Distributional Semantics

Lecture 3. Topic Modeling

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Lecture Plan

Lecture Plan

1. What Is Topic Modeling
2. Graphical Models
3. EM-algorithm
4. Probabilistic Generative Topic Model
5. Probabilistic LSA (PLSA)
6. Latent Dirichlet Allocation (LDA)
7. Additive Regularization of Topic Models (ARTM)
8. Seminar

What Is Topic Modeling

What Is Topic Modeling

What is this text about? **politics** 60%, **education** 20%, **culture** 10%, **sports** 10%, **economics** 0%.

What Is Topic Modeling

What is the difference between topic modeling and classification?

Unspecified in advance topics!

Topic Modeling in Social Media

Актуальные темы: Россия · Изменить

Доренко

Сталина
Твитов: 8 256

Через 10
Твитов: 2 008

Дудя

На Кубани

Самарской
Твитов: 1 480

Международному

запад
Твитов: 1 413

В Кремле

Сергей
Твитов: 4 158

Актуальные темы: США · Изменить

#FatTuesday
Твитов: 6 886

#TuesdayThoughts
Твитов: 43 тыс.

King Kong Bundy
Wrestling legend King Kong Bundy has died, aged 61

#PancakeDay
Твитов: 39,4 тыс.

#TuesdayMotivation
Твитов: 18,9 тыс.

#PaczkiDay

Ty Cobb

Good Tuesday
Твитов: 11,1 тыс.

McCarthyism
Твитов: 25 тыс.

Former Trump White House
Твитов: 2 317

Актуальные темы: Франция · Изменить

#MardiGras
Твитов: 14,6 тыс.

#MardiConseil
Твитов: 1 883

#lesplanetes
Твитов: 4 012

Hugo Clément
Твитов: 2 220

#GILyon2019
Твитов: 1 504

Vieilles Charrues

Condé-sur-Sarthe
Твитов: 1 836

Sonic
Твитов: 114 тыс.

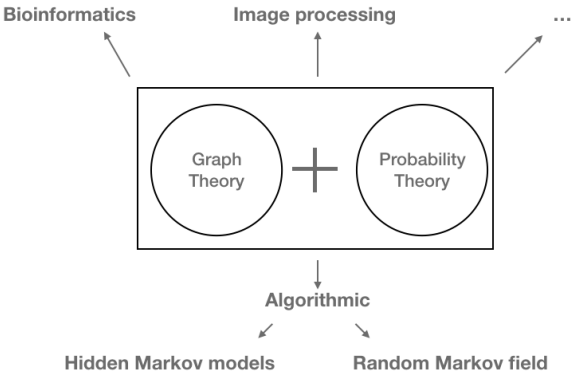
Black Eyed Peas
Твитов: 2 013

Bigard
Твитов: 11,4 тыс.

Graphical Models

Introduction to Graphical Models

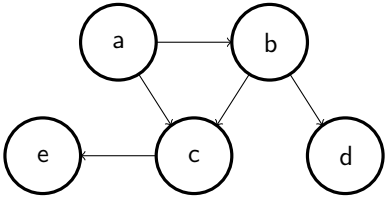
Graphical models bring together graph theory and probability theory in a powerful formalism for multivariate statistical modeling.



Directed Models

A **directed model** contains one factor for every random variable x_i in the distribution, and that factor consists of the conditional distribution over x_i given the parents of x_i, denoted $Pa_{\mathcal{G}}(x_i)$:

$$p(\mathbf{x}) = \prod_i p(x_i \mid Pa_{\mathcal{G}}(x_i)).$$

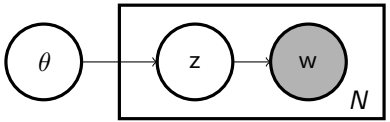


This graph corresponds to probability distributions that can be factored as follows:

$$p(a, b, c, d, e) = p(a)p(b \mid a)p(c \mid a, b)p(d \mid b)p(e \mid c).$$

Observable and Latent Variables

Observable variables are variables that can be observed and directly measured.
Latent (hidden) variables are variables that are not directly observed but are rather inferred from other variables that are observed (directly measured).



θ is a parameter, $z_1, \ldots, z_N$ are hidden (latent) variables, and $w_1, \ldots, w_N$ are observable variables.

Given θ, the joint probability for such model can be computes as follows:

$$p(\mathbf{w}, \mathbf{z} \mid \theta) = \prod_{i=1}^{N} p(z_i \mid \theta) p(w_i \mid z_i).$$

EM-algorithm

EM-algorithm

EM-algorithm is a powerful method that is widely used in probabilistic models. We will use $\mathbf{X}$ to denote the observable variables, $\mathbf{Z}$ to denote the latent variables, and $\mathbf{\Theta}$ to denote the model parameters (that we want to estimate).

Assume that we know how to compute a joint distribution over $\mathbf{X}$ and $\mathbf{Z}$ given $\mathbf{\Theta}$: $p(\mathbf{X}, \mathbf{Z} \mid \mathbf{\Theta})$. Let's write a log-likelihood function as an expectation over $\mathbf{Z}$:

$$\log p(\mathbf{X} \mid \mathbf{\Theta}) = \int q(\mathbf{Z}) \log p(\mathbf{X} \mid \mathbf{\Theta}) d\mathbf{Z}$$

where $q(\mathbf{Z})$ is an arbitrary probability density for $\mathbf{Z}$.

EM-algorithm

$$\log p(\mathbf{X} \mid \mathbf{\Theta}) = \int q(\mathbf{Z}) \log p(\mathbf{X} \mid \mathbf{\Theta}) d\mathbf{Z} = \int q(\mathbf{Z}) \log \frac{p(\mathbf{X}, \mathbf{Z} \mid \mathbf{\Theta})}{p(\mathbf{Z} \mid \mathbf{X}, \mathbf{\Theta})} \frac{q(\mathbf{Z})}{q(\mathbf{Z})} d\mathbf{Z}$$
$$= \int q(\mathbf{Z}) \log \frac{p(\mathbf{X}, \mathbf{Z} \mid \mathbf{\Theta})}{q(\mathbf{Z})} d\mathbf{Z} + \int q(\mathbf{Z}) \log \frac{q(\mathbf{Z})}{p(\mathbf{Z} \mid \mathbf{X}, \mathbf{\Theta})} d\mathbf{Z}$$
$$= \mathscr{L}(q, \mathbf{\Theta}) + D_{\mathrm{KL}}(q(\mathbf{Z}) \| p(\mathbf{Z} \mid \mathbf{X}, \mathbf{\Theta})) \geq \mathscr{L}(q, \mathbf{\Theta}).$$

A full log-likelihood function is hard to optimize. Instead of maximizing $\log p(\mathbf{X} \mid \mathbf{\Theta})$ by $\mathbf{\Theta}$, we are going to maximize $\mathscr{L}(q, \mathbf{\Theta})$ by $q, \mathbf{\Theta}$. Notice that $\log p(\mathbf{X} \mid \mathbf{\Theta})$ does not depend on q so we are not limited to choose q. Thus we can turn the last inequality into equality by putting $q(\mathbf{Z}) = p(\mathbf{Z} \mid \mathbf{X}, \mathbf{\Theta})$ so that $D_{\mathrm{KL}}(q(\mathbf{Z}) \| p(\mathbf{Z} \mid \mathbf{X}, \mathbf{\Theta})) = 0$.

EM-algorithm

- **E-step** creates a function for the expectation of the log-likelihood evaluated using the current estimate for the parameters:

$$q(\mathbf{Z})^{(n+1)} = \arg\max_{q'} \mathscr{L}(q', \mathbf{\Theta}^{(n)}) = p(\mathbf{Z} \mid \mathbf{X}, \mathbf{\Theta}^{(n)}).$$

- **M-step** computes parameters maximizing the expected log-likelihood found on the E-step:

$$\begin{aligned}
\mathbf{\Theta}^{(n+1)} &= \arg\max_{\mathbf{\Theta}'} \mathscr{L}(q^{(n+1)}, \mathbf{\Theta}') \\
&= \arg\max_{\mathbf{\Theta}'} \int q(\mathbf{Z})^{(n+1)} \log \frac{p(\mathbf{X}, \mathbf{Z} \mid \mathbf{\Theta}')}{q(\mathbf{Z})^{(n+1)}} d\mathbf{Z} \\
&= \arg\max_{\mathbf{\Theta}'} \int q(\mathbf{Z})^{(n+1)} \log p(\mathbf{X}, \mathbf{Z} \mid \mathbf{\Theta}') d\mathbf{Z} \\
&= \arg\max_{\mathbf{\Theta}'} \mathbb{E}_{\mathbf{Z} \sim q(\mathbf{Z})^{(n+1)}} [\log p(\mathbf{X}, \mathbf{Z} \mid \mathbf{\Theta}')].
\end{aligned}$$

Probabilistic Generative Topic Model

Probabilistic Generative Topic Model

The joint probability of a document d and a word w can be expressed with conditional probability formula as

$$P(d, w) = P(d)P(w \mid d).$$

Now we will define a model with hidden variables $\mathcal{Z} = \{z_1, \ldots, z_T\}$. We assume that each document has its on distribution over (latent) topics, and that each topic has its on distribution over (observable) words. So our goal is to find the matrices $\mathbf{\Phi}$ and $\mathbf{\Theta}$ such that

$$P(w \mid d) = \sum_{z \in \mathcal{Z}} P(w \mid z, d)P(z \mid d) = \sum_{z \in \mathcal{Z}} P(w \mid z)P(z \mid d) = \sum_{z \in \mathcal{Z}} \Phi_{w,z}\Theta_{z,d}.$$

In other words, we want to get a low-rank matrix factorization.

Probabilistic Generative Topic Model

A log-likelihood function for our model:

$$\begin{aligned}\log P(\mathcal{D}, \mathcal{W} \mid \mathbf{\Phi}, \mathbf{\Theta}) &= \log \prod_{d \in \mathcal{D}} \prod_{w \in \mathcal{W}} P(d, w)^{\#(w,d)} \\ &= \sum_{d \in \mathcal{D}} \sum_{w \in \mathcal{W}} \#(w, d) \log P(d, w) \\ &= \sum_{d \in \mathcal{D}} \sum_{w \in \mathcal{W}} \#(w, d) \log (P(d) P(w \mid d)) \\ &= \sum_{d \in \mathcal{D}} \sum_{w \in \mathcal{W}} \#(w, d) \log \left(P(d) \sum_{z \in \mathcal{Z}} \Phi_{w,z} \Theta_{z,d} \right) \longrightarrow \max_{\mathbf{\Phi}, \mathbf{\Theta}},\end{aligned}$$

which is an objective function with restrictions:

$$\sum_{w \in \mathcal{W}} \Phi_{w,z} = 1,\ \Phi_{w,z} \geq 0,\ \sum_{z \in \mathcal{Z}} \Theta_{z,d} = 1,\ \Theta_{z,d} \geq 0.$$

Probabilistic Generative Topic Model

If $P(d)$ does not depend on Φ or Θ, then the problem is equivalent to

$$\sum_{d\in\mathcal{D}}\sum_{w\in\mathcal{W}} \#(w,d)\log\sum_{z\in\mathcal{Z}}\Phi_{w,z}\Theta_{z,d} \longrightarrow \max_{\Phi,\Theta}.$$

A point (Φ,Θ) of a local extremum can be found as a solution of the system:

$$\begin{aligned} P(z \mid d,w) &\propto \Phi_{w,z}\Theta_{z,d}, \\ \Phi_{w,z} &\propto \sum_{d\in\mathcal{D}} \#(w,d)P(z \mid d,w), \\ \Theta_{z,d} &\propto \sum_{w\in\mathcal{W}} \#(w,d)P(z \mid d,w). \end{aligned}$$

It means that on an E-step we should compute $P(z \mid d,w)$, and on an M-step we should compute $P(w \mid z) = \Phi_{w,z}$ and $P(z \mid d) = \Theta_{z,d}$ according to the formulas obtained.

PLSA

PLSA

Probabilistic latent semantic analysis (PLSA) (1999) was the first technique in history for probabilistic topic modeling. The PLSA model is a latent variable model for co-occurrence data which associates an unobserved class variable $z \in \mathcal{Z} = \{z_1, \ldots, z_T\}$ with each observation. A joint probability model over $\mathcal{D} \times \mathcal{W}$ is defined by the mixture

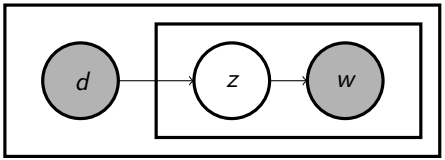
$$P(d, w) = P(d)P(w \mid d),\ P(w \mid d) = \sum_{z \in \mathcal{Z}} P(w \mid z)P(z \mid d).$$

PLSA

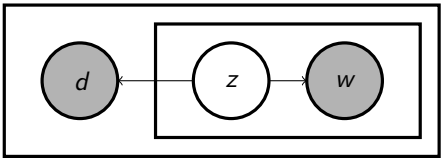
Since the cardinality of z is smaller than the number of documents/words in the collection, z acts as a bottleneck variable in predicting words. It is worth noticing that the model can be equivalently parameterized by:

$$P(d, w) = \sum_{z \in \mathcal{Z}} P(z)P(d \mid z)P(w \mid z)$$

which is perfectly symmetric in both entities, documents and words.



(a) non-symmetrical



(b) symmetrical

Two versions of the PLSA model.

PLSA

PLSA model uses EM algorithm for maximum likelihood estimation.

- **E-step**. For each $z \in \mathcal{Z}$, $d \in \mathcal{D}$, $w \in \mathcal{W}$ compute

$$P(z \mid d, w) = \frac{P(d, z, w)}{P(d, w)} = \frac{P(z)P(d \mid z)P(w \mid z)}{\sum\limits_{z' \in \mathcal{Z}} P(z')P(d \mid z')P(w \mid z')}.$$

- **M-step** For each $z \in \mathcal{Z}$, $d \in \mathcal{D}$, $w \in \mathcal{W}$ compute

$$P(w \mid z) \propto \sum_{d \in \mathcal{D}} \#(w, d)P(z \mid d, w),$$

$$P(d \mid z) \propto \sum_{w \in \mathcal{W}} \#(w, d)P(z \mid d, w),$$

$$P(z) \propto \sum_{d \in \mathcal{D}} \sum_{w \in \mathcal{W}} \#(w, d)P(z \mid d, w).$$

PLSA: Relation to LSA

Define matrices by $\hat{\boldsymbol{U}} : \hat{U}_{i,k} = P(w^{(i)} \mid z_k)$, $\hat{\boldsymbol{\Sigma}} : \hat{\Sigma}_{k,k} = P(z_k)$, $\hat{\boldsymbol{V}}^\top : \hat{V}^\top_{k,j} = P(d^{(j)} \mid z_k)$. The joint probability model $\boldsymbol{P}$ can then be written as a matrix product $\boldsymbol{P} = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{V}}^\top$.

The crucial difference between PLSA and LSA, however, is the objective function utilized to determine the optimal decomposition/approximation.

- In LSA, this is the L_2 or Frobenius norm, which corresponds to an implicit additive Gaussian noise assumption on counts.
- In contrast, PLSA relies on the likelihood function of multinomial sampling and aims at an explicit maximization of the predictive power of the model.

LDA

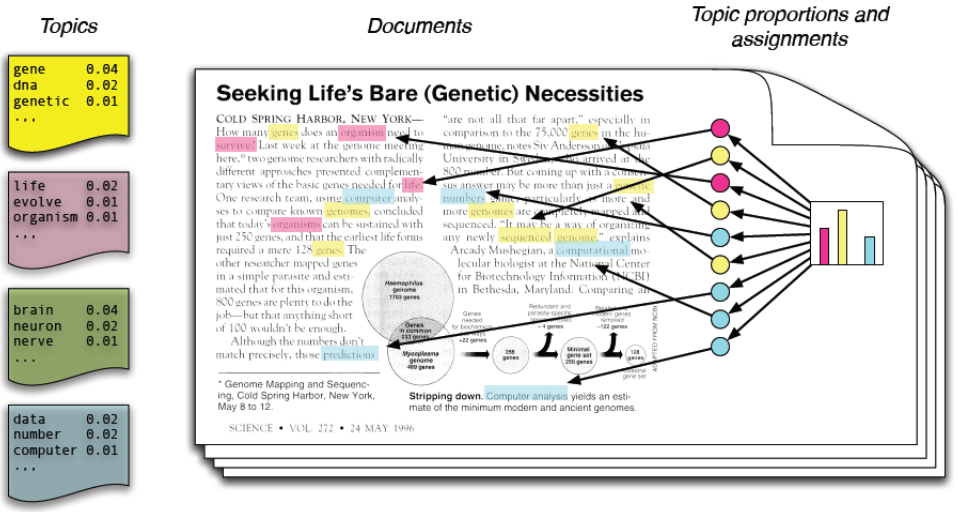
LDA

Latent Dirichlet allocation (LDA) (2003) is a topic modeling algorithm based on a generative graphical model. The concept of LDA is to consider all documents as composed of set of topics or clusters of words and to observe the distribution of them.

LDA

- We have a document collection $\mathcal{D}$ and a vocabulary $\mathcal{W}$.
- Each document $d \in \mathcal{D}$ consists of N_d words $w_{d,1}, \ldots, w_{d,N_d}$.
- The probability distributions over topics $\mathcal{Z}$ are sampled for each document $d \in \mathcal{D}$ from a Dirichlet distribution with parameter $\boldsymbol{\alpha}$.
- For each topic $z \in \mathcal{Z}$ we generate a probability distribution over all words $\mathcal{W}$ $\Phi_{:,z}$.
- After that, for each document d we generate N_d topics $z_{d,1}, \ldots, z_{d,N_d}$ from distribution $\Theta_{:,d}$, and for each of these topics $z_{d,k}$ generate per a single word from distribution $\Phi_{:,z_{d,k}}$.

LDA

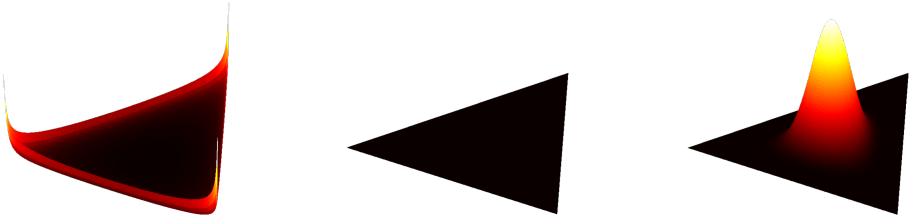

Topics
gene 0.04
dna 0.02
genetic 0.01
...
life 0.02
evolve 0.01
organism 0.01
...
brain 0.04
neuron 0.02
nerve 0.01
...
data 0.02
number 0.02
computer 0.01
...
Documents
Topic proportions and assignments
Seeking Life's Bare (Genetic) Necessities
COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.
Although the numbers don't match precisely, those predictions "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a matter of numbers, particularly as more and more genomes are completed and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an
* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.
Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.
SCIENCE • VOL. 272 • 24 MAY 1996

Dirichlet Distribution

A K-dimensional Dirichlet random variable $\boldsymbol{\theta}$ can take values in the $(K-1)$-simplex (a K-vector $\boldsymbol{\theta}$ lies in the $(K-1)$-simplex if $\theta_k \geq 0$ and $\sum_{k=1}^{K} \theta_k = 1$), and has the following probability density on this simplex:

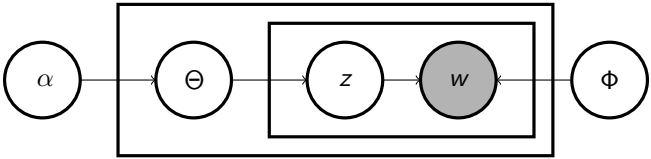
$$p(\boldsymbol{\theta} \mid \boldsymbol{\alpha}) = \frac{\Gamma(\sum_{k=1}^{K} \alpha_k)}{\prod_{k=1}^{K} \Gamma(\alpha_k)} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1}$$

where the parameter $\boldsymbol{\alpha}$ is a K-vector with components $\alpha_k > 0$, and where $\Gamma(x)$ is the Gamma function.



Dirichlet distribution plots for $\boldsymbol{\alpha} = (0.75, 0.75, 0.75), (1, 1, 1), (10, 10, 10)$

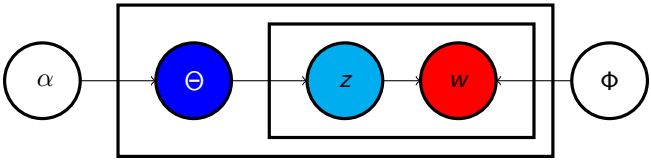
LDA Model



A graphical model for LDA.

In this model, $\boldsymbol{\alpha}$ and $\boldsymbol{\Phi}$ are parameters. $\boldsymbol{\Phi}$ is a matrix of size $|\mathcal{W}| \times |\mathcal{Z}|$, $\boldsymbol{\Theta}$ is a matrix of size $|\mathcal{Z}| \times |\mathcal{D}|$, $\boldsymbol{\alpha}$ is a vector of size $|\mathcal{Z}|$. Moreover, $\boldsymbol{\Theta}_{:,d} \sim Dir(\boldsymbol{\alpha})$.

LDA Model



A graphical model for LDA.

$$P(\mathcal{D}, \mathcal{W}, \mathcal{Z}, \Theta \mid \Phi, \alpha) = \underbrace{\prod_{d=1}^{|\mathcal{D}|}}_{1} \underbrace{P(\Theta_{:,d} \mid \alpha)}_{2} \underbrace{\prod_{n=1}^{N_d}}_{3} \underbrace{P(z_{d,n} \mid \Theta_{:,d})}_{4} \underbrace{P(w_{d,n} \mid z_{d,n}, \Phi)}_{5} \longrightarrow \max_{\Phi, \alpha}$$

- 1. for each document
- 2. generate topic probabilities
- 3. for each word
- 4. select topic
- 5. select word from topic

LDA Model

$$
\begin{aligned}
P(\mathcal{D}, \mathcal{W}, \mathcal{Z}, \boldsymbol{\Theta} \mid \boldsymbol{\Phi}, \boldsymbol{\alpha}) &= \prod_{d=1}^{|\mathcal{D}|} P(\boldsymbol{\Theta}_{:,d} \mid \boldsymbol{\alpha}) \prod_{n=1}^{N_d} P(z_{d,n} \mid \boldsymbol{\Theta}_{:,d}) P(w_{d,n} \mid z_{d,n}, \boldsymbol{\Phi}) \\
&= \prod_{d=1}^{|\mathcal{D}|} \frac{\Gamma(\sum_{t=1}^{|\mathcal{Z}|} \alpha_t)}{\prod_{t=1}^{|\mathcal{Z}|} \Gamma(\alpha_t)} \prod_{t=1}^{|\mathcal{Z}|} \Theta_{t,d}^{\alpha_t - 1} \prod_{n=1}^{N_d} \Theta_{z_{d,n},d} \Phi_{w_{d,n},z_{d,n}} \\
&= \prod_{d=1}^{|\mathcal{D}|} \frac{\Gamma(\sum_{t=1}^{|\mathcal{Z}|} \alpha_t)}{\prod_{t=1}^{|\mathcal{Z}|} \Gamma(\alpha_t)} \prod_{t=1}^{|\mathcal{Z}|} \Theta_{t,d}^{\alpha_t - 1} \prod_{n=1}^{N_d} \prod_{t=1}^{|\mathcal{Z}|} \Theta_{t,d}^{[z_{d,n}=t]} \Phi_{w_{d,n},t}^{[z_{d,n}=t]}.
\end{aligned}
$$

EM-algorithm for LDA

- **E-step**.

$$P(\boldsymbol{\Theta}, \mathcal{Z} \mid \mathcal{D}, \mathcal{W}, \boldsymbol{\Phi}, \boldsymbol{\alpha}) \approx q(\boldsymbol{\Theta}) q(\mathcal{Z}) = \arg \min_{q(\boldsymbol{\Theta}), q(\mathcal{Z})} D_{\mathrm{KL}}(q(\boldsymbol{\Theta}) q(\mathcal{Z}) || P(\boldsymbol{\Theta}, \mathcal{Z} \mid \mathcal{D}, \mathcal{W}, \boldsymbol{\Phi}, \boldsymbol{\alpha})).$$

- **M-step**.

$$\boldsymbol{\Phi} = \arg \max_{\boldsymbol{\Phi}} \mathbb{E}_{\boldsymbol{\Theta} \sim q(\boldsymbol{\Theta}), \mathcal{Z} \sim q(\mathcal{Z})} \log P(\boldsymbol{\Theta}, \mathcal{Z} \mid \mathcal{D}, \mathcal{W}, \boldsymbol{\Phi}, \boldsymbol{\alpha}).$$

Questions?

Seminar

Topic Modeling on Hillary Clinton Emails

1. Reading data
2. Preprocessing
3. LSA
4. LDA
5. Homework: PLSA

https://www.kaggle.com/s231644/topic-modeling-on-hillary-clinton-s-emails

Link