

## Division algorithm

**Integer division and remainders** (aka The Division Algorithm) Let  $n$  be an integer and  $d$  a positive integer. There are unique integers  $q$  and  $r$ , with  $0 \leq r < d$ , such that  $n = dq + r$ . In this case,  $d$  is called the divisor,  $n$  is called the dividend,  $q$  is called the quotient, and  $r$  is called the remainder.

Because these numbers are guaranteed to exist, the following functions are well-defined:

- **div** :  $\mathbb{Z} \times \mathbb{Z}^+ \rightarrow \mathbb{Z}$  given by **div** (  $(n, d)$  ) is the quotient when  $n$  is the dividend and  $d$  is the divisor.
- **mod** :  $\mathbb{Z} \times \mathbb{Z}^+ \rightarrow \mathbb{Z}$  given by **mod** (  $(n, d)$  ) is the remainder when  $n$  is the dividend and  $d$  is the divisor.

Because these functions are so important, we sometimes use the notation  $n \text{ div } d = \text{div} ( (n, d) )$  and  $n \text{ mod } d = \text{mod} ( (n, d) )$ .

**Pro-tip:** The functions **div** and **mod** are similar to (but not exactly the same as) the operators `/` and `%` in Java and python.

*Example calculations:*

20 **div** 4

20 **mod** 4

20 **div** 3

20 **mod** 3

−20 **div** 3

−20 **mod** 3