consider the following quantified statements

(i) $\forall x \in \mathbb{Z}\ (F(\ (1,x\ )))$

(ii) $\forall x \in \mathbb{Z}^{\neq 0}\ (F(\ (x,1)\ ))$

(iii) $\exists x \in \mathbb{Z}\ (F(\ (1,x)\ ))$

(iv) $\exists x \in \mathbb{Z}^{\neq 0}\ (F(\ (x,1)\ ))$

(v) $\forall x \in \mathbb{Z}^{\neq 0}\ \exists y \in \mathbb{Z}\ (F(\ (x,y)\ ))$

(vi) $\exists x \in \mathbb{Z}^{\neq 0}\ \forall y \in \mathbb{Z}\ (F(\ (x,y)\ ))$

(vii) $\forall y \in \mathbb{Z}\ \exists x \in \mathbb{Z}^{\neq 0}\ (F(\ (x,y)\ ))$

(viii) $\exists y \in \mathbb{Z}\ \forall x \in \mathbb{Z}^{\neq 0}\ (F(\ (x,y)\ ))$

(a) (*Graded for completeness*) Which of the statements (i) - (viii) is being **proved** by the following proof:

By universal generalization, **choose** e to be an **arbitrary** integer. We need to show that $F(\ (1,e)\ )$. By definition of the predicate F, we can rewrite this goal as $\exists c \in \mathbb{Z}\ (e = c \cdot 1)$. We pick the **witness** $c = e$, which is an integer and therefore in the domain. Calculating, $c \cdot 1 = e \cdot 1 = e$, as required. Since the predicate $F(1,e)$ evaluates to true for the arbitrary integer e, the claim has been proved. $\square$

Hint: it may be useful to identify the key words in the proof that indicate proof strategies.

Solution: From the keywords "By universal generalization", "arbitrary e," we know that the proof is trying to prove a universal quantification by universal generalization, so the statement should start with $\forall e$.

From the keyword "integer," we know the domain is the set of integers $\mathbb{Z}$.

From the next sentence "We need to show that $F(\ (1, e)\ )$," we know that the predicate is $F((1, e))$. The rest are just evaluating $F((1, e))$.

When combined together, the statement is $\forall e \in \mathbb{Z}\ F((1, e))$, which is equivalent to (i).

(b) (*Graded for completeness*) Which of the statements (i) - (viii) is being **disproved** by the following proof:

> To disprove the statement, we need to find a counterexample. We choose 2, a nonzero integer so in the domain. We need to show that $\neg F(\ (2, 1)\ )$. By definition of the predicate F, we can rewrite this goal as $1 \textbf{ mod } 2 \neq 0$. By definition of integer division, since $1 = 0 \cdot 2 + 1$ (and $0 \leq 1 < 2$), $1 \textbf{ mod } 2 = 1$ which is nonzero so the counterexample works to disprove the original statement. $\square$

Hint: it may be useful to identify the key words in the proof that indicate proof strategies.

Solution: From the keywords "find a counterexample." we infer that the proof is trying to disprove a universal quantification by counterexample, so the statement should start with $\forall e$. The counterexample e the proof picked is $e = 2$.

From the keyword "nonzero integer so in the domain," we infer the domain is the set of nonzero integers $\mathbb{Z}^{\neq 0}$.

From the next sentence "We need to show that $\neg F(\ (2, 1)\ )$," we infer that the predicate is $F((e, 1))$ (as $e = 2$). The rest are just evaluating $F((2, 1))$.

When combined together, the statement is $\forall e \in \mathbb{Z}^{\neq 0}\ F((e, 1))$, which is equivalent to (ii).

(c) (*Graded for correctness of evaluation of statement (is it true or false?) and fair effort completeness of the translation and proof*) Translate the statement to English, state whether is it true or false, and then justify your answer (by proving the statement or its negation).

$$\exists x \in \mathbb{Z}^{\neq 0}\ \exists y \in \mathbb{Z}^{\neq 0}\ (\ \neg(x = y) \land F(\ (x, y)\ ) \land F(\ (y, x)\ )\ )$$

Solution: The English statement is "There exists a nonzero integer x and a nonzero integer y such that $x \neq y$, x is a factor of y, and y is a factor of x". This statement is true. Consider the witness $x = 2, y = -2$, $2 \neq -2$. Since $2 = (-2) \cdot (-1)$ and $-2 = 2 \cdot (-1)$, $(\ \neg(x = y) \wedge F(\ (x, y)\ ) \wedge F(\ (y, x)\ )\ )$ is true.

(d) (*Graded for correctness of evaluation of statement (is it true or false?) and fair effort completeness of the translation and of the proof*) Translate the statement to English, state whether is it true or false, and then justify your answer (by proving the statement or its negation).

$$\forall x \in \mathbb{Z}^{\neq 0}\ \forall y \in \mathbb{Z}^{\neq 0}\ (\ F(\ (x, y)\ ) \rightarrow \neg F(\ (y, x)\ )\ )$$

Solution: The English statement is "For all nonzero integer x and for all nonzero integer y, if x is a factor of y, then y is not a factor of x". This statement is false. Consider the same $x = 2, y = -2$ from (c) as a counterexample. Since $2 = (-2) \cdot (-1)$ and $-2 = 2 \cdot (-1)$, $F((x, y))$ is true and $\neg F((y, x))$ is false. Hence, $(\ F(\ (x, y)\ ) \rightarrow \neg F(\ (y, x)\ )\ )$ is false.

(e) (*Graded for correctness of evaluation of statement (is it true or false?) and fair effort completeness of the translation and of the proof*) Translate the statement to English, state whether is it true or false, and then justify your answer (by proving the statement or its negation).

$$\exists x \in \mathbb{Z}^{\neq 0}\ \exists y \in \mathbb{Z}\ (\ F(\ (x, y)\ ) \wedge F(\ (x + 1, y)\ ) \wedge F(\ (x + 2, y)\ )\ )$$

Solution: The English statement is "There exists a nonzero integer x and an integer y such that x, $x + 1$, and $x + 2$ are all factors of y". This statement is true. Consider the witness $x = 1, y = 6$. 1, 2, and 3 are all factors of 6, since $6 = 1 \cdot 6 = 2 \cdot 3 = 3 \cdot 2$ therefore $F(\ (x, y)\ ) \wedge F(\ (x + 1, y)\ ) \wedge F(\ (x + 2, y)$ is true.

(f) (*Graded for correctness of evaluation of statement (is it true or false?) and fair effort completeness of the translation and of the proof*) Translate the statement to English, state whether is it true or false, and then justify your answer (by proving the statement or its negation).

$$\forall x \in \mathbb{Z}^{\neq 0}\ (\ F(\ (x, x^2)\ ) \wedge F(\ (x, x^3)\ )\ )$$

Solution: The English statement is "each nonzero integers are a factor of its square and its cube". This statement is true. By universal generalization, let x be a nonzero integer. $F((x, x^2))$ is true because $x \cdot x = x^2$ so the integer x witnesses that x is a factor of x^2. $F((x, x^3))$ is true because $x \cdot x^2 = x^3$ so the integer x^2 witnesses that x is a factor of x^3. Since $F((x, x^2)) \wedge F((x, x^3))$ is true for all x, the claim has been proven.

4. Structural induction. Recall that we define the set of bases as $B = \{\mathtt{A}, \mathtt{C}, \mathtt{U}, \mathtt{G}\}$. The set of RNA strands S is defined (recursively) by:

Basis Step:	$\mathtt{A} \in S, \mathtt{C} \in S, \mathtt{U} \in S, \mathtt{G} \in S$
Recursive Step:	If $s \in S$ and $b \in B$, then $sb \in S$

where sb is string concatenation. The function *rnalen* that computes the length of RNA strands in S is defined recursively by $rnalen : S \to \mathbb{Z}^+$

Basis step: If $b \in B$ then $rnalen(b) = 1$
Recursive step: If $s \in S$ and $b \in B$, then $rnalen(sb) = 1 + rnalen(s)$

The function *basecount* that computes the number of a given base b appearing in a RNA strand s is defined recursively:

		$basecount : S \times B$	$\to \mathbb{N}$	
Basis Step:	If $b_1 \in B, b_2 \in B$	$basecount(\ (b_1, b_2)\ )$	$=$	$\begin{cases} 1 & \text{when } b_1 = b_2 \\ 0 & \text{when } b_1 \neq b_2 \end{cases}$
Recursive Step:	If $s \in S, b_1 \in B, b_2 \in B$	$basecount(\ (sb_1, b_2)\ )$	$=$	$\begin{cases} 1 + basecount(\ (s, b_2)\ ) & \text{when } b_1 = b_2 \\ basecount(\ (s, b_2)\ ) & \text{when } b_1 \neq b_2 \end{cases}$

(a) (*Graded for correctness of evaluation of statement (is it true or false?) and fair effort completeness of the translation and proof*) Translate the statement to English, state whether is it true or false, and then justify your answer (by proving the statement or its negation).

$$\forall b \in B\ \exists s \in S\ (\ rnalen(s) = basecount(\ (s, b)\ )\ )$$

Solution: The English statement is "For all base b, there exists a RNA strand s such that the length of s equals the number of b appearing in s". This statement is true. By universal generalization, let b be an arbitrary base. Consider $s = b$ be a witness, which is in the domain as $b \in B$ and the basis step in the definition of S ensures $b \in S$. By applying the basis step of the two functions *basecount* and *rnalen*, we have $rnalen(s) = basecount(\ (s, b)\ ) = 1$.

(b) (*Graded for correctness of evaluation of statement (is it true or false?) and fair effort completeness of the translation and of the proof*) Translate the statement to English, state whether is it true or false, and then justify your answer (by proving the statement or its negation).

$$\forall s \in S\ (rnalen(s) > basecount(\ (s, \mathtt{A})\ ))$$

Solution: The English statement is "For all RNA strand s, the length of s is greater than the number of A appearing in s". This statement is false. Consider a counterexample $s = \mathtt{A}$, which is in the domain by the basis step in the definition of S. By applying the basis step of the two functions *basecount* and *rnalen*, we have $rnalen(s) = basecount(\ (s, \mathtt{A})\ ) = 1$, which implies $(rnalen(s) > basecount(\ (s, \mathtt{A})\ ))$ is false.

(c) (*Graded for correctness of evaluation of statement (is it true or false?) and fair effort completeness of the translation and of the proof*) Translate the statement to English, state whether is it true or false, and then justify your answer (by proving the statement or its negation).

$$\forall s \in S\ \forall b \in B\ (basecount(\ (s, b)\ ) > 0)$$

Solution: The English statement is "For all RNA strand s and for all base b, the number of b appearing in s is greater than 0". This statement is false. Consider a counterexample $s = \texttt{A}$ and $b = \texttt{C}$, which are both in the domain by the basis step in the definition of S and the set roster of B. By applying the basis step of function *basecount*, we have $basecount(\ (s, b)\ ) = basecount(\ (\texttt{A}, \texttt{C})\ )0$, which implies $(basecount(\ (s, b)\ )) > 0$ is false.