Division algorithm

Integer division and remainders (aka The Division Algorithm) Let n be an integer and d a positive integer. There are unique integers q and r, with $0 \leq r < d$, such that $n = dq + r$. In this case, d is called the divisor, n is called the dividend, q is called the quotient, and r is called the remainder.

Because these numbers are guaranteed to exist, the following functions are well-defined:

- **div** $: \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$ given by **div** $((n, d))$ is the quotient when n is the dividend and d is the divisor.
- **mod** $: \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$ given by **mod** $((n, d))$ is the remainder when n is the dividend and d is the divisor.

Because these functions are so important, we sometimes use the notation n **div** $d =$ **div** $((n, d))$ and n **mod** $d =$ **mod** $((n, d))$.

Pro-tip: The functions **div** and **mod** are similar to (but not exactly the same as) the operators / and % in Java and python.

Example calculations:

20 **div** 4

20 **mod** 4

20 **div** 3

20 **mod** 3

-20 **div** 3

-20 **mod** 3