Logical equivalence identities

(Some) logical equivalences

Can replace p and q with any compound proposition

$$\neg(\neg p) \equiv p$$

Double negation

$$p \lor q \equiv q \lor p \qquad \qquad p \land q \equiv q \land p$$

$$p \wedge q \equiv q \wedge p$$

Commutativity Ordering of terms

$$(p \lor q) \lor r \equiv p \lor (q \lor r)$$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

 $(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$ Associativity Grouping of terms

$$p \wedge F \equiv F$$

$$p \lor T \equiv T \quad p \land T \equiv p$$

$$p \vee F \equiv p$$

 $p \wedge F \equiv F$ $p \vee T \equiv T$ $p \wedge T \equiv p$ $p \vee F \equiv p$ **Domination** aka short circuit evaluation

$$\neg (p \land q) \equiv \neg p \lor \neg q \qquad \qquad \neg (p \lor q) \equiv \neg p \land \neg q$$

$$\neg (p \lor q) \equiv \neg p \land \neg q$$

DeMorgan's Laws

$$p \to q \equiv \neg p \lor q$$

$$p \to q \equiv \neg q \to \neg p$$
 Contrapositive

$$\neg(p \to q) \equiv p \land \neg q$$

$$\neg(p \leftrightarrow q) \equiv p \oplus q$$

$$p \leftrightarrow q \equiv q \leftrightarrow p$$

Extra examples:

 $p \leftrightarrow q$ is not logically equivalent to $p \land q$ because

 $p \to q$ is not logically equivalent to $q \to p$ because _____

Logical operators truth tables

Truth tables: Input-output tables where we use T for 1 and F for 0.

Input		Output				
		Conjunction	Exclusive or	Disjunction		
p	q	$p \wedge q$	$p\oplus q$	$p \lor q$		
\overline{T}	T	T	F	T		
T	F	F	T	T		
F	T	F	T	T		
F	F	F	F	F		
		AND	XOR	OR		

Input	Output Negation
p	$\neg p$
\overline{T}	F
F	T
	-NOT

Logical equivalence

Logical equivalence: Two compound propositions are logically equivalent means that they have the same truth values for all settings of truth values to their propositional variables.

Tautology: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Logical equivalence extra example

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		out	Output					
1	9	q	$\neg (p \land \neg q)$	$\neg (\neg p \lor \neg q)$	$(\neg p \lor q)$	$(\neg q \lor \neg p)$	$(p \land q)$	
\overline{I}	Γ	T						
7	Γ	F						
1	7	T						
I	7	F						