

# Distributional Semantics

## Lecture 4. Prediction-based Distributional Models

Florian Gouret

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# Lecture Plan

- 1 Introduction to Neural Networks
- 2 History
- 3 Word2Vec
- 4 Connection to Count-based Models
- 5 Analogical Reasoning with Word Embeddings
- 6 Seminar

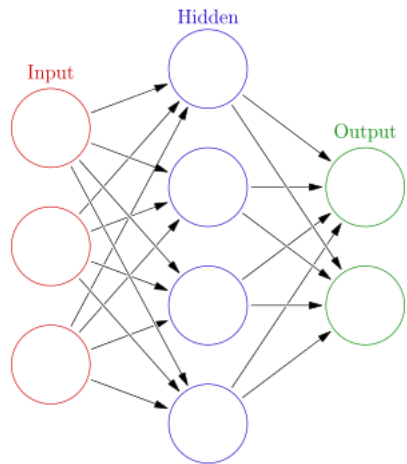
# Introduction

# Introduction: What do you think?

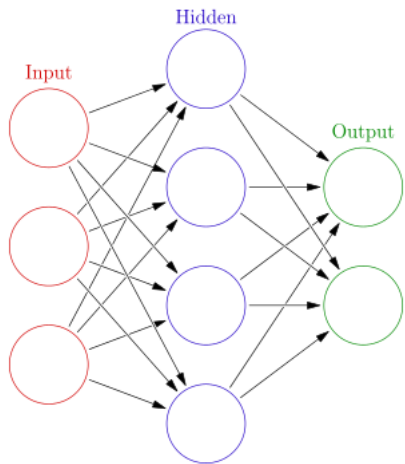


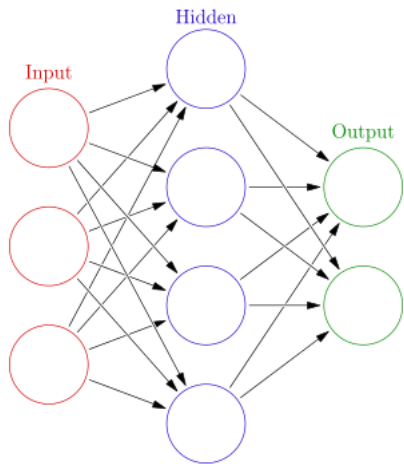
**Prediction-based Distributional Models?**

# Neural Networks



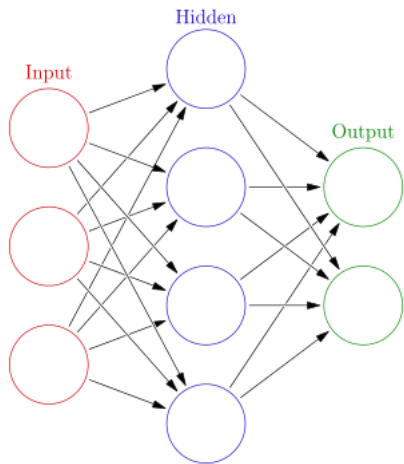
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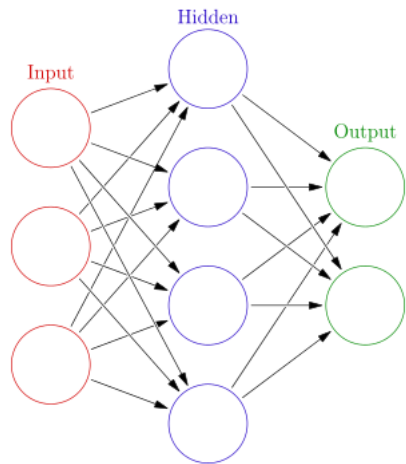


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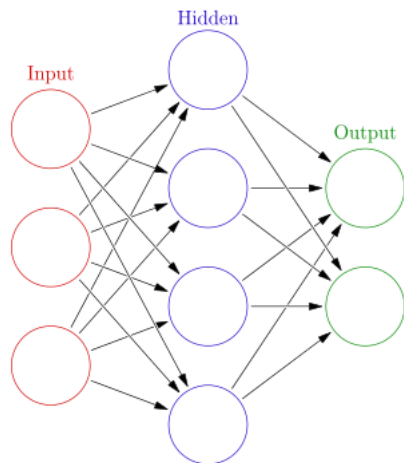




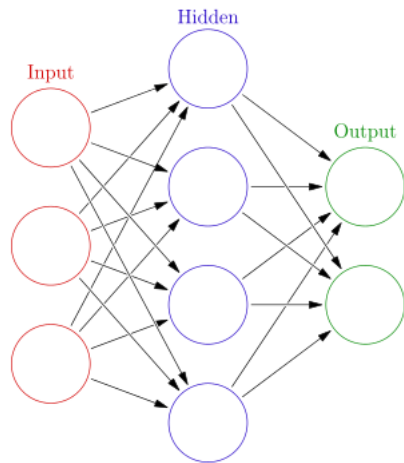
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- In a training process, model tries to minimize the **loss function** (MSE, log-likelihood, etc.) w.r.t. to its **parameters**.

# History

**To understand how much today prediction-based models evolved we have to first look at:**

- Statistical Language Modeling
- Neural Probabilistic Language Model (NPLM)
- SENNA
- HLBL

A **statistical model of language** can be represented by the **conditional probability** of the **next word** given all the previous ones, since

$$\hat{P}(w_1^T) = \prod_{t=1}^T \hat{P}(w_t \mid w_1^{t-1})$$

where  $w_t$  is the  $t$ -th word, and writing sub-sequence  $w_i^j = (w_i, w_{i+1}, \dots, w_{j-1}, w_j)$ .

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**$n$ -gram models** construct ( $n$ -dimensional) **tables of conditional probabilities** for the next word, for each one of a large number of *contexts*, i. e. combinations of the last  $n - 1$  words:

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Using this way of representing words and their probability, we would need tables with  $O(|\mathcal{W}|^n)$  elements... This is called **curse of dimensionality**.

In practice,  $n = 1, 2, 3$ .

# Distributed Representation of Words

A **distributed representation** is dense, low-dimensional, and real-valued. Distributed word representations are called **word embeddings**.

**Each dimension** of the embedding represents **a latent feature of a word**, hopefully capturing useful syntactic and semantic properties. A **distributed representation** is **compact**, in the sense that it can represent an exponential number of clusters in the number of dimensions.

**Facing the curse of dimensionality**

# A Neural Probabilistic Language Model

A **neural probabilistic language model (NPLM)** was proposed Bengio et al. in 2003.

The approach is as follows:

- 1 **associate** with **each word** in the vocabulary an **embedding** (a real-valued vector in  $\mathbb{R}^d$ );
- 2 **express** the **joint probability** function of word sequences in terms of the **embeddings** of these words in the sequence,
- 3 learn **simultaneously** the word **embeddings** and **parameters** of that probability function.

# A Neural Probabilistic Language Model

The aim is to learn a good model for  $f(w_t, \dots, w_{t-n+1}) = \hat{P}(w_t \mid w_1^{t-1})$ .

To do so, the function is decomposed into two parts:

- 1 A **mapping**  $C$  from **any element**  $w$  of  $\mathcal{W}$  to a **real vector**  $C(w) \in \mathbb{R}^d$  (the distributed feature vectors associated with each word in the vocabulary). In practice,  $C$  is represented by a  $|\mathcal{W}| \times d$  matrix of free parameters.
- 2 The **probability function over words** (expressed with  $C$ ):  
 $g : (i, C(w_{t-1}), \dots, C(w_{t-n+1})) \rightarrow \hat{P}(w_t = w^{(i)} \mid w_1^{t-1});$

$$f(w^{(i)}, w_{t-1}, \dots, w_{t-n+1}) = g(i, C(w_{t-1}), \dots, C(w_{t-n+1})).$$

# A Neural Probabilistic Language Model

Loss function:

$$L = \frac{1}{T} \sum_{t=1}^T \log f(w_t, w_{t-1}, \dots, w_{t-n+1} \mid \Theta) + R(\Theta) \longrightarrow \min_{\Theta},$$

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The probabilities are expressed by applying **SoftMax** to the **unnormalized log-probabilities**  $y_w$ :

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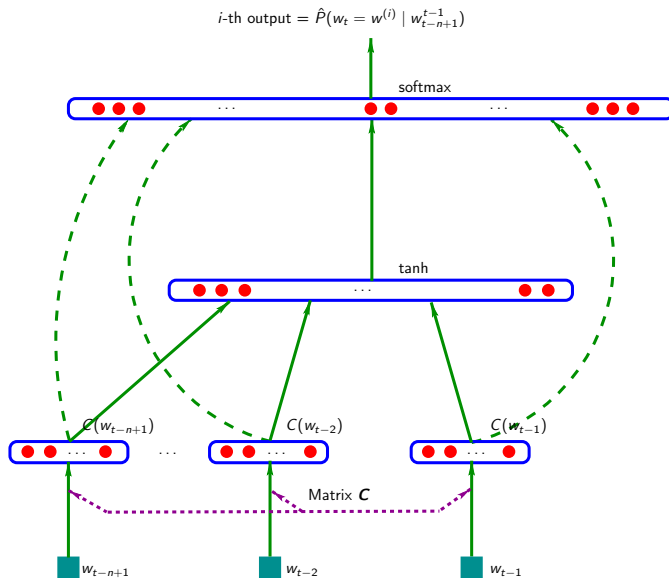
$$\hat{P}(w_t \mid w_1^{t-1}) = \frac{\exp y_{w_t}}{\sum_{w \in \mathcal{W}} \exp y_w},$$

$$\mathbf{y} = \mathbf{b} + \mathbf{W}\mathbf{x} + \mathbf{U} \tanh(\mathbf{d} + \mathbf{H}\mathbf{x}),$$

where the hyperbolic tangent  $\tanh$  is applied element by element,  $\mathbf{W}$  is optionally zero (no direct connections), and  $\mathbf{x}$  is the word features layer activation vector, which is the concatenation of the input word features from the matrix  $\mathbf{C}$ :

$$\mathbf{x} = [C(w_{t-1}), C(w_{t-2}), \dots, C(w_{t-n+1})].$$





Collebert and Weston 2008: a **neural language model** that could be **trained over billions of words**, because the **gradient of the loss** was computed **stochastically** over a small sample of possible outputs, in a spirit similar to Bengio.  
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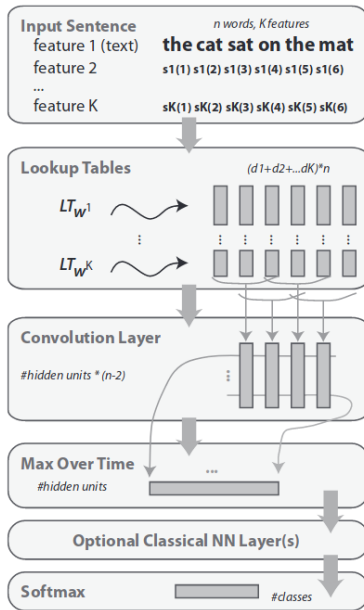
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- ➎ apply a layer that gives the output of size  $|\mathcal{W}|$  (e. g. linear layer):  $\mathbf{y} = \mathbf{b} + \mathbf{Hm}$





**Log-bilinear model (LBL)** is a probabilistic and linear neural model.

To **predict the next word**  $w_t$  given the context  $w_{t-n+1}, \dots, w_{t-1}$ , the model **computes the predicted feature vector**  $\hat{u}_t$  for the next word by **linearly combining the context word feature vectors**:

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Finally,

$$\hat{P}(w_t = w \mid w_{t-n+1}^{t-1}) = \frac{\exp(\hat{u}_t^\top \mathbf{u}_w + b_w)}{\sum_{w' \in \mathcal{W}} \exp(\hat{u}_t^\top \mathbf{u}_{w'} + b_{w'})}.$$

Here  $b_w$  is the bias for word  $w$ , which is used to capture the context-independent word frequency.

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The probability of each decision is given by

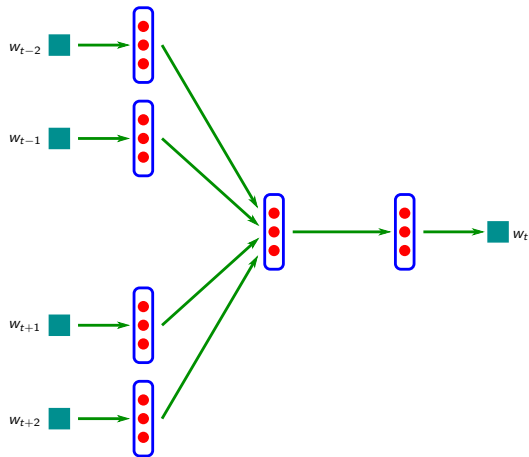
$$\hat{P}(d_i = 1 \mid \mathbf{q}_i, w_{t-n+1}^{t-1}) = \sigma(\hat{\mathbf{u}}_t^\top \mathbf{q}_i + b_i)$$

where  $\sigma(x) = 1/(1 + e^{-x})$  is the logistic function, and  $b_i$  is the node's bias that captures the context-independent tendency to visit the left child when leaving this node.

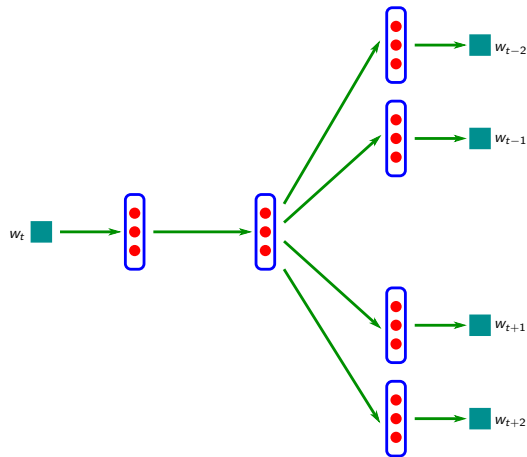
# Word2Vec

Two particular models for learning word representations that can be **efficiently trained** on large amounts of text data are **Skip-gram (SG)** and **Continuous bag-of-words model (CBOW)** introduced by Mikolov et al. in 2013.





Architecture of the CBOW model



Architecture of the Skip-gram model

# Continuous Skip-gram Model

Given a sequence of training words  $w_1, w_2, w_3, \dots, w_T$ , the objective of the Skip-gram model is to maximize the average log probability

$$L = \frac{1}{T} \sum_{t=1}^T \sum_{c \in \mathcal{C}_t} \log P(c \mid w_t),$$

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In the Skip-gram model, every word  $w$  is associated with two learnable parameter vectors,  $\mathbf{u}_w$  and  $\mathbf{v}_w$ . They are the “input” and “output” vectors of the  $w$  respectively. The probability of correctly predicting the context word  $c$  given the central word  $w_t$  is defined as

$$P(c \mid w_t) = \frac{\exp(\mathbf{u}_{w_t}^\top \mathbf{v}_c)}{\sum_{c' \in \mathcal{W}} \exp(\mathbf{u}_{w_t}^\top \mathbf{v}_{c'})}.$$

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This formulation is impractical because the cost of computing  $\nabla \log P(c \mid w_t)$  is proportional to the number of words in the vocabulary  $|\mathcal{W}|$  (which can be easily in order of millions).

The **hierarchical softmax** uses a **binary tree representation** of the output layer with the  $|\mathcal{W}|$  words as its leaves and, for each node, explicitly represents the relative probabilities of its child nodes. These define a **random walk** that assigns probabilities to words.

# Hierarchical Softmax

Each word  $w$  can be reached from the root. Let  $n(w, j)$  be the  $j$ -th node on the path from the root to  $w$ , and let  $L(w)$  be the length of this path, so  $n(w, 1) = \text{root}$  and  $n(w, L(w)) = w$ . For any inner node  $n$ , let  $\text{ch}(n)$  be an arbitrary fixed child of  $n$  and let  $\mathbb{I}[x]$  be 1 if  $x$  is true and  $-1$  otherwise. Then the hierarchical softmax defines  $P(c \mid w_t)$  as follows:

$$P(c \mid w_t) = \prod_{j=1}^{L(c)-1} \sigma(\mathbb{I}[n(c, j+1) = \text{ch}(n(c, j))] \mathbf{u}_{w_t}^\top \mathbf{v}_{n(c, j)})$$

where  $\sigma(x) = 1/(1 + e^{-x})$ .

This implies that the cost of computing  $\log P(c \mid w_t)$  and  $\nabla \log P(c \mid w_t)$  is proportional to  $L(c)$ , which on average is no greater than  $\log_2 |\mathcal{W}|$ .

The **noise contrastive estimation** (NCE) (2012) was introduced by Gutmann and Hyvarinen and applied to language modeling by Mnih and Teh. NCE follows the idea that a **good model** should be able to **differentiate data** from **noise** by means of **logistic regression**.

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$$\log \sigma(\mathbf{u}_w^\top \mathbf{v}_c) + \sum_{j=1}^k \mathbb{E}_{c_{n,j} \sim P_n} \log \sigma(-\mathbf{u}_w^\top \mathbf{v}_{c_{n,j}})$$



# Negative Sampling

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which is used to replace every  $\log P(c \mid w_t)$  term in the Skip-gram objective. Thus the task is to distinguish the target word  $c$  from draws  $c_n$  from the noise distribution  $P_n(w)$  using logistic regression, where there are  $k$  negative samples for each data sample.

# Subsampling of Frequent Words

In very large corpora, the most frequent words can easily occur hundreds of millions of times (e.g., “in”, “the”, and “a”). Such words usually provide less information value than the rare words.

To counter the imbalance between the rare and frequent words, the authors used a simple subsampling approach: each word  $w_t$  in the training set is discarded with probability computed by the formula

$$P(w_t) = 1 - \sqrt{\frac{r}{\#(w_t)}}$$

where  $r$  is a chosen threshold, typically around  $10^{-5}$ .

## Connection to Count-based Models

# Connection to Count-based Models

SGNS embeds both words and their contexts into a low-dimensional space  $\mathbb{R}^d$ , resulting in the word and context matrices  $\mathbf{U}$  and  $\mathbf{V}$ . Let  $\mathbf{M} = \mathbf{UV}^\top$ . An implicit matrix  $\mathbf{M}$  of dimensions  $|\mathcal{W}| \times |\mathcal{W}|$  is factorized into two smaller matrices, and  $M_{w,c} = \mathbf{u}_w^\top \mathbf{v}_c$ .

# Connection to Count-based Models

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Levy and Goldberg (2014) has shown that  $M_{w,c} = f(w, c) = \log 2 \cdot PMI_{w,c} - \log k$ , where  $k$  is a number of negative samples.

# Analical Reasoning with Word Embeddings

To compare the quality of different versions of word vectors, researchers typically used a table showing example words and their most similar words, and understand them intuitively.

Somewhat surprisingly, these questions can be answered by performing simple algebraic operations with the vector representation of words. To find a word that is similar to *small* in the same sense as *biggest* is similar to *big*, we can simply compute vector

$$v^* = v_{biggest} - v_{big} + v_{small}.$$

The relationship is defined by subtracting two word vectors, and the result is added to another word. Thus for example,

$$v_{Paris} - v_{France} + v_{Italy} = v_{Rome}.$$

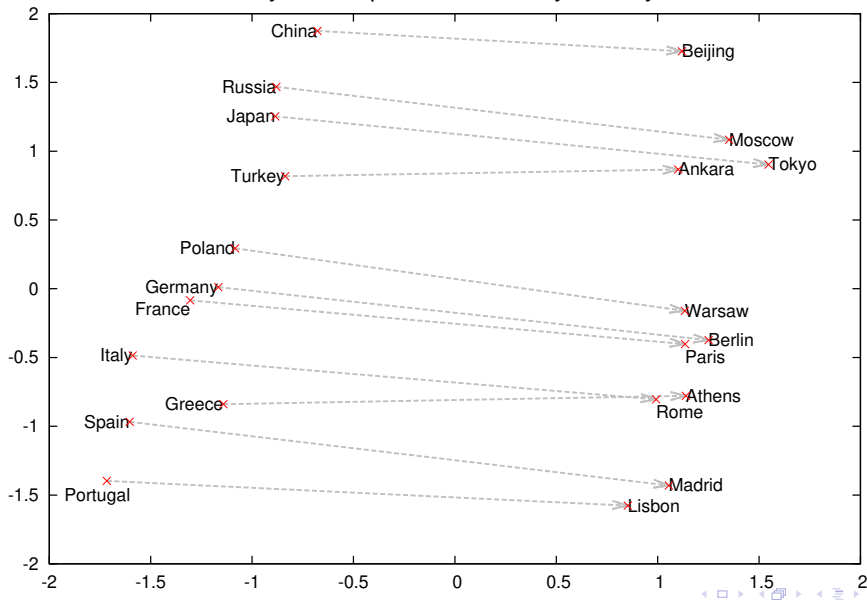


# Examples of the Learned Relationships

| Relationship         | Example 1           | Example 2         | Example 3            |
|----------------------|---------------------|-------------------|----------------------|
| France - Paris       | Italy: Rome         | Japan: Tokyo      | Florida: Tallahassee |
| big - bigger         | small: larger       | cold: colder      | quick: quicker       |
| Miami - Florida      | Baltimore: Maryland | Dallas: Texas     | Kona: Hawaii         |
| Einstein - scientist | Messi: midfielder   | Mozart: violinist | Picasso: painter     |
| Sarkozy - France     | Berlusconi: Italy   | Merkel: Germany   | Koizumi: Japan       |
| copper - Cu          | zinc: Zn            | gold: Au          | uranium: plutonium   |
| Berlusconi - Silvio  | Sarkozy: Nicolas    | Putin: Medvedev   | Obama: Barack        |
| Microsoft - Windows  | Google: Android     | IBM: Linux        | Apple: iPhone        |
| Microsoft - Ballmer  | Google: Yahoo       | IBM: McNealy      | Apple: Jobs          |
| Japan - sushi        | Germany: bratwurst  | France: tapas     | USA: pizza           |

Examples of the word pair relationships, using the best word vectors from Skip-gram model trained on 783M words with 300 dimensionality.

## Country and Capital Vectors Projected by PCA



Questions?

# Seminar

# Word2Vec for Text Classification

- 1 TODO
- 2 Homework: implementation of CBOW or SG

TODO

▶ [Link](#)