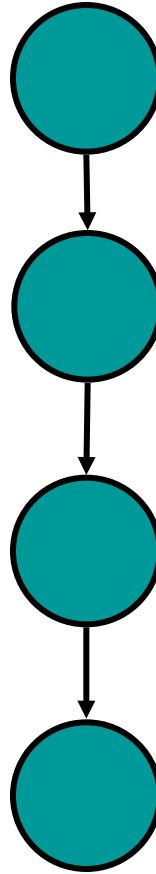


# CS2040C Data Structures and Algorithms

## Trees

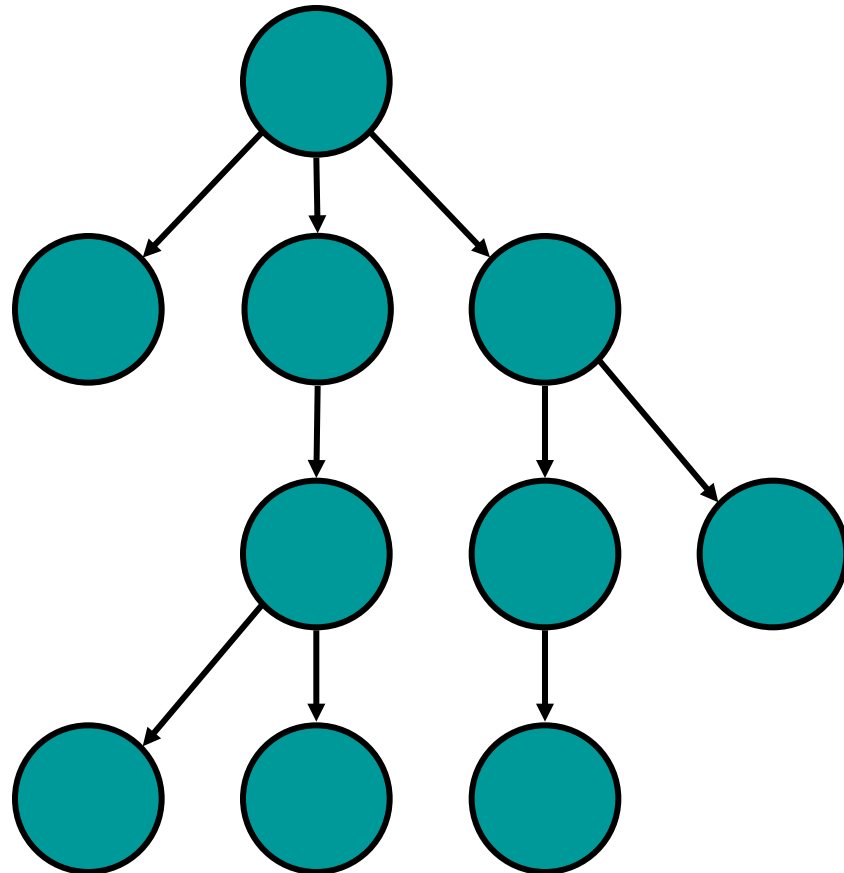
**Introduction to Trees (more on Trees after the break)**

# Linked list (linear data structure)

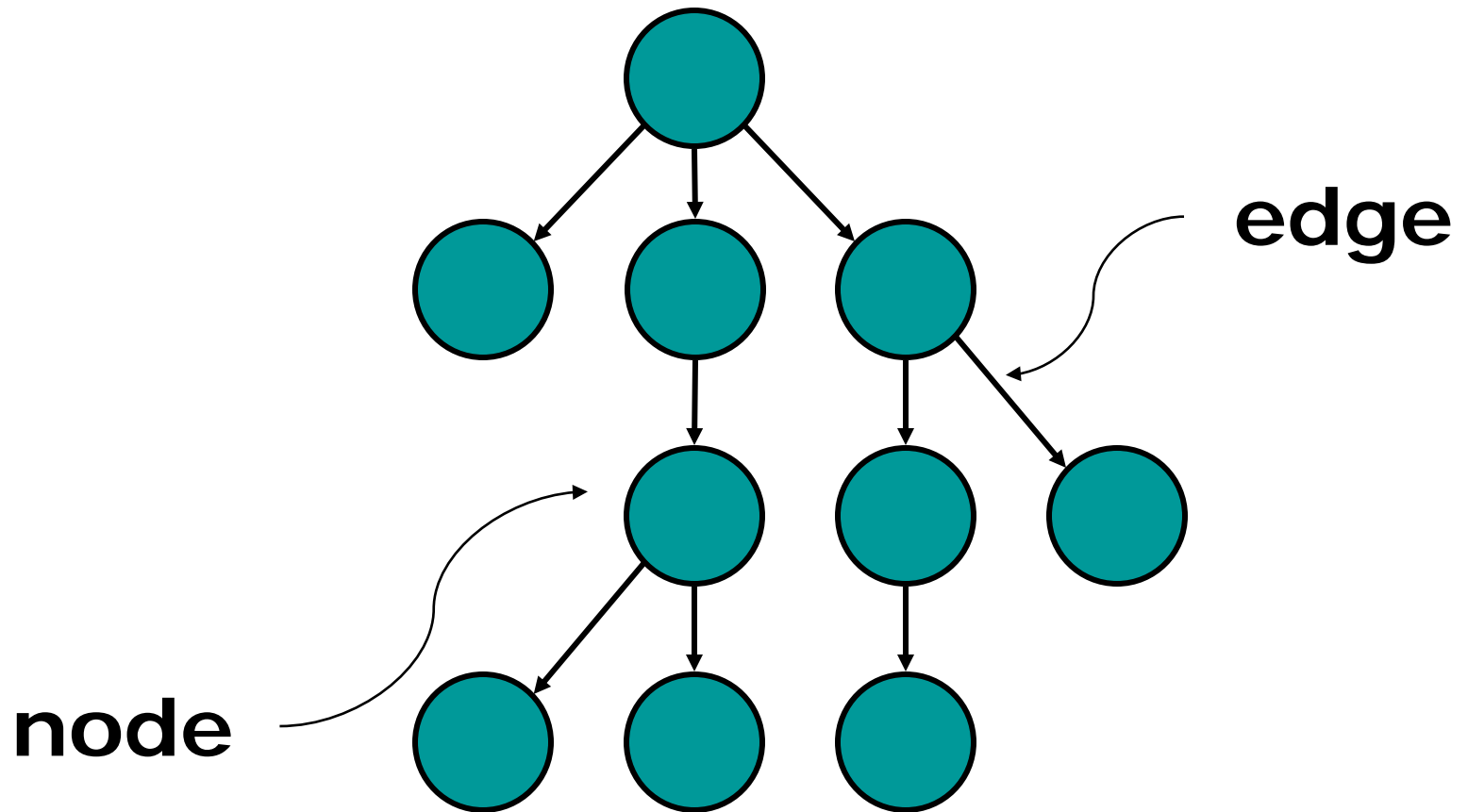


# A Tree as a Data Structure (non-linear)

- shown upside down
- Used to represent relationships
- has a hierarchy



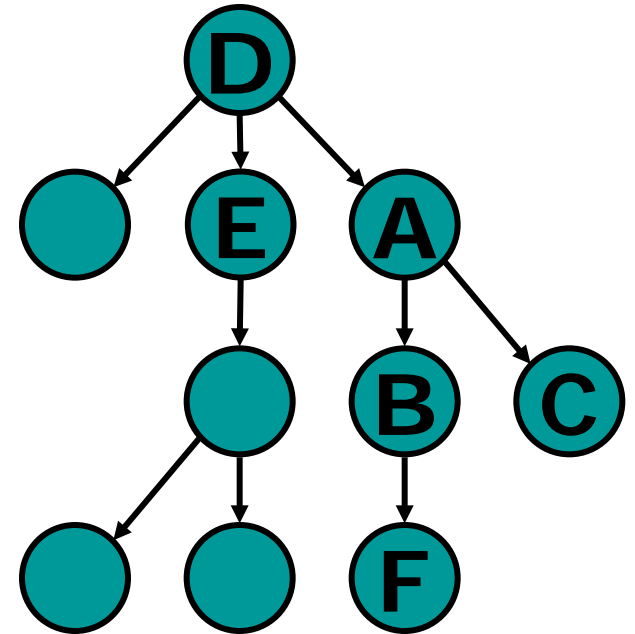
# Definitions



- Data objects (the circles) in a tree are called **nodes or vertices** (vertex for singular)
- Links between nodes are called **edges**

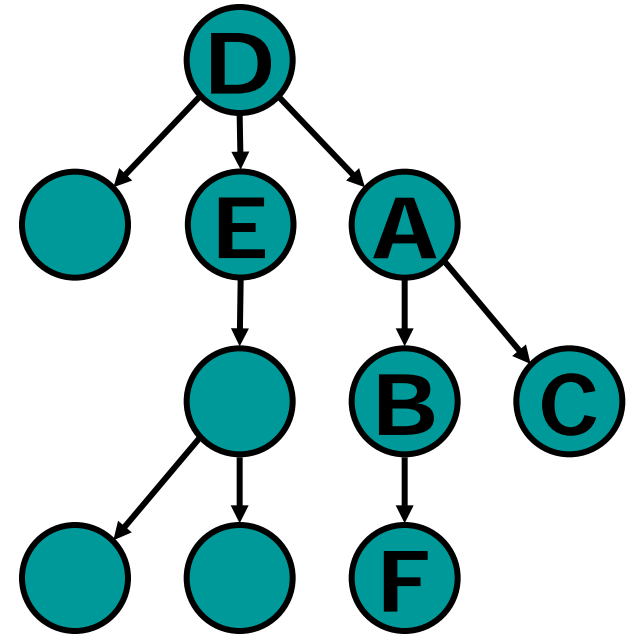
# Relationship

- A is a **parent** of B and C
- B and C are **children** of A
- B and C are **siblings** (with the same parent A)

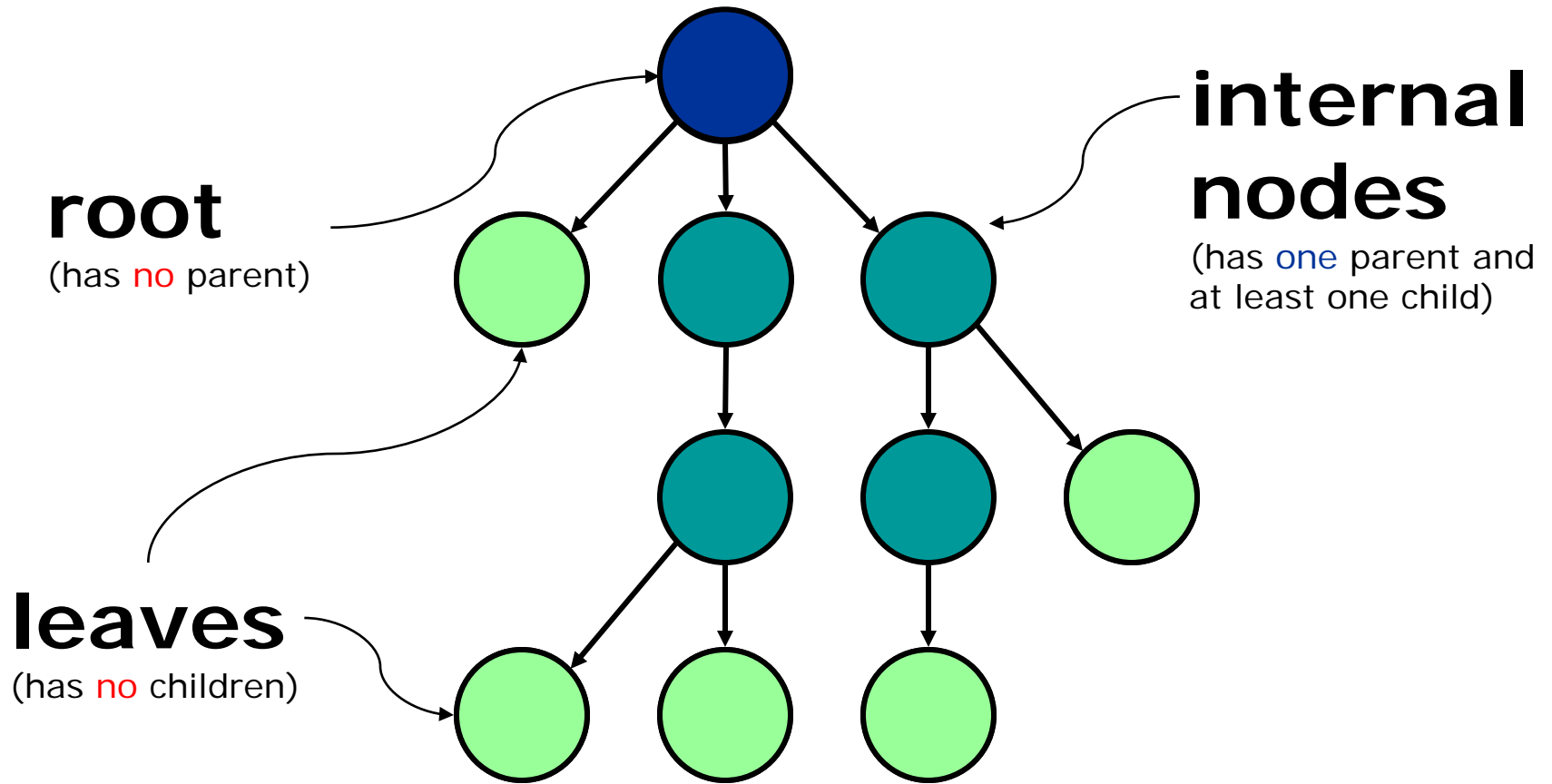


# Relationship

- D is an **ancestor** of B
- B is a **descendant** of A and D
- **Definition:** Node X is an **ancestor** of node Y if
  - X is a parent of Y, or
  - X is a parent of some node Z and Z is an ancestor of Y



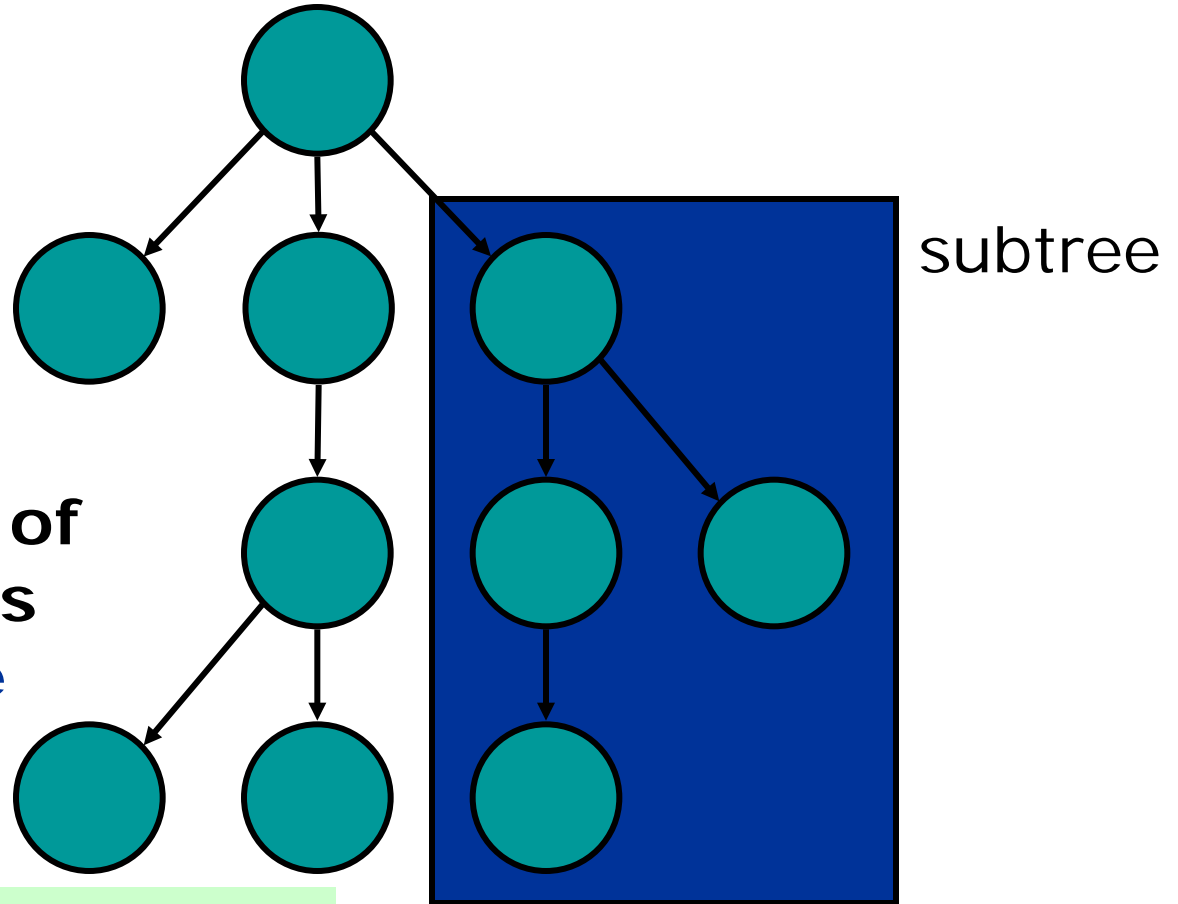
# Tree Nodes



- Every node (except the root) of a tree has **one** parent
- A node with no children is a **leaf** node

# Subtree

A node and all of its descendants form a **subtree**



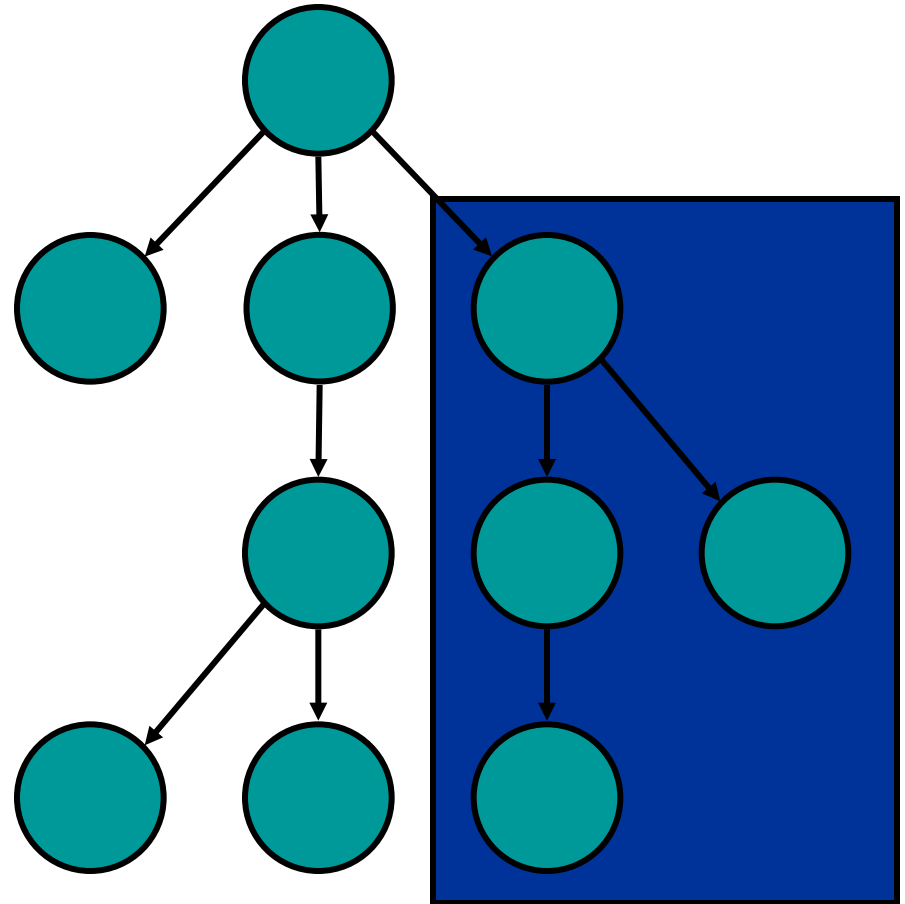
Q: Can a leaf be a subtree?



# Tree is recursive!

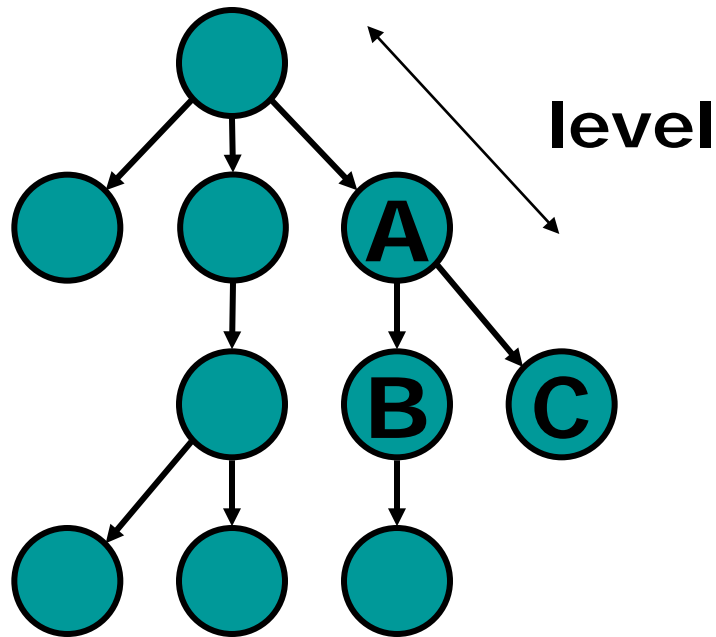
A **tree** is either

- nothing, or
- A node, with some set of subtrees, each of which is a **tree**...



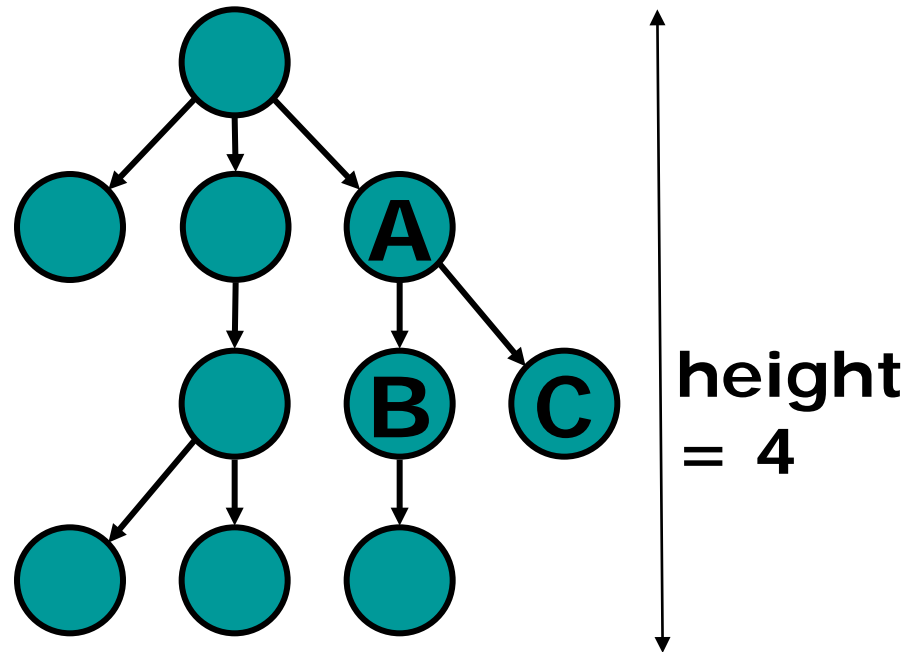
# Level of a node

- **Number of nodes on the path from the root to the node**
  - level of root is 1
  - level of A is 2



# Height of a tree

- **Maximum** level of the nodes in the tree is the **height** of the tree

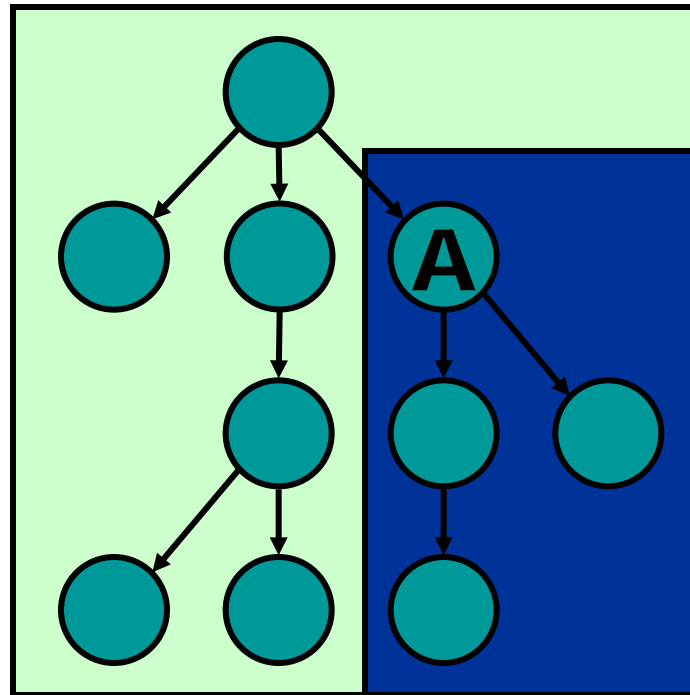


**Note:** there are different definitions of height, e.g. max no of edges from leaf to root (in this case, height is 3) (Visualgo uses this definition)

# Size of a tree

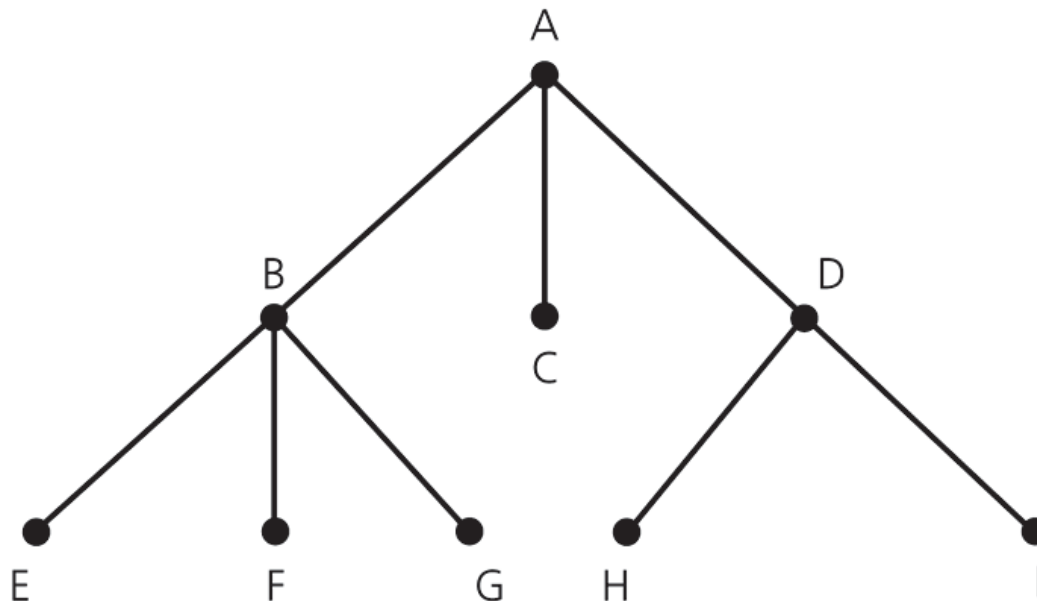
- Number of nodes in the tree is the **size** of the tree

- the size of this tree is 10
- the size of the subtree rooted at A is 4



# General Trees

- An  $n$ -ary tree
  - A tree whose nodes each can have no more than  $n$  children



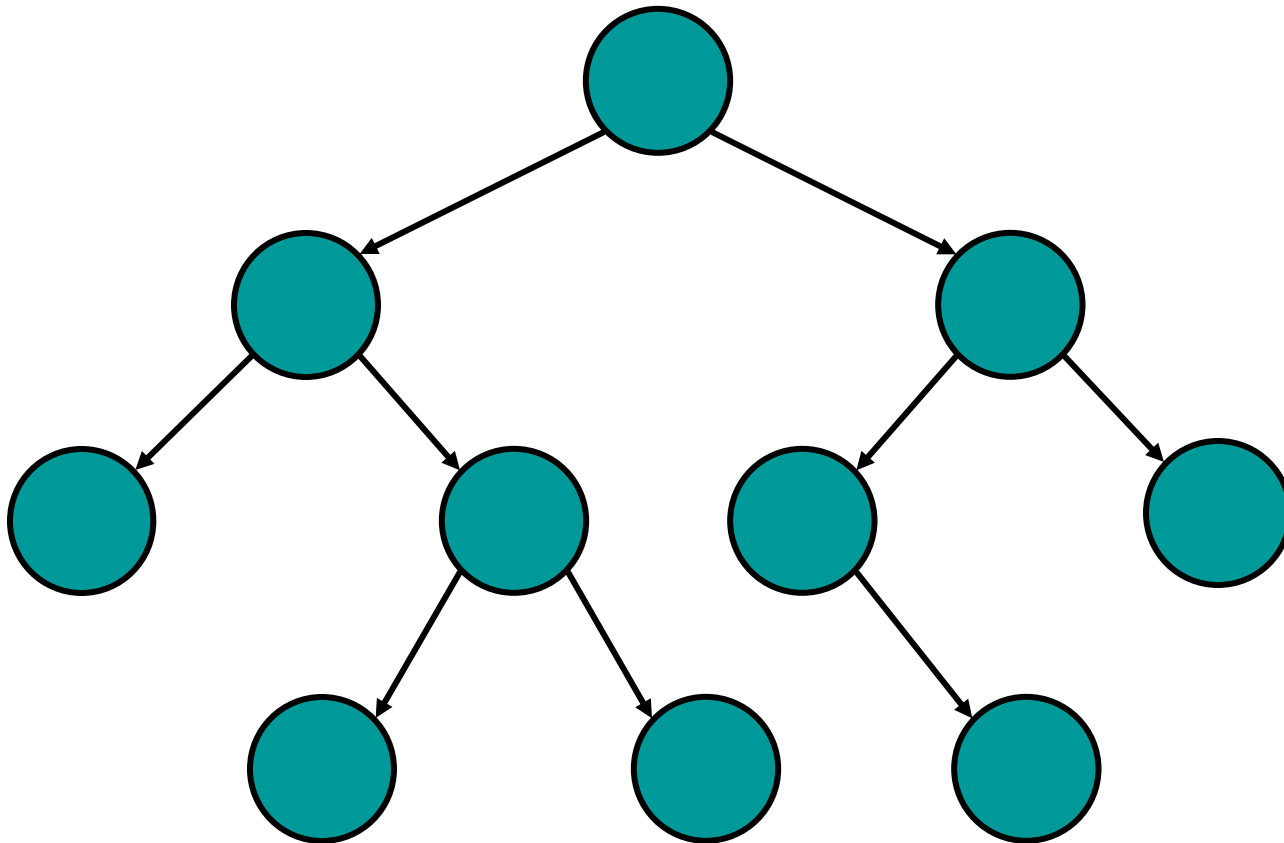
# Binary Trees

Each node has at most **2 ordered** children

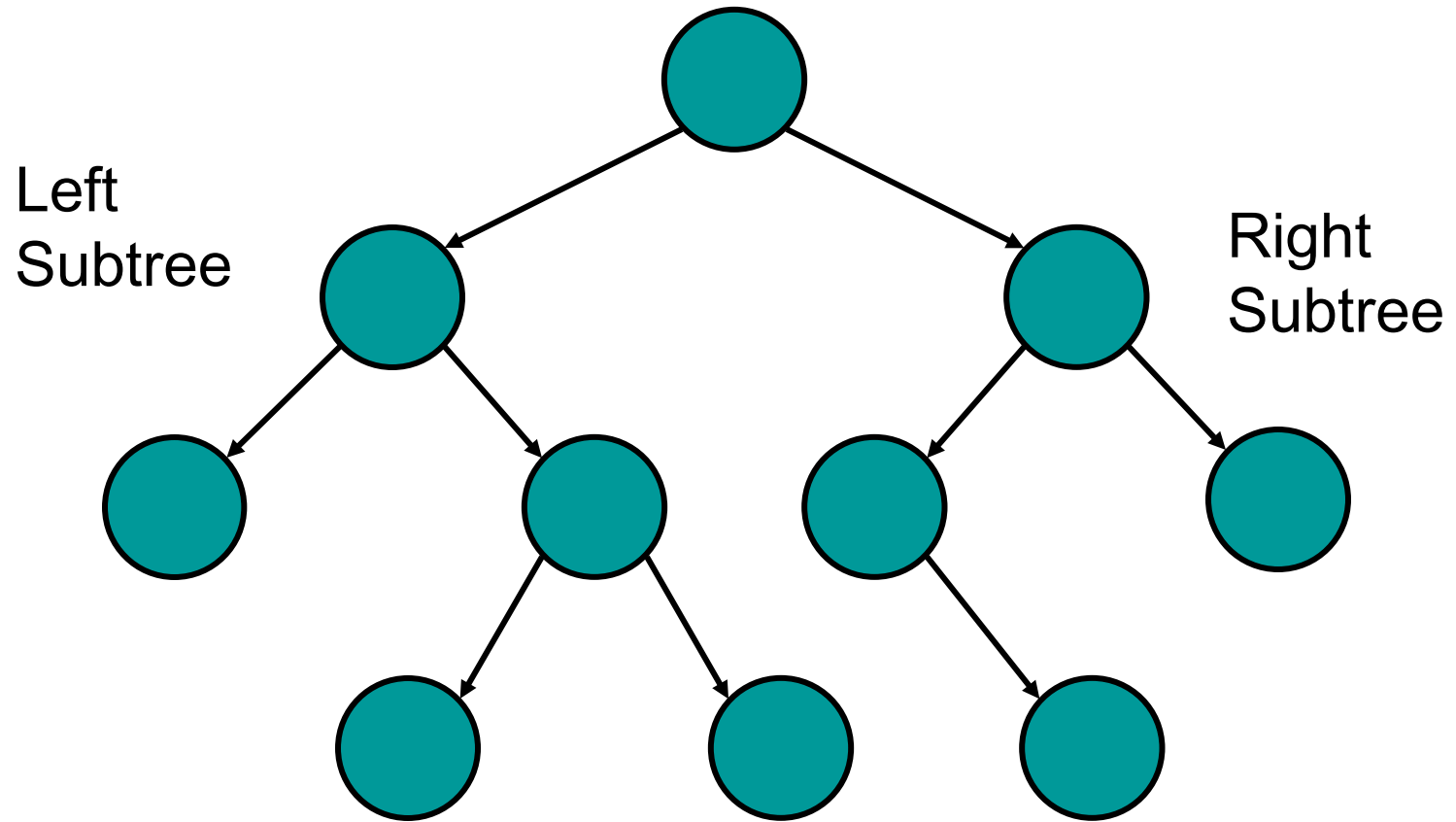
# Binary Tree

Each node has **at most 2 ordered** children

**Q:** What is the meaning of “**order**”?



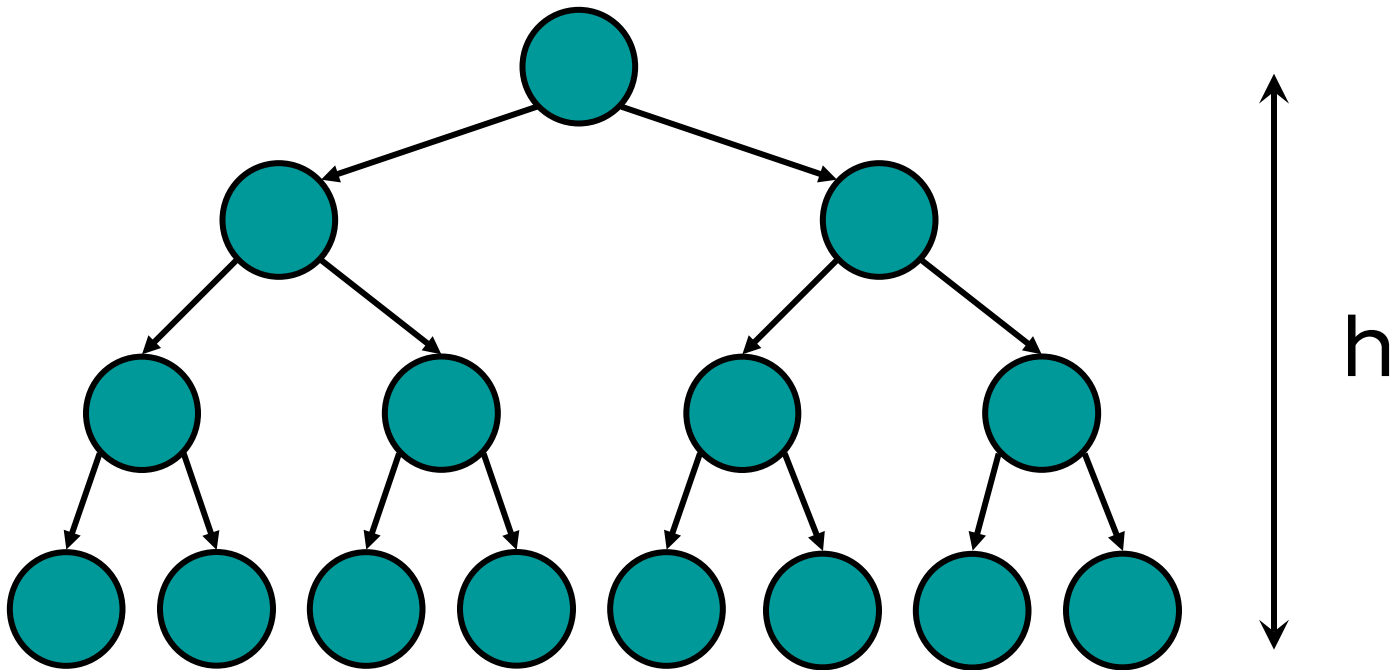
# Binary Tree is Recursive





# Full Binary Tree

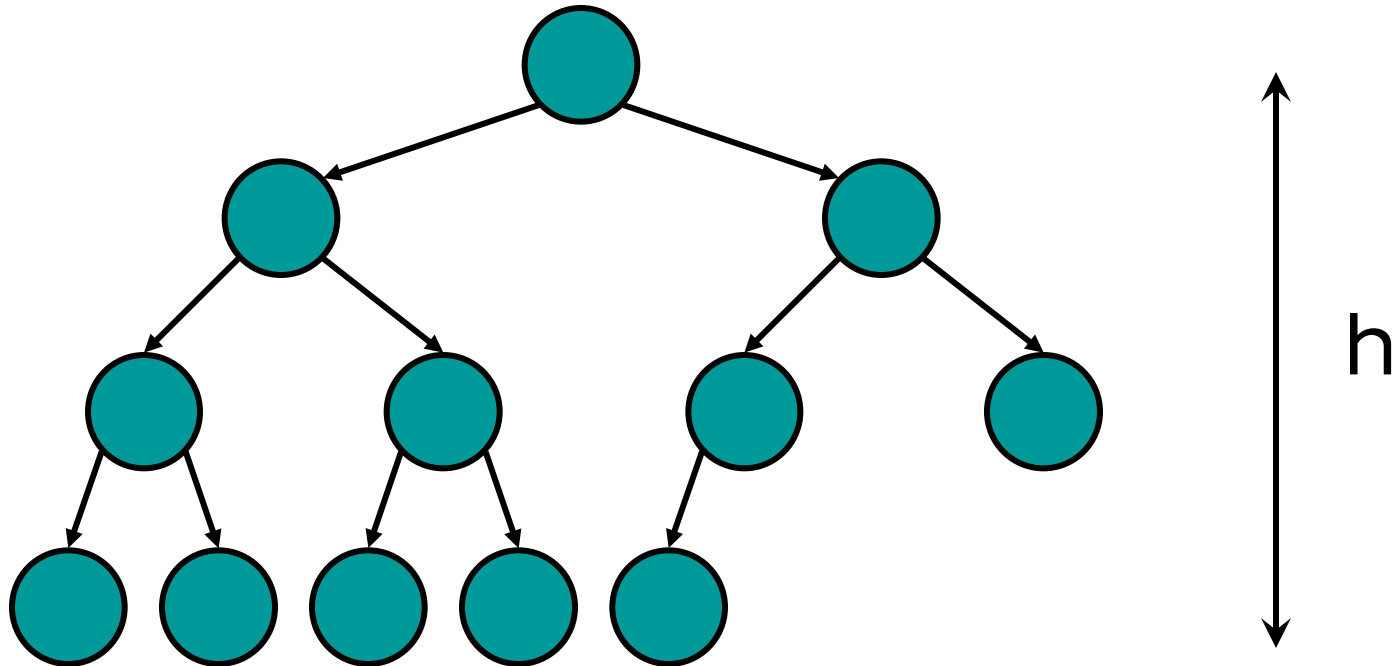
- All nodes at a level  $< h$  have **two** children (where  $h$  is the **height** of the tree)



**Q:** Is this definition the same as “all nodes except the leaf nodes have 2 children”?

# Complete Binary Tree

- Full down to level  $h-1$
- level  $h$  filled in from left to right

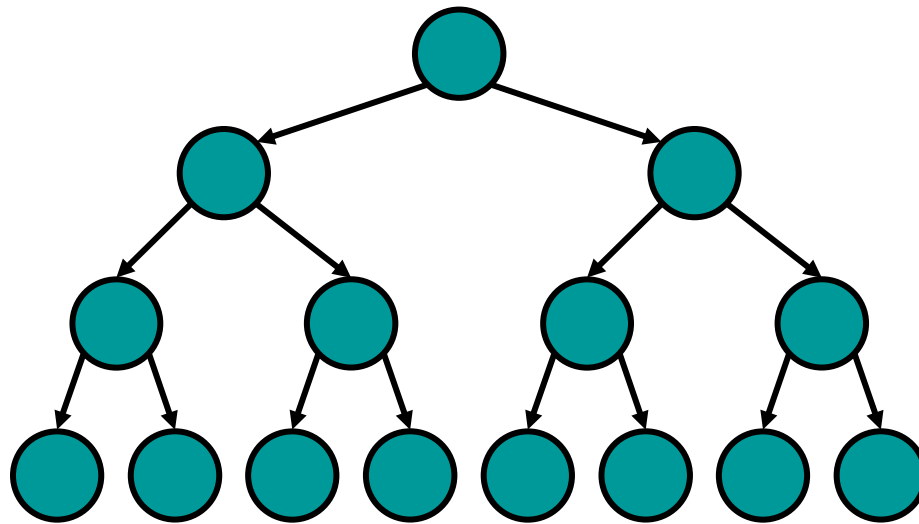


# Full Binary Tree Property

Number of nodes in a full binary tree of height  $h$  is

$$2^h - 1$$

Therefore the height of a full binary tree is  $O(\log N)$

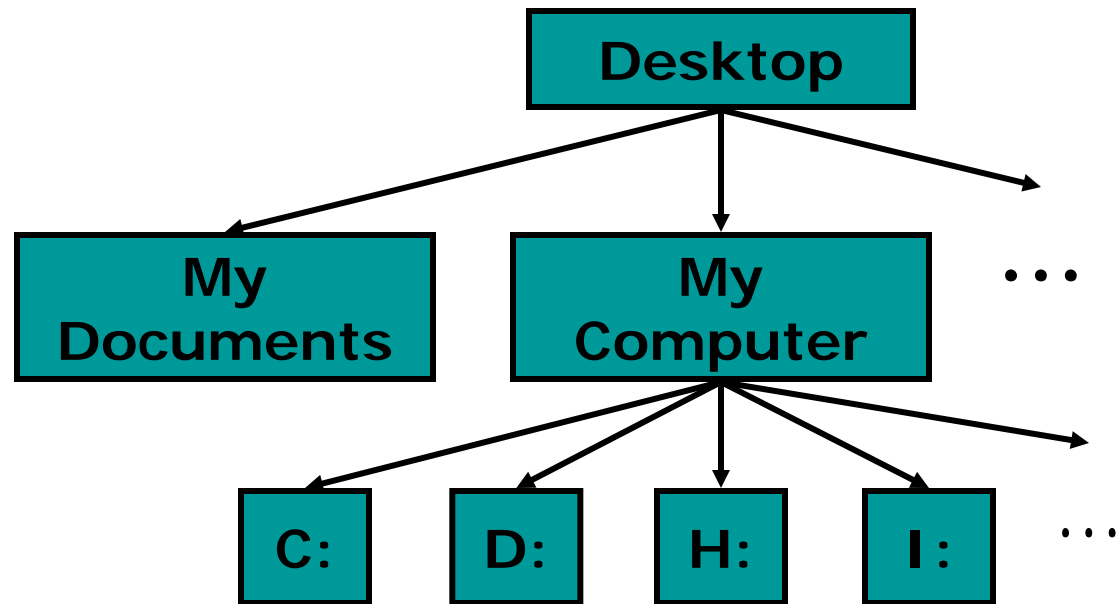
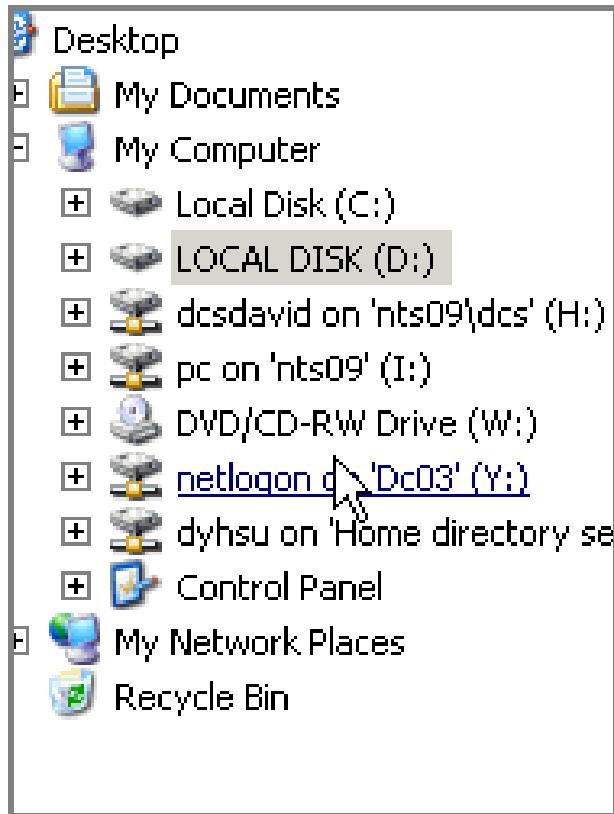


**Q:** How many nodes in a **complete** binary tree of height  $h$ ?

# Applications of Trees

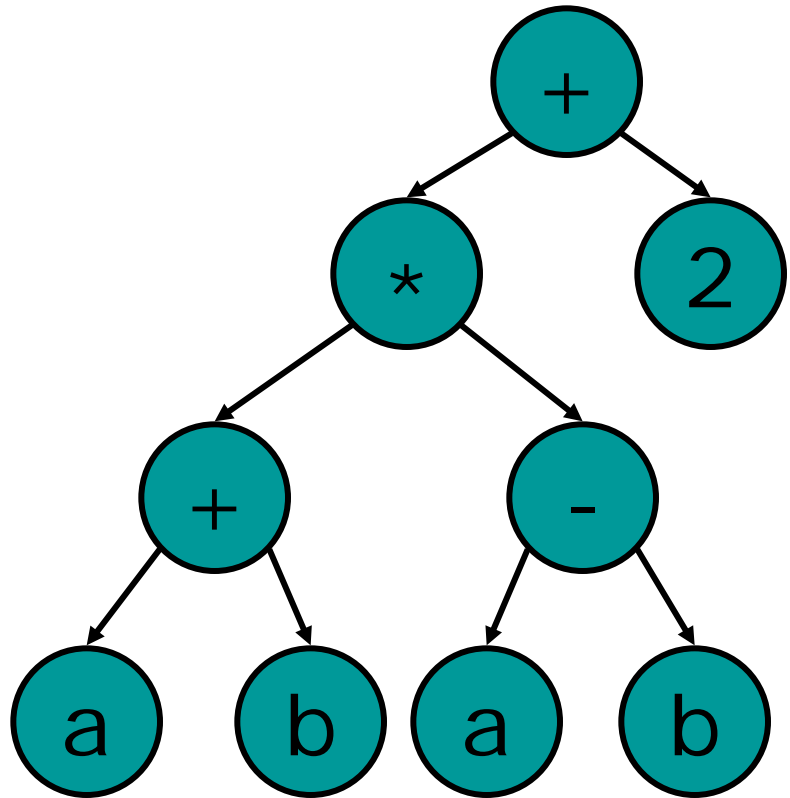
A tree can be used to represent data that is **hierarchical** in nature

# File systems



# Arithmetic Expressions

$(a+b) * (a-b) + 2$



**Q:** How do you construct such a tree from a given arithmetic expression?