

# Base expansion definition

**Definition** For  $b$  an integer greater than 1 and  $n$  a positive integer, the **base  $b$  expansion of  $n$**  is

$$(a_{k-1} \cdots a_1 a_0)_b$$

where  $k$  is a positive integer,  $a_0, a_1, \dots, a_{k-1}$  are (symbols for) nonnegative integers less than  $b$ ,  $a_{k-1} \neq 0$ , and

$$n = \sum_{i=0}^{k-1} a_i b^i$$

Notice: *The base  $b$  expansion of a positive integer  $n$  is a string over the alphabet  $\{x \in \mathbb{N} \mid x < b\}$  whose leftmost character is nonzero.*

Base $b$	Collection of possible coefficients in base $b$ expansion of a positive integer
Binary ( $b = 2$ )	$\{0, 1\}$
Ternary ( $b = 3$ )	$\{0, 1, 2\}$
Octal ( $b = 8$ )	$\{0, 1, 2, 3, 4, 5, 6, 7\}$
Decimal ( $b = 10$ )	$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Hexadecimal ( $b = 16$ )	$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$ letter coefficient symbols represent numerical values $(A)_{16} = (10)_{10}$ $(B)_{16} = (11)_{10}$ $(C)_{16} = (12)_{10}$ $(D)_{16} = (13)_{10}$ $(E)_{16} = (14)_{10}$ $(F)_{16} = (15)_{10}$

# Base expansion examples

*Examples:*

$(1401)_2$

$(1401)_{10}$

$(1401)_{16}$

## Base expansion review

Find and fix any and all mistakes with the following:

(a)  $(1)_2 = (1)_8$

(b)  $(142)_{10} = (142)_{16}$

(c)  $(20)_{10} = (10100)_2$

(d)  $(35)_8 = (1D)_{16}$

## Base expansion final review

Convert  $(2A)_{16}$  to

- binary (base \_\_\_\_)
- decimal (base \_\_\_\_)
- octal (base \_\_\_\_)
- ternary (base \_\_\_\_)