

# Logical equivalence identities

## (Some) logical equivalences

Can replace  $p$  and  $q$  with any compound proposition

$$\neg(\neg p) \equiv p$$

**Double negation**

$$p \vee q \equiv q \vee p$$

$$p \wedge q \equiv q \wedge p$$

**Commutativity** Ordering of terms

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

**Associativity** Grouping of terms

$$p \wedge F \equiv F$$

$$p \vee T \equiv T$$

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

**Domination** aka short circuit evaluation

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

**DeMorgan's Laws**

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

**Contrapositive**

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

$$\neg(p \leftrightarrow q) \equiv p \oplus q$$

$$p \leftrightarrow q \equiv q \leftrightarrow p$$




*Extra examples:*


$p \leftrightarrow q$  is not logically equivalent to  $p \wedge q$  because \_\_\_\_\_

$p \rightarrow q$  is not logically equivalent to  $q \rightarrow p$  because \_\_\_\_\_

# Logical operators truth tables

Truth tables: Input-output tables where we use  $T$  for 1 and  $F$  for 0.

| Input |     | Output                                                                            |                                                                                   |                                                                                   |
|-------|-----|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|       |     | Conjunction                                                                       | Exclusive or                                                                      | Disjunction                                                                       |
| $p$   | $q$ | $p \wedge q$                                                                      | $p \oplus q$                                                                      | $p \vee q$                                                                        |
| $T$   | $T$ | $T$                                                                               | $F$                                                                               | $T$                                                                               |
| $T$   | $F$ | $F$                                                                               | $T$                                                                               | $T$                                                                               |
| $F$   | $T$ | $F$                                                                               | $T$                                                                               | $T$                                                                               |
| $F$   | $F$ | $F$                                                                               | $F$                                                                               | $F$                                                                               |
|       |     |  |  |  |

| Input                                                                               | Output   |
|-------------------------------------------------------------------------------------|----------|
| Negation                                                                            |          |
| $p$                                                                                 | $\neg p$ |
| $T$                                                                                 | $F$      |
| $F$                                                                                 | $T$      |
|  |          |

## Logical equivalence

**Logical equivalence** : Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.

**Tautology**: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated  $T$ .

**Contradiction**: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated  $F$ .

**Contingency**: A compound proposition that is neither a tautology nor a contradiction.

## Logical equivalence extra example

*Extra Example*: Which of the compound propositions in the table below are logically equivalent?

| Input |     | Output                  |                            |                   |                        |                |
|-------|-----|-------------------------|----------------------------|-------------------|------------------------|----------------|
| $p$   | $q$ | $\neg(p \wedge \neg q)$ | $\neg(\neg p \vee \neg q)$ | $(\neg p \vee q)$ | $(\neg q \vee \neg p)$ | $(p \wedge q)$ |
| $T$   | $T$ |                         |                            |                   |                        |                |
| $T$   | $F$ |                         |                            |                   |                        |                |
| $F$   | $T$ |                         |                            |                   |                        |                |
| $F$   | $F$ |                         |                            |                   |                        |                |