

hw1-definitions-and-notation

CSE20S24

Due: 4/9/24 at 5pm (no penalty late submission until 8am next morning)

In this assignment,

You will practice reading and applying definitions to get comfortable working with mathematical language.

Relevant class material: Week 1.

You will submit this assignment via Gradescope (<https://www.gradescope.com>) in the assignment called “hw1-definitions-and-notation”.

For all HW assignments: These homework assignments may be done individually or in groups of up to 3 students. Please ensure your name(s) and PID(s) are clearly visible on the first page of your homework submission, start each question on a new page, and upload the PDF to Gradescope. If you’re working in a group, *submit only one submission per group*: one partner uploads the submission through their Gradescope account and then adds the other group member(s) to the Gradescope submission by selecting their name(s) in the “Add Group Members” dialog box. You will need to re-add your group member(s) every time you resubmit a new version of your assignment.

Each homework question will be graded either for **correctness** (including clear and precise explanations and justifications of all answers) or **fair effort completeness**. You may collaborate on “graded for correctness” questions only with CSE 20 students in your group; if your group has questions about a problem, you may ask in drop-in help hours or post a private post (visible only to the Instructors) on Piazza. For “graded for completeness” questions: collaboration is allowed with any CSE 20 students this quarter; if your group has questions about a problem, you may ask in drop-in help hours or post a public post on Piazza.

All submitted homework for this class must be typed. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions.

Integrity reminders

- Problems should be solved together, not divided up between the partners. The homework is designed to give you practice with the main concepts and techniques of the course, while getting to know and learn from your classmates.
- You may not collaborate on homework questions graded for correctness with anyone other than your group members. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza (as private notes viewable only to the Instructors). You *cannot* use any online resources about the course content other than the class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions (aligned with the textbook) and also to protect the learning experience you will have when the ‘aha’ moments of solving the problem authentically happen.
- Do not share written solutions or partial solutions for homework with other students in the class who are not in your group. Doing so would dilute their learning experience and detract from their success in the class.

Assigned questions

1. Modeling

- (a) (*Graded for completeness*)¹ In class, we used 4-tuples to represent the user ratings for four movies. This representation is memory-efficient because we use the order of the components in the 4-tuples to represent which movie is being rated. However, it is not easily extended when we want to add new movies to the database. Define a new model that would allow us to represent the user ratings of movie databases where we allow for new movies to be added. Use only the data types we have talked about in class: sets, n -tuples, and strings. Explain the design choices that you used to define your model by referencing properties of the data-type(s) you choose. Demonstrate your model by showing how the rating of the user who dislikes Dune and Oppenheimer and likes Barbie and Nimona is represented.
- (b) (*Graded for completeness*) Colors can be described as amounts of red, green, and blue mixed together² Mathematically, a color can be represented as a 3-tuple (r, g, b) where r represents the red component, g the green component, b the blue component and where each of r , g , b must be a value from this collection of numbers:

¹This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer **each** part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

²This RGB representation is common in web applications. Many online tools are available to play around with mixing these colors, e.g. https://www.w3schools.com/colors/colors_rgb.asp.

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255}

(This is the same definition as in the Week 1 Review quiz.)

Can you find two different 3-tuples that represent colors that are indistinguishable to your eye? You can use the website in the footnote to play around with different choices of red, green, and blue levels to see if you can distinguish between the resulting colors. Why or why not?

A complete answer will include the specific example 3-tuples that work, along with a description of the colors that they represent and why they are indistinguishable, or an explanation of why there can't be such an example.

2. Sets and functions

- (a) (*Graded for correctness*)³ Each of the sets below is described using set builder notation or recursion or as a result of set operations applied to other known sets. Rewrite each of the sets using the roster method.

Remember our discussions of data-types: use clear notation that is consistent with our class notes and definitions to communicate the data-types of the elements in each set.

Sample response that can be used as reference for the detail expected in your answer:

The set $\{A\} \circ \{AU, AC, AG\}$ can be written using the roster method as

$$\{AAU, AAC, AAG\}$$

because set-wise concatenation gives a set whose elements are all possible results of concatenating an element of the left set with an element of the right set. Since the left set in this example only has one element, namely A , each of the elements of the set we described starts with A . There are three elements of this set, one for each of the distinct elements of the right set.

³This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

i.

$$\{n \in \mathbb{Z}^+ \mid n \leq 3\} \times \{m \in \mathbb{N} \mid m \leq 3\}$$

(Note: typo fixed Apr 3)

ii. The set X defined recursively as

Basis Step: $1 \in X, 3 \in X, 5 \in X$

Recursive Step: If the integer $n \in X$, then the result of multiplying n by -1 is in X

iii.

$$\{x \in S \mid rnalen(x) = 2\} \circ \{x \in S \mid rnalen(x) = 0\}$$

where S is the set of RNA strands and $rnalen$ is the recursively defined function that we discussed in class,

$$\begin{array}{ll} rnalen : S & \rightarrow \mathbb{Z}^+ \\ \text{Basis Step:} & \text{If } b \in B \text{ then } rnalen(b) = 1 \\ \text{Recursive Step:} & \text{If } s \in S \text{ and } b \in B, \text{ then } rnalen(sb) = 1 + rnalen(s) \end{array}$$

iv.

$$\{(r, g, b) \in C \mid r + g + b = 2 \text{ and } g = 1\}$$

where $C = \{(r, g, b) \mid 0 \leq r \leq 255, 0 \leq g \leq 255, 0 \leq b \leq 255, r \in \mathbb{N}, g \in \mathbb{N}, b \in \mathbb{N}\}$ is the set that you worked with in Monday's review quiz.

(b) (*Graded for correctness*) Recall the function which takes an ordered pair of ratings 4-tuples and returns a measure of the difference between them

$$d_0 : \{-1, 0, 1\}^4 \times \{-1, 0, 1\}^4 \rightarrow \mathbb{R}$$

given by

$$d_0((x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4)) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 + (x_4 - y_4)^2}$$

Define a **new** function which we'll call d_{new} with the same domain $\{-1, 0, 1\}^4 \times \{-1, 0, 1\}^4$ and codomain \mathbb{R} but where there is some example pair of ratings 4-tuples

$$((x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4))$$

where

$$d_0((x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4)) \neq d_{new}((x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4))$$

Your answer should include **both** a precise and clear definition for the rule defining d_{new} which unambiguously specifies output for each input of the function **and** the example ordered pair of ratings 4-tuples that demonstrate that the functions are not equal. Also include a justification of your answers with (clear, correct, complete) calculations for each of the function applications and/or references to definitions and connecting them with the desired conclusion.

- (c) (*Graded for correctness*) A function *basecount* that computes the number of a given base b appearing in a RNA strand s is defined recursively:

$$\text{basecount} : S \times B \rightarrow \mathbb{N}$$

Basis Step:

$$\text{If } b_1 \in B, b_2 \in B \quad \text{basecount}((b_1, b_2)) = \begin{cases} 1 & \text{when } b_1 = b_2 \\ 0 & \text{when } b_1 \neq b_2 \end{cases}$$

Recursive Step:

$$\text{If } s \in S, b_1 \in B, b_2 \in B \quad \text{basecount}((sb_1, b_2)) = \begin{cases} 1 + \text{basecount}((s, b_2)) & \text{when } b_1 = b_2 \\ \text{basecount}((s, b_2)) & \text{when } b_1 \neq b_2 \end{cases}$$

Consider the function application

$$\text{basecount}((\text{ACAU}, \text{A}))$$

What is the input? What is the output? Give an example of a different choice of input that gives the same output.

Your answer should include clearly labeled answers to each of the three parts of the question, along with a justification for the values of the applications that makes specific reference to the parts of the recursive definition of the *basecount* function used to calculate it.

Project Part 1 due 10/14/21; Part 2 due 11/4/21; Part 3 due 12/2/21

The project component of this class will be an opportunity for you to extend your work on assignments and explore applications of your choosing.

Why? To go deeper and explore the material from discrete math and how it relates to Computer Science.

How? During emergency remote instruction last academic year, we discovered that video assessment and some open-ended personalized projects help ensure fairness and can be less stressful for students than in-person midterm exams. Asynchronous project submission also gives flexibility and allows more physical distancing.

Your videos: We will delete all the videos we receive from you after assigning final grades for the course, and they will be stored in a university-controlled Google Drive directory only accessible to the course staff during the quarter. Please send an email to the instructor (minnes@eng.ucsd.edu) if you have concerns about the video / screencast components of this project or cannot complete projects in this style for some reason.

You may produce screencasts with any software you choose. One option is to record yourself with Zoom; a tutorial on how to use Zoom to record a screencast (courtesy of Prof. Joe Politz) is here:

https://drive.google.com/open?id=1KROMAQuTCk40zwrEFotlYSJJQdcG_GUU.

The video that was produced from that recording session in Zoom is here:

<https://drive.google.com/open?id=1MxJN6CQcXqIb0ekDYMxjh7mTt1TyRVM1>

What resources can you use?

This project must be completed individually, without any help from other people, including the course staff (other than logistics support if you get stuck with screencast).

You can use any of this quarter's CSE 20 offering (notes, readings, class videos, homework feedback). These resources should be more than enough. If you are struggling to get started and want to look elsewhere online, you must acknowledge this by listing and citing any resources you consult (even if you do not explicitly quote them). Link directly to them and include the name of the author / video creator and the reason you consulted this reference. The work you submit for the project needs to be your own. Again, you shouldn't need to look anywhere other than this quarter's material and doing so may result in definitions or notations that conflict with our norms in this class so think carefully before you go down this path.

The project has three parts.

- Part 1 of Project: due Thursday October 14
- Part 2 of Project: due Thursday November 4
- Part 3 of Project: due Thursday December 2

Part 1: due Thursday October 14

Written component

1. In Week 1, we discussed the mathematical definition of a function, namely that a **function** is defined by its (1) domain, (2) codomain, and (3) rule assigning each element in the domain exactly one element in the codomain.
 - (a) Write out the definition for an example function you make up. You can choose any example you like, so long as it is your own independent effort and it is not a function from a class example, homework, or Review Quiz in this class so far. Use the notation defined in class. Label and define the domain, codomain, and rule clearly.
 - (b) Calculate the result of applying your function to an element in its domain. Just like in homework, include (clear, correct, complete) calculations and/or references to definitions to present your function application.
2. In CSE 20 this quarter, we will be exploring the applications of discrete mathematics for core CS topics. The following videos introduce some of these topics and the work happening here at UCSD to explore them. Pick one of the following videos, watch it, and then write a few sentences answering the following:
 - (a) Which video did you watch? Why did you choose the video you watched?
 - (b) What followup question(s) would you like to ask the person in the video about their work? At least one of your followup questions should be about a technical aspect of the work that you would like to learn more about.

(Click on the video titles below for the links)

Bioinformatics and virology: *Niema Moshiri - Genome Sequence Alignment*

Human robotics interaction: *Angelique Taylor - Improving Human-Robot Interaction*

Computer vision: *Manmohan Chandraker: Giving Computers the Gift of Vision*

Data centers and energy efficiency: *Max Mellette: Improving Data Centers with Photonics*

Programming languages and data structures: *Nadia Polikarpova: Creating New Languages for Programming*

Machine learning (and surfing) for climate science: *Studying Climate Change Through Surfing with Smartfin - We Are CSE: Jasmine Simmons*

Video component

Presenting your reasoning and demonstrating it via screenshare are important skills that also show us a lot of your learning. Getting practice with this style of presentation is a good thing for you to learn in general and a rich way for us to assess your skills.

Prepare a 3-5 minute screencast video that starts with your face and your student ID for a few seconds at the beginning, and introduce yourself audibly while on screen. You don't have to be on camera for the rest of the video, though it's fine if you are. We are looking for a brief confirmation that it's you creating the video and doing the work submitted for the project.

Then, explain your work in question 1 of the written component. Discuss at least one potential mistake that someone solving a similar question should avoid (this could be a mistake you made while thinking about this problem or something you anticipate a classmate might struggle with); explain why the mistake is wrong and how to fix it.

Finally, explain any differences between your pre-survey description of "what a function is" and the mathematical definition of what a function is. You should have an email copy of your responses to the pre-survey, and you can refer to what you wrote in your explanation.

Use this Google form

<https://forms.gle/SLd8SrJdXR5HCLQr7> (click to follow link)

to directly upload a video file for this assignment. It should be a file that you can easily play on your system. One way you can determine if this is true is if you can store it on your Google Drive and play it from there, since that's how we will watch it.

Checklist (this is how we will grade Part 1 of the project)

- Question 1
 - The function definition is complete and uses correct notation, and is different from class, homework, and quiz examples.
 - The calculation of the function application is correct and is supported by clear, correct, complete justification.
- Question 2
 - (At least) one of the videos is mentioned and a reason for selecting it is included.
 - At least one technical question is described that is connected to the video selected.
- Video
 - Video loads correctly and is between 3 and 5 minutes. It includes your face and your student ID, and you introduce yourself audibly while on screen.
 - Video presents your solution for Question 1.
 - A potential mistake is presented and discussed.
 - The mathematical definition of function is compared to your response in the pre-survey.

Part 2: due Thursday November 4

Written component

1. In this part of the project, you will select one question from one of the review quizzes from 10/4/2021 (Monday of Week 1) to 10/29/2021 (Friday of Week 5) to revisit. Include the problem statement, why you picked this question (e.g. what is interesting about it, what is hard about it, or why you wanted to take a second look at it), and your solution.
 - Question selection: you can pick any **one question** listed in the Review sections of the relevant notes documents, and you must address all of its parts.
 - For each part of your chosen question: prepare a complete solution (you can use the homework solutions we post for guidance about the style). Your submission will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Your goal should be to convince the reader that your results and methods are sound. Imagine you are preparing these solutions for someone else taking CSE 20 who missed that week and is “catching up”.
2. In this part of the project, you’ll consider the importance of data types and precision in Computer Science. Read three articles
 - (1) This discussion of the causes of a wide-spread problem in published genomics papers <https://www.nature.com/articles/d41586-021-02211-4> (Click to follow link) from the journal Nature;
 - (2) this IEEE profile of Katherine Johnson, <https://spectrum.ieee.org/the-institute/ieee-history/katherine-johnson-the-hidden-figures-mathematician-who-got-astronaut-john-glenn-into-space> (Click to follow link) a NASA “computer” who calculated trajectories for early space exploration and who passed away in 2020; and
 - (3) this NASA report about the unsuccessful 1999 Mars Climate Orbiter mission <https://solarsystem.nasa.gov/missions/mars-climate-orbiter/in-depth/>In one or two sentences, summarize the main lesson you draw from each article.

Thinking back to your own experiences, give an example of when you used computers or Computer Science to help you **avoid** an error. Also, give an example when your use of computers or Computer Science **caused** an error.

What measures do you take to increase your confidence in the results of your own human and digital (i.e. machine) computation? Why do you think these are sufficient?

Video component

Presenting your reasoning and demonstrating it via screenshare are important skills that also show us a lot of your learning. Getting practice with this style of presentation is a good thing for you to learn in general and a rich way for us to assess your skills.

Prepare a 3-5 minute screencast video explaining your work in question 1 of the written component. During your solution presentation, point out at least one potential mistake that someone solving a similar question should avoid (this could be a mistake you made while thinking about this problem or something you anticipate a classmate might struggle with); explain why the mistake is wrong and how to fix it.

You do not need to include complete details of every part of your solution. It is up to you to choose what is most important so that you can stick to the timing guidelines and still have time to include discussing potential mistakes.

Include your face and your student ID (we'd like a photo ID that includes your name and picture if possible) for a few seconds at the beginning, and introduce yourself audibly while on screen. You don't have to be on camera the whole time, though it's fine if you are. We are looking for a brief confirmation that it's you creating the video/doing the work attached to the video.

Use this Google form

<https://forms.gle/SLd8SrJdXR5HCLQr7> (click to follow link)

to directly upload a video file for this assignment. It should be a file that you can easily play on your system. One way you can determine if this is true is if you can store it on your Google Drive and play it from there, since that's how we will watch it.

Checklist (this is how we will grade Part 2 of the project)

- Question 1
 - Selected review quiz question is labelled clearly, including the day it belongs to and the statement of the question.
 - Solution is complete: it addresses each part of the review quiz question selected.
 - Solution is correct: it clearly and correctly justifies the correct answer for each part of the question. You are welcome to check your answers with the Gradescope autograder (we will be reopening the review quizzes for this purpose). We will evaluate your submissions for the quality of your justification.
- Question 2

- A key lesson from each of the three references is stated clearly and is relevant to the message of the articles. Supporting explanations are included.
 - A specific example of an instance where using computers/ CS *caused* an error is described.
 - A specific example of an instance where using computers/ CS helped *avoid* an error is described.
 - Lesson(s) are drawn from the previous experiences.
 - Specific strategies for increasing confidence in computation are described and justified.
- Video
 - Video loads correctly and is between 3 and 5 minutes. It includes your face and your student ID, and you introduce yourself audibly while on screen.
 - Video presents your solution for Question 1.
 - A potential mistake is presented and discussed.

Part 3: due Thursday December 2

Written component

1. In this part of the project, you will analyze a quantified statement about RNA strands. The definitions for RNA strands are available in the class notes. Example quantified statements about RNA strands are in the homework. Complete the following:

- (a) Write a quantified statement symbolically. Your quantified statement should satisfy **all** of the following requirements:
 - Have a nesting of quantifiers with at least one forall quantification and at least one existential quantification.
 - Have at least one negation and at least one binary logic operation (and, or, xor, if, iff).
 - Negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
 - Use **exactly one** of the predicates Mut, Ins, Del.
 - Not be a statement we have analyzed already in class materials.
- (b) Translate your statement from part a. to English.
- (c) Negate the whole statement from part a. and rewrite this negated statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
- (d) Prove or disprove your statement from part a.

2. In this part of the project, you'll consider the impact Computer Science has on society. Read two articles:

(1) This policy piece about facial recognition software

<https://thehill.com/policy/technology/569543-federal-agencies-planning-to-expand-use-of-facial-recognition> (Click to follow link);
and

(2) this exploration of accessibility for visually impaired website users

<https://www.wsj.com/articles/colorblind-users-push-technology-designers-to-use-signals-beyond-color-11591351201> (Click to follow link).

Update (November 18): the Wall Street Journal about web design is now behind a pay wall. If you do not have access to it, please read the following two articles instead:

<https://webaim.org/articles/visual/colorblind> (Click to follow link) and <https://webaim.org/resources/designers/> (Click to follow link)

In one or two sentences, summarize the main lesson you draw from each article.

Give an example of an algorithm or computer system that you use or have been used by others to make some decision that affects you. The algorithms may be institutional or personal, formal or heuristic, and should output a specific result or decision. Explain your

answer, either by reference to your knowledge of the algorithm itself or by observations you make about outputs of the algorithm.

Give an example of how someone different from you might have a different experience with this algorithm. Support your example with lessons you learned in the readings, citing what you learned from the articles.

Video component

Presenting your reasoning and demonstrating it via screenshare are important skills that also show us a lot of your learning. Getting practice with this style of presentation is a good thing for you to learn in general and a rich way for us to assess your skills.

Prepare a 3-5 minute screencast video explaining your work in question 1 parts (c) and (d) of the written component (i.e. the negation and proof). During your solution presentation, point out at least one potential mistake that someone solving a similar question should avoid (this could be a mistake you made while thinking about this problem or something you anticipate a classmate might struggle with); explain why the mistake is wrong and how to fix it.

You do not need to include complete details of every part of your solution to these parts. It is up to you to choose what is most important so that you can stick to the timing guidelines and still have time to include discussing potential mistakes.

Include your face and your student ID (we'd like a photo ID that includes your name and picture if possible) for a few seconds at the beginning, and introduce yourself audibly while on screen. You don't have to be on camera the whole time, though it's fine if you are. We are looking for a brief confirmation that it's you creating the video/doing the work attached to the video.

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to directly upload a video file for this assignment. It should be a file that you can easily play on your system. One way you can determine if this is true is if you can store it on your Google Drive and play it from there, since that's how we will watch it.

Checklist (this is how we will grade Part 3 of the project)

- Question 1
 - Quantified statement is clearly stated, is well-defined, syntactically correct, and meets all requirements.
 - Translation to English is clear, correct, and complete.

- The negation of the quantified statement is clearly stated, is well-defined, syntactically correct, and meets all requirements.
- The proof or disproof of the original statement is clear, correct, and complete.
- Question 2
 - A key lesson from each of the two references is stated clearly and is relevant to the message of the articles. Supporting explanations are included.
 - A specific example of an algorithm or computer systems that impacts you is described.
 - A description of how this algorithm or computer system might impact someone different from you differently is included, and is supported with references to one or both of the articles.
- Video
 - Video loads correctly and is between 3 and 5 minutes. It includes your face and your student ID, and you introduce yourself audibly while on screen.
 - Video presents your solution for Question 1 parts (c) and (d).
 - A potential mistake is presented and discussed.