Algorithm definition

New! An algorithm is a finite sequence of precise instructions for solving a problem.

Algorithms can be expressed in English or in more formalized descriptions like pseudocode or fully executable programs.

Sometimes, we can define algorithms whose output matches the rule for a function we already care about. Consider the (integer) logarithm function

$$logb : \{b \in \mathbb{Z} \mid b > 1\} \times \mathbb{Z}^+ \quad \rightarrow \quad \mathbb{N}$$

defined by

$$logb(\ (b, n)\ ) = \text{greatest integer } y \text{ so that } b^y \text{ is less than or equal to } n$$

Calculating integer part of base b logarithm

```
1   procedure logb(b,n: positive integers with b>1)
2   i := 0
3   while n > b-1
4     i := i+1
5     n := n div b
6   return i {i holds the integer part of the base b logarithm of n}
```

Trace this algorithm with inputs $b = 3$ and $n = 17$

	b	n	i	$n > b - 1$?
Initial value	3	17		
After 1 iteration				
After 2 iterations				
After 3 iterations				

Compare: does the output match the rule for the (integer) logarithm function?

Base expansion algorithms

Two algorithms for constructing base b expansion from decimal representation

Most significant first: Start with left-most coefficient of expansion (highest value)

Informally: Build up to the value we need to represent in "greedy" approach, using units determined by base.

Calculating base b expansion, from left

```
1  procedure baseb1(n,b: positive integers with b>1)
2  v := n
3  k := 1+ output of logb algorithm with inputs b and n
4  for i := 1 to k
5     a_{k-i} := 0
6     while v ≥ b^{k-i}
7        a_{k-i} := a_{k-i} + 1
8        v := v - b^{k-i}
9  return (a_{k-1},...,a_0){(a_{k-1}...a_0)_b is the base b expansion of n}
```

Least significant first: Start with right-most coefficient of expansion (lowest value)

Idea: (when $k > 1$)

$$\begin{aligned} n &= a_{k-1}b^{k-1} + \cdots + a_1 b + a_0 \\ &= b(a_{k-1}b^{k-2} + \cdots + a_1) + a_0 \end{aligned}$$

so $a_0 = n$ **mod** b and $a_{k-1}b^{k-2} + \cdots + a_1 = n$ **div** b.

Calculating base b expansion, from right

```
1  procedure baseb2(n, b: positive integers with b > 1)
2  q := n
3  k := 0
4  while q ≠ 0
5     a_k := q mod b
6     q := q div b
7     k := k + 1
8  return (a_{k-1}, ..., a_0){(a_{k-1} ... a_0)_b is the base b expansion of n}
```

Base conversion algorithm

Practice: write an algorithm for converting from base b_1 expansion to base b_2 expansion: