Polytopes in LEAN

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0.1 H-polytopes

Definition 1.

Proposition 2. *Every H-polytope is a V-polytope.*

0.1.1 Primitive Spaces

Definition 3.

Lemma 4.

Proof. □

Lemma 5.

Proof. □

Lemma 6. *The intersection of a primspace in A with an affine subspace E of A is itself a primspace in E (but not necessarily in A).*

Proof. □

0.1.2 Bounded

Definition 7.

0.2 V-Polytopes

Definition 8. A V-polytope is the convex hull of finitely many points.

Proposition 9.

0.2.1 Convex Hulls

Definition 10. A set S is convex if for any $x, y \in S$, and any $t \in [0, 1]$, S also contains $tx + (1-t)y$.

Lemma 11. *The empty set is convex.*

Proof. □

Lemma 12. ^{n} *is convex.*

Proof. □

Definition 13. The convex hull of a set A is the intersection of all convex subsets containing A.

Proposition 14.

Proof. □

Lemma 15.

Proof. □

0.2.2 Polar duals

Definition 16.

Proposition 17.

0.3 The Main theorem

Theorem 18. *Every H-polytope is a V-polytope and vice-versa.*