

**Supplementary Materials for ‘Bayesian nowcasting for delay  
adjustments using time-varying parametric functions of  
cumulative reporting probability’ by Erick A.  
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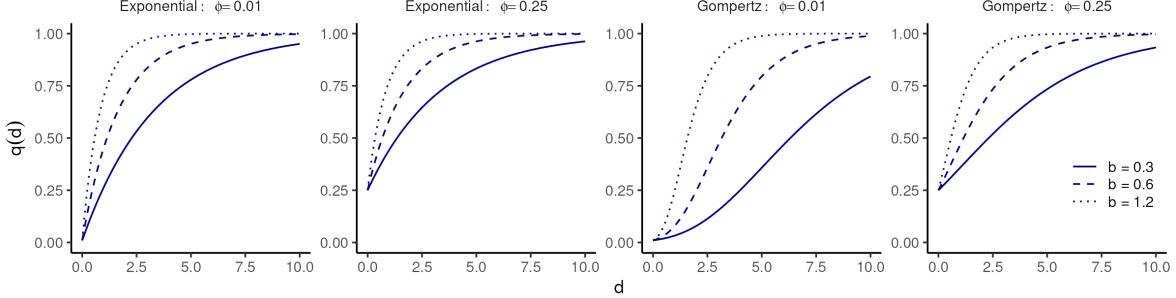
**Abstract**

These supplementary materials provide additional details about our Bayesian nowcasting model with time-varying parametric reporting probabilities. Web Appendix A presents illustrative curves of the parametric reporting probabilities and additional results on the marginal distribution of reported cases. Web Appendix B describes the scenarios and results of the simulation study. Web Appendix C contains additional figures and tables related to the real-world applications discussed in the main manuscript. Finally, Web Appendix D explores possible extensions of the parametric reporting probabilities.

**Keywords:** Cumulative reporting probability, Diagnostics, Nowcasting

## Web Appendix A. Methods

### A.1. Parametric cumulative reporting probability



**Web Figure 1:** Cumulative reporting probability function  $q(d)$  for delay  $d$  under the Exponential and Gompertz models, with different values for the immediate reporting probability ( $\phi = 0.01, 0.25$ ) and the rate parameter ( $b = 0.3, 0.6, 1.2$ ).

### A.2. Marginal model: Integrate actual number of cases $N_t$

We want to find the marginal distribution  $P(Y_{(t,d)} \mid \lambda_t, q_d)$ , this applies to all cumulative reporting probability  $q_d$ ,  $q(d)$  and  $q_t(d)$ . By definition,

$$P(Y_{(t,d)} \mid \lambda_t, q_d) = \sum_{N_t=Y_{(t,d)}}^{\infty} P(N_t \mid \lambda_t) P(Y_{(t,d)} \mid N_t, q_d).$$

Substituting the Poisson and Binomial PMFs:

$$P(N_t \mid \lambda_t) = \frac{\lambda_t^{N_t} e^{-\lambda_t}}{N_t!}, \quad P(Y_{(t,d)} \mid N_t, q_d) = \binom{N_t}{Y_{(t,d)}} q_d^{Y_{(t,d)}} (1 - q_d)^{N_t - Y_{(t,d)}}.$$

Hence,

$$P(Y_{(t,d)} \mid \lambda_t, q_d) = \sum_{N_t=Y_{(t,d)}}^{\infty} \frac{\lambda_t^{N_t} e^{-\lambda_t}}{N_t!} \binom{N_t}{Y_{(t,d)}} q_d^{Y_{(t,d)}} (1 - q_d)^{N_t - Y_{(t,d)}}.$$

Recall that

$$\binom{N_t}{Y_{(t,d)}} = \frac{N_t!}{Y_{(t,d)}!(N_t - Y_{(t,d)})!}.$$

This cancels  $N_t!$  in the denominator:

$$P(Y_{(t,d)} \mid \lambda_t, q_d) = \sum_{N_t=Y_{(t,d)}}^{\infty} \frac{\lambda_t^{N_t} e^{-\lambda_t}}{Y_{(t,d)}!(N_t - Y_{(t,d)})!} q_d^{Y_{(t,d)}} (1 - q_d)^{N_t - Y_{(t,d)}}.$$

Let  $m_t = N_t - Y_{(t,d)}$ , so that  $N_t = Y_{(t,d)} + m_t$ . As  $N_t$  ranges from  $Y_{(t,d)}$  to  $\infty$ ,  $m_t$  ranges from 0 to  $\infty$ . Substituting,

$$P(Y_{(t,d)} \mid \lambda_t, q_d) = \frac{q_d^{Y_{(t,d)}} e^{-\lambda_t}}{Y_{(t,d)}!} \sum_{m_t=0}^{\infty} \frac{\lambda_t^{Y_{(t,d)}+m_t} (1 - q_d)^{m_t}}{m_t!}.$$

Factoring out  $\lambda_t^{Y_{(t,d)}}$ :

$$P(Y_{(t,d)} \mid \lambda_t, q_d) = \frac{q_d^{Y_{(t,d)}} \lambda_t^{Y_{(t,d)}} e^{-\lambda_t}}{Y_{(t,d)}!} \sum_{m_t=0}^{\infty} \frac{(\lambda_t(1-q_d))^{m_t}}{m_t!}.$$

Recall:

$$\sum_{m=0}^{\infty} \frac{x^m}{m!} = e^x,$$

with  $x = \lambda_t(1 - q_d)$ . Hence,

$$\sum_{m_t=0}^{\infty} \frac{(\lambda_t(1-q_d))^{m_t}}{m_t!} = e^{\lambda_t(1-q_d)}.$$

Substituting, we obtain:

$$P(Y_{(t,d)} \mid \lambda_t, q_d) = \frac{(q_d \lambda_t)^{Y_{(t,d)}} e^{-\lambda_t}}{Y_{(t,d)}!} e^{\lambda_t(1-q_d)} = \frac{(q_d \lambda_t)^{Y_{(t,d)}}}{Y_{(t,d)}!} e^{-\lambda_t q_d}.$$

Thus, we arrive at:

$$\boxed{P(Y_{(t,d)} \mid \lambda_t, q_d) = \text{Poisson}(q_d \lambda_t),}$$

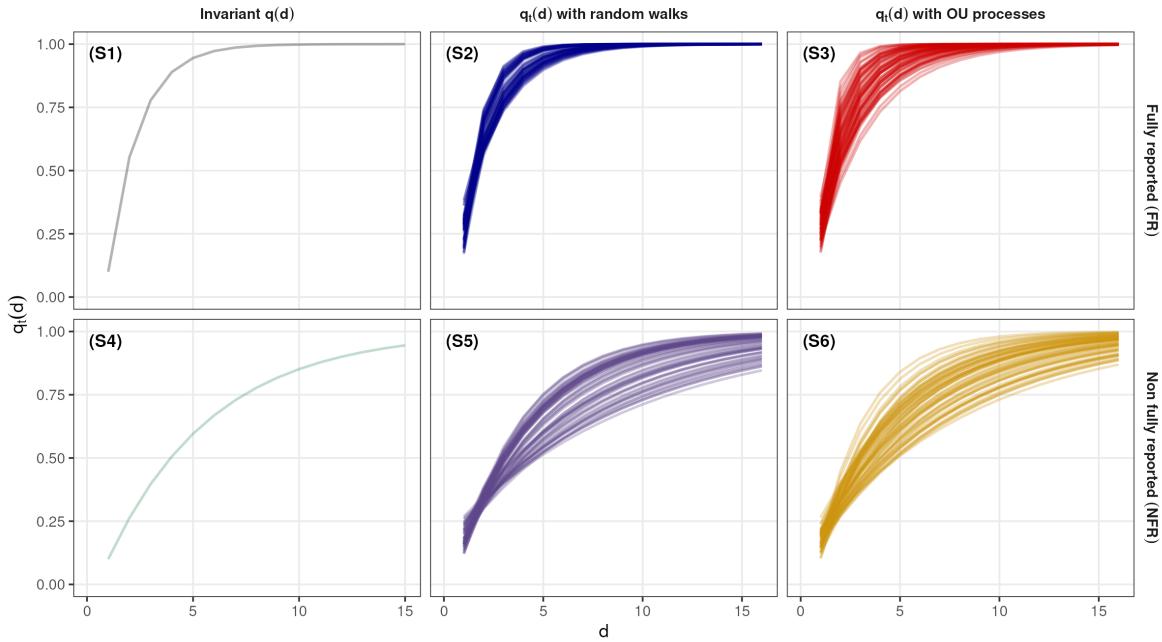
i.e.,  $Y_{(t,d)}$  given  $\lambda_t$  and  $q_d$  follows a Poisson distribution with mean  $q_d \lambda_t$ .

## Web Appendix B. Simulation study

### B.1. Data simulation

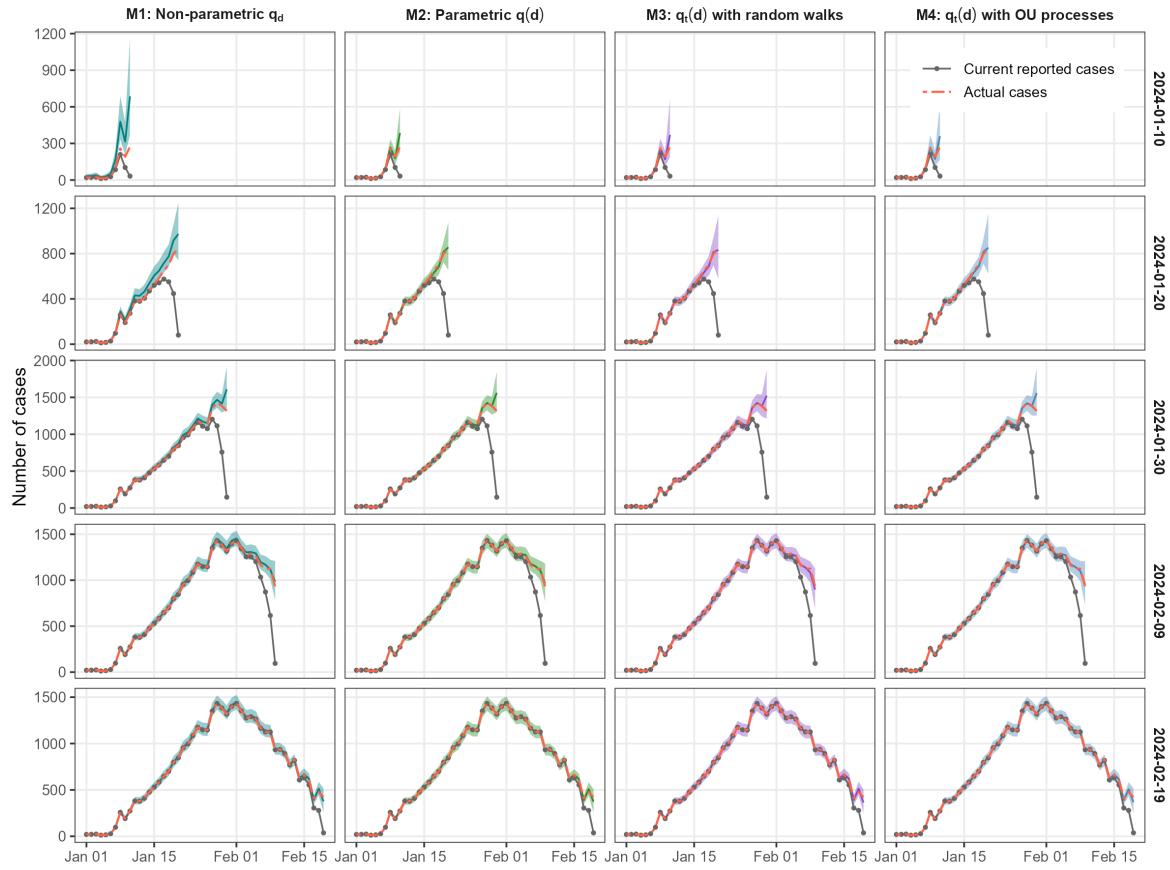
Assumption	Fully reported (FR)		Non fully reported (NFR)	
	Scenario	Parameters	Scenario	Parameters
Time-invariant $q(d)$	S1	$\phi = 0.1$ $b = 0.7$	S4	$\phi = 0.1$ $b = 0.2$
Time-varying $q_t(d)$ with random walks	S2	$\mu_\phi = 1.0$ $\sigma_\phi = 0.1$ $\mu_b = \log(0.7)$ $\sigma_b = 0.1$	S5	$\mu_\phi = 1.5$ $\sigma_\phi = 0.1$ $\mu_b = \log(0.2)$ $\sigma_b = 0.1$
Time-varying $q_t(d)$ with OU processes	S3	$\text{logit}(\phi_1) = 1.0$ $\mu_\phi = 1.0$ $\theta_\phi = 0.3$ $\sigma_\phi = 0.2$ $\log(b_1) = \log(0.7)$ $\mu_b = \log(0.7)$ $\theta_b = 0.3$ $\sigma_b = 0.2$	S6	$\text{logit}(\phi_1) = 1.5$ $\mu_\phi = 1.5$ $\theta_\phi = 0.2$ $\sigma_\phi = 0.15$ $\log(b_1) = \log(0.2)$ $\mu_b = \log(0.2)$ $\theta_b = 0.2$ $\sigma_b = 0.15$

**Web Table 1:** Parameter values for the exponential cumulative reporting probability  $q_t(d)$  across the six study scenarios.

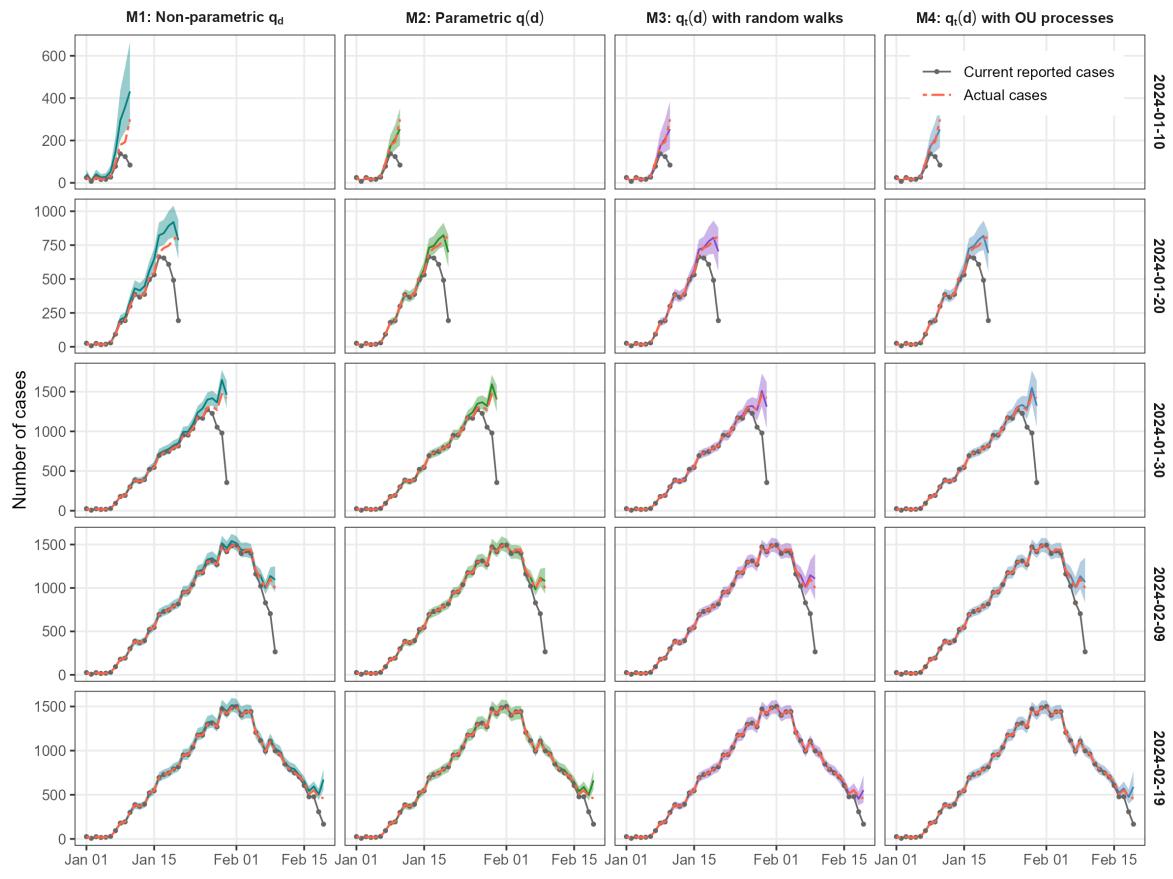


**Web Figure 2:** Exponential cumulative reporting probability  $q_t(d)$  across the six study scenarios. A single function is shown for the time-invariant scenarios (S1 and S4), while 60 functions are displayed for the time-varying scenarios (S2, S3, S5, and S6).

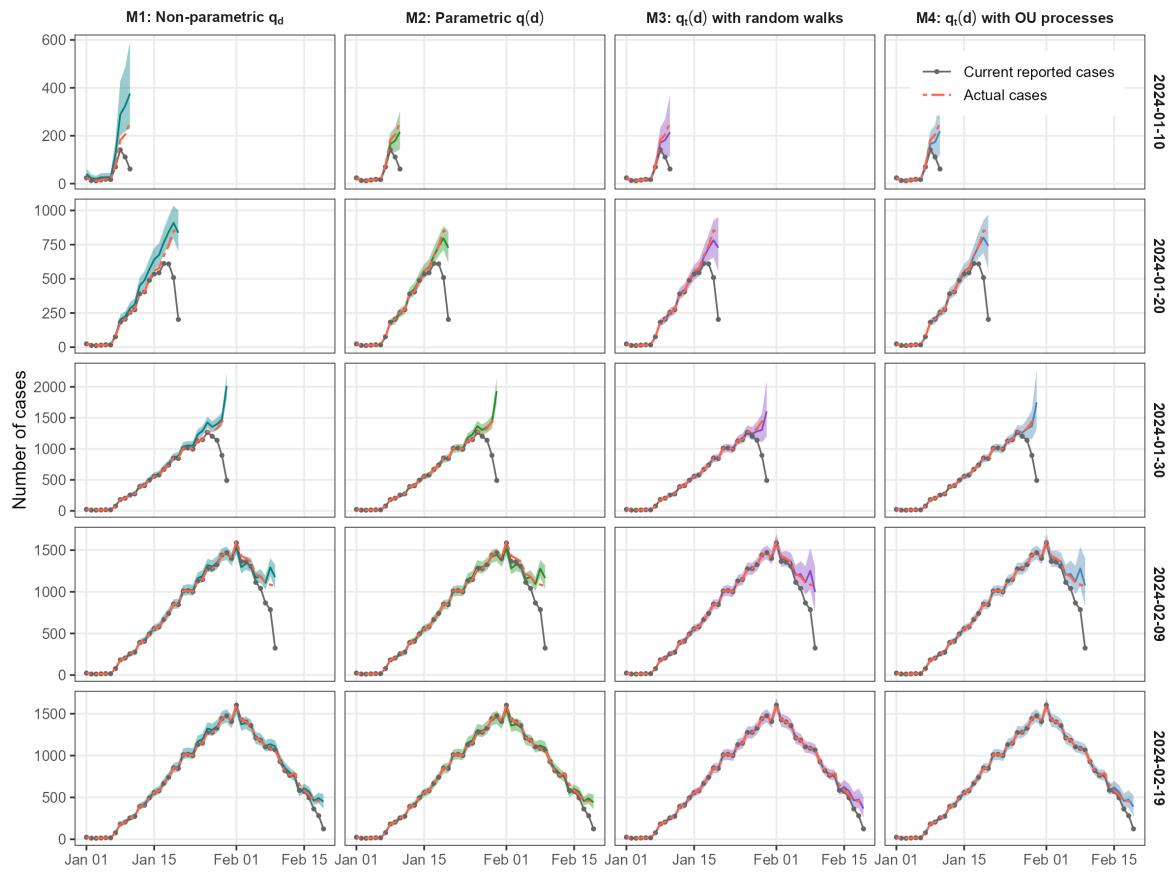
## B.2. Fully reported scenarios



**Web Figure 3:** Comparison of four models for scenario S1, fully reported with constant cumulative reporting probability, at five current time-points.



**Web Figure 4:** Comparison of four models for scenario S2, fully reported with time-varying cumulative reporting probability with random walks, at five current time-points.



**Web Figure 5:** Comparison of four models for scenario S3, fully reported with time-varying cumulative reporting probability with OU processes, at five current time-points.

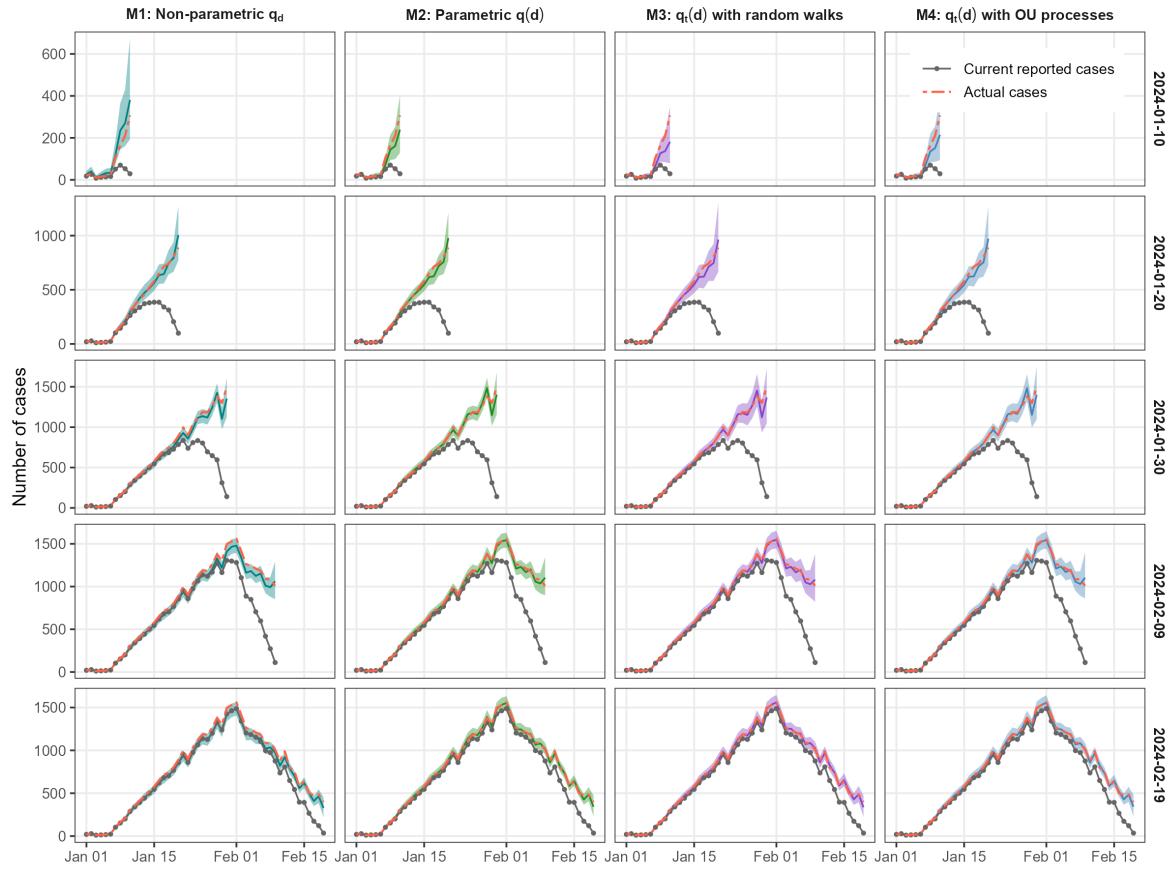
Scenario	Nowcasting time	RMSE	RMSPE	MAE	MAPE	MCIW	MCR	Method
S1	(1) Initial	95.44	69.61	63.34	67.99	147.08	0.54	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>18.46</b>	9.31	<b>9.29</b>	6.93	<b>65.07</b>	0.99	M2: Parametric $q(d)$
	Completed days: 0	18.76	<b>9.25</b>	9.42	<b>6.84</b>	77.72	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	19.53	9.52	9.88	7.07	74.05	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	67.31	12.02	55.40	11.65	147.49	0.82	M1: Non-parametric $q_d$
	Now: $t = 20$	21.79	2.80	10.34	1.68	<b>113.39</b>	<b>1.00</b>	M2: Parametric $q(d)$
	Completed days: 5	22.09	2.82	10.23	1.64	130.86	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>21.54</b>	<b>2.73</b>	<b>9.97</b>	<b>1.59</b>	125.73	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	43.48	3.55	32.99	3.00	176.80	0.99	M1: Non-parametric $q_d$
	Now: $t = 30$	32.14	2.26	15.01	1.17	<b>168.64</b>	<b>1.00</b>	M2: Parametric $q(d)$
	Completed days: 15	32.48	2.28	14.74	1.14	186.70	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>31.86</b>	<b>2.23</b>	<b>14.36</b>	<b>1.10</b>	180.74	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	31.17	2.80	23.51	1.95	179.95	<b>1.00</b>	M1: Non-parametric $q_d$
	Now: $t = 40$	25.79	2.47	13.78	1.23	<b>176.20</b>	<b>1.00</b>	M2: Parametric $q(d)$
	Completed days: 25	25.76	2.48	13.38	1.20	184.85	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>25.32</b>	<b>2.43</b>	<b>13.11</b>	<b>1.17</b>	181.58	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(5) Final	18.53	3.91	12.77	2.12	135.10	<b>1.00</b>	M1: Non-parametric $q_d$
	Now: $t = 50$	16.42	3.75	9.29	1.76	<b>133.65</b>	<b>1.00</b>	M2: Parametric $q(d)$
	Completed days: 35	16.26	<b>3.71</b>	9.02	1.73	136.50	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>16.18</b>	<b>3.71</b>	<b>8.94</b>	<b>1.72</b>	135.16	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
S2	(1) Initial	92.53	69.55	62.61	68.16	132.92	0.51	M1: Non-parametric $q_d$
	Now: $t = 10$	15.33	8.68	8.20	6.77	<b>55.63</b>	0.99	M2: Parametric $q(d)$
	Completed days: 0	<b>15.06</b>	<b>8.51</b>	<b>8.04</b>	<b>6.58</b>	65.61	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	15.48	8.66	8.25	6.72	63.13	0.99	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	83.87	13.86	64.14	12.81	138.17	0.69	M1: Non-parametric $q_d$
	Now: $t = 20$	41.45	5.30	22.12	3.41	<b>103.31</b>	0.93	M2: Parametric $q(d)$
	Completed days: 5	<b>29.48</b>	<b>3.75</b>	<b>15.08</b>	<b>2.33</b>	128.17	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	34.90	4.40	17.79	2.68	125.37	0.98	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	88.03	6.77	62.38	5.29	163.19	0.79	M1: Non-parametric $q_d$
	Now: $t = 30$	76.17	5.60	46.56	3.71	<b>155.04</b>	0.84	M2: Parametric $q(d)$
S3	Completed days: 15	<b>47.78</b>	<b>3.41</b>	<b>24.41</b>	<b>1.88</b>	202.04	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	51.42	3.65	25.92	1.98	203.10	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	86.94	7.87	64.36	5.44	168.22	0.76	M1: Non-parametric $q_d$
	Now: $t = 40$	80.07	7.39	54.93	4.74	<b>164.77</b>	0.80	M2: Parametric $q(d)$
	Completed days: 25	<b>35.80</b>	<b>3.42</b>	<b>20.12</b>	<b>1.81</b>	201.56	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	40.44	3.88	21.98	2.00	202.66	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
	(5) Final	54.21	10.36	42.41	6.90	127.74	0.78	M1: Non-parametric $q_d$
	Now: $t = 50$	51.84	10.12	39.20	6.52	<b>126.25</b>	0.79	M2: Parametric $q(d)$
	Completed days: 35	<b>19.40</b>	<b>4.05</b>	<b>12.23</b>	<b>2.22</b>	142.24	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	22.08	4.72	13.31	2.49	142.64	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
S3	(1) Initial	93.60	69.95	62.15	67.79	131.73	0.54	M1: Non-parametric $q_d$
	Now: $t = 10$	21.51	10.84	10.92	7.92	<b>54.61</b>	0.97	M2: Parametric $q(d)$
	Completed days: 0	<b>20.84</b>	<b>10.49</b>	<b>10.50</b>	<b>7.62</b>	65.26	<b>0.98</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	21.14	10.68	10.71	7.74	63.57	<b>0.98</b>	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	89.49	14.36	64.68	12.74	136.14	0.68	M1: Non-parametric $q_d$
	Now: $t = 20$	55.06	7.17	29.54	4.68	<b>102.53</b>	0.90	M2: Parametric $q(d)$
	Completed days: 5	<b>43.79</b>	<b>5.53</b>	<b>22.04</b>	<b>3.32</b>	147.07	<b>0.98</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	44.77	5.64	22.24	<b>3.32</b>	143.83	<b>0.98</b>	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	112.40	8.49	73.74	6.15	162.61	0.75	M1: Non-parametric $q_d$
	Now: $t = 30$	104.63	7.71	63.14	5.08	<b>154.46</b>	0.78	M2: Parametric $q(d)$
S3	Completed days: 15	77.21	5.42	37.53	2.83	254.97	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>74.97</b>	<b>5.26</b>	<b>35.99</b>	<b>2.70</b>	245.53	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	91.36	8.08	68.41	5.70	169.21	0.71	M1: Non-parametric $q_d$
	Now: $t = 40$	84.01	7.56	59.12	5.00	<b>165.11</b>	0.78	M2: Parametric $q(d)$
	Completed days: 25	<b>49.81</b>	<b>4.73</b>	<b>26.28</b>	<b>2.37</b>	244.44	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
(5) Final	Incomplete days: 15	51.13	4.89	26.42	2.40	234.56	0.99	M4: Time-varying $q_t(d)$ - OU
	Now: $t = 50$	44.15	8.13	33.93	5.43	<b>126.00</b>	0.84	M1: Non-parametric $q_d$
	Completed days: 35	27.17	5.98	15.54	2.99	161.35	<b>0.99</b>	M2: Parametric $q(d)$
	Incomplete days: 15	<b>26.83</b>	<b>5.91</b>	<b>15.22</b>	<b>2.96</b>	155.67	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
								M4: Time-varying $q_t(d)$ - OU

**Web Table 2:** Nowcasting performance based on 100 simulations under fully reported scenarios: time-invariant (S1), time-varying with random walks (S2), and time-varying with OU processes (S3) cumulative reporting probabilities.

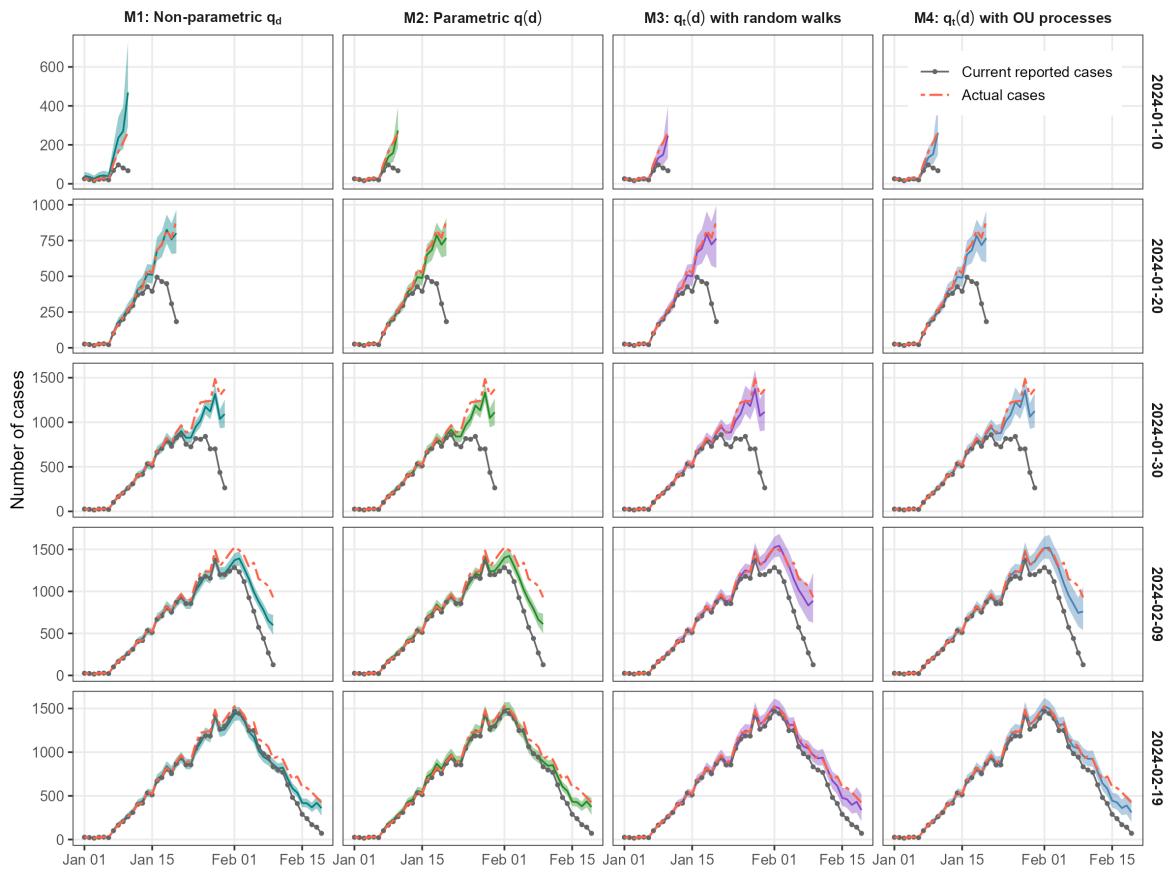
Scenario	Nowcasting time	RMSE	RMSPE	MAE	MAPE	MCIW	MCR	Method
S1	(1) Initial	134.10	71.47	113.19	69.59	260.23	0.29	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>26.07</b>	11.47	<b>17.73</b>	<b>9.26</b>	<b>111.50</b>	<b>0.99</b>	M2: parametric $q(d)$
	Completed days: 0	26.50	<b>11.45</b>	18.04	9.27	136.70	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	27.59	11.94	18.94	9.73	129.33	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	98.66	12.65	91.91	11.97	257.75	0.70	M1: Non-parametric $q_d$
	Now: $t = 20$	37.15	4.42	25.51	3.14	<b>199.25</b>	0.99	M2: parametric $q(d)$
	Completed days: 5	37.76	4.48	25.77	3.17	250.02	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>36.86</b>	<b>4.37</b>	<b>25.16</b>	<b>3.09</b>	234.37	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	63.61	4.51	48.94	3.56	265.22	0.98	M1: Non-parametric $q_d$
	Now: $t = 30$	54.75	3.80	35.96	2.55	<b>254.35</b>	0.99	M2: parametric $q(d)$
	Completed days: 15	55.51	3.84	36.19	2.56	305.68	0.99	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>54.48</b>	<b>3.77</b>	<b>35.35</b>	<b>2.50</b>	287.54	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	44.54	4.29	32.70	3.07	224.40	0.99	M1: Non-parametric $q_d$
	Now: $t = 40$	43.08	4.19	30.05	2.85	<b>220.72</b>	0.99	M2: parametric $q(d)$
	Completed days: 25	43.17	4.21	29.85	2.84	243.80	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>42.56</b>	<b>4.14</b>	<b>29.47</b>	<b>2.80</b>	234.38	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(5) Final	27.48	6.49	19.53	4.37	146.78	<b>0.99</b>	M1: Non-parametric $q_d$
	Now: $t = 50$	26.97	6.39	19.15	4.29	<b>145.32</b>	<b>0.99</b>	M2: parametric $q(d)$
	Completed days: 35	26.87	<b>6.33</b>	19.09	4.26	152.53	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>26.77</b>	6.34	<b>18.97</b>	<b>4.25</b>	148.85	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
S2	(1) Initial	130.03	71.44	112.00	70.08	232.36	0.23	M1: Non-parametric $q_d$
	Now: $t = 10$	21.62	10.09	15.40	8.48	<b>92.76</b>	0.98	M2: parametric $q(d)$
	Completed days: 0	<b>21.24</b>	<b>9.94</b>	<b>15.14</b>	<b>8.31</b>	112.56	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	21.83	10.12	15.54	8.47	107.61	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	127.13	15.99	114.79	14.74	229.08	0.50	M1: Non-parametric $q_d$
	Now: $t = 20$	70.60	8.58	55.96	6.99	<b>168.95</b>	0.81	M2: parametric $q(d)$
	Completed days: 5	<b>50.36</b>	<b>6.08</b>	<b>38.56</b>	<b>4.79</b>	240.62	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	59.82	7.26	46.38	5.77	231.54	0.95	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	138.15	9.90	116.67	8.52	224.31	0.57	M1: Non-parametric $q_d$
	Now: $t = 30$	125.92	8.99	103.93	7.57	<b>213.61</b>	0.60	M2: parametric $q(d)$
S3	Completed days: 15	<b>81.56</b>	<b>5.75</b>	<b>61.42</b>	<b>4.42</b>	345.93	<b>0.97</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	88.04	6.20	66.47	4.78	347.25	<b>0.97</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	129.99	12.38	110.81	10.32	189.90	0.53	M1: Non-parametric $q_d$
	Now: $t = 40$	125.67	12.02	105.87	9.90	<b>186.78</b>	0.55	M2: parametric $q(d)$
	Completed days: 25	<b>59.94</b>	<b>5.80</b>	<b>46.31</b>	<b>4.39</b>	286.33	<b>0.97</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	68.56	6.64	53.33	5.06	288.13	0.96	M4: Time-varying $q_t(d)$ - OU
	(5) Final	76.16	16.62	65.04	13.64	125.80	0.58	M1: Non-parametric $q_d$
	Now: $t = 50$	75.01	16.40	63.86	13.40	<b>124.43</b>	0.57	M2: parametric $q(d)$
	Completed days: 35	<b>31.30</b>	<b>6.84</b>	<b>25.14</b>	<b>5.32</b>	163.66	<b>0.98</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	36.45	8.03	29.22	6.21	163.77	<b>0.98</b>	M4: Time-varying $q_t(d)$ - OU
S3	(1) Initial	131.52	71.64	111.05	69.02	230.11	0.28	M1: Non-parametric $q_d$
	Now: $t = 10$	30.38	13.39	20.85	10.70	<b>90.81</b>	0.94	M2: parametric $q(d)$
	Completed days: 0	<b>29.43</b>	<b>12.95</b>	<b>20.07</b>	<b>10.34</b>	112.00	<b>0.97</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	29.86	13.20	20.48	10.53	108.54	0.96	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	138.46	<b>17.47</b>	119.92	15.44	224.26	0.48	M1: Non-parametric $q_d$
	Now: $t = 20$	93.30	11.35	72.15	9.04	<b>166.75</b>	0.71	M2: parametric $q(d)$
	Completed days: 5	<b>75.00</b>	<b>9.06</b>	<b>56.91</b>	<b>7.06</b>	295.39	<b>0.95</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	76.81	9.30	58.23	7.23	284.53	0.94	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	180.46	12.87	148.10	10.76	224.08	0.48	M1: Non-parametric $q_d$
	Now: $t = 30$	171.61	12.21	138.76	10.06	<b>212.91</b>	0.51	M2: parametric $q(d)$
S3	Completed days: 15	132.25	9.20	96.35	6.85	497.51	0.96	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>128.62</b>	<b>8.96</b>	<b>94.33</b>	<b>6.72</b>	466.87	<b>0.97</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	130.30	12.30	109.37	10.14	191.11	0.50	M1: Non-parametric $q_d$
	Now: $t = 40$	125.40	11.86	104.16	9.67	<b>187.03</b>	0.53	M2: parametric $q(d)$
	Completed days: 25	<b>84.01</b>	<b>8.06</b>	<b>63.38</b>	<b>5.96</b>	406.40	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
(5) Final	Completed days: 35	45.25	10.21	<b>35.17</b>	<b>7.61</b>	375.10	0.97	M4: Time-varying $q_t(d)$ - OU
	Incomplete days: 15	<b>44.98</b>	<b>10.12</b>	35.52	7.68	198.03	0.96	M1: Non-parametric $q_d$
	Now: $t = 50$	59.33	12.77	50.00	10.45	<b>123.17</b>	0.68	M2: parametric $q(d)$
	Completed days: 35	45.25	10.21	<b>35.17</b>	<b>7.61</b>	216.14	<b>0.98</b>	M3: Time-varying $q_t(d)$ - RW

**Web Table 3:** Nowcasting performance, for the last 5 time units, based on 100 simulations under fully reported scenarios: time-invariant (S1), time-varying with random walks (S2), and time-varying with OU processes (S3) cumulative reporting probabilities.

### B.3. Non-fully reported scenarios



**Web Figure 6:** Comparison of four models for scenario S4, non-fully reported with constant cumulative reporting probability, at five current time-points.



**Web Figure 7:** Comparison of four models for scenario S5, non-fully reported with time-varying cumulative reporting probability with random walks, at five current time-points.

Scenario	Nowcasting time	RMSE	RMSPE	MAE	MAPE	MCIW	MCR	Method
S4	(1) Initial	43.91	34.61	27.01	29.79	124.02	<b>0.97</b>	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>38.27</b>	<b>25.95</b>	<b>23.80</b>	<b>23.09</b>	<b>70.66</b>	0.87	M2: Parametric $q(d)$
	Completed days: 0	44.28	28.08	27.69	25.27	78.99	0.89	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	42.59	27.78	26.72	25.03	74.35	0.87	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	<b>33.20</b>	<b>5.33</b>	<b>20.75</b>	<b>4.10</b>	151.80	0.99	M1: Non-parametric $q_d$
	Now: $t = 20$	35.30	5.59	22.43	4.35	<b>132.74</b>	0.98	M2: Parametric $q(d)$
	Completed days: 5	39.50	5.87	24.35	4.47	183.56	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	36.54	5.55	22.56	4.23	164.77	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	60.21	5.12	48.67	4.46	<b>183.84</b>	0.93	M1: Non-parametric $q_d$
	Now: $t = 30$	41.28	3.21	27.35	2.33	185.10	0.99	M2: Parametric $q(d)$
	Completed days: 15	46.17	3.52	29.88	2.50	248.86	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>40.85</b>	<b>3.13</b>	<b>26.43</b>	<b>2.22</b>	224.03	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	70.47	5.73	63.53	5.03	<b>185.52</b>	0.82	M1: Non-parametric $q_d$
	Now: $t = 40$	40.75	3.66	28.37	2.39	189.36	0.98	M2: Parametric $q(d)$
	Completed days: 25	40.51	3.63	28.16	2.37	229.11	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>39.20</b>	<b>3.54</b>	<b>26.56</b>	<b>2.25</b>	214.88	0.99	M4: Time-varying $q_t(d)$ - OU
	(5) Final	45.16	6.47	40.09	5.37	<b>137.67</b>	0.92	M1: Non-parametric $q_d$
	Now: $t = 50$	25.31	4.71	19.03	3.03	141.35	<b>0.99</b>	M2: Parametric $q(d)$
	Completed days: 35	25.03	4.66	18.80	3.01	156.26	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>24.13</b>	<b>4.60</b>	<b>17.82</b>	<b>2.90</b>	150.86	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
S5	(1) Initial	52.01	38.85	32.08	34.17	117.35	<b>0.91</b>	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>38.67</b>	<b>26.96</b>	<b>24.54</b>	<b>24.29</b>	<b>62.73</b>	0.80	M2: Parametric $q(d)$
	Completed days: 0	41.21	27.95	26.35	25.36	71.01	0.80	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	40.20	27.84	25.76	25.26	67.75	0.79	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	63.97	10.57	44.44	8.81	141.89	0.86	M1: Non-parametric $q_d$
	Now: $t = 20$	64.41	10.00	43.07	8.10	<b>124.69</b>	0.82	M2: Parametric $q(d)$
	Completed days: 5	<b>58.13</b>	<b>8.69</b>	<b>37.68</b>	<b>6.79</b>	196.10	<b>0.97</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	60.82	9.08	39.64	7.16	178.81	0.94	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	140.78	11.36	104.68	9.10	<b>172.05</b>	0.54	M1: Non-parametric $q_d$
	Now: $t = 30$	145.59	11.35	102.66	8.65	173.81	0.62	M2: Parametric $q(d)$
S6	Completed days: 15	<b>97.82</b>	<b>7.33</b>	<b>62.49</b>	<b>5.07</b>	322.63	<b>0.96</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	109.49	8.17	69.81	5.63	307.94	0.92	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	153.75	12.85	130.69	10.56	<b>174.45</b>	0.42	M1: Non-parametric $q_d$
	Now: $t = 40$	152.47	12.89	125.91	10.26	177.53	0.47	M2: Parametric $q(d)$
	Completed days: 25	<b>79.36</b>	<b>7.02</b>	<b>57.44</b>	<b>4.85</b>	315.47	<b>0.95</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	85.71	7.58	61.42	5.20	314.01	0.93	M4: Time-varying $q_t(d)$ - OU
	(5) Final	98.77	15.01	86.75	12.26	<b>129.91</b>	0.46	M1: Non-parametric $q_d$
	Now: $t = 50$	97.04	15.16	84.42	12.16	132.56	0.47	M2: Parametric $q(d)$
	Completed days: 35	<b>47.19</b>	<b>8.79</b>	<b>36.15</b>	<b>5.97</b>	203.02	<b>0.96</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	49.62	9.33	37.67	6.28	200.21	0.94	M4: Time-varying $q_t(d)$ - OU
S6	(1) Initial	47.19	36.79	29.33	31.92	115.61	<b>0.94</b>	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>39.88</b>	<b>26.31</b>	<b>24.69</b>	<b>23.42</b>	<b>63.16</b>	0.85	M2: Parametric $q(d)$
	Completed days: 0	43.89	27.84	27.38	24.96	70.61	0.85	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	42.52	27.61	26.56	24.77	67.90	0.84	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	72.16	11.41	47.92	9.08	143.59	0.83	M1: Non-parametric $q_d$
	Now: $t = 20$	72.59	11.21	46.96	8.73	<b>124.15</b>	0.78	M2: Parametric $q(d)$
	Completed days: 5	67.34	<b>10.02</b>	<b>43.14</b>	<b>7.67</b>	205.58	<b>0.95</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>67.26</b>	10.10	43.31	7.74	188.52	0.93	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	137.57	11.43	103.69	9.21	172.30	0.53	M1: Non-parametric $q_d$
	Now: $t = 30$	133.39	10.80	97.34	8.46	<b>172.02</b>	0.59	M2: Parametric $q(d)$
S6	Completed days: 15	<b>110.68</b>	<b>8.39</b>	71.73	5.87	361.90	<b>0.95</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	110.94	8.41	<b>71.25</b>	<b>5.83</b>	334.56	0.94	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	143.72	11.64	118.39	9.42	<b>175.66</b>	0.48	M1: Non-parametric $q_d$
	Now: $t = 40$	140.93	11.58	113.99	9.15	178.49	0.50	M2: Parametric $q(d)$
	Completed days: 25	90.06	7.95	63.76	5.36	363.99	<b>0.97</b>	M3: Time-varying $q_t(d)$ - RW
(5) Final	Incomplete days: 15	<b>86.26</b>	<b>7.56</b>	<b>61.78</b>	<b>5.18</b>	341.77	<b>0.97</b>	M4: Time-varying $q_t(d)$ - OU
	Now: $t = 50$	82.06	12.00	68.99	9.56	134.08	0.56	M1: Non-parametric $q_d$
	Completed days: 35	49.38	9.02	37.55	6.11	226.83	<b>0.97</b>	M2: Parametric $q(d)$
	Incomplete days: 15	<b>47.32</b>	<b>8.62</b>	<b>35.79</b>	<b>5.78</b>	216.21	<b>0.97</b>	M3: Time-varying $q_t(d)$ - RW

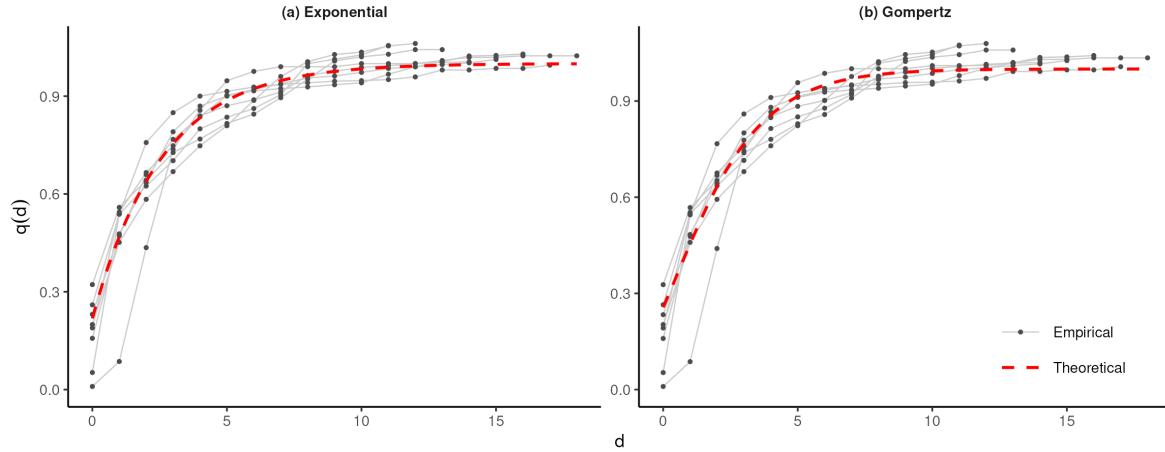
**Web Table 4:** Nowcasting performance based on 100 simulations under non-fully reported scenarios: time-invariant (S4), time-varying with random walks (S5), and time-varying with OU processes (S6) cumulative reporting probabilities.

Scenario	Nowcasting time	RMSE	RMSPE	MAE	MAPE	MCIW	MCR	Method
S4	(1) Initial	61.55	33.42	48.18	29.76	219.12	<b>0.95</b>	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>53.89</b>	<b>27.84</b>	<b>43.64</b>	<b>25.93</b>	<b>122.65</b>	0.77	M2: parametric $q(d)$
	Completed days: 0	62.42	31.46	51.34	29.81	139.07	0.80	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	60.01	30.64	49.34	28.98	130.00	0.76	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	<b>53.03</b>	<b>6.60</b>	<b>41.70</b>	<b>5.31</b>	266.23	0.99	M1: Non-parametric $q_d$
	Now: $t = 20$	56.65	7.08	45.56	5.83	<b>236.97</b>	0.95	M2: parametric $q(d)$
	Completed days: 5	64.90	8.09	53.00	6.77	364.23	<b>1.00</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	59.56	7.43	47.87	6.11	308.92	0.99	M4: Time-varying $q_t(d)$ - OU
	3) Peak	84.94	6.16	72.26	5.30	<b>280.55</b>	0.89	M1: Non-parametric $q_d$
	Now: $t = 30$	<b>64.40</b>	<b>4.61</b>	<b>50.73</b>	<b>3.69</b>	285.26	0.97	M2: parametric $q(d)$
S5	Completed days: 15	73.80	5.29	59.46	4.33	430.27	0.99	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	64.74	4.64	51.35	3.73	361.64	<b>1.00</b>	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	75.66	7.01	64.60	5.92	<b>236.48</b>	0.86	M1: Non-parametric $q_d$
	Now: $t = 40$	61.34	5.79	48.87	4.53	244.05	0.96	M2: parametric $q(d)$
	Completed days: 25	61.43	5.77	49.29	4.55	316.68	<b>0.99</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>60.22</b>	<b>5.68</b>	<b>47.95</b>	<b>4.44</b>	280.89	0.98	M4: Time-varying $q_t(d)$ - OU
	(5) Final	39.56	8.46	33.35	6.93	<b>154.12</b>	0.95	M1: Non-parametric $q_d$
	Now: $t = 50$	34.09	7.46	27.33	5.78	159.40	0.98	M2: parametric $q(d)$
	Completed days: 35	33.68	<b>7.35</b>	27.13	5.73	181.21	0.98	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>33.47</b>	<b>7.35</b>	<b>26.87</b>	<b>5.69</b>	170.09	<b>0.99</b>	M4: Time-varying $q_t(d)$ - OU
S5	(1) Initial	72.92	37.34	57.31	33.82	205.57	<b>0.85</b>	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>54.42</b>	<b>29.03</b>	<b>44.87</b>	<b>27.31</b>	<b>107.30</b>	0.67	M2: parametric $q(d)$
	Completed days: 0	58.01	30.61	48.44	29.22	123.72	0.69	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	56.58	30.20	47.20	28.74	117.25	0.67	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	98.04	12.32	83.66	10.73	239.76	0.76	M1: Non-parametric $q_d$
	Now: $t = 20$	102.17	12.80	87.61	11.22	<b>212.40</b>	0.67	M2: parametric $q(d)$
	Completed days: 5	<b>94.09</b>	<b>11.77</b>	<b>81.12</b>	<b>10.37</b>	390.15	<b>0.94</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	97.97	12.29	84.58	10.83	339.66	0.89	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	206.93	14.99	180.10	13.22	<b>246.81</b>	0.41	M1: Non-parametric $q_d$
	Now: $t = 30$	223.11	16.16	193.50	14.20	250.89	0.40	M2: parametric $q(d)$
S6	Completed days: 15	<b>158.86</b>	<b>11.40</b>	<b>135.84</b>	<b>9.90</b>	596.44	<b>0.89</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	178.24	12.81	153.05	11.18	545.12	0.82	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	186.87	17.27	169.63	15.48	<b>206.83</b>	0.37	M1: Non-parametric $q_d$
	Now: $t = 40$	197.33	18.20	180.08	16.40	212.07	0.36	M2: parametric $q(d)$
	Completed days: 25	<b>118.18</b>	<b>10.95</b>	<b>105.04</b>	<b>9.62</b>	482.45	<b>0.89</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	128.88	11.90	115.40	10.53	462.71	0.83	M4: Time-varying $q_t(d)$ - OU
	(5) Final	97.88	20.42	88.87	18.15	<b>133.35</b>	0.44	M1: Non-parametric $q_d$
	Now: $t = 50$	100.65	20.99	91.05	18.60	136.89	0.44	M2: parametric $q(d)$
	Completed days: 35	<b>67.19</b>	<b>14.07</b>	<b>60.22</b>	<b>12.36</b>	250.31	<b>0.90</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	71.63	15.00	64.04	13.14	235.49	0.85	M4: Time-varying $q_t(d)$ - OU
S6	(1) Initial	66.15	35.36	52.28	31.63	202.21	<b>0.89</b>	M1: Non-parametric $q_d$
	Now: $t = 10$	<b>56.16</b>	<b>28.32</b>	<b>45.45</b>	<b>26.50</b>	<b>107.97</b>	0.73	M2: parametric $q(d)$
	Completed days: 0	61.84	30.90	50.75	29.32	122.69	0.74	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 10	59.89	30.20	49.04	28.56	117.41	0.72	M4: Time-varying $q_t(d)$ - OU
	(2) Increasing	111.94	13.97	93.51	11.92	243.34	0.72	M1: Non-parametric $q_d$
	Now: $t = 20$	114.83	14.27	95.25	12.10	<b>211.92</b>	0.63	M2: parametric $q(d)$
	Completed days: 5	108.51	13.59	93.45	11.92	417.21	<b>0.90</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>108.02</b>	<b>13.52</b>	<b>92.56</b>	<b>11.81</b>	365.64	0.87	M4: Time-varying $q_t(d)$ - OU
	(3) Peak	193.25	14.09	165.66	12.20	<b>246.98</b>	0.45	M1: Non-parametric $q_d$
	Now: $t = 30$	195.20	14.26	168.05	12.39	248.55	0.44	M2: parametric $q(d)$
S6	Completed days: 15	<b>177.30</b>	<b>12.82</b>	152.27	11.16	694.70	<b>0.90</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	178.49	12.95	<b>151.63</b>	<b>11.13</b>	604.39	0.88	M4: Time-varying $q_t(d)$ - OU
	(4) Decreasing	154.24	14.19	133.50	12.18	<b>210.21</b>	0.48	M1: Non-parametric $q_d$
	Now: $t = 40$	164.55	15.10	142.71	12.99	215.54	0.45	M2: parametric $q(d)$
	Completed days: 25	131.95	12.31	112.49	10.35	586.80	<b>0.95</b>	M3: Time-varying $q_t(d)$ - RW
S6	Incomplete days: 15	<b>126.19</b>	<b>11.68</b>	<b>108.46</b>	<b>9.93</b>	511.04	0.93	M4: Time-varying $q_t(d)$ - OU
	(5) Final	73.68	15.33	64.12	13.06	<b>136.52</b>	0.61	M1: Non-parametric $q_d$
	Now: $t = 50$	75.52	15.69	65.34	13.31	140.35	0.60	M2: parametric $q(d)$
	Completed days: 35	67.00	14.14	59.16	12.16	294.88	<b>0.94</b>	M3: Time-varying $q_t(d)$ - RW
	Incomplete days: 15	<b>63.94</b>	<b>13.47</b>	<b>54.84</b>	<b>11.27</b>	257.92	<b>0.94</b>	M4: Time-varying $q_t(d)$ - OU

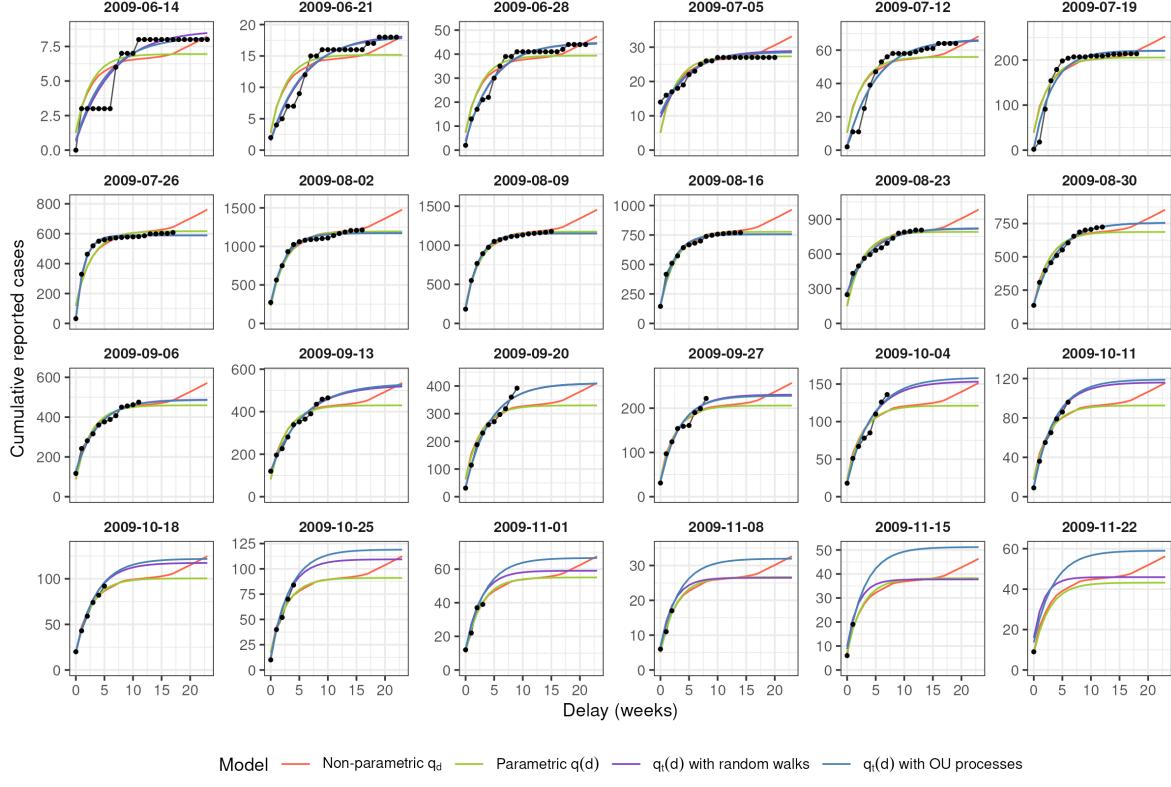
**Web Table 5:** Nowcasting performance, for the last 5 time units, based on 100 simulations under non-fully reported scenarios: time-invariant (S4), time-varying with random walks (S5), and time-varying with OU processes (S6) cumulative reporting probabilities.

## Web Appendix C. Case studies

### C.1. Weekly severe acute respiratory infections in Paraná, Brazil (2009)



**Web Figure 8:** Empirical and fitted cumulative reporting probabilities  $q(d)$  for SARI infections from the week of 14 June to 22 November 2009. Theoretical exponential (a) and Gompertz (b) curves assume time-invariant reporting probabilities. Empirical curves are computed based on the estimated actual number of cases for each week.

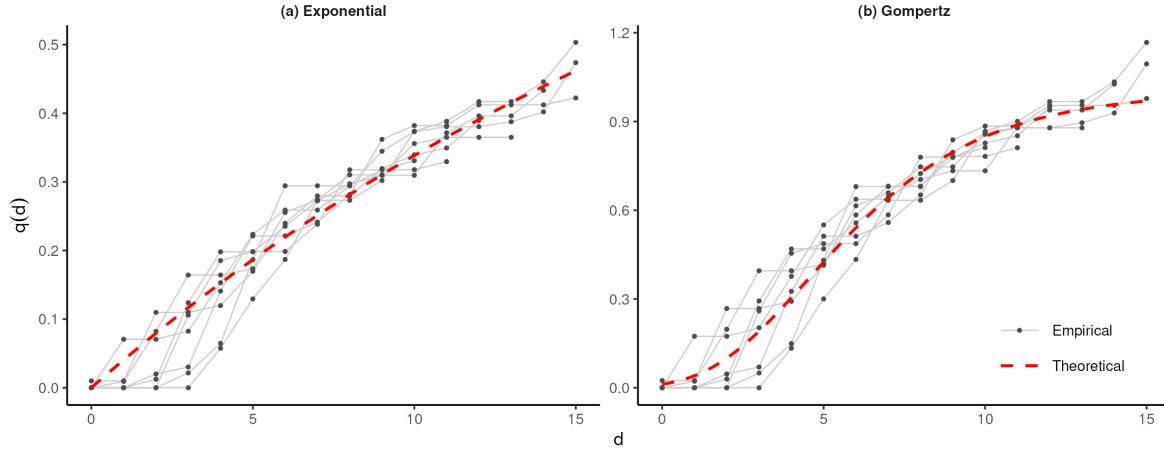


**Web Figure 9:** Cumulative reported SARI cases from 14 June to 22 November 2009, with reporting delays up to 26 weeks, along with theoretical curves from 4 fitted models.

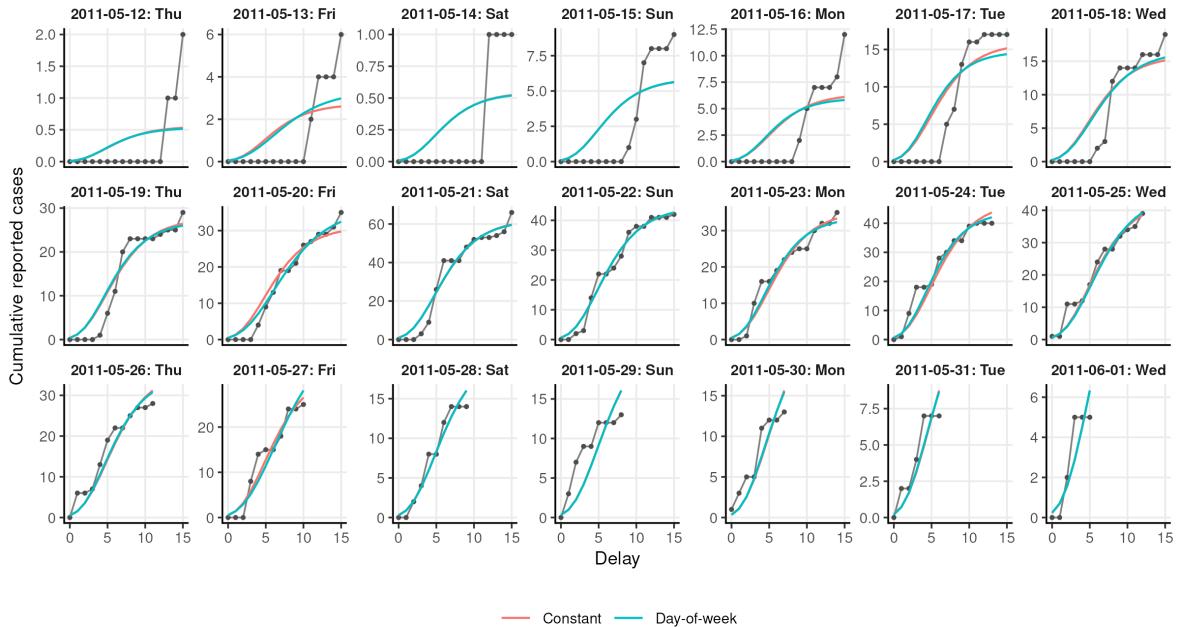
Time (in weeks)	RMSE	RMSPE	MAE	MAPE	MCIW	MCR	Method
(1) Initial	4560.45	361.59	1829.28	207.03	2713.42	0.62	M1: Non-parametric $q_d$
Now: $t = 8$	889.51	80.14	344.79	60.08	519.13	0.38	M2: Parametric $q(d)$
Completed weeks: 0	486.13	53.95	235.90	48.39	1833.53	0.88	M3: Time-varying $q_t(d)$ - RW
Incomplete weeks: 8	127.67	35.71	72.70	32.69	853.53	0.88	M4: Time-varying $q_t(d)$ - OU
(2) Increasing	1319.30	149.93	941.10	136.79	1114.28	0.17	M1: Non-parametric $q_d$
Now: $t = 12$	171.05	23.42	105.06	19.36	204.42	0.75	M2: Parametric $q(d)$
Completed weeks: 0	220.10	67.51	149.85	52.65	348.43	0.83	M3: Time-varying $q_t(d)$ - RW
Incomplete weeks: 12	186.25	48.41	126.34	36.53	418.68	0.92	M4: Time-varying $q_t(d)$ - OU
(3) Peak	428.75	54.42	298.41	50.37	446.88	0.38	M1: Non-parametric $q_d$
Now: $t = 16$	134.82	26.89	100.80	22.75	84.75	0.38	M2: Parametric $q(d)$
Completed weeks: 0	179.89	31.39	137.34	25.17	138.63	0.44	M3: Time-varying $q_t(d)$ - RW
Incomplete weeks: 16	168.66	28.51	129.71	22.39	155.94	0.44	M4: Time-varying $q_t(d)$ - OU
(4) Decreasing	200.71	29.24	131.22	26.49	245.55	0.55	M1: Non-parametric $q_d$
Now: $t = 20$	114.84	31.42	91.72	26.45	70.60	0.35	M2: Parametric $q(d)$
Completed weeks: 0	107.32	30.92	86.29	22.64	101.95	0.40	M3: Time-varying $q_t(d)$ - RW
Incomplete weeks: 20	103.57	27.72	82.66	21.06	108.40	0.40	M4: Time-varying $q_t(d)$ - OU
(5) Final	102.62	16.99	64.85	15.39	148.96	0.67	M1: Non-parametric $q_d$
Now: $t = 24$	82.13	24.02	61.15	20.71	64.17	0.46	M2: Parametric $q(d)$
Completed weeks: 0	66.74	17.45	46.02	13.80	93.79	0.62	M3: Time-varying $q_t(d)$ - RW
Incomplete weeks: 24	66.45	15.55	45.02	12.44	94.08	0.67	M4: Time-varying $q_t(d)$ - OU

**Web Table 6:** Comparison metrics of SARI nowcasting for four models at five current weekly time-points (2 August, 30 August, 27 September, 25 October, and 22 November).

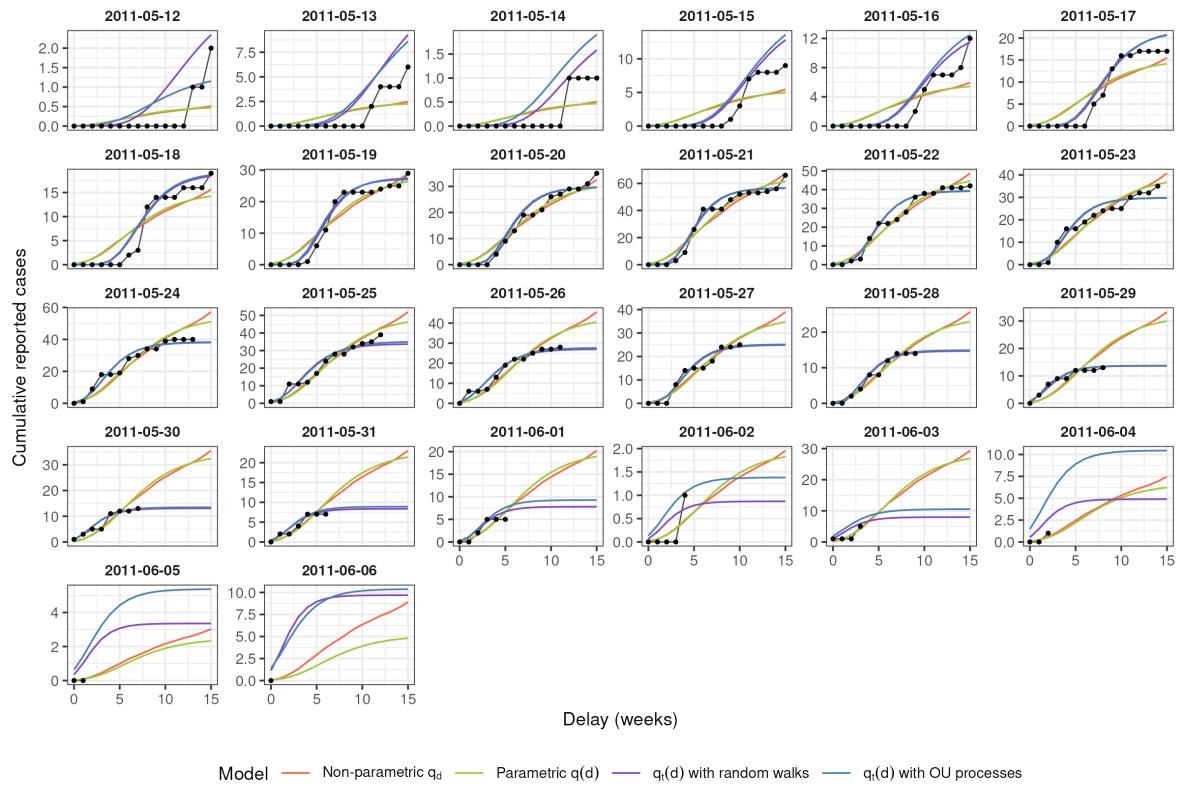
## C.2. Daily hospitalizations for the hemolytic-uremic syndrome (HUS) in Germany (2011)



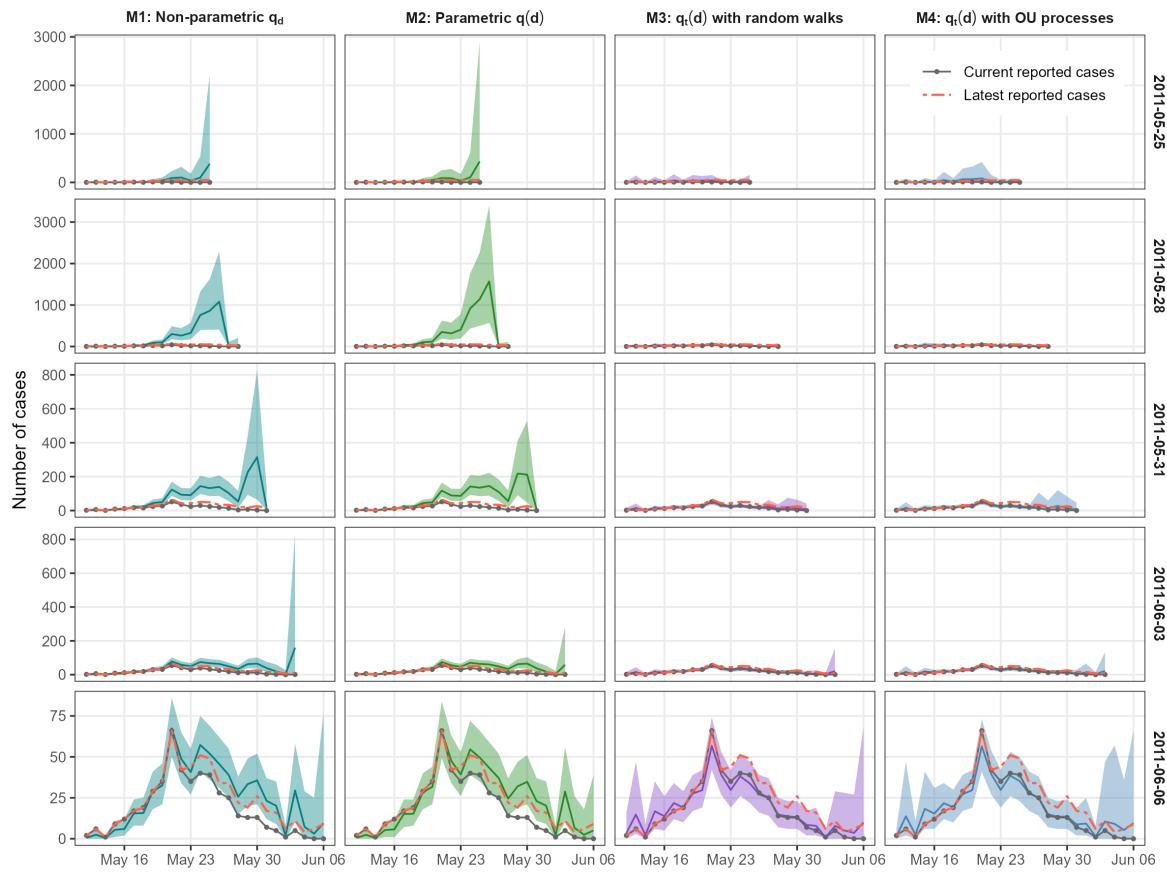
**Web Figure 10:** Empirical and fitted cumulative reporting probabilities  $q(d)$  for HUS-O104 hospitalizations from 20 May to 27 May 2011. Theoretical exponential (a) and Gompertz (b) curves assume time-invariant reporting probabilities. Empirical curves are computed based on the estimated actual number of cases for each day.



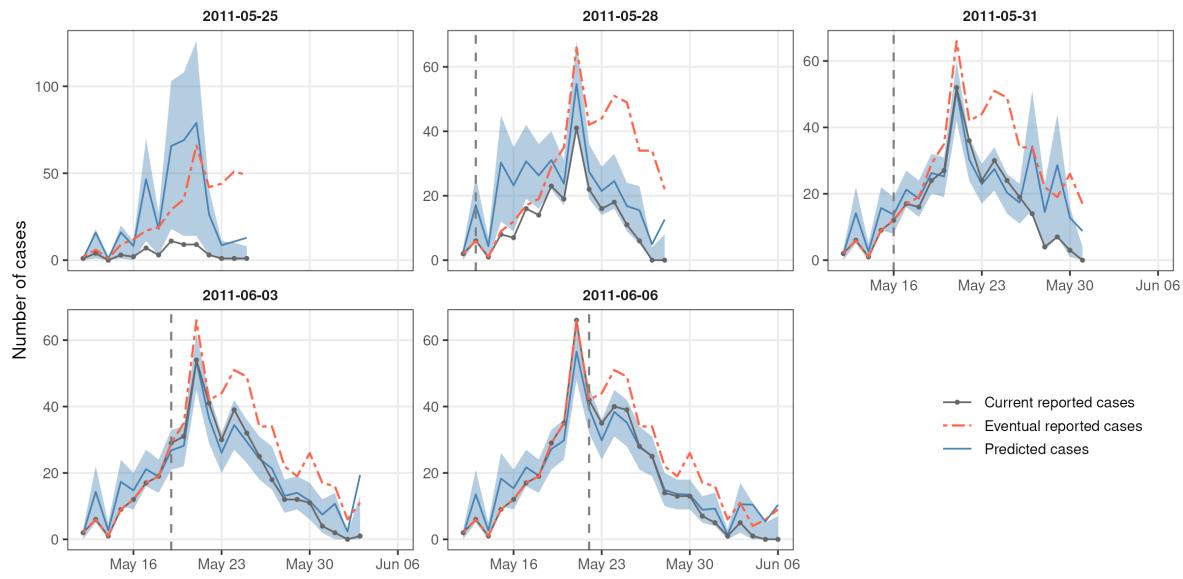
**Web Figure 11:** Cumulative reported HUS-O104 hospitalizations between May 12 - June 1 2011, with delays up to 15 days, along with fitted values for models with time-invariant (constant) and day-of-week varying cumulative reporting probability.



**Web Figure 12:** Cumulative reported HUS-O104 hospitalizations from 12 May to 6 June 2011, with reporting delays up to 15 days, along with theoretical curves from 4 fitted models.



**Web Figure 13:** Comparison of four models for nowcasting HUS-O104 hospitalizations at five current time-points (25 May, 28 May, 31 May, 3 June, and 6 June).



**Web Figure 14:** Nowcasting of daily HUS-O104 hospitalizations at 70% credibility using model M4, which employs a Gompertz cumulative reporting probability with OU processes, at five time points (25 May, 28 May, 31 May, 3 June, and 6 June).

## Web Appendix D. Extension of parametric cumulative reporting probabilities

In some applications, the cumulative reporting mechanism may exhibit more complex patterns than those captured by a single parametric function. A straightforward extension is to define nested functions that account for different rates of convergence. For example, a nested exponential reporting probability can be defined as

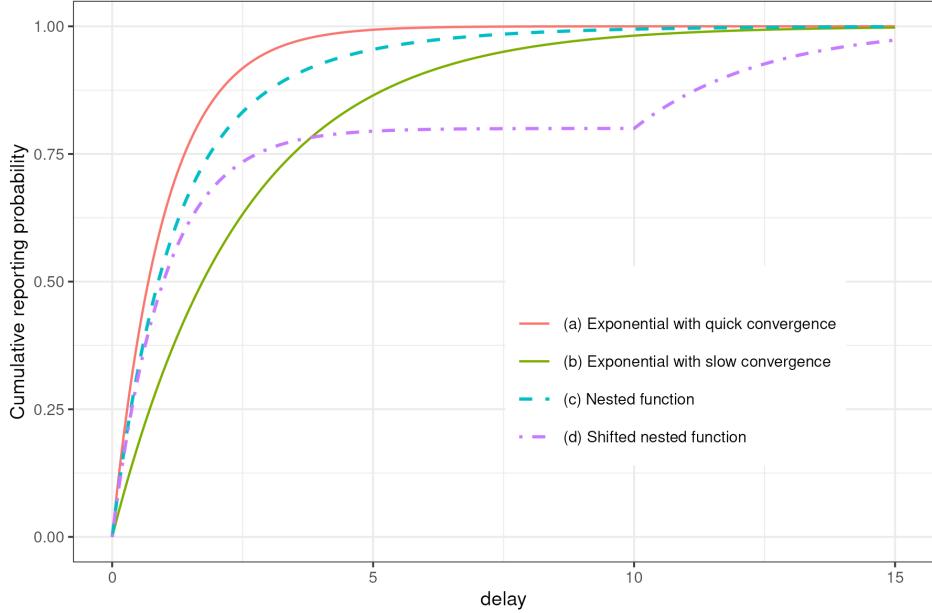
$$q(d) = 1 - \alpha \exp(-b_1 d) - (1 - \alpha) \exp(-b_2 d), \quad (1)$$

where  $\alpha \in (0, 1)$  defines the relative contribution of each component, and  $b_1$  and  $b_2$  are the rate parameters of the nested functions. Figure 15 illustrates the shape of this function, which lies between the two nested components; however, it is not drastically different from the original single-rate functions.

A more flexible extension is to introduce a shifted nested function that captures a change in the reporting pattern starting at a delay  $d_0$ :

$$q(d) = 1 - \alpha \exp(-b_1 d) - (1 - \alpha) \exp(-b_2 \max(0, d - d_0)), \quad (2)$$

where  $\alpha \in (0, 1)$  defines the contribution of the first component, and  $b_1$  and  $b_2$  are the respective rate parameters. Figure 15 shows that this function can capture scenarios where reporting initially appears to converge, but additional cases continue to be reported after a certain delay  $d_0$ .



**Web Figure 15:** Exponential cumulative reporting probabilities with fast (a) and slow (b) convergence, along with (c) nested cumulative reporting probabilities and (d) shifted nested probabilities starting at delay  $d_0 = 10$ .

Although this second extension is promising, it should only be applied when the data provide evidence of such behavior. Moreover, it increases model complexity by introducing additional parameters, including  $d_0$ . We consider extending these parameters to vary over time as future work. Similar approaches could involve additional nested functions or alternative parametric forms to capture more complex reporting mechanisms.