Definitions set prereqs

Term	Notation Example(s)	We say in English ...
all reals	$\mathbb{R}$	The (set of all) real numbers (numbers on the number line)
all integers	$\mathbb{Z}$	The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	$\mathbb{Z}^+$	The (set of all) strictly positive integers
all natural numbers	$\mathbb{N}$	The (set of all) natural numbers. **Note**: we use the convention that 0 is a natural number.

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sequence	$x_1, \ldots, x_n$	A sequence x_1 to x_n
summation	$\sum_{i=1}^{n} x_i$ or $\displaystyle\sum_{i=1}^{n} x_i$	The sum of the terms of the sequence x_1 to x_n
piecewise rule definition	$f(x) = \begin{cases} \text{rule 1 for } x & \text{when COND 1} \\ \text{rule 2 for } x & \text{when COND 2} \end{cases}$	Define f of x to be the result of applying rule 1 to x when condition COND 1 is true and the result of applying rule 2 to x when condition COND 2 is true. This can be generalized to having more than two conditions (or cases).
function application	$f(7)$	f of 7 **or** f applied to 7 **or** the image of 7 under f
	$f(z)$	f of z **or** f applied to z **or** the image of z under f
	$f(g(z))$	f of g of z **or** f applied to the result of g applied to z
absolute value	$|-3|$	The absolute value of -3
square root	$\sqrt{9}$	The non-negative square root of 9

Pro-tip: the meaning of two vertical lines $|\ \ |$ depends on the data-types of what's between the lines. For example, when placed around a number, the two vertical lines represent absolute value. We've seen a single vertial line $|$ used as part of set builder definitions to represent "such that". Again, this is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.

Defining functions

New! Defining functions A function is defined by its (1) domain, (2) codomain, and (3) rule assigning each element in the domain exactly one element in the codomain.

The domain and codomain are nonempty sets.
The rule can be depicted as a table, formula, piecewise definition, or English description.
The notation is

> "Let the function FUNCTION-NAME: DOMAIN $\to$ CODOMAIN be given by
> FUNCTION-NAME(x) = ... for every $x \in DOMAIN$".

or

> "Consider the function FUNCTION-NAME: DOMAIN $\to$ CODOMAIN defined as
> FUNCTION-NAME(x) = ... for every $x \in DOMAIN$".

Example: The absolute value function

Domain

Codomain

Rule

Defining functions ratings

Recall our representation of Netflix users' ratings of movies as n-tuples, where n is the number of movies in the database. Each component of the n-tuple is -1 (didn't like the movie), 0 (neutral rating or didn't watch the movie), or 1 (liked the movie).

Consider the ratings $P_1 = (-1, 0, 1, 0)$, $P_2 = (1, 1, -1, 0)$, $P_3 = (1, 1, 1, 0)$, $P_4 = (0, -1, 1, 0)$

Which of P_1, P_2, P_3 has movie preferences most similar to P_4?

One approach to answer this question: use **functions** to quantify difference among user preferences.

For example, consider the function $d_0 : \{-1, 0, 1\}^4 \times \{-1, 0, 1\}^4 \to \mathbb{R}$ given by

$$d_0(\ (\ (x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4)\ )\ ) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 + (x_4 - y_4)^2}$$

Defining functions recursively

When the domain of a function is a *recursively defined set*, the rule assigning images to domain elements (outputs) can also be defined recursively.

Recall: The set of RNA strands S is defined (recursively) by:

Basis Step: $\quad\quad \texttt{A} \in S, \texttt{C} \in S, \texttt{U} \in S, \texttt{G} \in S$

Recursive Step: $\quad$ If $s \in S$ and $b \in B$, then $sb \in S$

where sb is string concatenation.

Definition (Of a function, recursively) A function *rnalen* that computes the length of RNA strands in S is defined by:

$$rnalen : S \;\rightarrow \mathbb{Z}^+$$

Basis Step: $\quad\quad$ If $b \in B$ then $\quad\quad\quad$ $rnalen(b) \;= 1$

Recursive Step: $\quad$ If $s \in S$ and $b \in B$, then $\quad$ $rnalen(sb) \;= 1 + rnalen(s)$

The domain of *rnalen* is

The codomain of *rnalen* is

Example function application:

$$rnalen(\texttt{ACU}) =$$

Example: A function *basecount* that computes the number of a given base b appearing in a RNA strand s is defined recursively:

$$basecount : S \times B \quad\;\; \rightarrow \mathbb{N}$$

Basis Step: $\quad\quad$ If $b_1 \in B, b_2 \in B$ $\quad\quad$ $basecount(\,(b_1, b_2)\,) \;\; = \begin{cases} 1 & \text{when } b_1 = b_2 \\ 0 & \text{when } b_1 \neq b_2 \end{cases}$

Recursive Step: $\quad$ If $s \in S, b_1 \in B, b_2 \in B$ $\quad$ $basecount(\,(sb_1, b_2)\,) \;\; = \begin{cases} 1 + basecount(\,(s, b_2)\,) & \text{when } b_1 = b_2 \\ basecount(\,(s, b_2)\,) & \text{when } b_1 \neq b_2 \end{cases}$