# Logical equivalence identities

#### (Some) logical equivalences

Can replace p and q with any compound proposition

$$\neg(\neg p) \equiv p$$

Double negation

$$p \lor q \equiv q \lor p \qquad \qquad p \land q \equiv q \land p$$

$$p \wedge q \equiv q \wedge p$$

Commutativity Ordering of terms

$$(p \lor q) \lor r \equiv p \lor (q \lor r)$$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

 $(p \lor q) \lor r \equiv p \lor (q \lor r)$   $(p \land q) \land r \equiv p \land (q \land r)$  Associativity Grouping of terms

$$p \wedge F \equiv F$$

$$p \lor T \equiv T \quad p \land T \equiv p$$

$$p \vee F \equiv p$$

 $p \wedge F \equiv F$   $p \vee T \equiv T$   $p \wedge T \equiv p$   $p \vee F \equiv p$  **Domination** aka short circuit evaluation

$$\neg (p \land q) \equiv \neg p \lor \neg q$$

$$\neg (p \land q) \equiv \neg p \lor \neg q \qquad \qquad \neg (p \lor q) \equiv \neg p \land \neg q$$

DeMorgan's Laws

$$p \to q \equiv \neg p \lor q$$

$$p \to q \equiv \neg q \to \neg p$$
 Contrapositive

$$\neg(p \to q) \equiv p \land \neg q$$

$$\neg(p \leftrightarrow q) \equiv p \oplus q$$

$$p \leftrightarrow q \equiv q \leftrightarrow p$$

Extra examples:

 $p \leftrightarrow q$  is not logically equivalent to  $p \land q$  because

 $p \to q$  is not logically equivalent to  $q \to p$  because \_\_\_\_\_

## Logical operators example truth table

Input		t	Output					
p	q	r	$\mid (p \wedge q) \oplus (\ (p \oplus q) \wedge r\ ) \mid (p \wedge q) \lor (\ (p \oplus q) \wedge r\ )$					
$\overline{T}$	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

## Logical equivalence

Logical equivalence: Two compound propositions are logically equivalent means that they have the same truth values for all settings of truth values to their propositional variables.

**Tautology**: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

**Contingency**: A compound proposition that is neither a tautology nor a contradiction.

#### Logical equivalence extra example

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output						
p	q	$\neg (p \land \neg q)$	$\neg (\neg p \lor \neg q)$	$(\neg p \lor q)$	$(\neg q \lor \neg p)$	$(p \wedge q)$		
$\overline{T}$	T							
T	F							
F	T							
F	F							