## Definitions set prereqs

Term	Notation Example(s)	We say in English
all reals	$\mathbb{R}$	The (set of all) real numbers (numbers on the number
		line)
all integers	$\mathbb{Z}$	The (set of all) integers (whole numbers including neg-
		atives, zero, and positives)
all positive integers	$\mathbb{Z}^+$	The (set of all) strictly positive integers
all natural numbers	N	The (set of all) natural numbers. <b>Note</b> : we use the
		convention that 0 is a natural number.

## Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form "The set of all x from the universe U such that x is ..." by writing

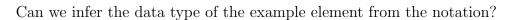
$$\{x \in U \mid ...x...\}$$

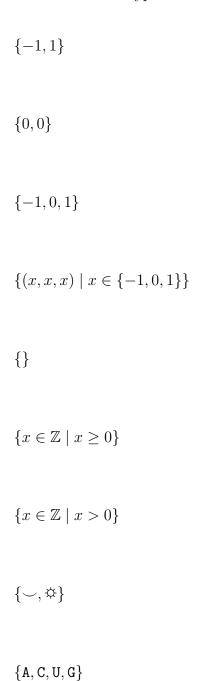
or form "the collection of all outputs of some operation when the input ranges over the universe U" by writing

$$\{...x...\mid x\in U\}$$

We use the symbol  $\in$  as "is an element of" to indicate membership in a set.

**Example sets**: For each of the following, identify whether it's defined using the roster method or set builder notation and give an example element.





{AUG, UAG, UGA, UAA}

## Set operations

To define a set we can use the roster method, set builder notation, a recursive definition, and also we can apply a set operation to other sets.

New! Cartesian product of sets and set-wise concatenation of sets of strings

**Definition**: Let X and Y be sets. The **Cartesian product** of X and Y, denoted  $X \times Y$ , is the set of all ordered pairs (x, y) where  $x \in X$  and  $y \in Y$ 

$$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

Conventions: (1) Cartesian products can be chained together to result in sets of n-tuples and (2) When we form the Cartesian product of a set with itself  $X \times X$  we can denote that set as  $X^2$ , or  $X^n$  for the Cartesian product of a set with itself n times for a positive integer n.

**Definition**: Let X and Y be sets of strings over the same alphabet. The **set-wise concatenation** of X and Y, denoted  $X \circ Y$ , is the set of all results of string concatenation xy where  $x \in X$  and  $y \in Y$ 

$$X \circ Y = \{xy \mid x \in X \text{ and } y \in Y\}$$

**Pro-tip**: the meaning of writing one element next to another like xy depends on the data-types of x and y. When x and y are strings, the convention is that xy is the result of string concatenation. When x and y are numbers, the convention is that xy is the result of multiplication. This is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.

Fill in the missing entries in the table:

Set	Example elements in this set and their data type:
B	A C G U
	(A,C) $(U,U)$
$B \times \{-1, 0, 1\}$	
$\{-1,0,1\} \times B$	
	(0, 0, 0)
$\{\mathtt{A},\mathtt{C},\mathtt{G},\mathtt{U}\}\circ\{\mathtt{A},\mathtt{C},\mathtt{G},\mathtt{U}\}$	
	GGGG

## Definitions functions prereqs

Term	Notation Example(s)	We say in English
sequence	$x_1, \ldots, x_n$	A sequence $x_1$ to $x_n$
summation	$x_1, \dots, x_n$ $\sum_{i=1}^n x_i \text{ or } \sum_{i=1}^n x_i$	The sum of the terms of the sequence $x_1$ to $x_n$
piecewise rule definition	$f(x) = \begin{cases} \text{rule 1 for } x & \text{when COND 1} \\ \text{rule 2 for } x & \text{when COND 2} \end{cases}$	Define $f$ of $x$ to be the result of applying rule 1 to $x$ when condition COND 1 is true and the result of applying rule 2 to $x$ when condition COND 2 is true. This can be generalized to having more than two conditions (or cases).
function applica-	f(7)	f of 7 <b>or</b> $f$ applied to 7 <b>or</b> the image of 7 under $f$
01011	f(z)	f of $z$ or $f$ applied to $z$ or the image of $z$ under $f$
	f(g(z))	f of $g$ of $z$ or $f$ applied to the result of $g$ applied to $z$
absolute value	-3	The absolute value of $-3$
square root	$\sqrt{9}$	The non-negative square root of 9

**Pro-tip**: the meaning of two vertical lines | | depends on the data-types of what's between the lines. For example, when placed around a number, the two vertical lines represent absolute value. We've seen a single vertial line | used as part of set builder definitions to represent "such that". Again, this is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.