Logical equivalence identities

(Some) logical equivalences

Can replace p and q with any compound proposition

$\neg(\neg p) \equiv p$ **Double negation**

$p \vee q \equiv q \vee p$ $\qquad p \wedge q \equiv q \wedge p$ **Commutativity** Ordering of terms

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $\qquad (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ **Associativity** Grouping of terms

$p \wedge F \equiv F$ $\qquad p \vee T \equiv T$ $\quad p \wedge T \equiv p$ $\qquad p \vee F \equiv p$ **Domination** aka short circuit evaluation

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\qquad \neg(p \vee q) \equiv \neg p \wedge \neg q$ **DeMorgan's Laws**

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$ $\qquad$ **Contrapositive**

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$\neg(p \leftrightarrow q) \equiv p \oplus q$

$p \leftrightarrow q \equiv q \leftrightarrow p$

Extra examples:

$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because _____

$p \rightarrow q$ is not logically equivalent to $q \rightarrow p$ because _____

Logical operators truth tables

Truth tables: Input-output tables where we use T for 1 and F for 0.

Input		Output				Input		Output
		Conjunction	**Exclusive or**	**Disjunction**				**Negation**
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$		p		$\neg p$
T	T	T	F	T		T		F
T	F	F	T	T		F		T
F	T	F	T	T				
F	F	F	F	F				

Logical equivalence

Logical equivalence : Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.

Tautology: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Logical equivalence extra example

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					