CS2040C Data Structures and Algorithms

Stack ADT

Outline

- Stack
  - Introduction
  - Specification
  - Implementations
    - Linked List
    - STL vector
  - Applications
    - Bracket Matching
    - Towers of Hanoi
    - Maze Exploration

Stack: A specialised list

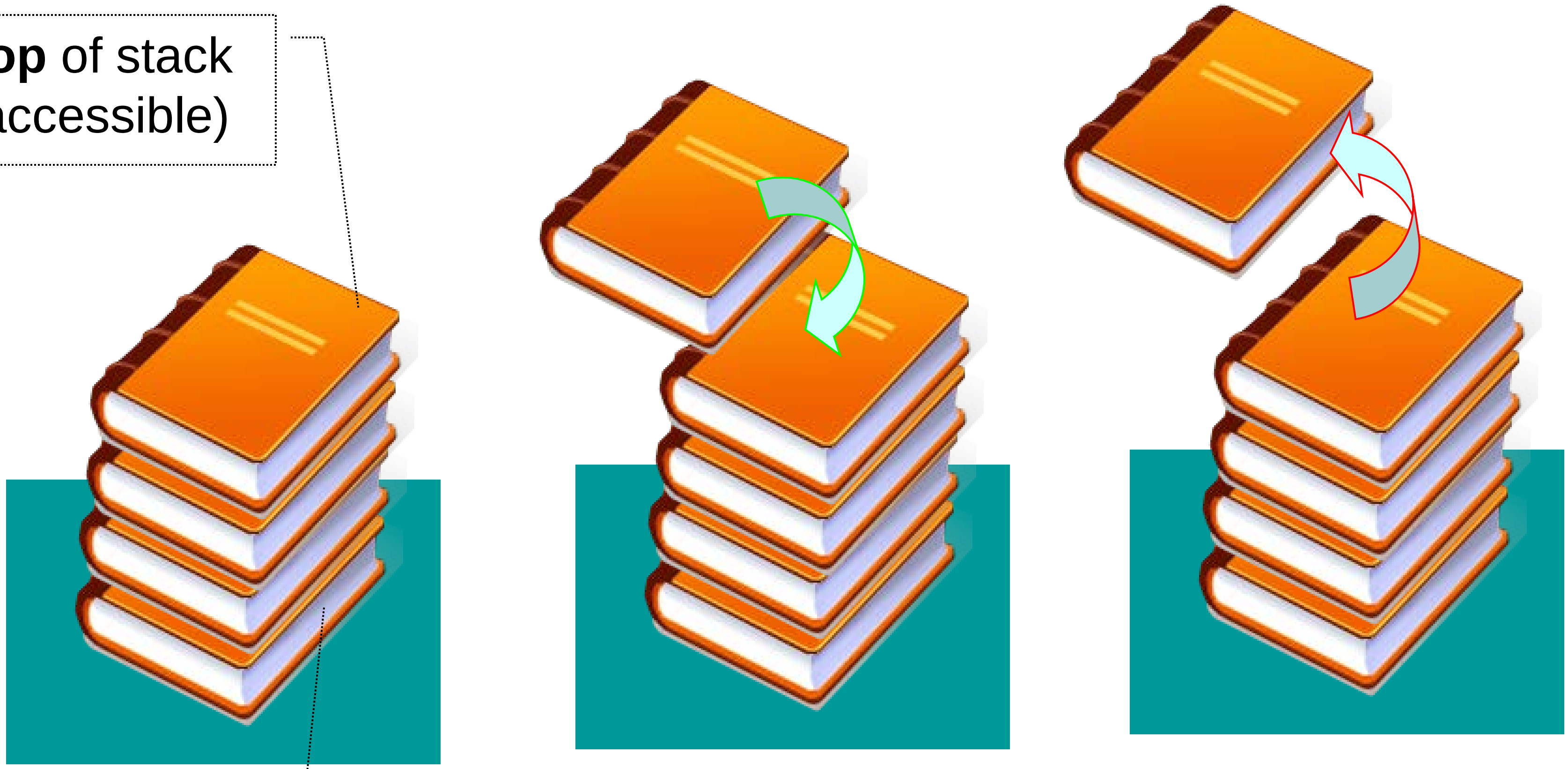
- List ADT allows user to manipulate (insert/retrieve/remove) item at **any position within the sequence of items**
- There are cases where we only want to consider a few specific positions only
  - E.g. only the first/last position
  - Can be considered as special cases of list
- Stack is one such example:
  - Only manipulation at the **last position** is allowed
- Queue (to be covered later) is another example
  - Only manipulation at the **first** and **last position** are allowed

What is a stack?

- Real life example:
  - A stack of books, a stack of plates, etc.
- It is easier to add/remove item to/from the **top of the stack**
- The latest item added is the first item you can get out from the stack
  - Known as **L**ast **I**n **F**irst **O**ut (LIFO) order
- Major Operations:
  - Push: Place item on top of the stack
  - Pop: Remove item from the top of the stack
- It is also common to provide:
  - Top: Take a look at the topmost item without removing it

Stack : Illustration

Top of stack (accessible)



Bottom of stack (inaccessible)

A **stack** of four books

Push a new book on top

Pop a book from top

Stack ADT: C++ Specification

```
template <typename T>
class Stack {
   public:
     Stack();

     bool isEmpty() const;
     int size() const;

     void push(const T& newItem) throw (SimpleException);

     void pop() throw (SimpleException);
     void pop(T& stackTop) throw (SimpleException);


     void getTop(T& stackTop) const
               throw (SimpleException);
private:
     //Implementation dependent
     //See subsequent implementation slides
};
```

Stack ADT is a template class just like List ADT

Note the 2 versions of **pop()**

Stack ADT: Implementations

- Many ways to implement Stack ADT, we will cover:
  - Linked List implementation:
    - Study the best way to make use of linked list
  - STL vector implementation:
    - Make use of STL container vector
- Learn how to weigh the pros and cons for each implementation

Stack ADT: Design Consideration

- How to choose appropriate implementation?
  - Concentrate on the major operations in ADT
  - Match with data structures you have learned
    - Pick one to be the internal (underlying) data structure of an ADT
    - Can the internal data structure support what you need?
    - Is the internal data structure efficient in those operations?

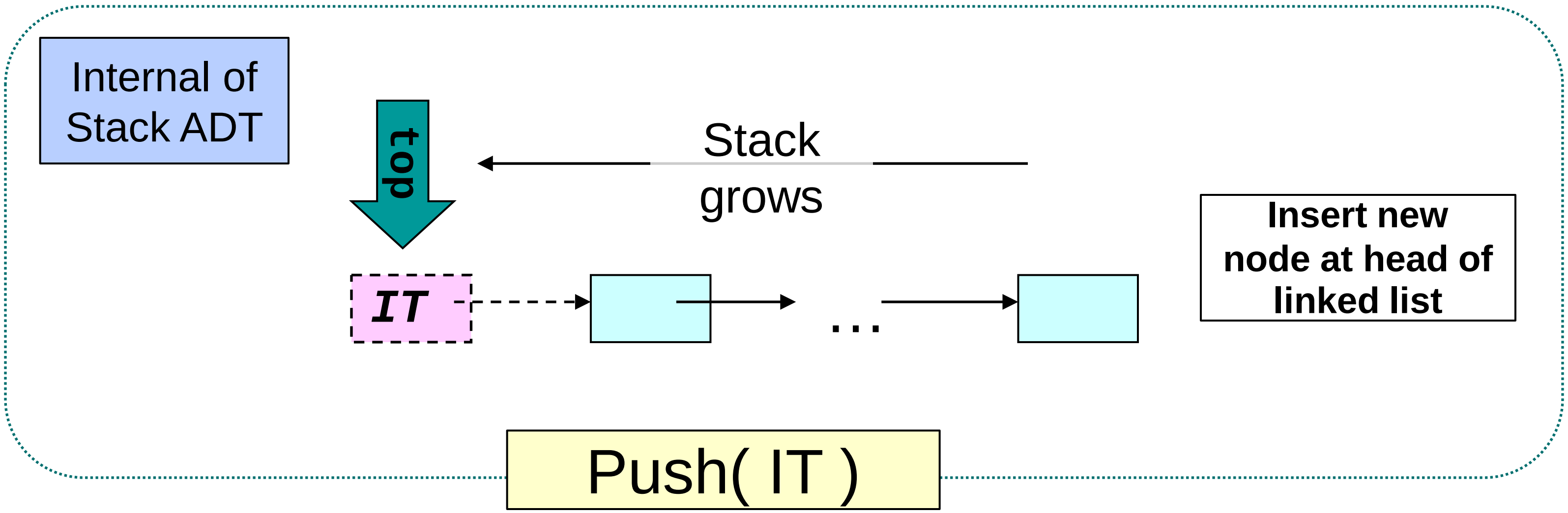
- Internal data structure like array, linked list etc are usually very flexible:
  - Make sure you use them in the best possible way

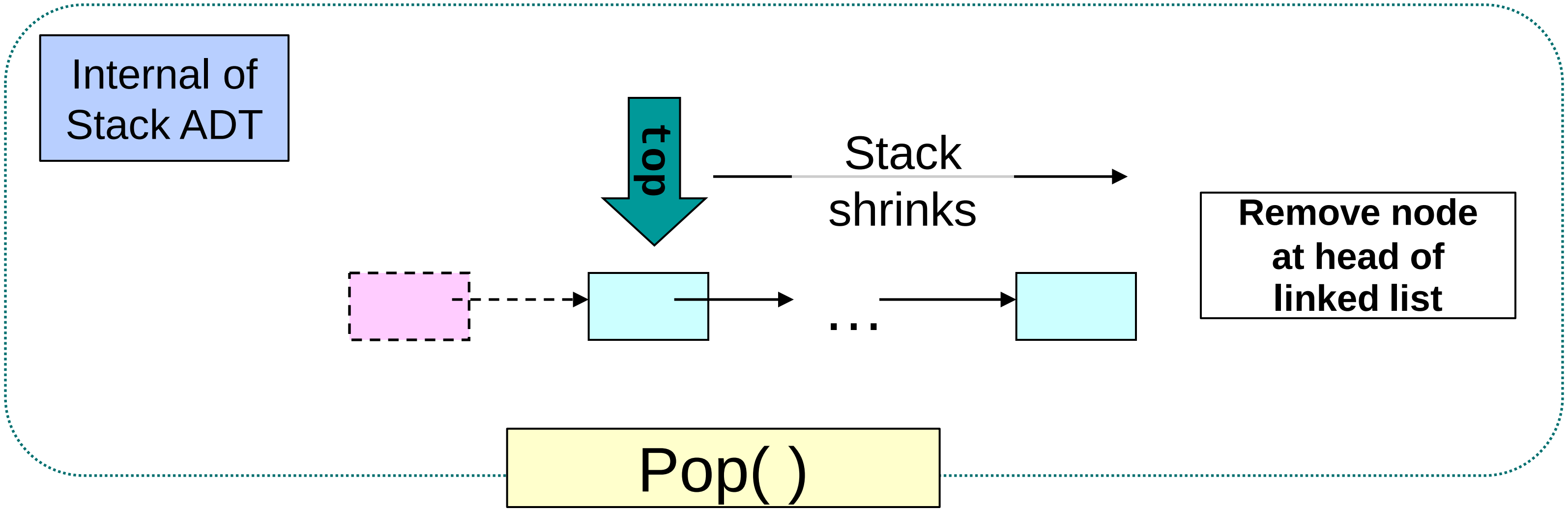
Stack ADT using Linked List

Stack ADT: Using Linked List

- Characteristics of singly linked list:
  - Efficient manipulation of 1st Node:
    - Has a **head** pointer directly pointing to it
    - No need to traverse the list
  - Manipulation of other locations is possible:
    - Need to first traverse the list, less efficient
- Hence, best way to use singly linked list:
  - Use 1st Node as the top of stack
- Question:
  - How would you use **other variations of linked list?**

Stack ADT: Using Linked List (Illustration)





Stack ADT (Linked List): C++ Specification

```
template <typename T>
class Stack {
  public:
    Stack();
    ~Stack()

    bool isEmpty() const;
    int size() const;

    void push(const T& newItem) throw (SimpleException);

    void pop() throw (SimpleException);
    void pop(T& stackTop) throw (SimpleException);


    void getTop(T& stackTop) const
                    throw (SimpleException);


private:
    struct ListNode {
        T item;
        ListNode* next;
    };

    ListNode* _head;
    int _size;
};
```

Need destructor as we allocate memory dynamically

Methods from slide 6. No change.

Similar to Linked List implementation of List ADT

StackP.h

Implement Stack ADT (Linked List): 1/3

```
template<typename T>
Stack<T>::Stack()
: _size(0), _head(NULL) { }

template<typename T>
Stack<T>::~Stack()
{
    while (!isEmpty())
        pop();
}

template<typename T>
bool Stack<T>::isEmpty() const
{
    return _size == 0;
}

template<typename T>
int Stack<T>::size() const
{
    return _size;
}
```

Make use of own methods to clear up the nodes

StackP.cpp

Implement Stack ADT (Linked List): 2/3

```
template<typename T>
void Stack<T>::pop(T& stackTop)
    throw (SimpleException)
{
    ListNode* cur;

    if ( isEmpty() )
        throw SimpleException("Stack is empty on pop()");
    else {

        stackTop = _head->item;

        cur = _head;
        _head = _head->next;

        delete cur;
        cur = NULL;
        _size-- ;

    }
}
```

As we only remove from head position. General removal code not needed.

StackP.cpp

Implement Stack ADT (Linked List): 3/3

```
template<typename T>
void Stack<T>::push(const T& newItem)
    throw (SimpleException)
{
    ListNode* newPtr = new ListNode;

    newPtr->item = newItem;
    newPtr->next = _head;
    _head = newPtr;
    _size++;

}

template<typename T>
void Stack<T>::getTop(T& stackTop) const
    throw (SimpleException)
{
    if ( isEmpty() )
        throw SimpleException("Stack is empty on getTop()");
    else
        stackTop = _head->item;
}
```

As we only insert at head position. General insertion code not needed.

StackP.cpp

Stack ADT using STL vector

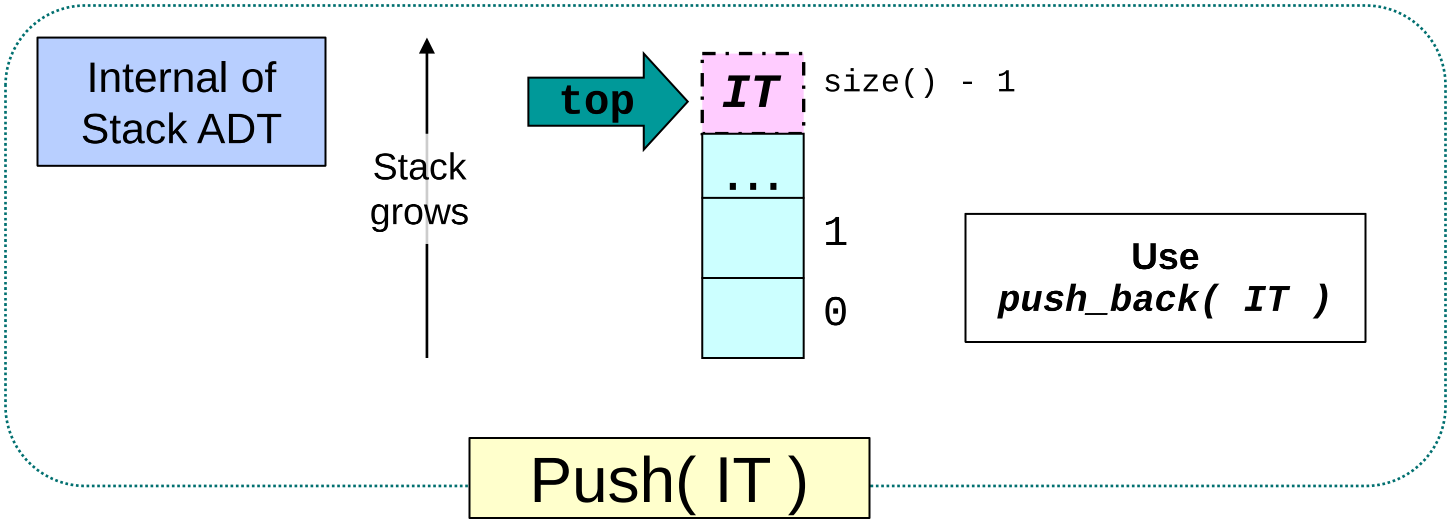
STL Vector can simplify your code

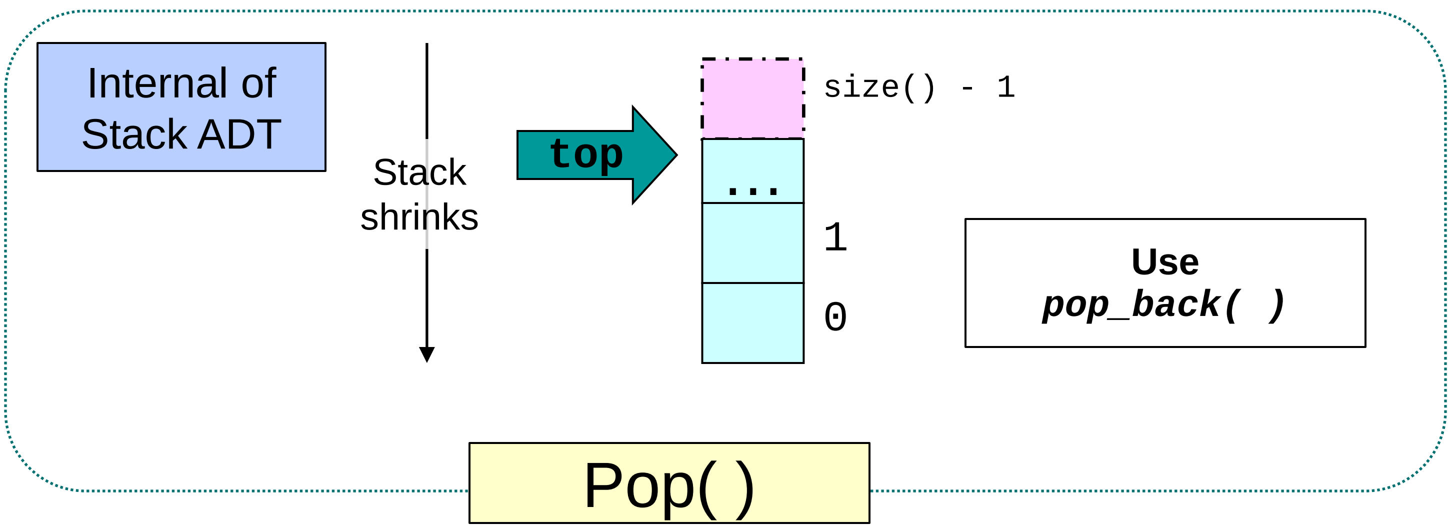
Stack ADT: Using STL Vector

- STL Vector has the following capabilities:
  - Add/remove the last item
    - ***push_back()*** and ***pop_back()***
    - Very efficient
  - Use iterator to add/remove item from any location
    - Not efficient
    - Quite cumbersome (need to setup and move iterator)

- What Stack ADT needs:
  - Add/Remove from **top of stack**
    - **No manipulation of other locations**
  - Hence, to make the best use of STL Vector:
    - Use the **back of vector** as the **top of stack**

Stack ADT: Using STL Vector (Illustration)





Stack ADT (STL vector) : C++ Specification

```
#include <vector>
using namespace std;

template <typename T>
class Stack {
   public:
     Stack();

     bool isEmpty() const;
     int size() const;

     void push(const T& newItem) throw (SimpleException);

     void pop() throw (SimpleException);
     void pop(T& stackTop) throw (SimpleException);


     void getTop(T& stackTop) const
                          throw (SimpleException);


private:
      vector<T> _items;
};
```

We need STL Vector.

Methods from slide 6. No change.

The only private declaration.

StackV.h

Implement Stack ADT (STL vector) : 1/2

```
template<typename T>
Stack<T>::Stack()
{ }

template<typename T>
bool Stack<T>::isEmpty() const
{
    return _items.empty();
}

template<typename T>
int Stack<T>::size() const
{
    return _items.size();
}

template<typename T>
void Stack<T>::push( const T& newItem )
    throw (SimpleException)
{
    _items.push_back(newItem);
}
```

No need to initialize anything.

We use **methods from vector** class to help us

StackV.cpp

Implement Stack ADT (STL vector) : 2/2

```
template<typename T>
void Stack<T>::getTop(T& stackTop) const
    throw (SimpleException)
{
    if ( _items.empty() )
        throw SimpleException("Stack is empty on getTop()");
    else
        stackTop = _items.back();
}

template<typename T>
void Stack<T>::pop(T& stackTop)
    throw (SimpleException)
{
    if ( _items.empty() )
        throw SimpleException("Stack is empty on pop()");
    else {
        stackTop = _items.back();
        _items.pop_back();
    }
}
```

Code for the other version of **`pop()`** is not shown as it is very similar.

`StackV.cpp`

STL Stack

STL has a built-in stack ADT

STL Stack: Specification

```
template <typename T>
class stack {
public:
      bool empty() const;
      size_type size() const;
      T& top();
      void push(const T& t);
      void pop();
};
```

- Very close to our own specification ☺
- See example which highlights the differences
  - Especially the `top()` method

STL Stack: Example Usage

```
#include <stack>
#include <iostream>
using namespace std;

int main()
{
  stack<int> s;

   s.push(5);
   int &j = s.top();

   s.push(3);

   j++;
   cout << "top: " << s.top() << endl;

   s.pop();
   cout << "After pop, top: " << s.top() << endl;
     ... ...
}
```

Output:

```
top: 3
After pop, top: 6
```

Stack Applications

LIFO? Is it good for anything?

Stack Applications

- Many useful applications for stack:
  - Bracket Matching
  - Towers of Hanoi
  - Maze Exploration
- More "Computer Science" inclined examples (for your own reading):
  - Base-N number conversion
  - Postfix evaluation
  - Infix to postfix conversion

Stack Application 1

Bracket Matching

Bracket Matching: Description

- Mathematical expression can get quite convoluted:
  - E.g. { [x+2(i-4!)]^e+4π/5*(φ-7.28) ……

- We are interested in checking whether all brackets are matched correctly (with), [with] and { with }

- Bracket matching is equally useful for checking programming code

Bracket Matching: Pseudo-Code

- Go through the input string character by character
  - Non-bracket characters
    - Ignore
  - Open bracket { , [or (
    - Push into stack
  - Close bracket },] or)
    - Pop from stack and check
    - If stack is empty or the stack top bracket does not agree with the closing bracket, complain and exit
    - Else continue
- If the stack is not empty after we read through the whole string
  - The input is incorrect

Bracket Matching: Implementation (1)

```
bool check_bracket( string input )
{
     stack<char> sc;
     char current;
     bool ok = true;

     for (unsigned int pos = 0;
                              ok && pos < input.size(); pos++){
        current = input[pos];
        switch (current){
            case '{':
                sc.push('}'); //Question: Why are we pushing the
                break;        //         closing bracket here??
            case '[':
                sc.push(']');
                break;
            case '(':
                sc.push(')');
                break;
            ... ...
```

Bracket Matching: Implementation (2)

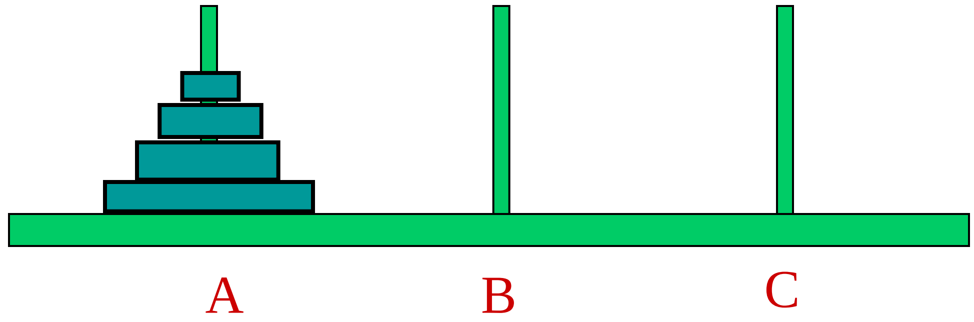
```
            case '}':
            case ']':
            case ')':
                if (sc.empty())      //missing open bracket
                   ok = false;
                else {
                     if (sc.top() == current)//matched!
                        sc.pop();
                     else                   //mismatched!
                         ok = false;
                }
                break;
        }
    }

    if (sc.empty() && ok){      //make sure no leftover
       return true;
    } else
       return false;
}
```

Stack Application 2

Towers of Hanoi

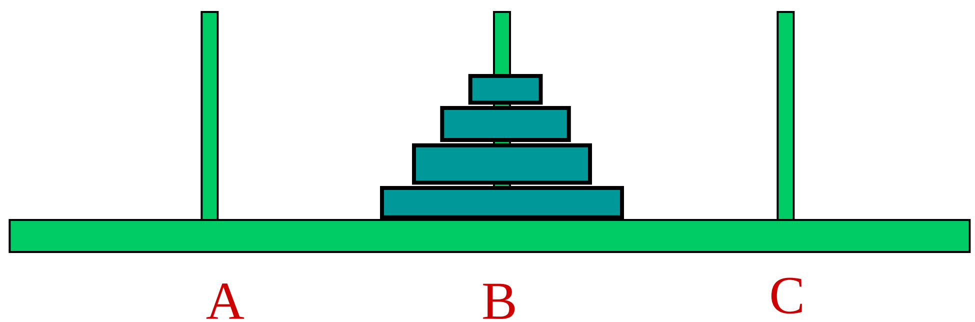
Towers of Hanoi : Description

initial state



- How do we move all the disks from pole "**A**" to pole "**B**", using pole "**C**" as temporary storage
  - One disk at a time
  - Disk must rest on top of larger disk

final state



Each pole is a stack…

- We are not writing a program to solve the puzzle automatically
- Just want a simple program to allow a user to play the puzzle instead
  - Keep track of the discs
  - Check movement
  - Display the current state
  - etc
- Since we can only
  - Remove the topmost disc from a pole, then
  - Place the disc on top of other pole
- Clearly, each pole is a stack

Towers of Hanoi: Header File

```
#include <stack>

using namespace std;

class ToH
{
public:
    ToH(unsigned int nDiscs = 10);

    bool move(int from, int to);

    void display();

private:
    stack<int> _poles[3];
    unsigned int _nDiscs;

};
```

ToH.h

Towers of Hanoi: Implementation 1/2

```
ToH::ToH(unsigned int nDiscs)
{
    _nDiscs = nDiscs;

    for (int i = _nDiscs; i > 0; i--)
        _poles[0].push(i);
}

ToH::display()
{
   //code not shown, left as a challenge
}
```

ToH.cpp

Towers of Hanoi: Implementation 2/2

```
bool ToH::move(int from, int to)
{
     if ( from == to )
        return true;     //pretend we have moved

     if ( from < 0 || from > 2 || to < 0 || to > 2 )
        return false;

     if ( _poles[from].empty() )
        return false;

     if ( (!_poles[to].empty()) &&
              ( _poles[from].top() > _poles[to].top()) )
        return false;

     _poles[to].push( _poles[from].top() );
     _poles[from].pop();

     return true;
}
```

ToH.cpp

Towers of Hanoi: Sample Usage

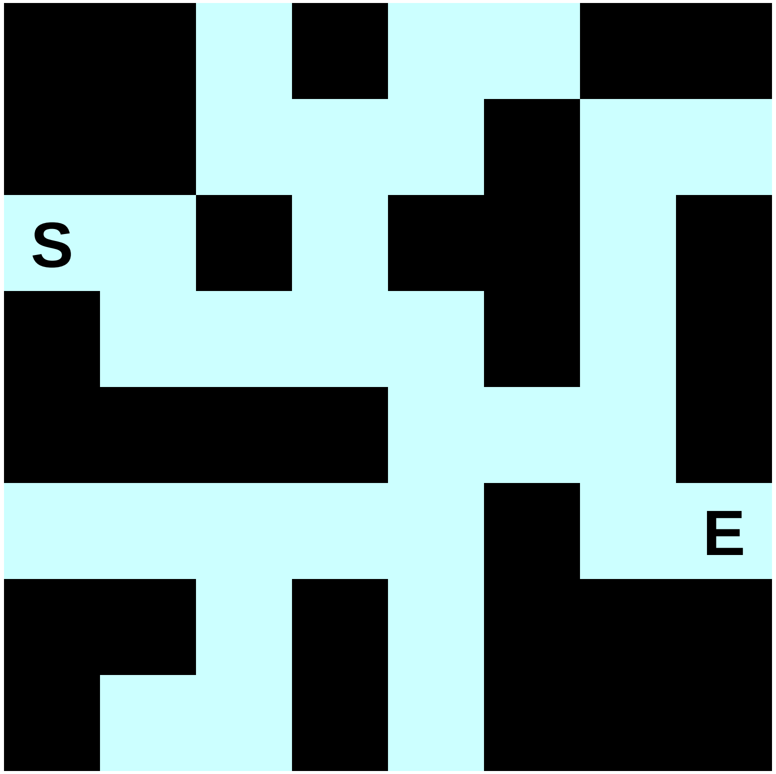
A simple program to allow a human player to play the puzzle.

```
... ...
int main()
{
    ToH t(3);
    int from, to;

    t.display();
    do {
        cout << "Move from: ";
        cin >> from;
        cout << "To: ";
        cin >> to;

        if (from != -1 && to != -1){
            if (t.move(from, to)){
               cout << "Move ok!" << endl;
            } else
               cout << "Cannot move!!" << endl;
        }
        t.display();
    } while (from != -1 && to != -1);
    ... ...
}
```

Stack Application 3

Exploring a Maze

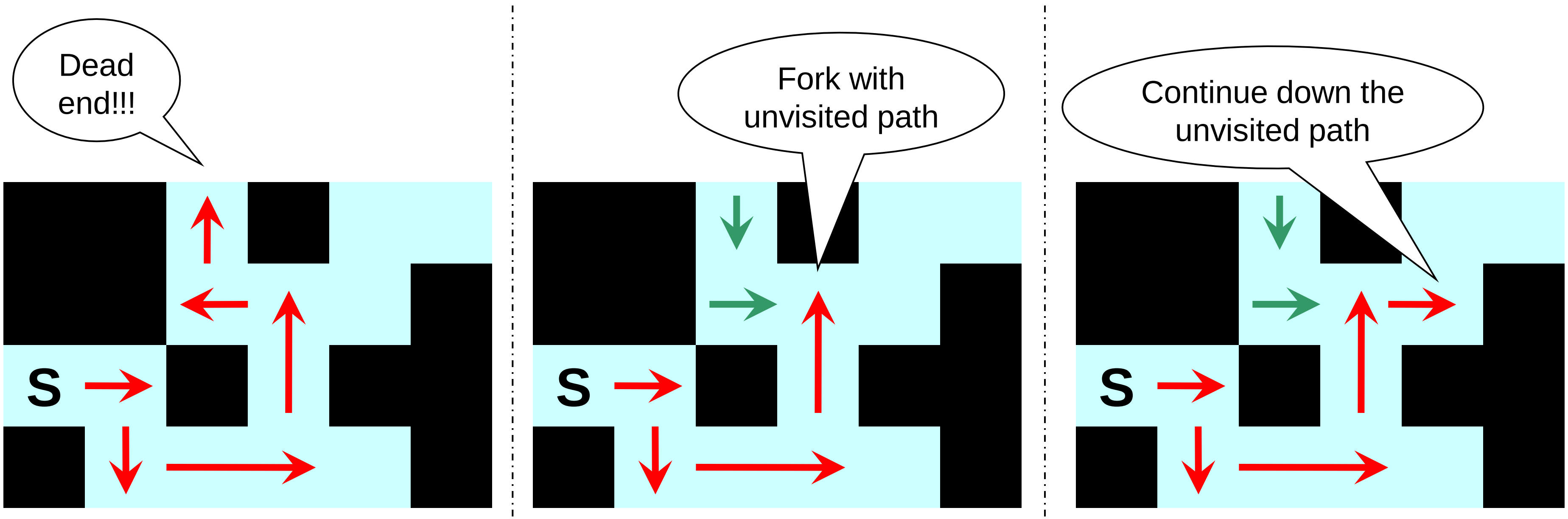
Exploring Maze: Description



- How to define an algorithm that **always** get you from **S** to **E** (as long as there is a path)?
  - What should you do when you reach a dead end?

Exploring Maze: Some ideas

- When we reach a dead end
  - Restarting from **S** is usually not a good idea
- Instead, we retrace our steps until:
  - the most recent fork which has an unvisited path
  - take the unvisited path and continue exploration



Exploring Maze: Some design issues

- The maze is represented as an N x N 2d array
  - Each square has a unique **`(row, column)`** coordinate
- Let's encode the directions of movement as an enumeration:

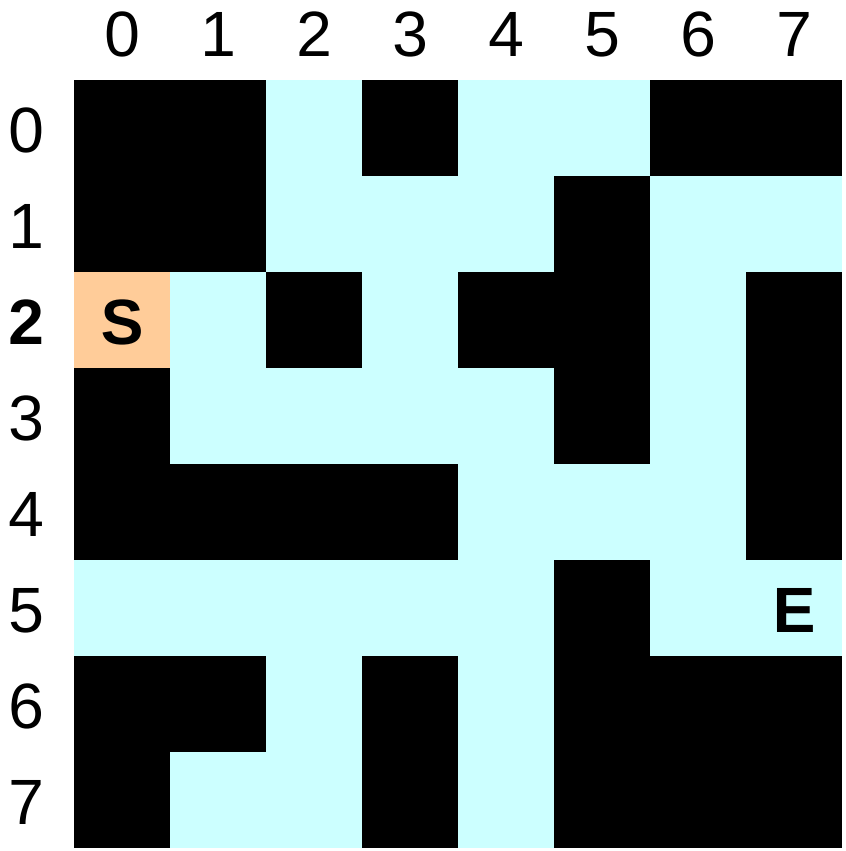
```
enum Direction {Up, Left, Down, Right, NoDir};
```

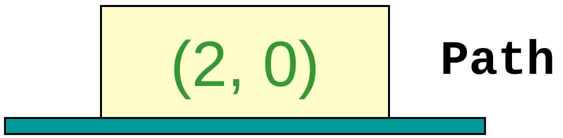
- Each square will know about:
  - Which direction is unvisited
  - Assume a method **`getUnvisitedDir()`** is implemented for this purpose
- When a square has multiple unvisited exits:
  - We visit them in the order of the **`enum`** above
- The path travelled is kept as a **stack of coordinates**

Exploring Maze: Pseudo Code

```
1.  Path = empty
2.  done = false      //are we at the end yet?
3.  Path.push(coordinate of S)
4.  while (Path is not empty && not done)
      CurSq = Path.top( )   //where are we now?
      NewDir = CurSq.getUnvisitedDir( )
      if (NewDir == NoDir)        //dead end!
         Path.pop( )              //move back one square
      else                        //there is an exit
         NewSq = CurSq.move(NewDir)
         Path.push(coordinate of NewSq)
         if (NewSq == E)          //Yes! We reached the end!
            done = true
```

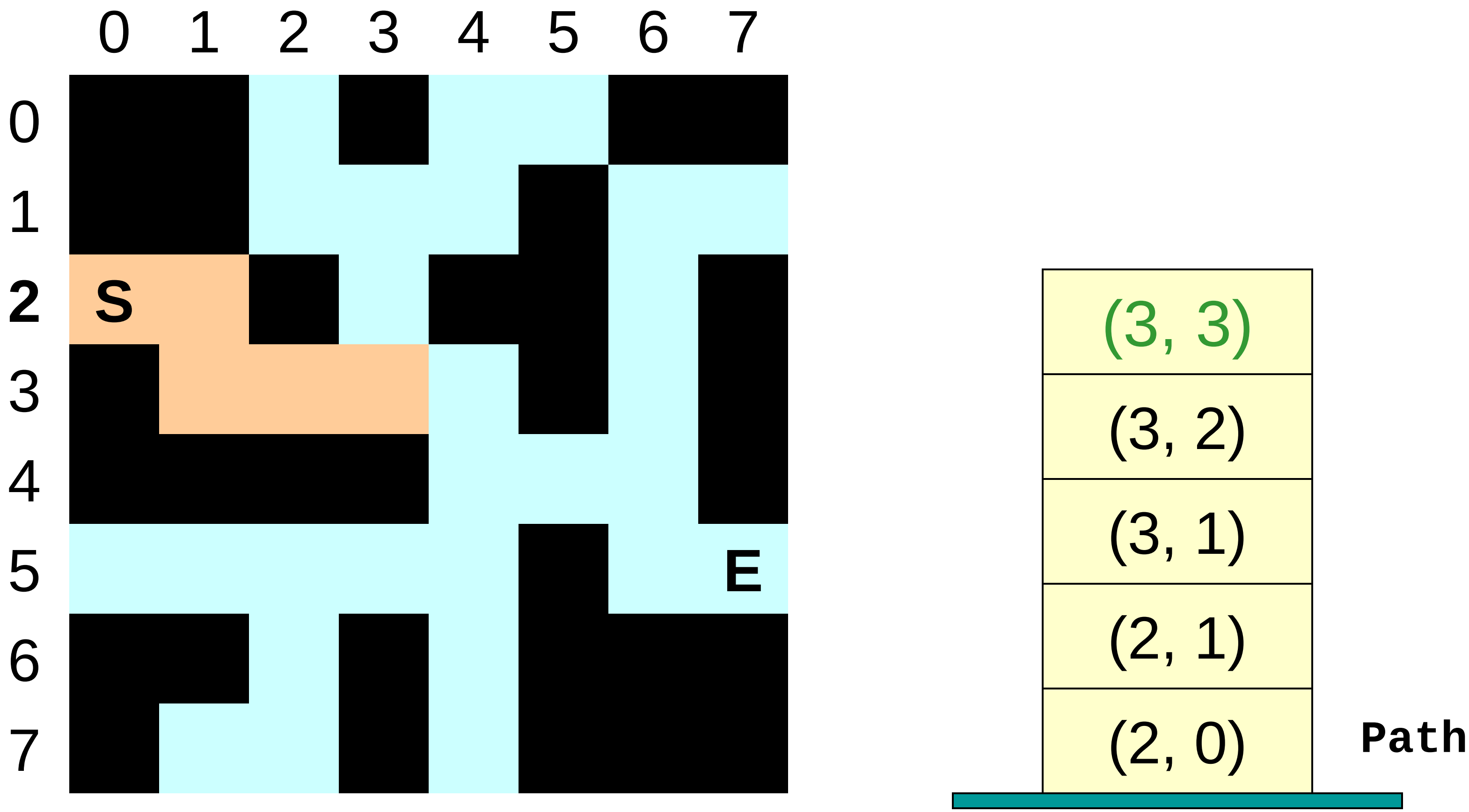
Exploring Maze: Test Run …. (1)





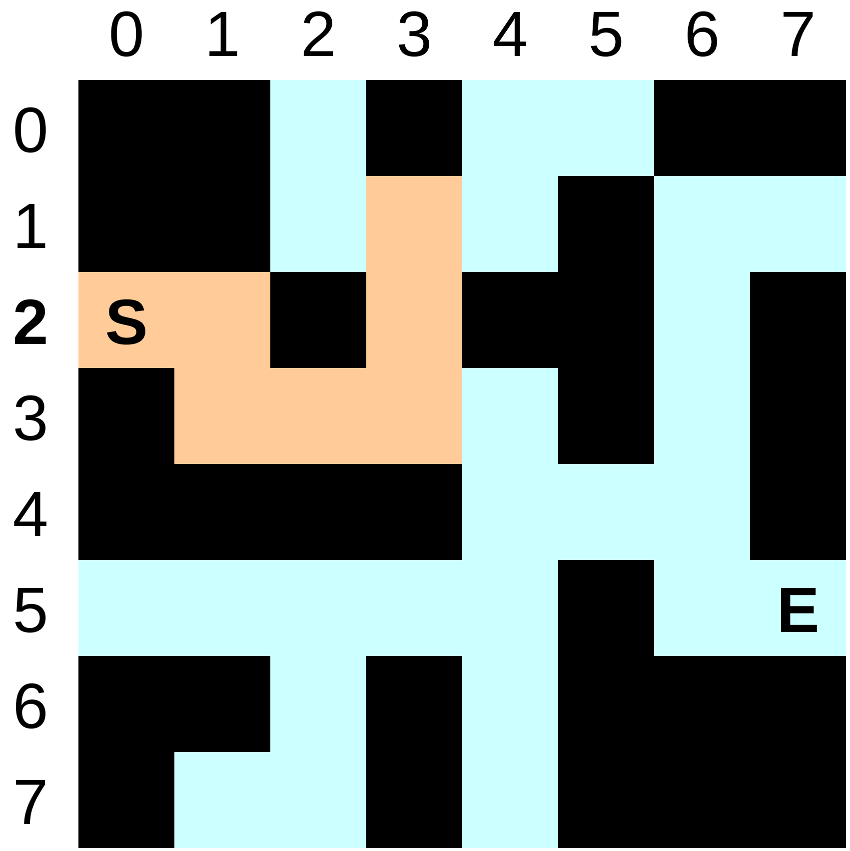
- Just started at (2, 0)

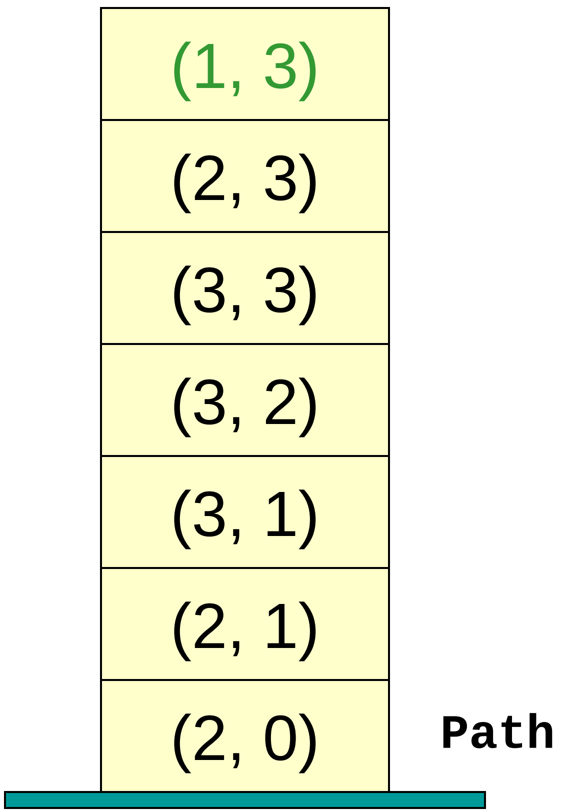
Exploring Maze: Test Run ….(2)



- (3, 3) is the first square with multiple exits
  - As stated, we will first try to go Up

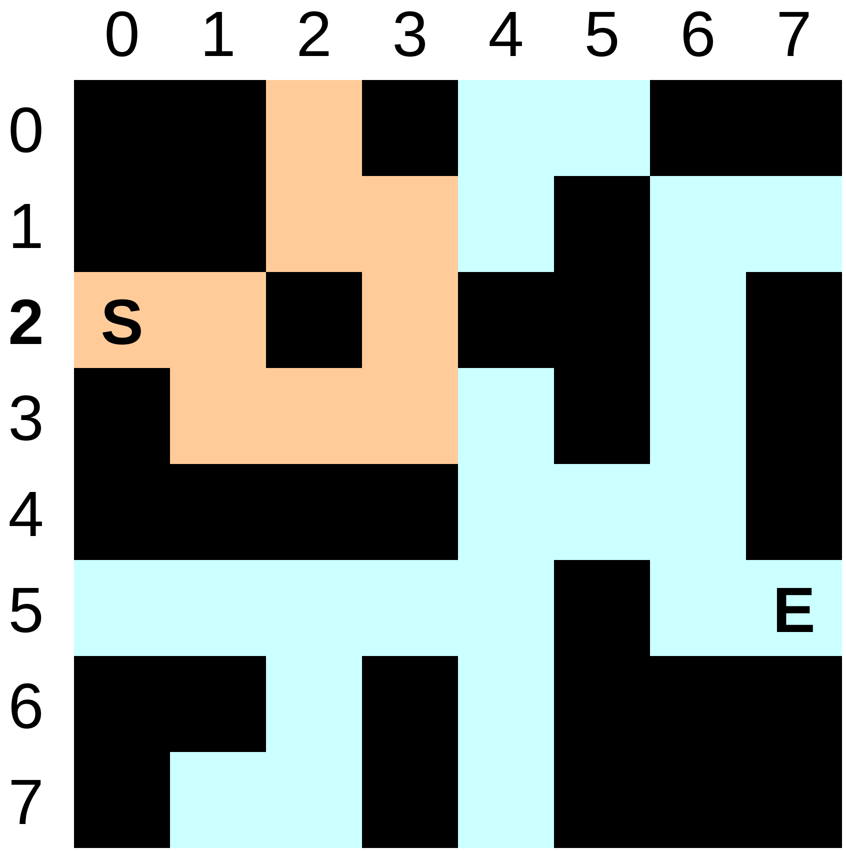
Exploring Maze: Test Run ….(3)

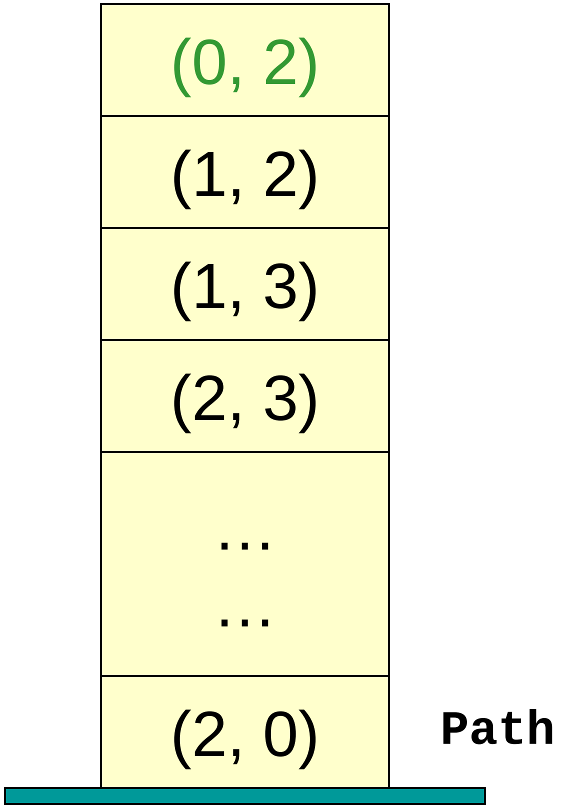




- Multiple exits at (1, 3)
  - go Up is impossible, so go Left is the 2nd choice

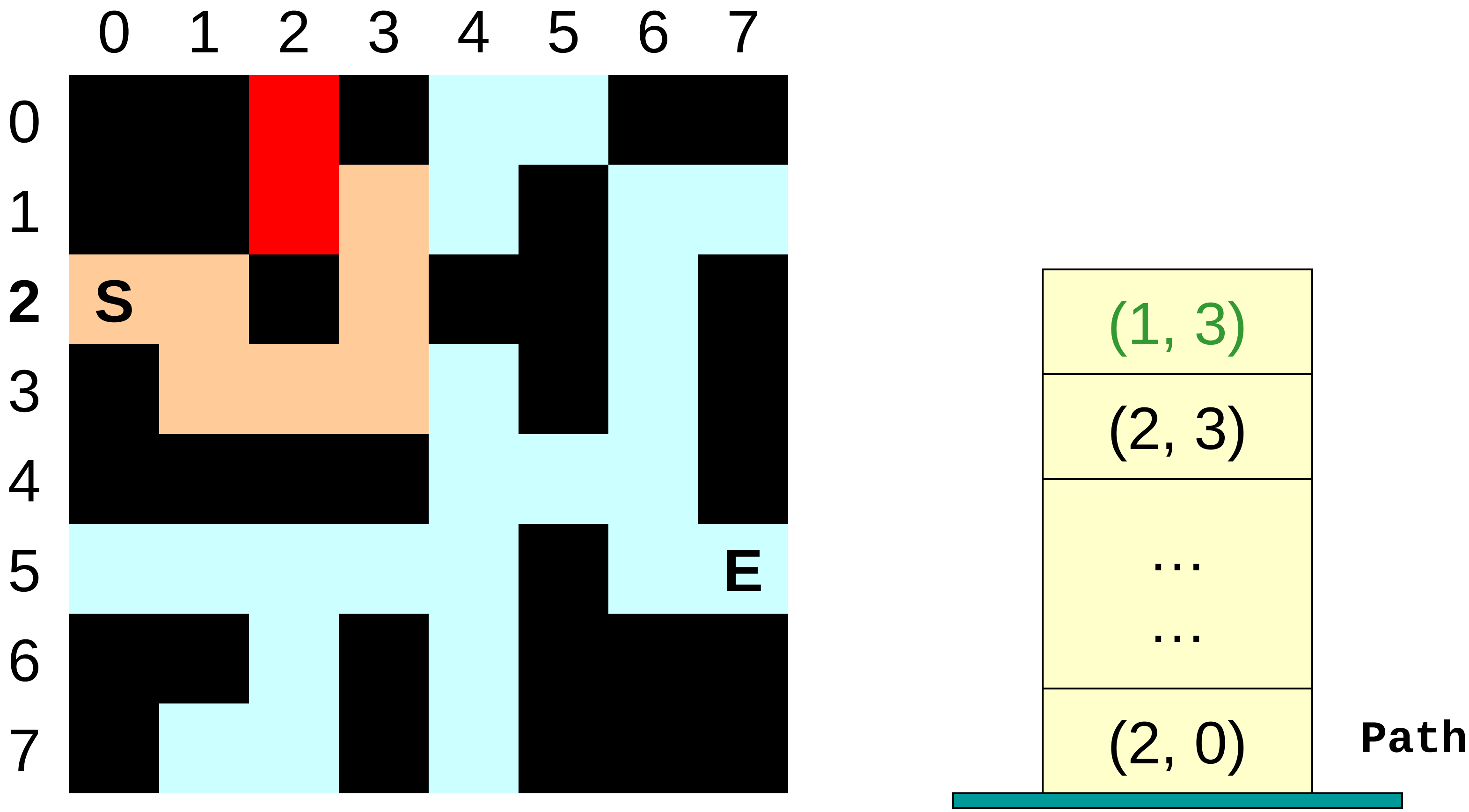
Exploring Maze: Test Run ….(4)





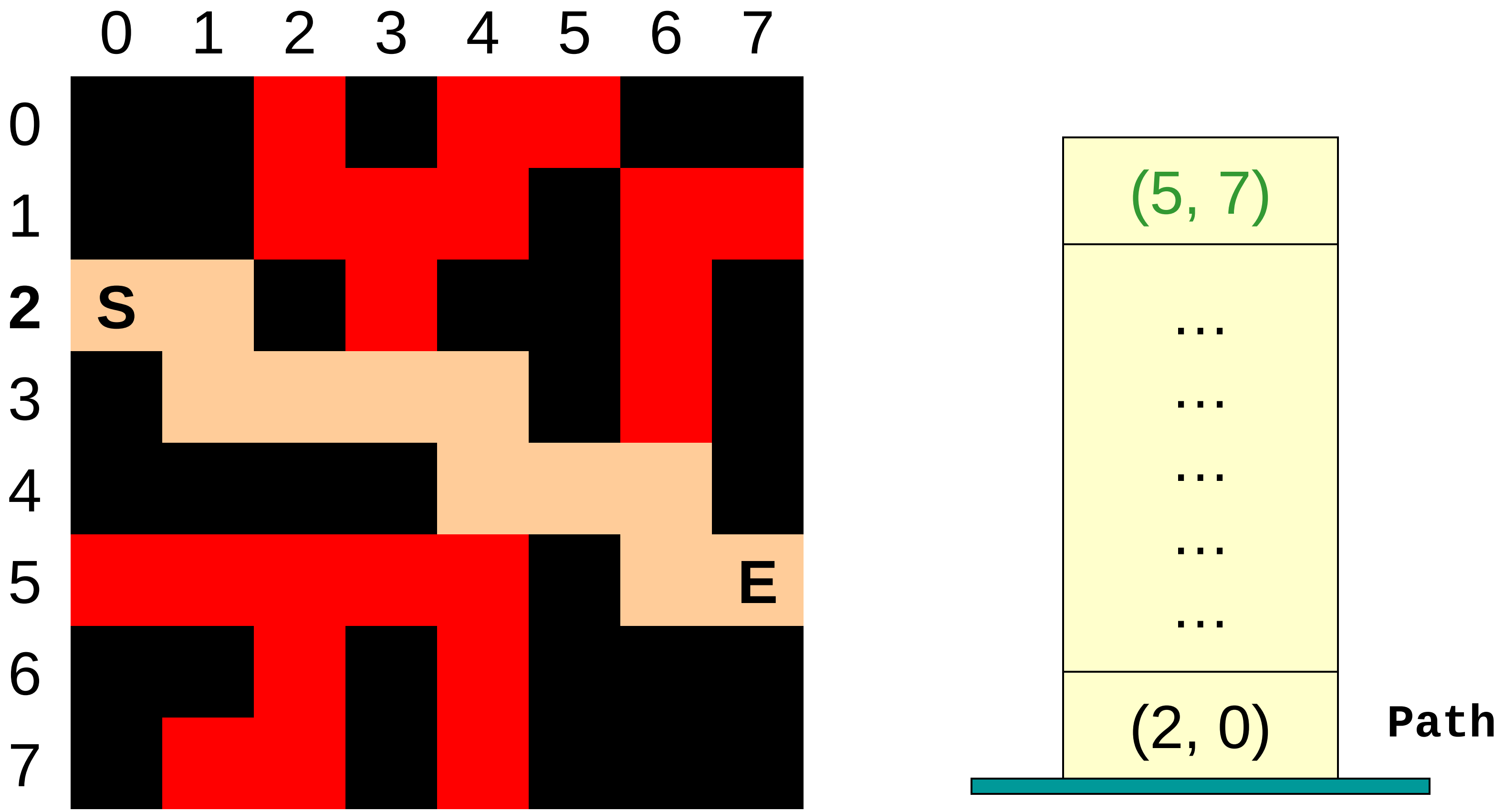
- No exits from (0, 2)
  - Back trace: pop until a square with unvisited exits

Exploring Maze: Test Run ….(5)



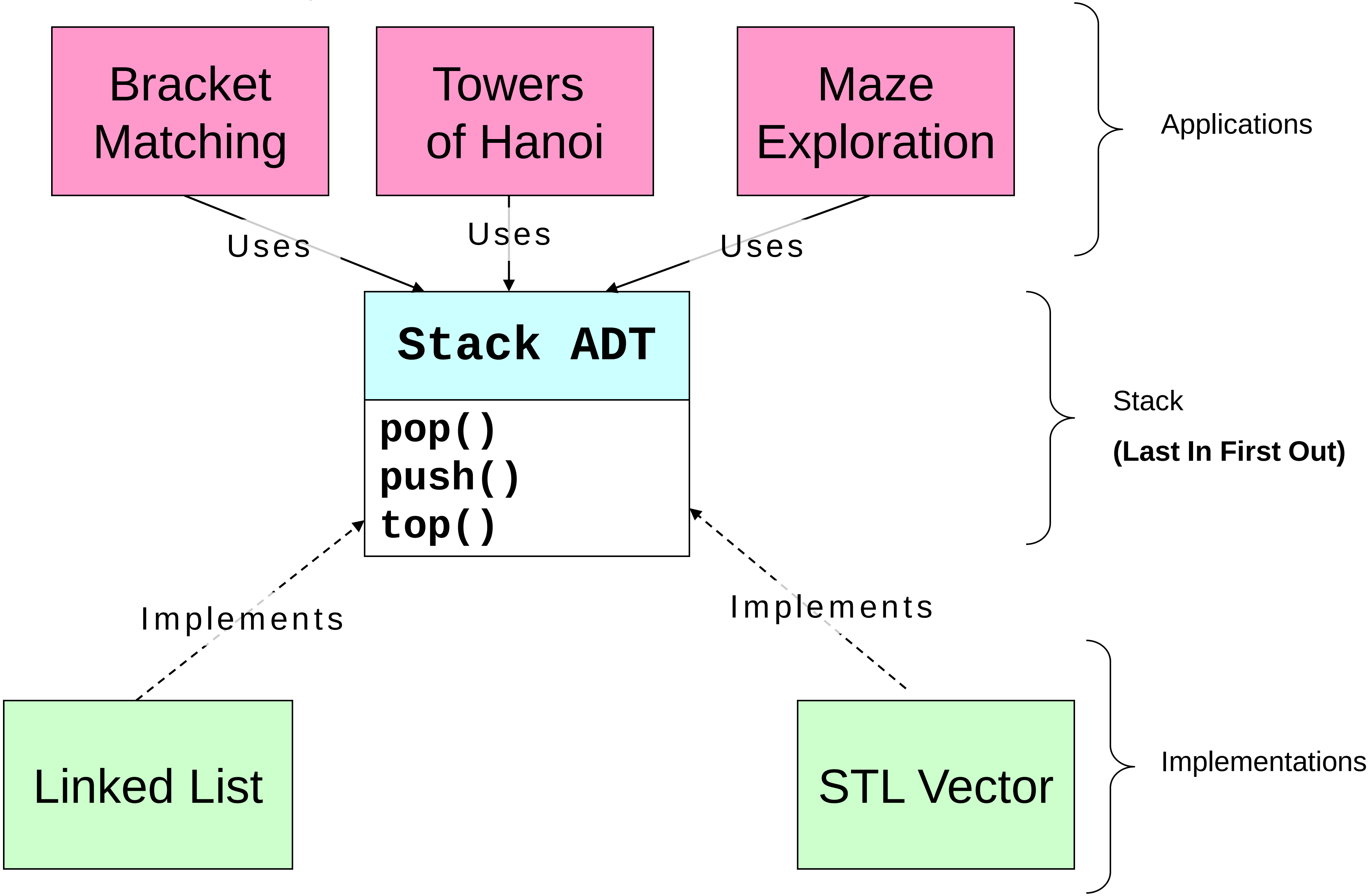
- Back to (1,3) after several pops
  - Up, Left, Down all impossible, going Right to (1,4)

Exploring Maze: Test Run (much later)



Note: algorithm travelled the whole maze to find the exit

Summary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