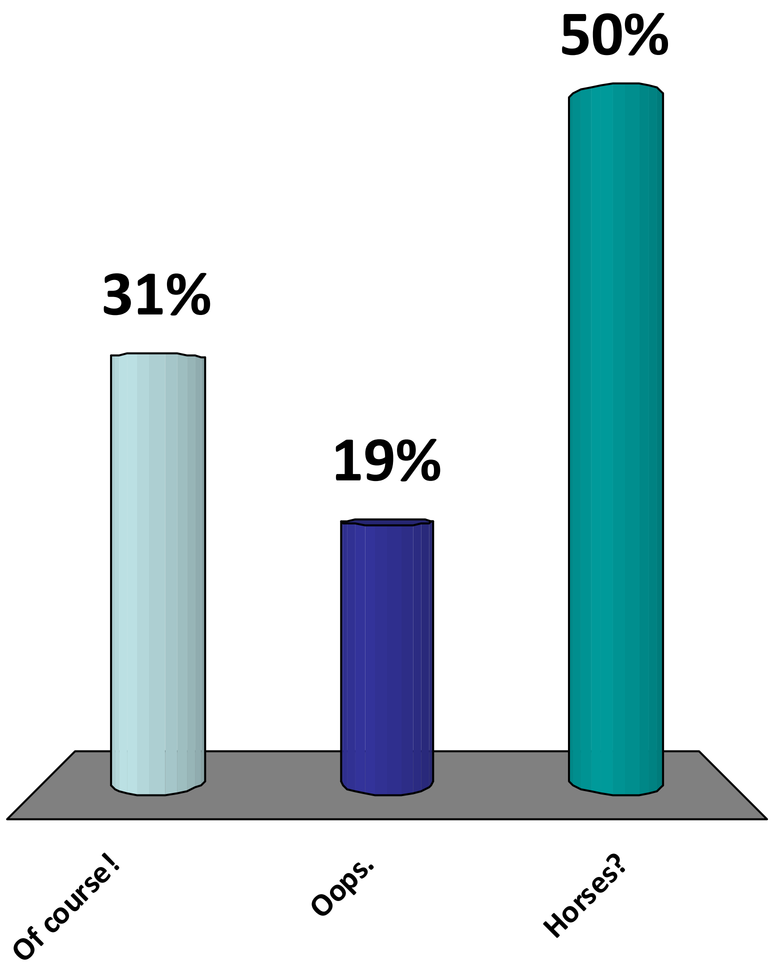
CS2020
Data Structures and Algorithms

Welcome!

I remembered to bring my clicker to class?

1. Of course!
2. Oops.
3. Horses?



Administrativia

Problem Set 2

- Due last night

Problem Set 3

Problem Set 3

Exercise: Sorting

Problem Set 3

Problem 1: Sorting Detective

- Six suspicious sorting algorithms
  - Investigate the mysterious sorting code.
  - Identify each sorting algorithm.
  - Find the criminal: Dr. Evil!

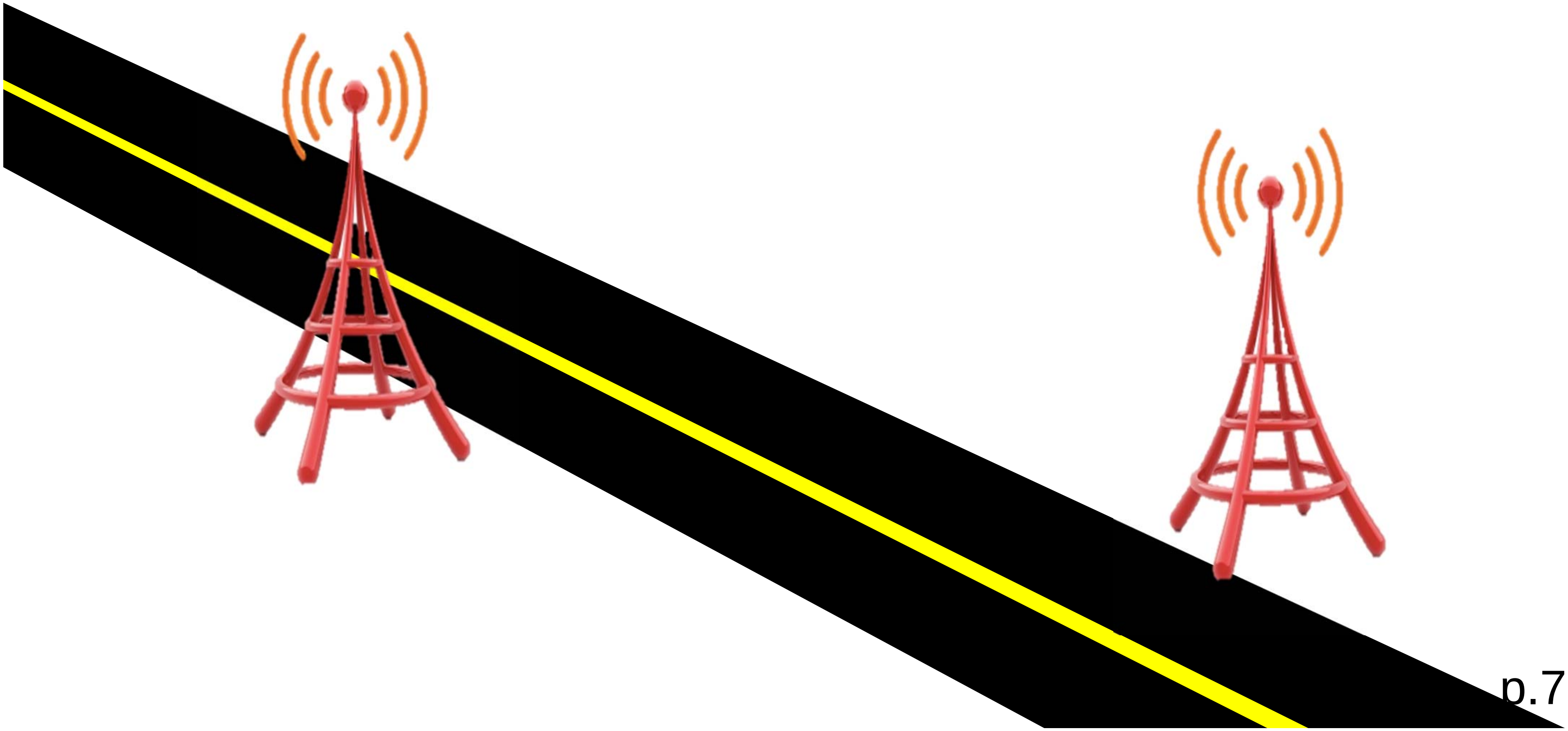
- Focus on the properties:
  - Asymptotic performance
  - Stability
  - Performance on special inputs
- Absolute speed is not a good reason…

(Material covered for this problem on Friday)

Problem Set 3

Problem 2: Cellular Coverage

- Build a data structure to help choose the best locations to deploy cellular towers.



Problem Set 3

Problem 2: Cellular Coverage

- Build a data structure to help choose the best locations to deploy cellular towers.
  - Given a set of towers, what is the coverage?
- Try small examples:
  - What about two towers that don't overlap?
  - What about two towers that do overlap?
- Hint (in the question):
  - Why should Tower implement Comparable?

Two Weeks of CS2020

Two Weeks of CS2020

Java

- Basic syntax
- Classes
- Interfaces
- Objects
- Access control (public/private/protected)
- Generics
- Etc.

Two Weeks of CS2020

Object-oriented programming

- Encapsulation
- Modularity
- Importance of interfaces
- Inheritance

Two Weeks of CS2020

Algorithms

- Theory:
  - Big-O notation
  - Sequential model of computation
  - Intro to algorithm analysis

- Binary search:
  - Basic approach
  - 1d Peak Finding
  - 2d Peak finding
  - Herbert

Why peak finding?

Why peak finding?

1. Common optimization technique: find a locally optimal solution.

Why peak finding?

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2. Binary search example: how can you apply techniques *similar* to binary search to other problems.

Why peak finding?

1. Common optimization technique: find a locally optimal solution.

2. Binary search example: how can you apply techniques *similar* to binary search to other problems.

3. Neat progression of algorithms: shows the steps you might go through to solve a difficult problem (e.g., d-dimensional peak finding).

Two Week Survey

[How are things going so far?]

Topics to cover

Generating effective test cases and debugging

Loop invariants

More searching

Proper documentation / Javadocs

Inner classes

Topics to cover

Generating effective test cases and debugging

Problem Set 3

Loop invariants

Today (and onwards)

More searching Yes

Proper documentation / Javadocs

Slowly over the semester.

Inner classes In 2 weeks.

Technical Questions

Important Question!

When should I use binary search?

- Look for monotonicity:
  - As I increase something, there is something else that always increases.
  - As I decrease something, there is something else that always decreases.
- Look for maximization/minimization/search problems.
- Look for problems where you can bound the thing you are searching for.

Java Questions

"Coming to Java from Python, I feel quite out of place. … I find myself constantly referring to javadocs or google for ways to do trivial things [that are easy in Python]."

"Working with arrays and strings because [what I expect to work always] fails in Java"

"Why is Java so painful to work with compared to Python?"

Problem Set Questions (from survey)

How long should I spend on them?

- Huge variance: 2-8 hours
- Talk to tutors for advice on how to go faster.

Will you go over problem set solutions?

- Go over problem sets in Discussion Groups. (Ask your tutor questions.)
- Discuss solutions on Nota Bene (after the due date).
- It's ok if you are still confused by some problems!

Technical Questions

Important Question!

How are classes and interfaces used differently in Java?

Classes

- Contain implementation
- Contains state
- Mixed public and private
- Objects instantiate classes.
- Classes implement interfaces.
- An object can inherit from only one parent class.

Interfaces

- **NO** implementation
- No state
- All public
- Cannot be instantiated.
- Defines how to interact with an object.
- An object may implement many interfaces.

Technical Questions

What exactly is a package or a project?

- A collection of classes / interfaces / etc. that are stored together in a common place.
- Put all your CS2020 classes in one package.
- In Eclipse, a project is an organizational construct that holds related code.
- An Eclipse project can have code from different packages.

Two Weeks of CS2020

And on to Week 3!

Today: Sorting

- Writing a sorting algorithm in Java
- Sorting algorithms
  - BubbleSort
  - SelectionSort
  - InsertionSort
  - MergeSort
- Properties
  - Running time
  - Space usage
  - Stability

Sorting

Problem definition:

Input: array A[1..n] of words / numbers

Output: array B[1..n] that is a permutation of A such that:

$B[1] \leq B[2] \leq \ldots \leq B[n]$

Example:

$A = [9, 3, 6, 6, 6, 4] \rightarrow [3, 4, 6, 6, 6, 9]$

Sorting

```java
public interface ISort{

    public void sort(int[] dataArray);

}
```

Sorting Widgets

```
public interface ISortWidgets{

    public void sort(Widget[] dataArray);

}
```

```
public void sort(Widget[] dataArray) {

   Widget x = dataArray[0];
   Widget y = dataArray[1];
   if (x < y) {
```

Generic Types

```
public interface ISort<TypeA>{

    public void sort(TypeA[] dataArray);

}
```

What goes wrong?

```
public void sort(TypeA[] dataArray) {

    TypeA x = dataArray[0];
    TypeA y = dataArray[1];

    if (x < y) {
        ...
        ...
    }
```

What goes wrong?

```
public void sort(TypeA[] dataArray) {

   TypeA x = dataArray[0];
   TypeA y = dataArray[1];

   if (x < y) {
       ...
       ...
   }
```

Illegal comparison!

What if: TypeA == Student?

```
class Student {
    double m_CAP;
    String m_name;
    Matric m_id;
}
```

Comparable Interface

```
class Student implements Comparable<Student> {
...
...
}
```

```
interface Comparable<TypeA> {

    int compareTo(TypeA other);

}
```

Comparable Interface

x.compareTo(y) :

-1: if (x<y)

0: if (x == y)

1: if (x>y)

Must define a total ordering
Must be transitive.

```
interface Comparable<TypeA> {

    int compareTo(TypeA other);

}
```

Sorting Students, again

```
class Student implements Comparable<Students> {
...
...
}
```

```
public void sort(Student[] dataArray) {

   Student x = dataArray[0];
   Student y = dataArray[1];

   if (x.compareTo(y) < 0) {  // if (x<y)
```

Implementing Comparable

```
class Student implements Comparable<Student> {
// compare students by CAP
int compareTo(Student other){
   if (this.getCAP() < other.getCAP())
        return -1;
   else if (this.getCAP() > other.getCAP())
        return 1;
   else // equal CAP
        return 0;
   }
}
```

Almost works…

```
public interface ISort{

    public void sort(Comparable[] dataArray);

}
```

Comparable to what?

```
public interface ISort{

    public void sort(Comparable<ZZZ>[] dataArray);

}
```

Generic Sorting

```
public interface ISort<TypeA extends Comparable<TypeA>>
{
    public void sort(TypeA[] dataArray);
}
```

Generic Sorting

```
public interface ISort<TypeA extends Comparable<TypeA>>
{
    public void sort(TypeA[] dataArray);
}
```

extends, not **implements**!!

weird… no good reason... a mystery…

Generic Sorting

```
public interface ISort<TypeA extends Comparable<TypeA>>
{
    public void sort(TypeA[] dataArray);
}
```

Generic Sorting

```
public interface ISort{

    public <TypeA extends Comparable<TypeA>>
    void sort(TypeA[] dataArray);

}
```

Sorting

```
public <TypeA extends Comparable<TypeA>>
void sort(TypeA[] dataArray) {
    for (int i=0; i<dataArray.length; i++){
        for (int j=0; j<dataArray.length-1; j++){
            TypeA first = dataArray[j];
            TypeA second = dataArray[j+1];
            if (first.compareTo(second) > 0)
                swap(dataArray, j, j+1);
        }
    }
}
```

Generic Sorting

```
Student[] dataArray = new Student[100];
sort(dataArray);
```

```
class Student implements Comparable<Student> {
    int compareTo(Student other) {
        …
    }
}
```

Generic Sorting

```
Emotion[] dataArray = new Emotion[100];
sort(dataArray);
```

Error!

```
class Emotion {
    int compareTo(Emotion other) {
        …
    }
}
```

Which of the following correctly implements:

boolean stringsEqual(String A, String B)

A. `return (A == B);`

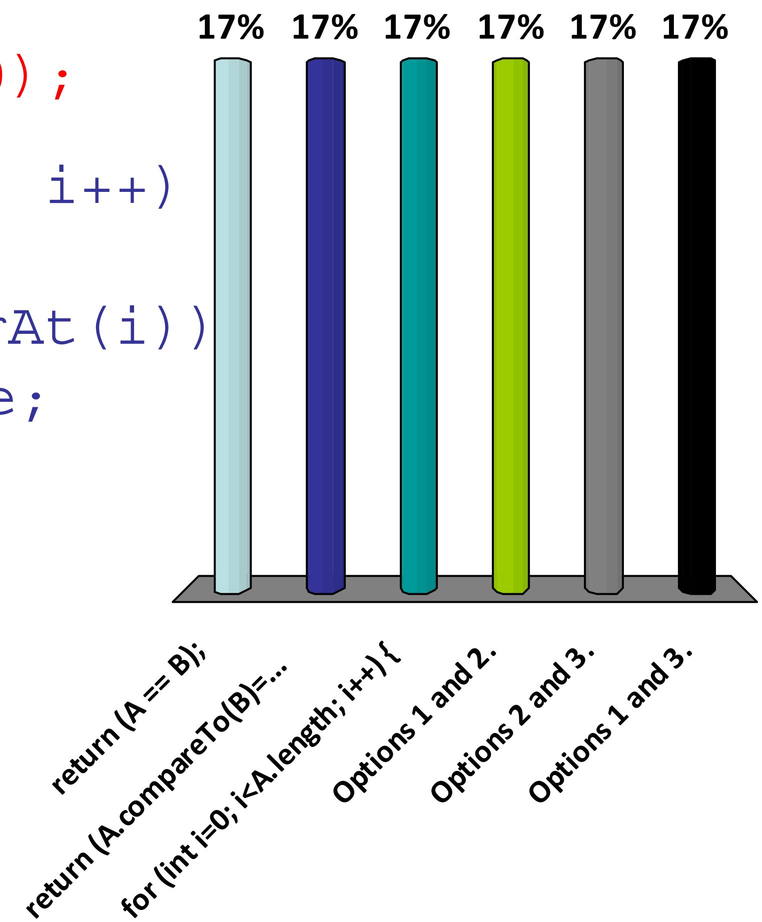
✔ B. `return (A.compareTo(B)==0);`

C.
```
for (int i=0; i<A.length; i++)
{
if (A.charAt(i) != B.charAt(i))
                return false;
}
return true;
```

D. Options 1 and 2.

E. Options 2 and 3.

F. Options 1 and 3.



Comparable Interface

Most Java classes support Comparable

- Integer, Float, etc.
- BigInteger
- String
- Date
- Time
- …

Generic Array

Problem:

```
class Widget<TypeA> {
    void buildArray(int size){
            TypeA[] array = new TypeA[size];
            ...
    }
}
```

Cannot instantiate generic arrays!

(How big should it be? Without knowing sizeof(TypeA), Java cannot decide.)

Generic Array

Solution: use ArrayList

```
class Widget<TypeA> {
    void buildArray(int size){
            ArrayList<TypeA> array = new ArrayList<TypeA>(size);
            …
    }
}
```

Comparing Students

```
class Student implements Comparable<Student> {
...
...
}
```

```
interface Comparable<TypeA> {

    int compareTo(TypeA other);

}
```

Generic Sorting

```
public interface ISort{

    public <TypeA extends Comparable<TypeA>>
    void sort(TypeA[] dataArray);

}
```

Today: Sorting

- Writing a sorting algorithm in Java
- Sorting algorithms
  - BubbleSort
  - SelectionSort
  - InsertionSort
  - MergeSort
- Properties
  - Running time
  - Space usage
  - Stability

Aside: BogoSort

```
BogoSort(A[1..n])
```

Repeat:

a) Choose a random permutation of the array A.

b) If A is sorted, return A.

What is the expected running time of BogoSort?

Aside: BogoSort

`QuantumBogoSort(A[1..n])`

a) Choose a random permutation of the array A.
b) If A is sorted, return A.
c) If A is not sorted, destroy the universe.

What is the expected running time of Quantum BogoSort?

(Remember QuantumBogoSort when you learn about non-deterministic Turing Machines.)

Today: Sorting

- Writing a sorting algorithm in Java
- Sorting algorithms
  - BubbleSort
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- Properties
  - Running time
  - Space usage
  - Stability

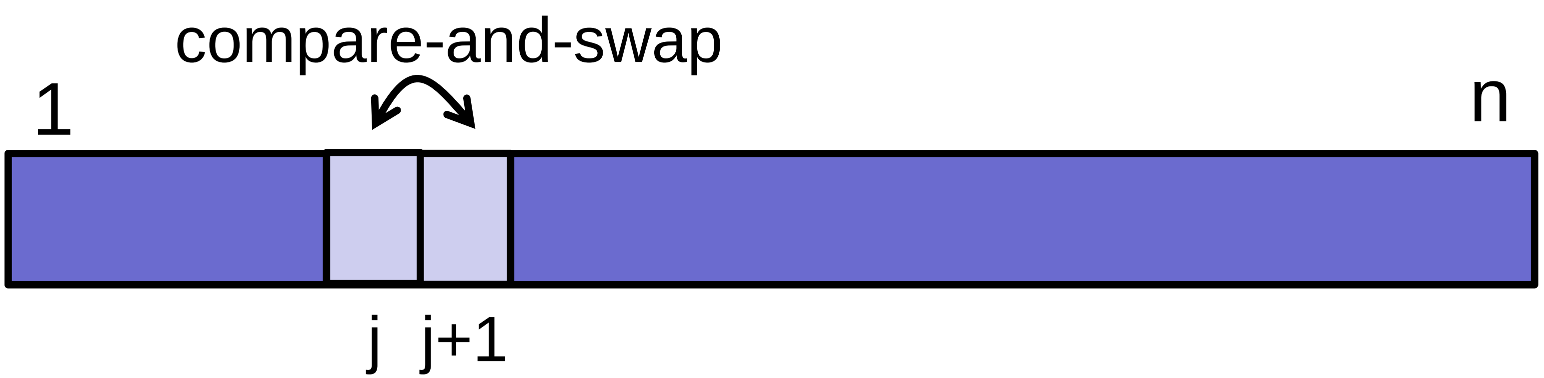
BubbleSort

BubbleSort(A, n)

repeat n **times:**

for j ← 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])



BubbleSort

Example: 8 2 4 9 3 6

BubbleSort

Example: **8** **2** 4 9 3 6

2 **8** 4 9 3 6

BubbleSort

Example:

	8	2	4	9	3	6
	2	**8**	**4**	9	3	6
	2	**4**	**8**	9	3	6

BubbleSort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	**8**	**9**	3	6
2	4	**8**	**9**	3	6

BubbleSort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	**9**	**3**	6
2	4	8	**3**	**9**	6

BubbleSort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	4	8	3	**9**	**6**
2	4	8	3	**6**	**9**

BubbleSort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	4	8	3	9	6
2	**4**	**8**	**3**	**6**	**9**

BubbleSort

Example:

$\boxed{2 \quad 4}$		8	3	6	9
2	$\boxed{4 \quad 8}$		3	6	9
2	4	$\boxed{8 \quad 3}$		6	9
2	4	3	$\boxed{8 \quad 6}$		9
2	4	3	6	$\boxed{8 \quad 9}$	
2	**4**	**3**	**6**	**8**	**9**

BubbleSort

Example:

2	4	3	6	8	9
2	4	3	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	**3**	**4**	**6**	**8**	**9**

BubbleSort

Example:

2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	**3**	**4**	**6**	**8**	**9**

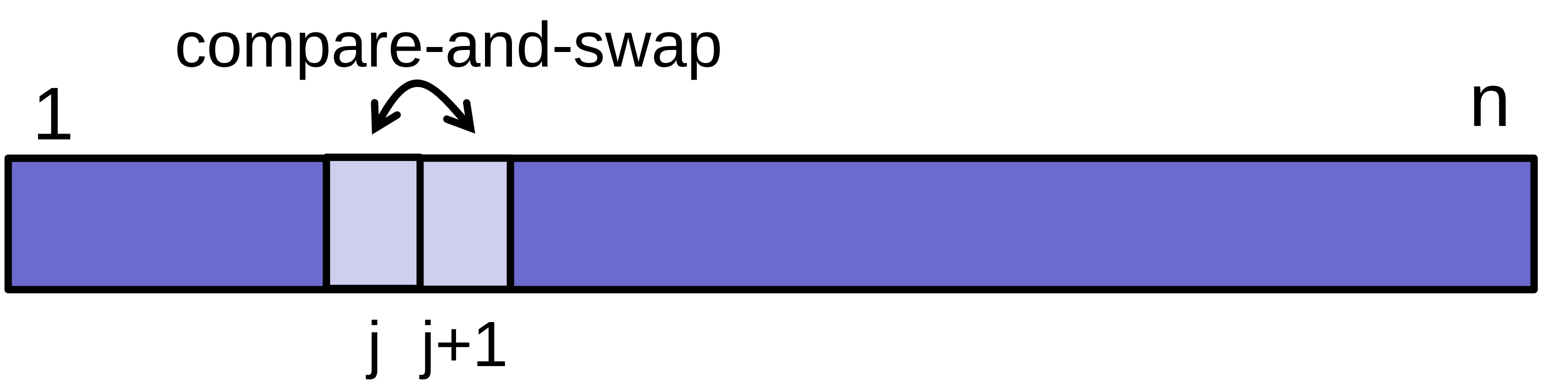
BubbleSort

BubbleSort(A, n)

repeat n **times:**

for j ← 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])



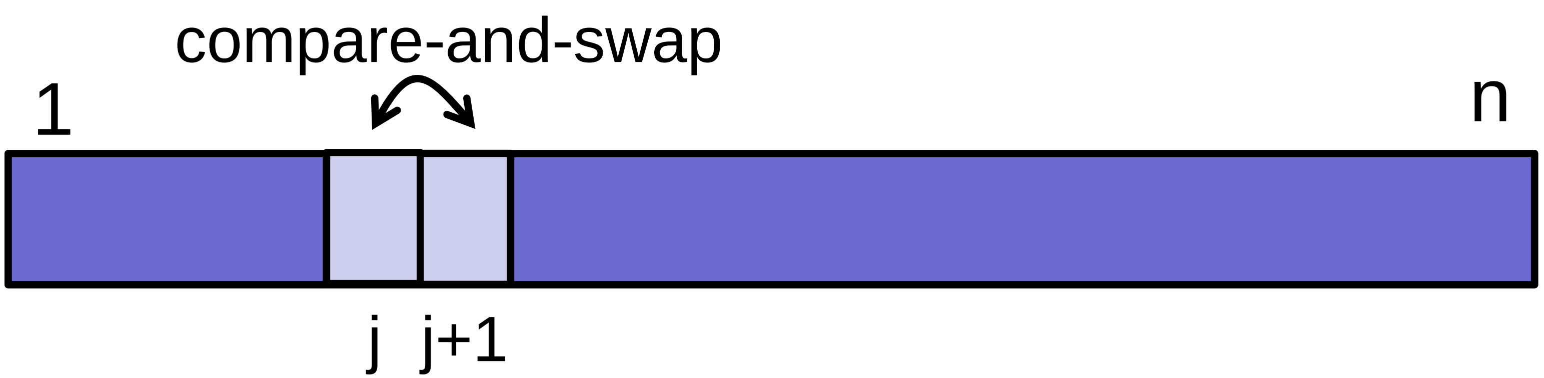
BubbleSort

BubbleSort(A, n)

repeat (until no swaps) **:**

for $j \leftarrow 1$ **to** $n-1$

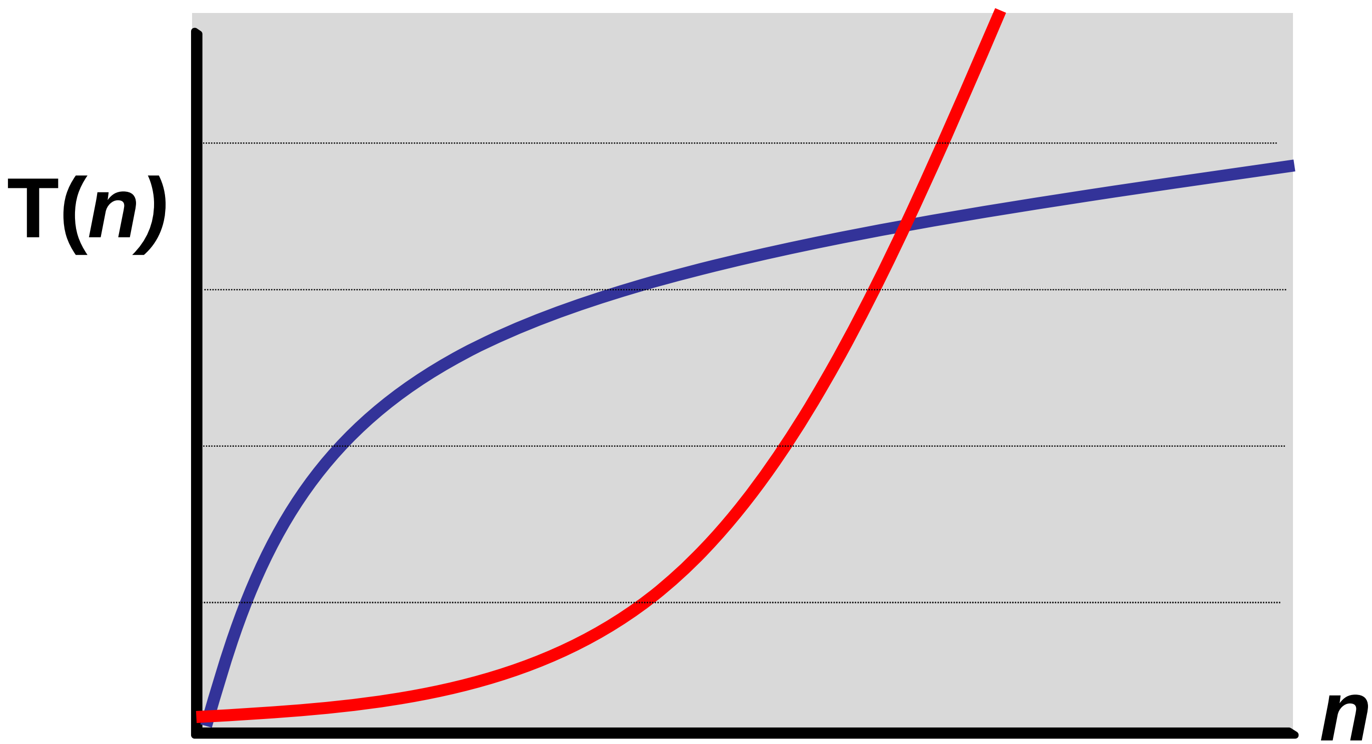
if $A[j] > A[j+1]$ **then** swap(A[j], A[j+1])



Big-O Notation

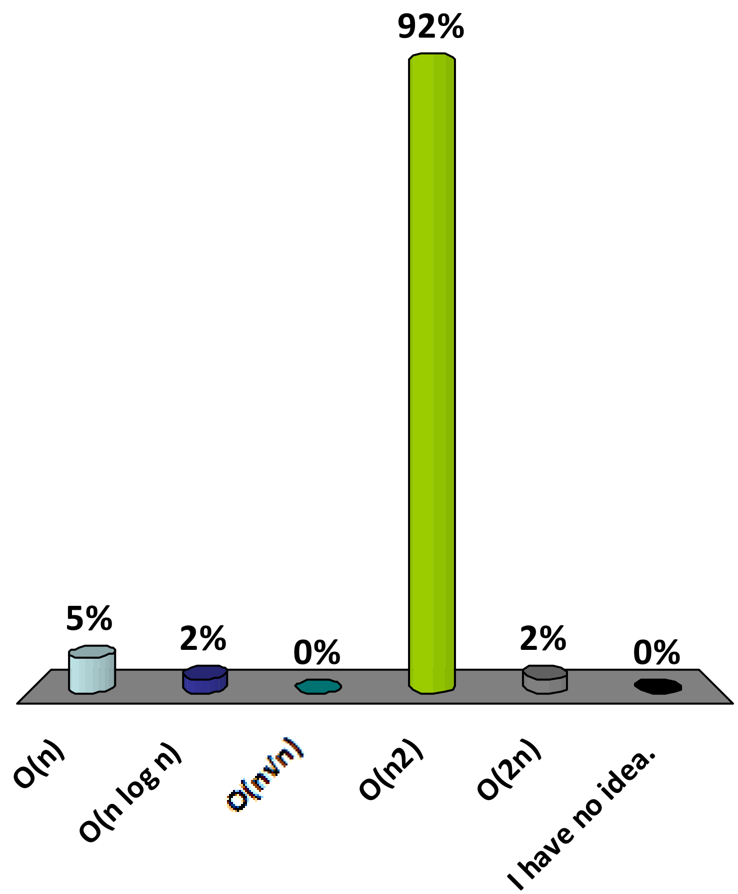
How does an algorithm scale?

- For large inputs, what is the running time?
- T(n) = running time on inputs of size n



What is the running time of BubbleSort?

A. O(n)
B. O(n log n)
C. O(n√n)
D. O(n^2)
E. O(2^n)
F. I have no idea.



BubbleSort

Running time:

- Depends on the input!

BubbleSort

Example:					
$\boxed{2 \quad 3}$		4	6	8	9
2	$\boxed{3 \quad 4}$		6	8	9
2	3	$\boxed{4 \quad 6}$		8	9
2	3	4	$\boxed{6 \quad 8}$		9
2	3	4	6	$\boxed{8 \quad 9}$	
2	**3**	**4**	**6**	**8**	**9**

BubbleSort

Running time:

- Depends on the input!

Best-case:

- Already sorted: O(n)

BubbleSort

Best-case:

- Already sorted: O(n)

Average-case:

- Assume inputs are chosen at random…

Worst-case:

- Bound how long it takes.

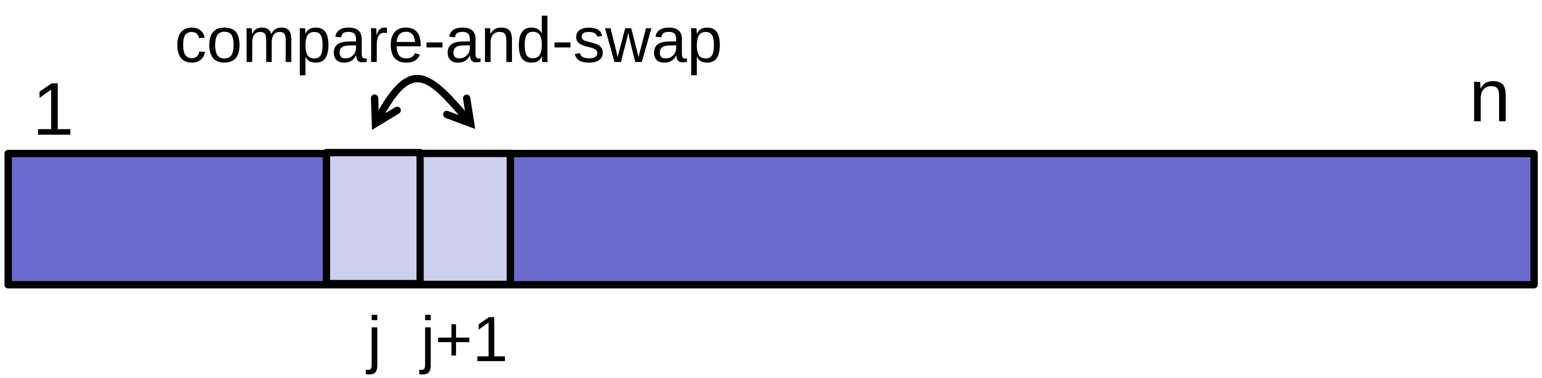
BubbleSort Analysis

BubbleSort(A, n)

repeat (until no swaps) **:**

for j ← 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])



BubbleSort Analysis

BubbleSort(A, n)

repeat (until no swaps) **:**

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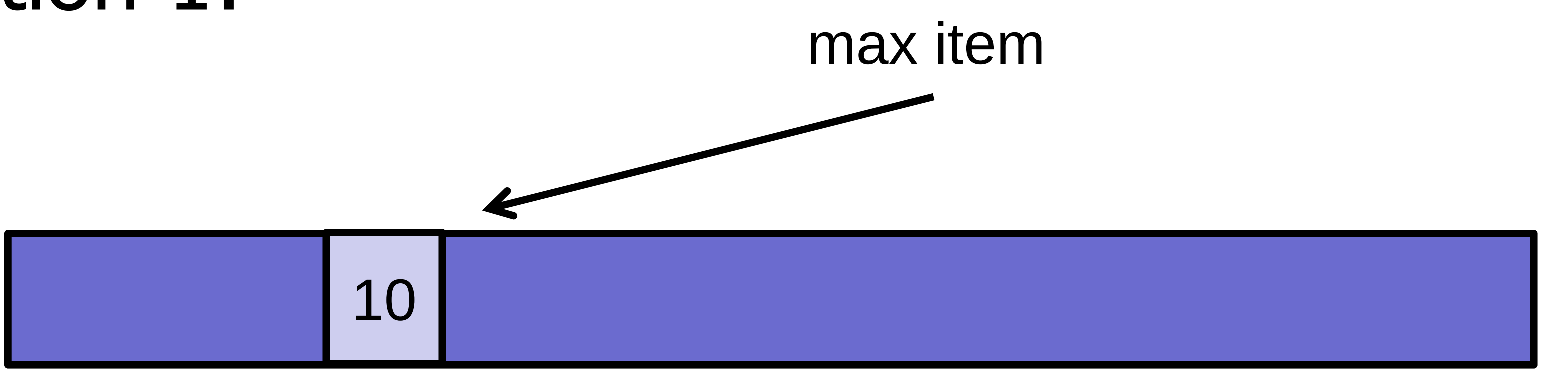
if A[j] > A[j+1] **then** swap(A[j], A[j+1])

Iteration 1:



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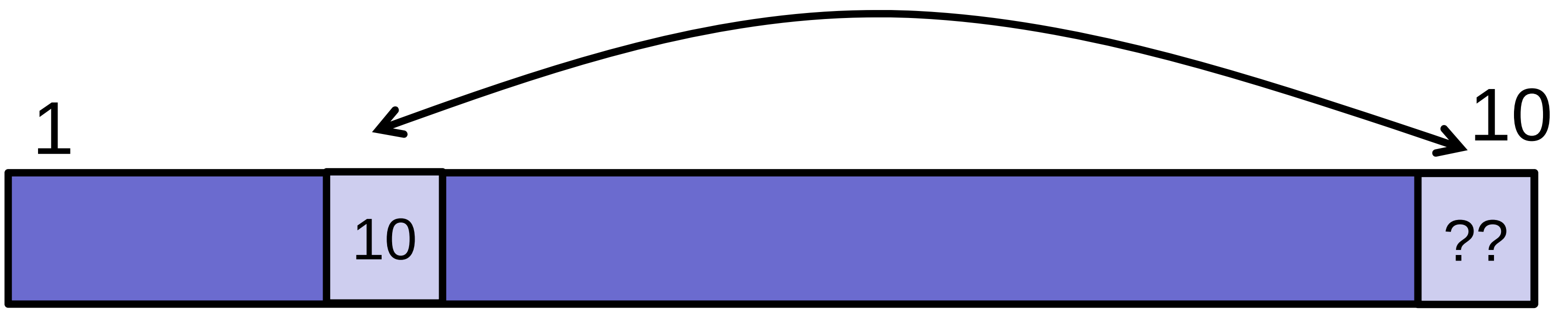


BubbleSort Analysis

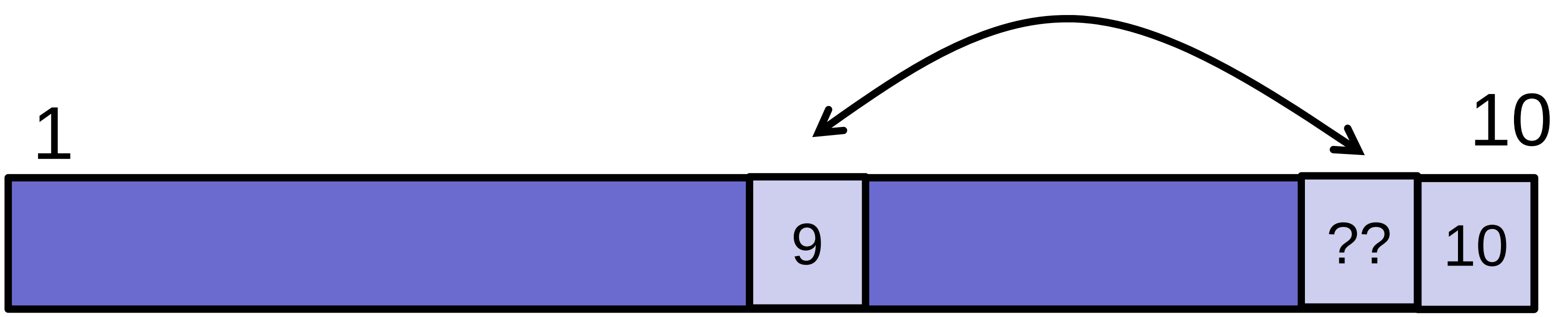
BubbleSort(A, n)

repeat (until no swaps) **:**

for j $\leftarrow$ 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])

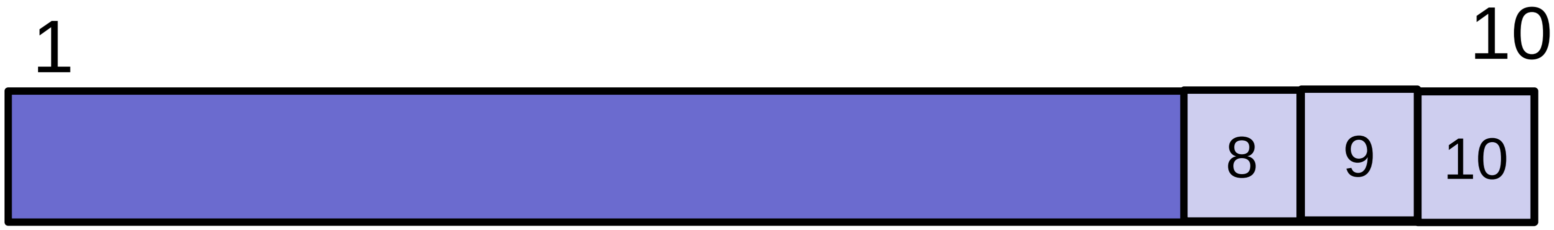
Iteration 2:



BubbleSort Analysis

Loop invariant:

At the end of iteration j: ??



BubbleSort Analysis

Loop invariant:

At the end of iteration j, the biggest j items are correctly sorted in the final j positions of the array.

BubbleSort Analysis

Loop invariant:

At the end of iteration j, the biggest j items are correctly sorted in the final j positions of the array.

Worst case: n iterations ➔ $O(n^2)$ time

BubbleSort

Best-case: $O(n)$

- Already sorted

Average-case: $O(n^2)$

- Assume inputs are chosen at random…

Worst-case: $O(n^2)$

- Bound on how long it takes.

Today: Sorting

- Writing a sorting algorithm in Java
- Sorting algorithms
  - BubbleSort
  - SelectionSort
  - InsertionSort
  - MergeSort
- Properties
  - Running time
  - Space usage
  - Stability

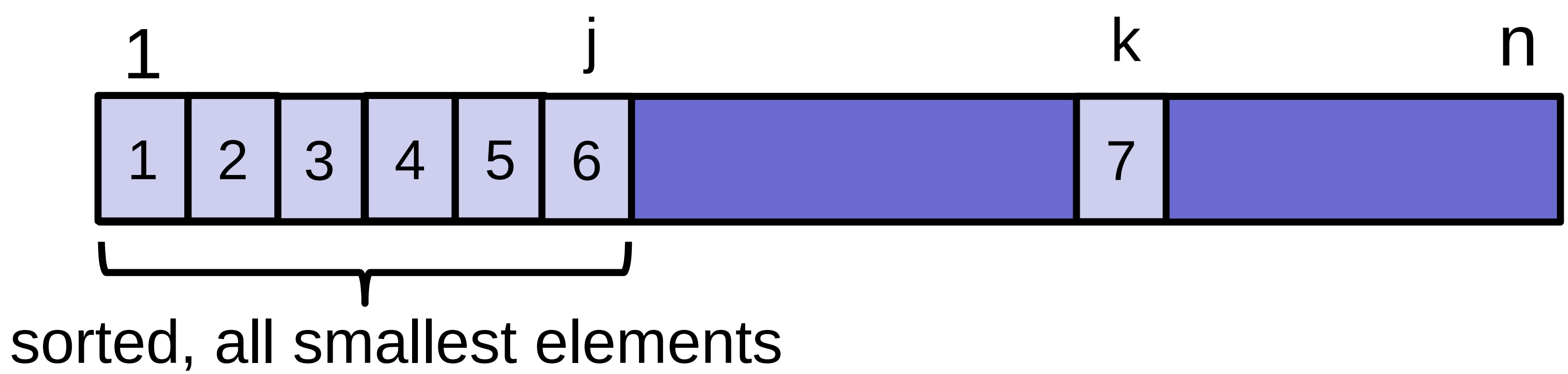
SelectionSort

SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])



SelectionSort

Example: 8 2 4 9 3 6

SelectionSort

Example:	8	**2**	4	9	3	6
	2	8	4	9	3	6

SelectionSort

Example:	8	**2**	4	9	3	6
	2	8	4	9	**3**	6
	2	3	4	9	8	6

SelectionSort

Example:

Example:	8	**2**	4	9	3	6
	2	8	4	9	**3**	6
	2	3	**4**	9	8	6
	2	3	4	9	8	6

SelectionSort

Example:					
8	**2**	4	9	3	6
2	8	4	9	**3**	6
2	3	**4**	9	8	6
2	3	4	9	8	**6**
2	3	4	6	8	9

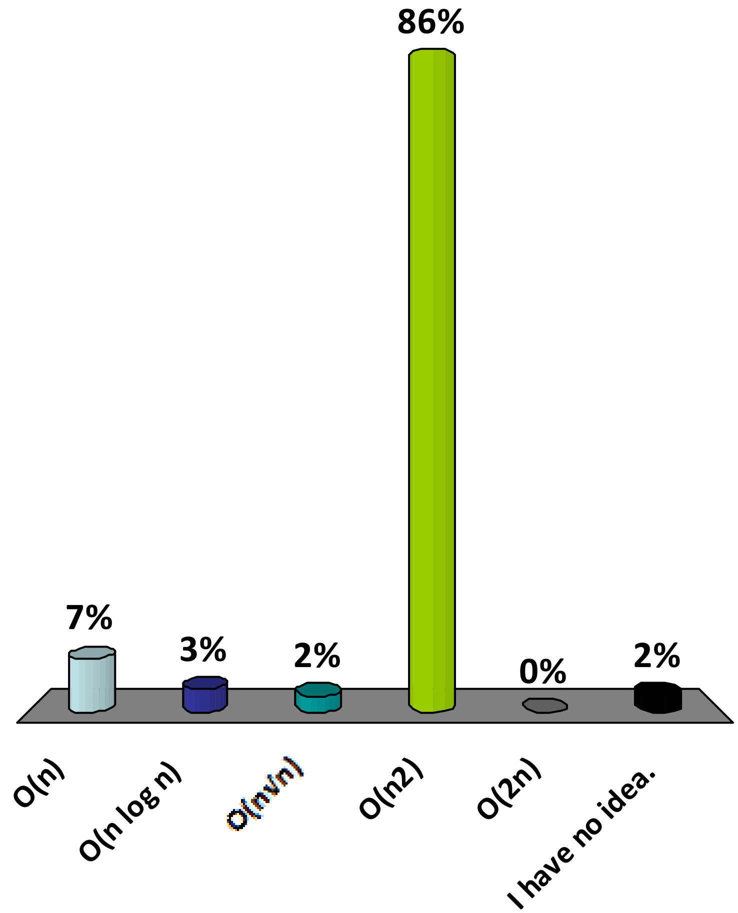
SelectionSort

Example:

Example:	8	**2**	4	9	3	6
	2	8	4	9	**3**	6
	2	3	**4**	9	8	6
	2	3	4	9	8	**6**
	2	3	4	6	**8**	9
	2	3	4	6	8	9

What is the (worst-case) running time of SelectionSort?

A. $O(n)$
B. $O(n \log n)$
C. $O(n\sqrt{n})$
✓ D. $O(n^2)$
E. $O(2^n)$
F. I have no idea.



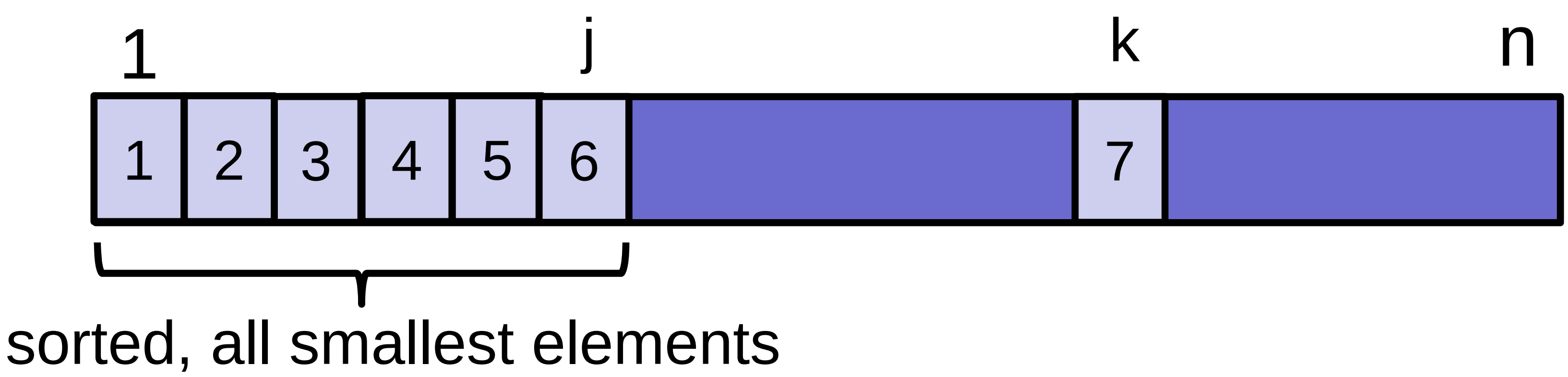
SelectionSort

SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])



SelectionSort

Time: (n – j)

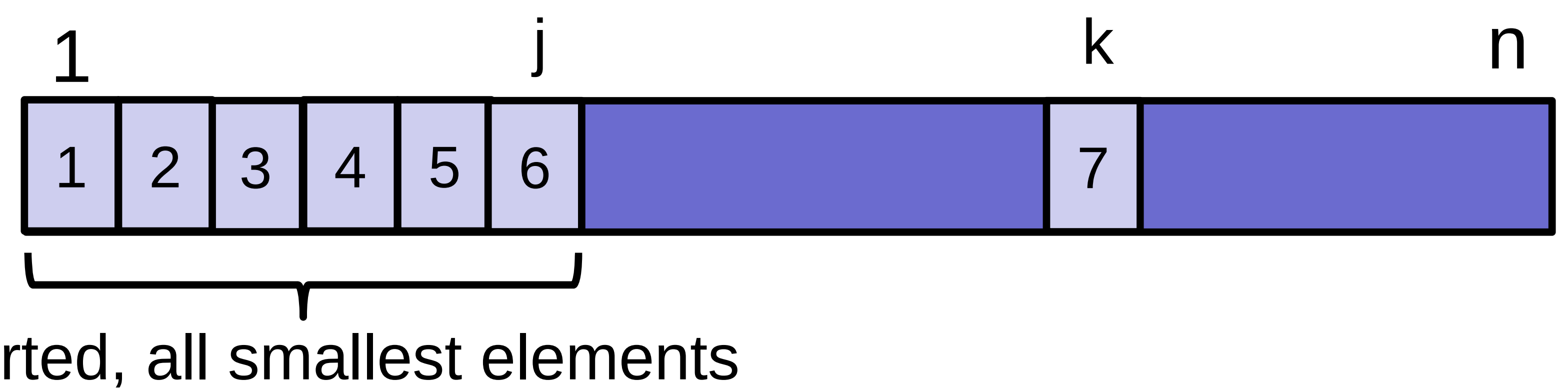
SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

Running time: n + (n-1) + (n-2) + (n-3) + ...



Basic facts

$$n + (n - 1) + (n - 2) + (n - 3) + \ldots + 1 \qquad = (n)(n+1)/2$$

$$= \Theta(n^2)$$

SelectionSort

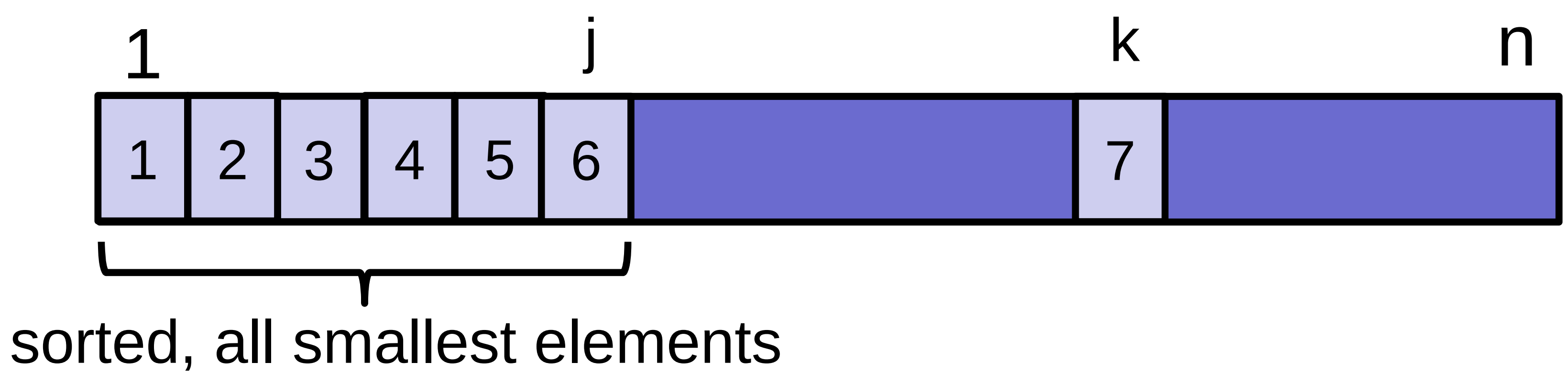
SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

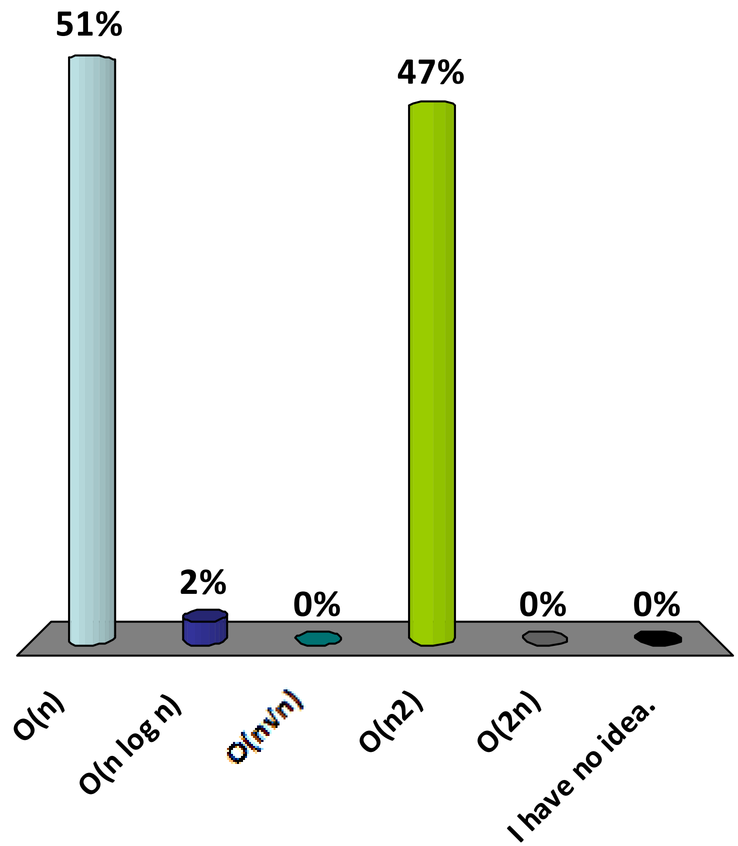
swap(A[j], A[k])

Running time: $O(n^2)$



What is the BEST CASE running time of SelectionSort?

A. $O(n)$
B. $O(n \log n)$
C. $O(n\sqrt{n})$
D. $O(n^2)$
E. $O(2^n)$
F. I have no idea.



SelectionSort

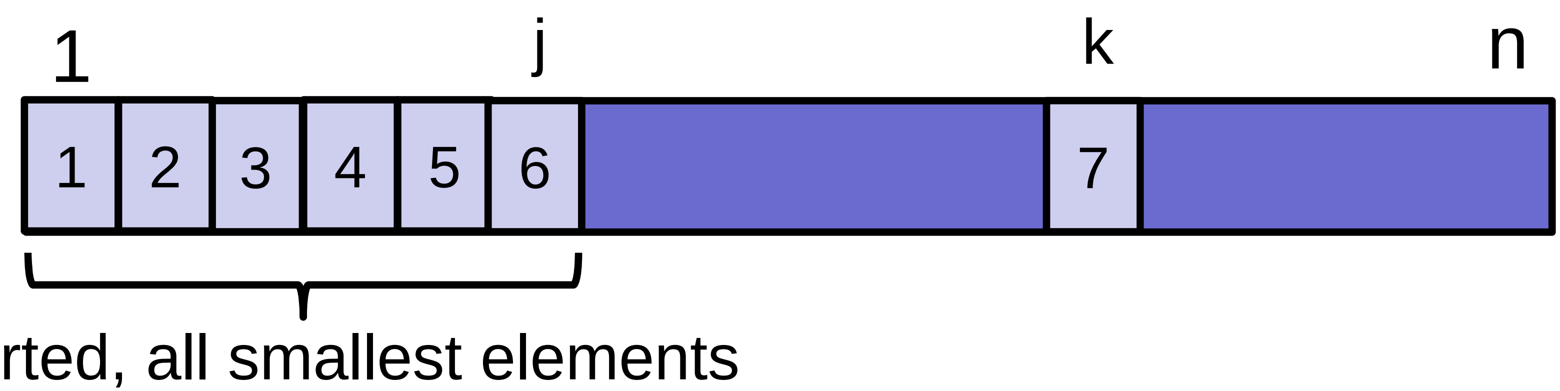
SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

Running time: $O(n^2)$ and $\Omega(n^2)$



SelectionSort Analysis

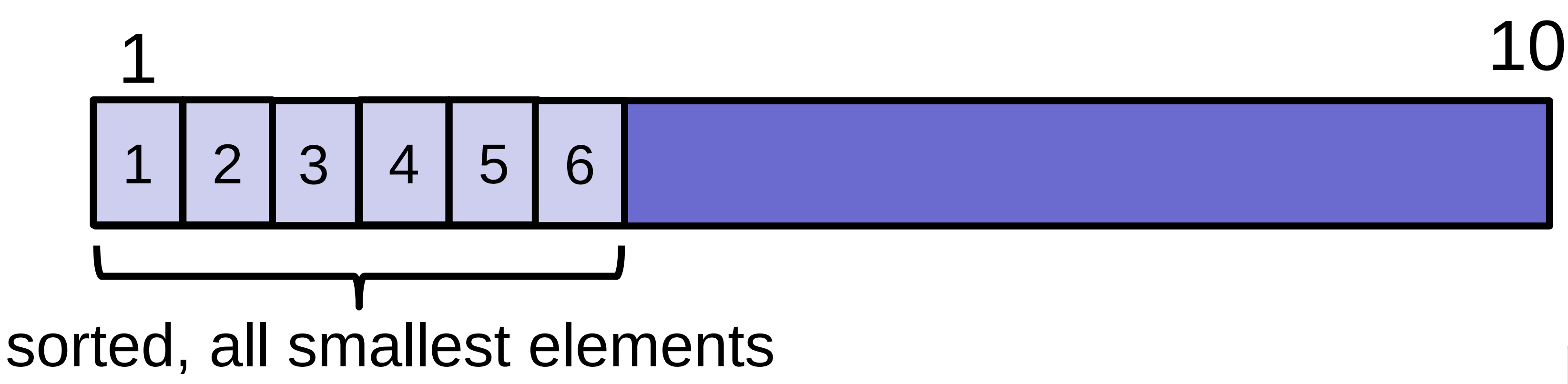
Loop invariant:

At the end of iteration j: ??

SelectionSort Analysis

Loop invariant:

At the end of iteration j: the smallest j items are correctly sorted in the first j positions of the array.



Today: Sorting

- Writing a sorting algorithm in Java
- Sorting algorithms
  - BubbleSort
  - SelectionSort
  - InsertionSort
  - MergeSort
- Properties
  - Running time
  - Space usage
  - Stability

Insertion Sort

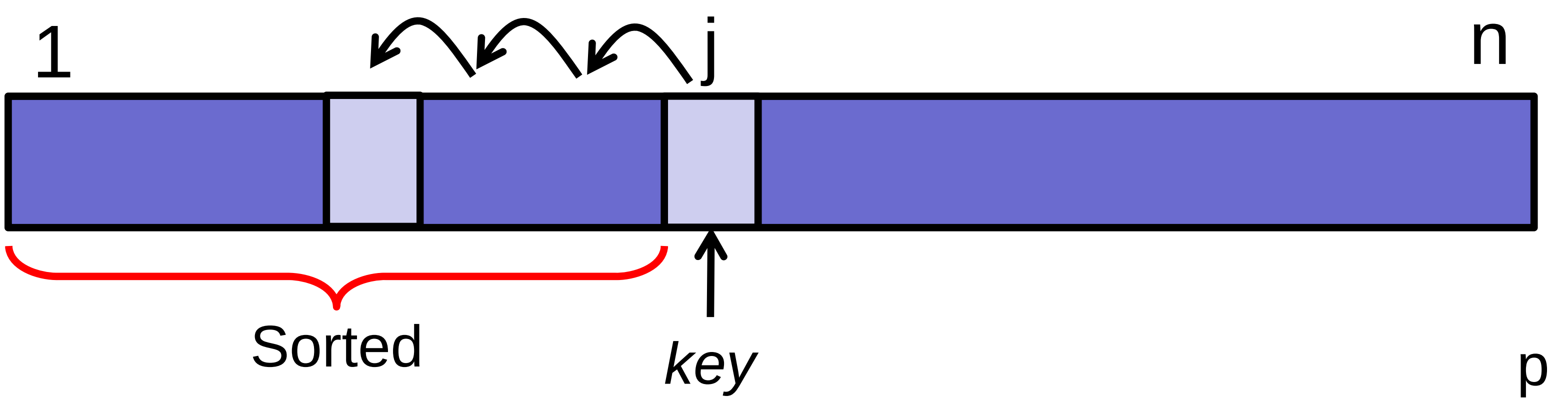
InsertionSort(A, n)

for j ← 2 **to** n | **Invariant:** A[1..j-1] is sorted

key ← A[j]

Insert key into the sorted array A[1..j-1]

Illustration:



Insertion Sort

InsertionSort(A, n)

for j ← 2 **to** n

Invariant: A[1..j-1] is sorted

key ← A[j]

i ← j-1

while (i > 0) **and** (A[i] > *key*)

A[i+1] ← A[i]

i ← i-1

A[i+1] ← *key*

Insertion Sort

Example: **8 2 4 9 3 6**

Insertion Sort

Example: 8 2 4 9 3 6

2 8 4 9 3 6

Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	**4**	**8**	**9**	**3**	**6**

Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	**4**	**8**	**9**	**3**	**6**

Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	**3**	**4**	**8**	**9**	**6**

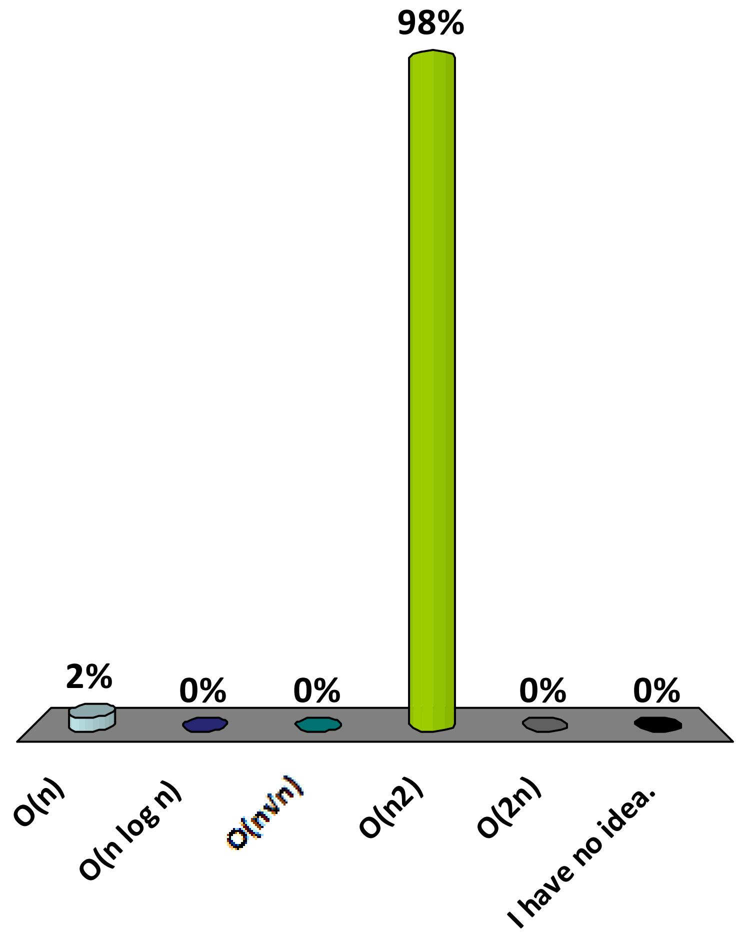
Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	3	4	8	9	6
2	**3**	**4**	**6**	**8**	**9**

What is the (worst-case) running time of InsertionSort?

A. $O(n)$
B. $O(n \log n)$
C. $O(n\sqrt{n})$
D. $O(n^2)$
E. $O(2^n)$
F. I have no idea.



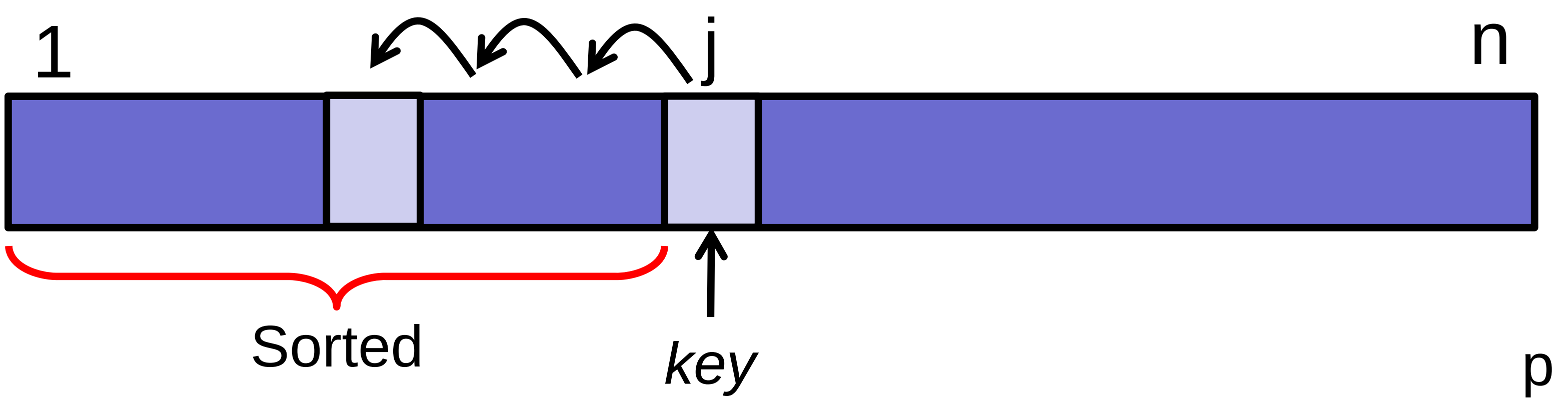
Insertion Sort

Insertion-Sort(A, n)

for j ← 2 **to** n

key ← A[j]

Insert key into the sorted array A[1..j-1]



Insertion Sort Analysis

Insertion-Sort(A, n)

```
for  j ← 2 to n
        key ← A[j]
        i ← j–1
        while (i > 0) and (A[i] > key)
                A[i+1] ← A[i]
                i ← i–1
        A[i+1] ← key
```

Repeat at most j times.

Basic facts

$$1 + 2 + 3 + \ldots + (n-2) + (n-1) + n \quad = (n)(n+1)/2$$

$$= \Theta(n^2)$$

Insertion Sort

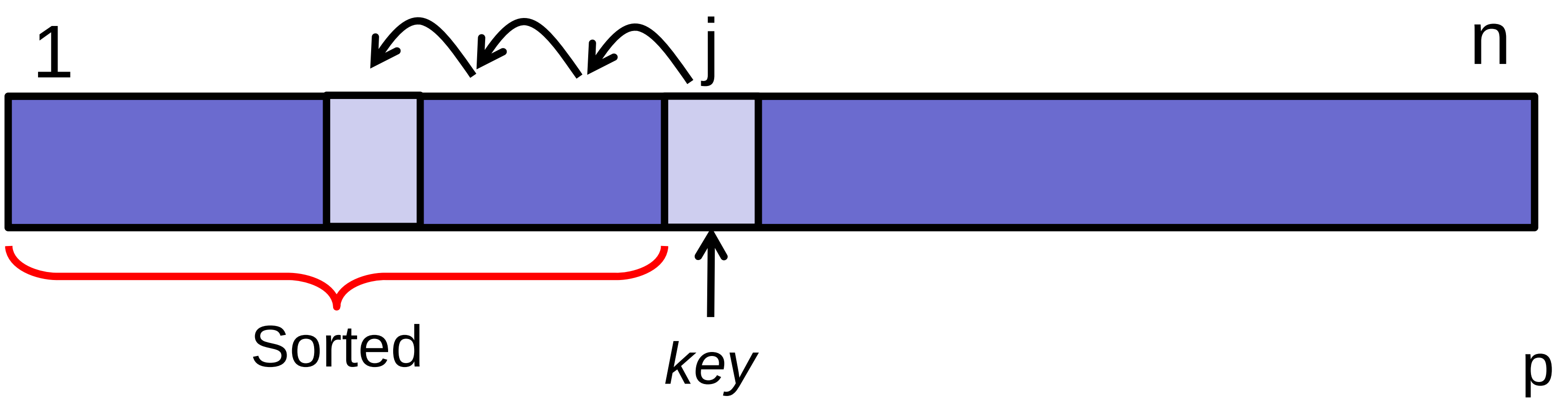
Insertion-Sort(A, n)

for j $\leftarrow$ 2 **to** n

key $\leftarrow$ A[j]

Insert key into the sorted array A[1..j-1]

Running time: $O(n^2)$



Insertion Sort

Best-case:

Average-case:

- Random permutation

Worst-case:

Insertion Sort

Best-case:

- Already sorted: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Average-case:

- Random permutation?

Worst-case:

- Inverse sorted: [10, 9, 8, 7, 6, 5, 4, 3, 2, 1]

Insertion Sort

Very fast!

Best-case: $O(n)$

– Already sorted: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Average-case:

– Random permutation?

Worst-case: $O(n^2)$

– Inverse sorted: [10, 9, 8, 7, 6, 5, 4, 3, 2, 1]

Performance Profiling, V2

(*Dracula* vs. *Lewis & Clark*)

Step	Function	Running Time
Create vectors:	Read each file	1.09s
	Parse each file	3.68s
	Sort words in each file	332.13s
	Count word frequencies	0.30s
Dot product:		6.06s
Norm:		3.80s
Angle:		6.06s
Total:		**11minutes ≈ 680.49s**

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Today: Sorting

- Writing a sorting algorithm in Java
- Sorting algorithms
  - BubbleSort
  - SelectionSort
  - InsertionSort
  - MergeSort
- Properties
  - Running time
  - Space usage
  - Stability

Properties of Sorting Algorithms

Time complexity

- Worst case: $O(n^2)$
- Sorted list: BubbleSort

  SelectionSort

  InsertionSort

- Almost sorted list?

How expensive is it to sort:

[1, 2, 3, 4, 5**, 7, 6**, 8, 9, 10]

How expensive is it to sort:

[1, 2, 3, 4, 5**, 7, 6**, 8, 9, 10]

BubbleSort and InsertionSort are fast.

SelectionSort is slow.

NB Challenge of the Day:

Find a permutation of [1..n] where:

- BubbleSort is slow.
- InsertionSort is fast.

Or explain why no such sequence exists.

Properties of Sorting Algorithms

Space complexity

- Worst case: $O(n)$

- In-place sorting algorithm:
  - Only $O(1)$ extra space needed.
  - All manipulation happens within the array.

So far:

All sorting algorithms we have seen are in-place.

Properties of Sorting Algorithms

Stability

What happens with repeated elements?

Key	1	2	**5**	3	4	**5**	6	7	8	9
Data	a	b	**C**	g	h	**D**	j	k	l	m

Properties of Sorting Algorithms

Stability

What happens with repeated elements?

Key	1	2	**5**	3	4	**5**	6	7	8	9
Data	a	b	**C**	g	h	**D**	j	k	l	m

UNSTABLE

Key	1	2	3	4	**5**	**5**	6	7	8	9
Data	a	b	g	h	**D**	**C**	j	k	l	m

Properties of Sorting Algorithms

Stability: preserves order of equal elements

What happens with repeated elements?

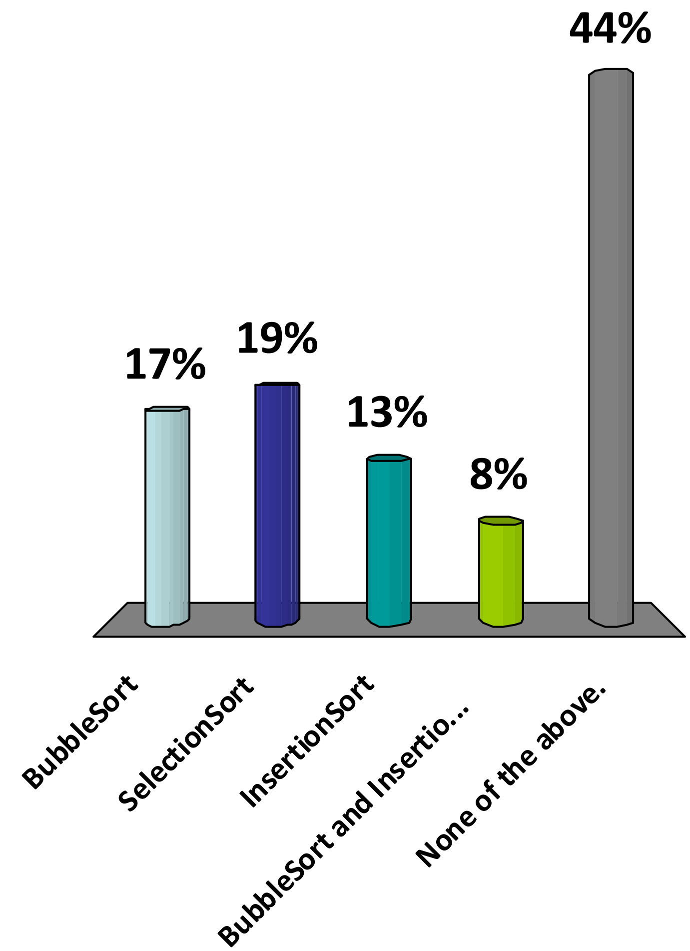
Key	1	2	**5**	3	4	**5**	6	7	8	9
Data	a	b	**C**	g	h	**D**	j	k	l	m

⇩ STABLE

Key	1	2	3	4	**5**	**5**	6	7	8	9
Data	a	b	g	h	**C**	**D**	j	k	l	m

Which are NOT stable?

A. BubbleSort
B. SelectionSort
C. InsertionSort
D. BubbleSort and InsertionSort
E. None of the above.



InsertionSort

```
Insertion-Sort(A, n)
    for  j ← 2 to n
         key ← A[j]
         i ← j-1
         while(i > 0) and(A[i] > key)
               A[i+1] ← A[i]
               i ← i-1
               A[i+1] ← key
```

SelectionSort

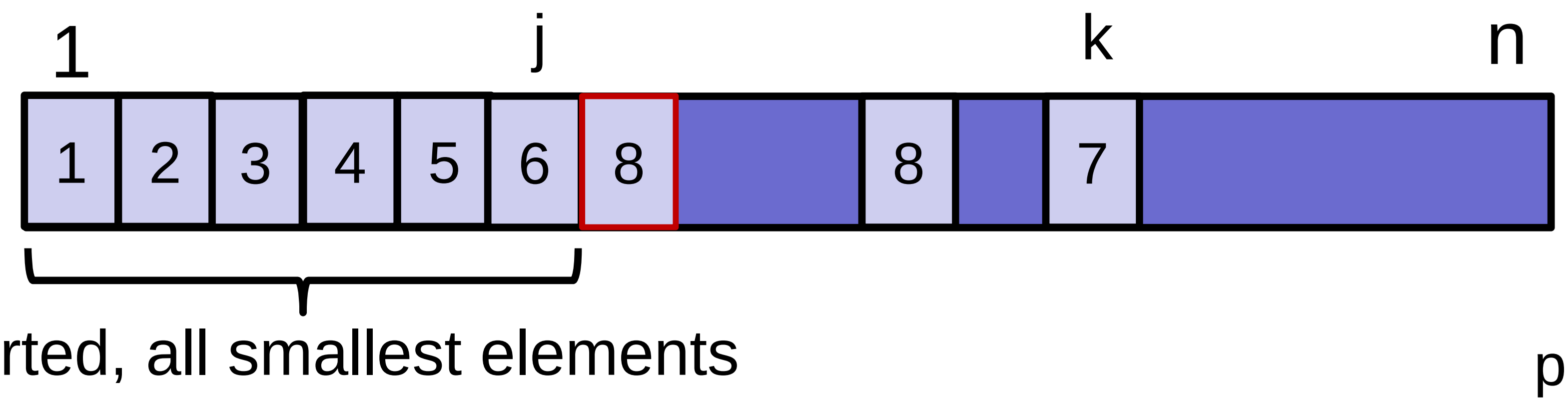
SelectionSort(A, n)

for j ← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

Stable: No: swap changes order



Today: Sorting

- Writing a sorting algorithm in Java
- Sorting algorithms
    - BubbleSort
    - SelectionSort
    - InsertionSort
    - MergeSort
- Properties
    - Running time
    - Space usage
    - Stability

MergeSort

MergeSort(A, n)

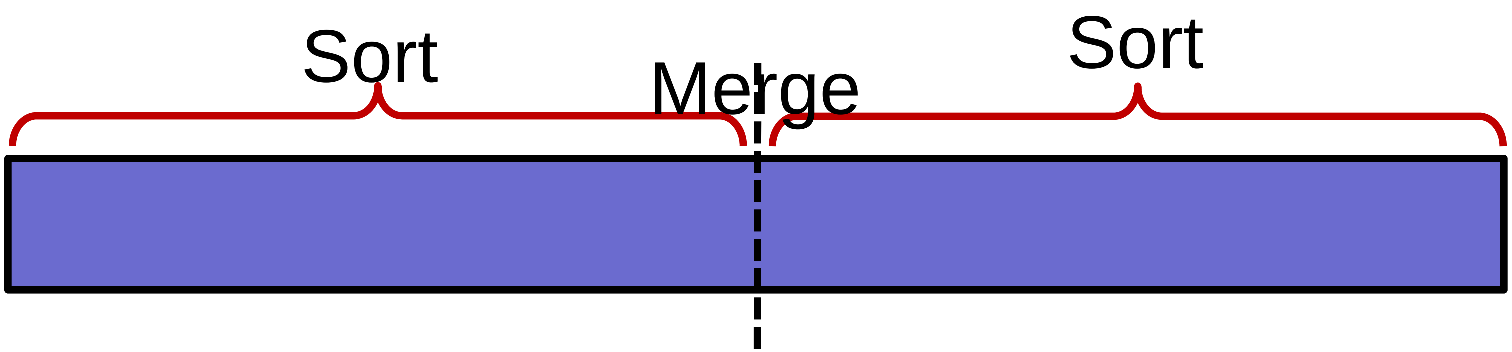
if (n=1) **then return;**

else: **recurse**

X ← MergeSort**(**A[1..n/2], n/2**);**

Y ← MergeSort**(**A[n/2+1, n], n/2**);**

return Merge **(**X,Y, n/2**);**



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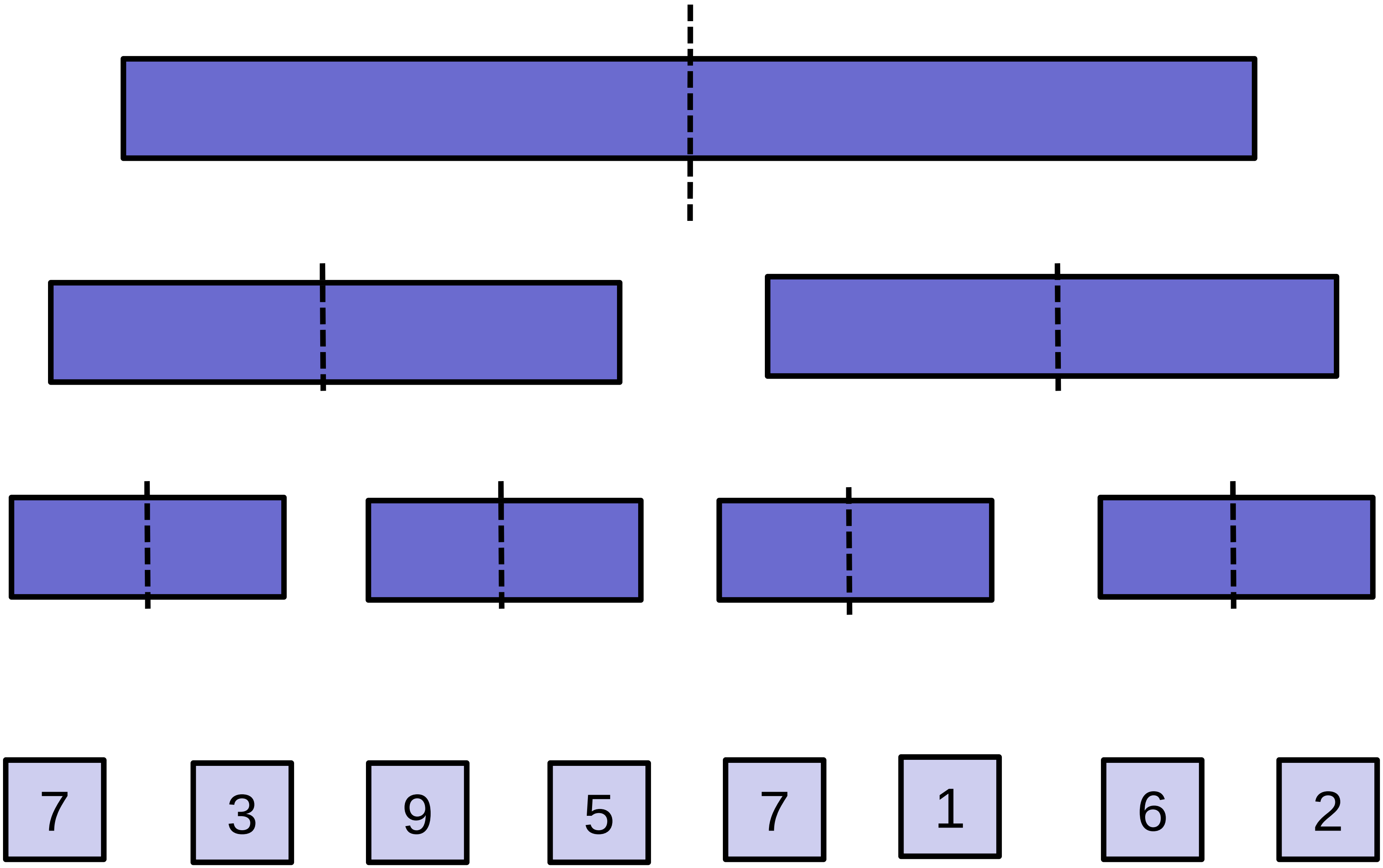
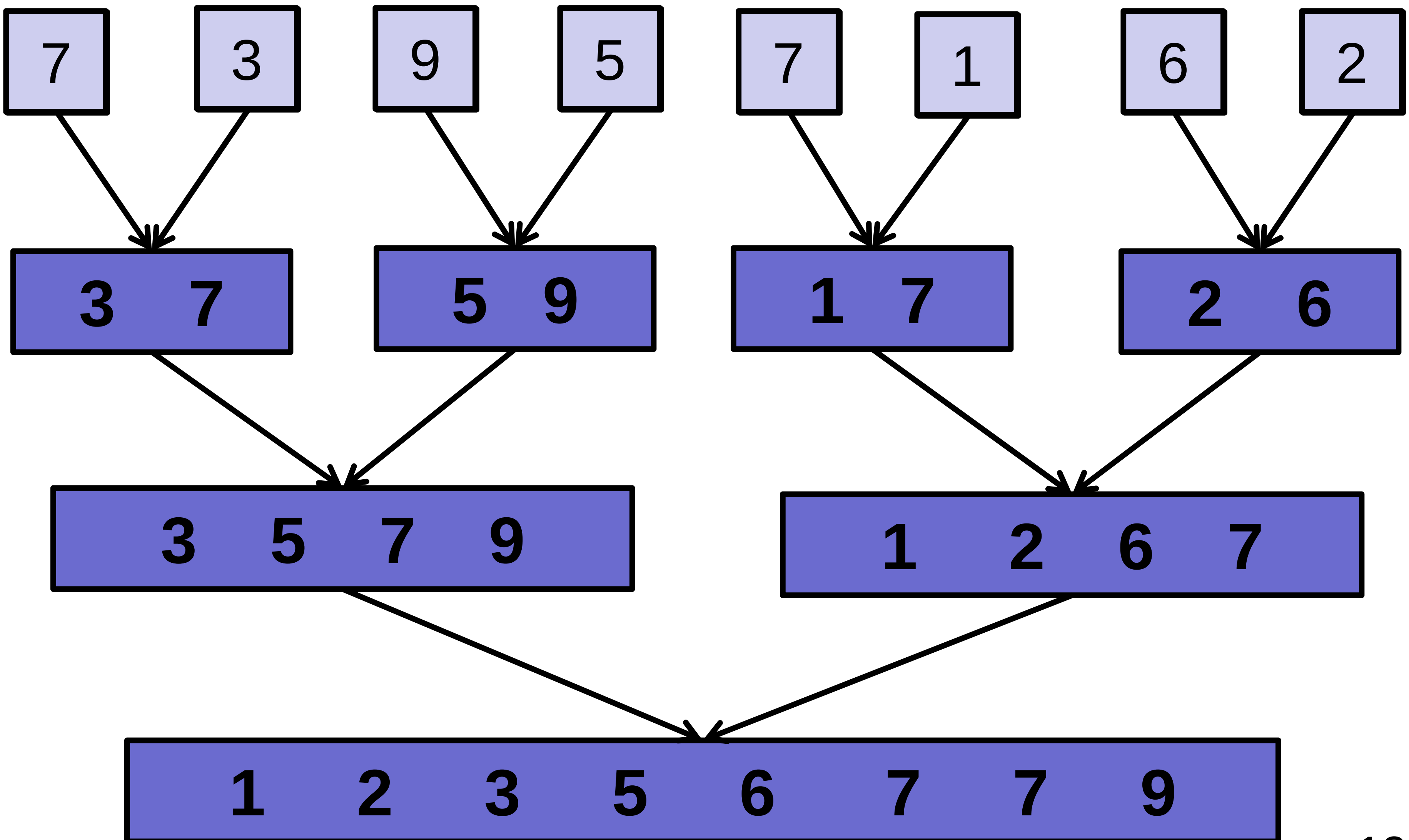
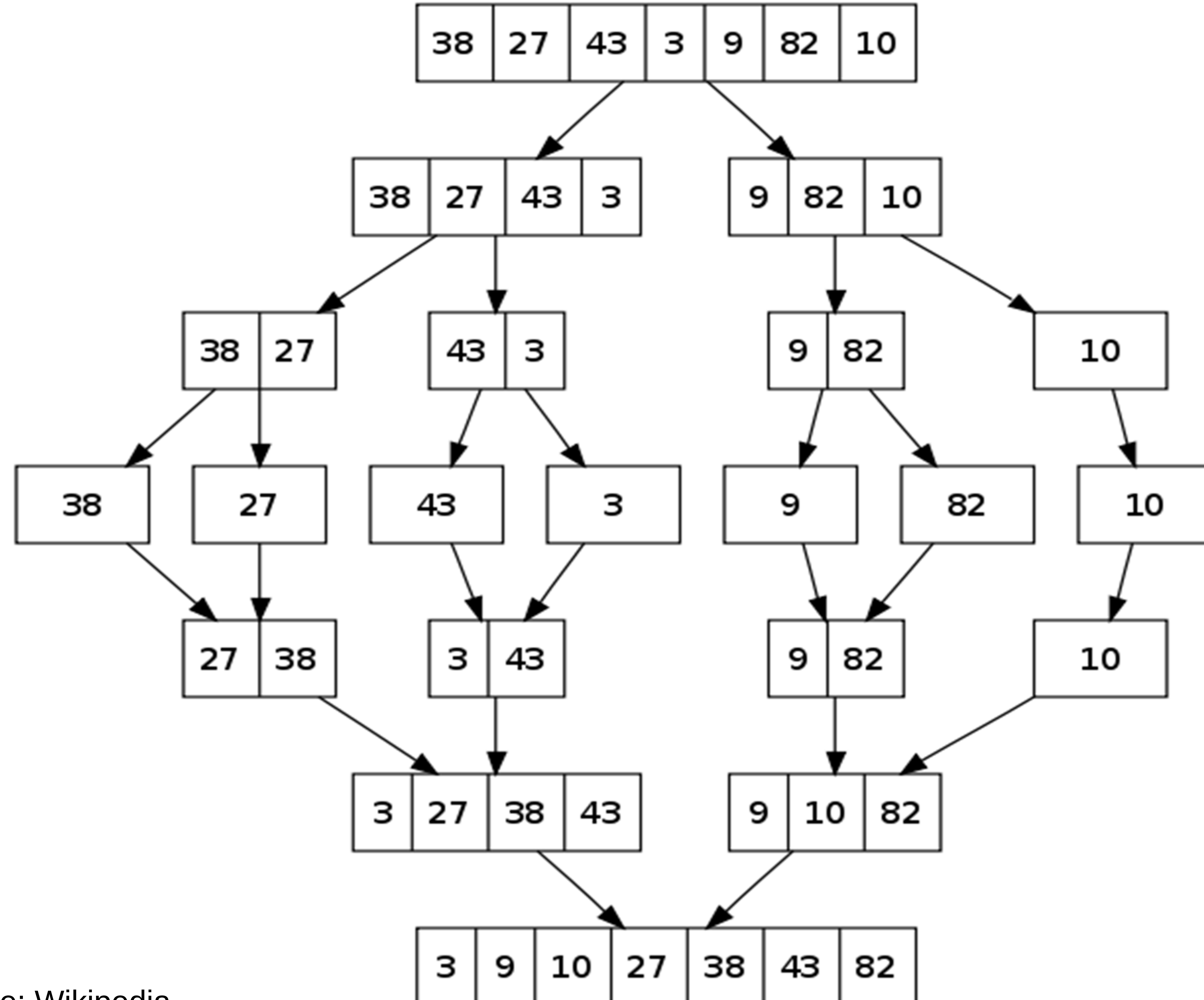
Divide-and-Conquer



Merging





Source: Wikipedia

Merging Two Sorted Lists

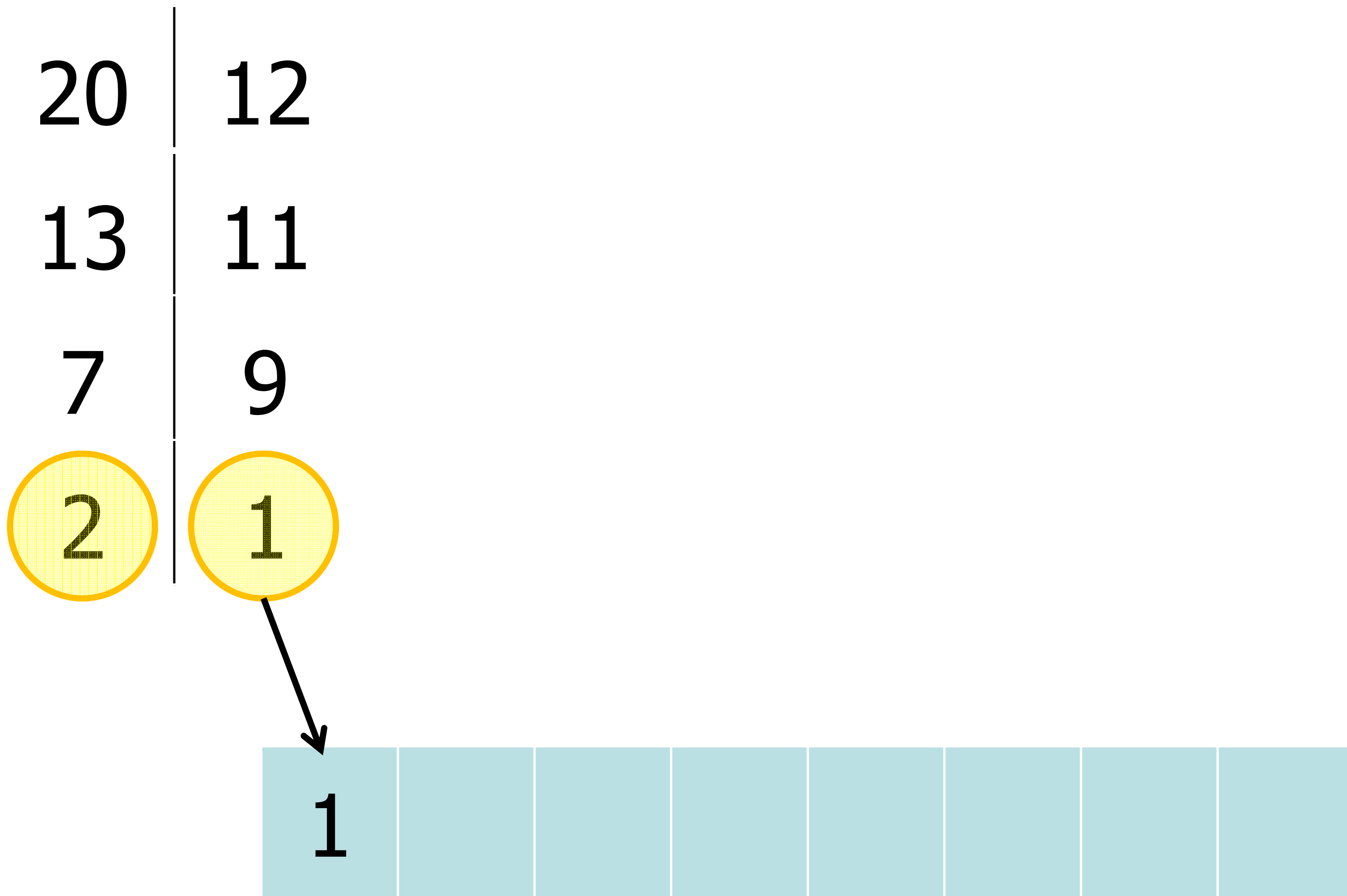
Key subroutine: Merge

- How?
- How fast??

Merging Two Sorted Lists

20	12
13	11
7	9
2	1

Merging Two Sorted Lists



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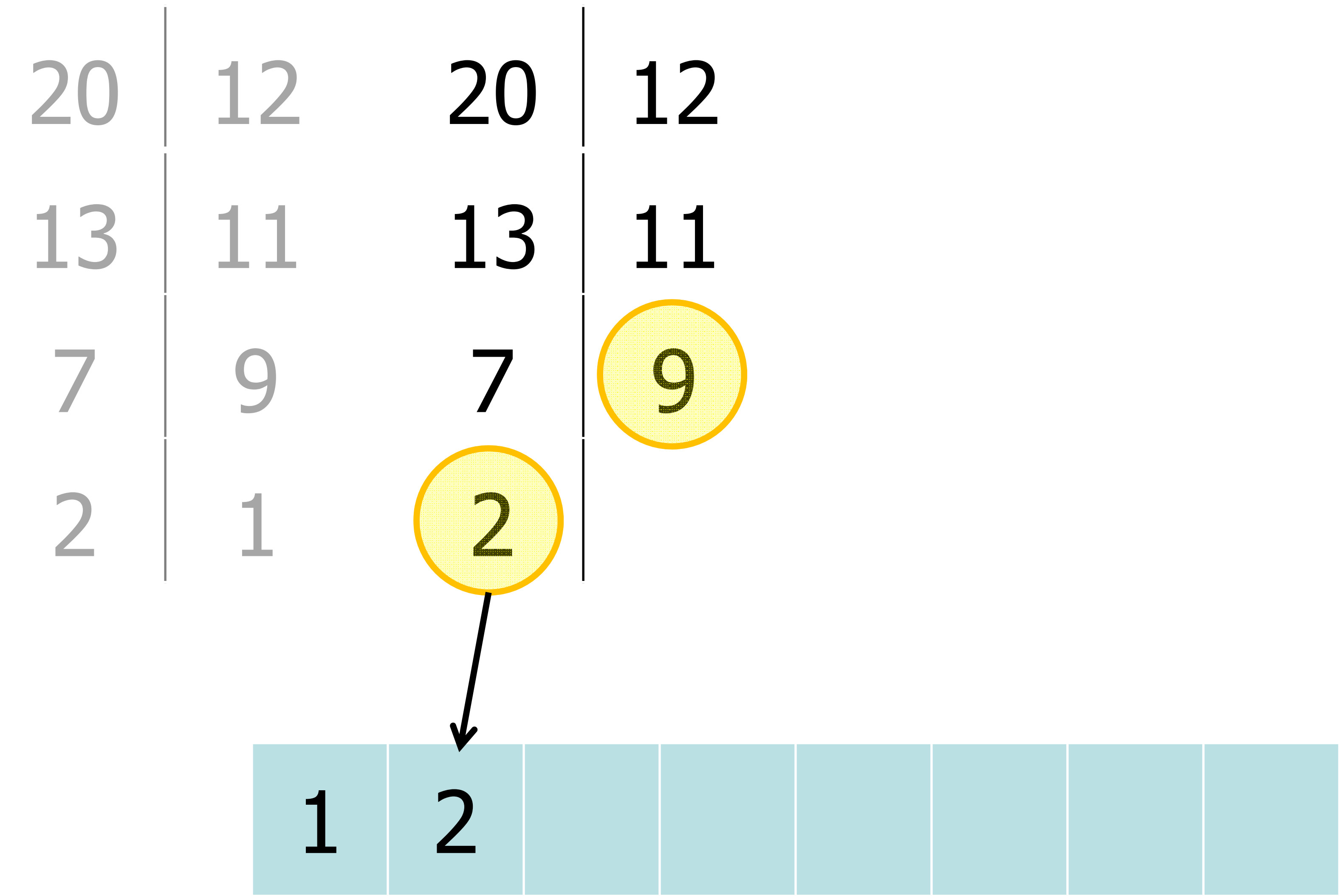
Merging Two Sorted Lists



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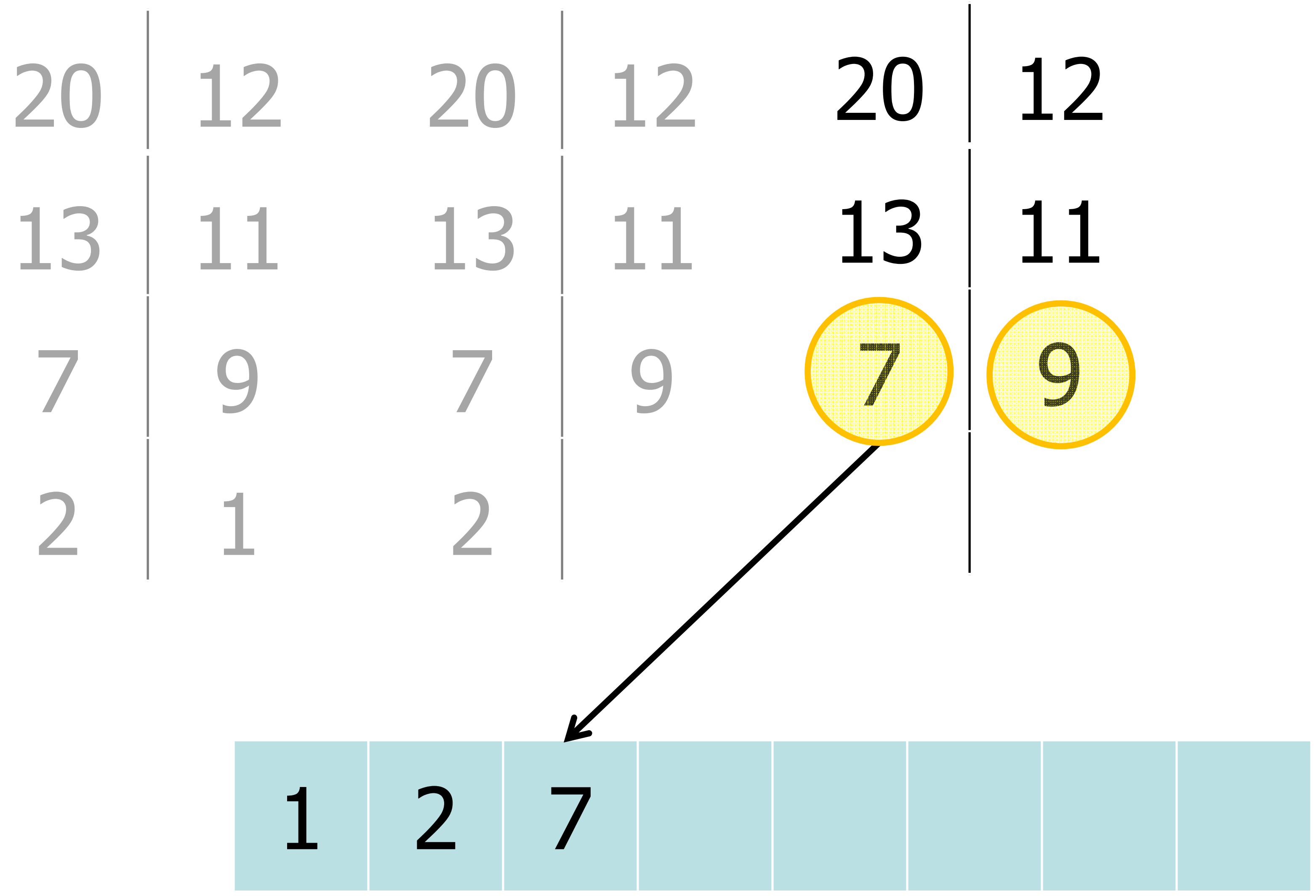
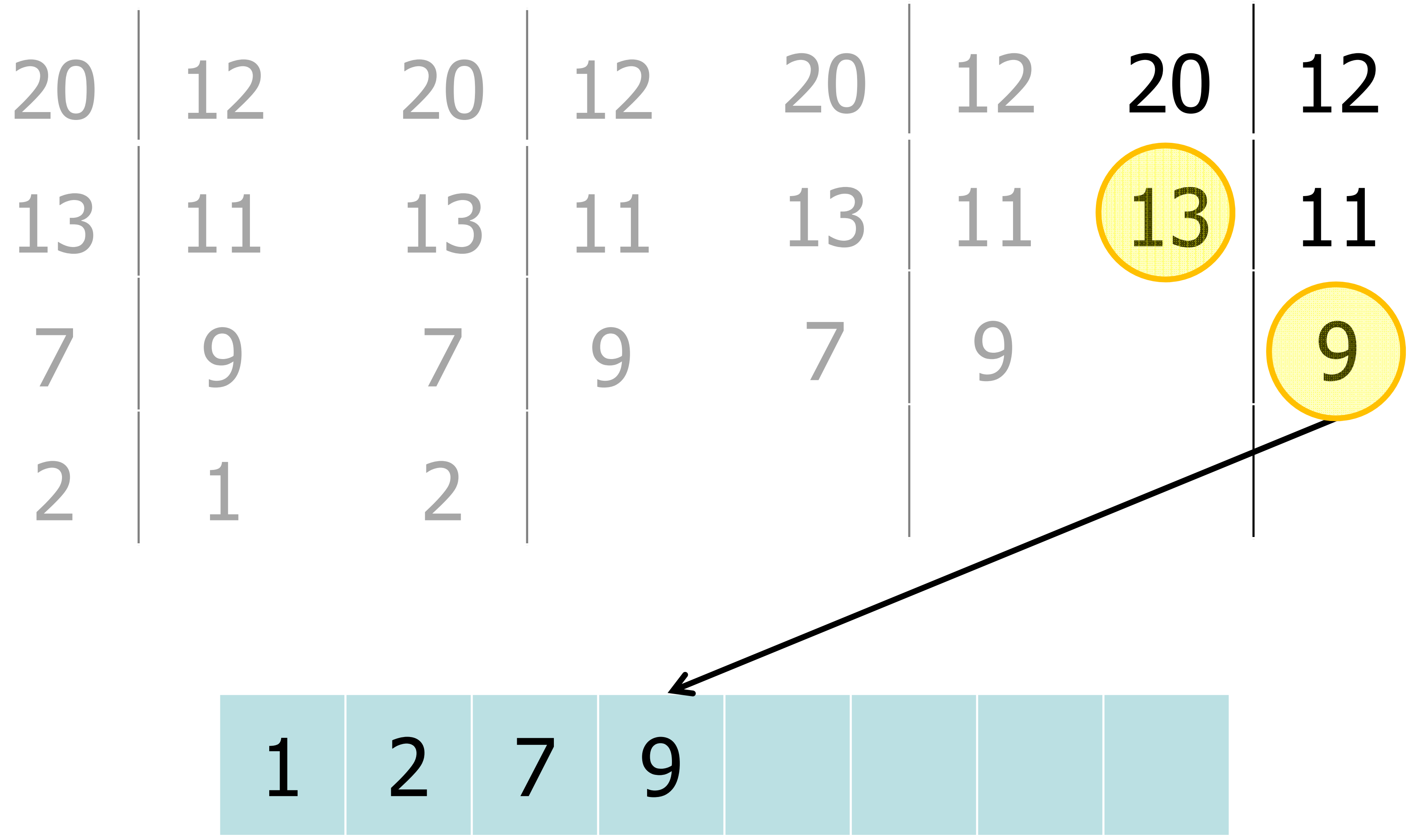
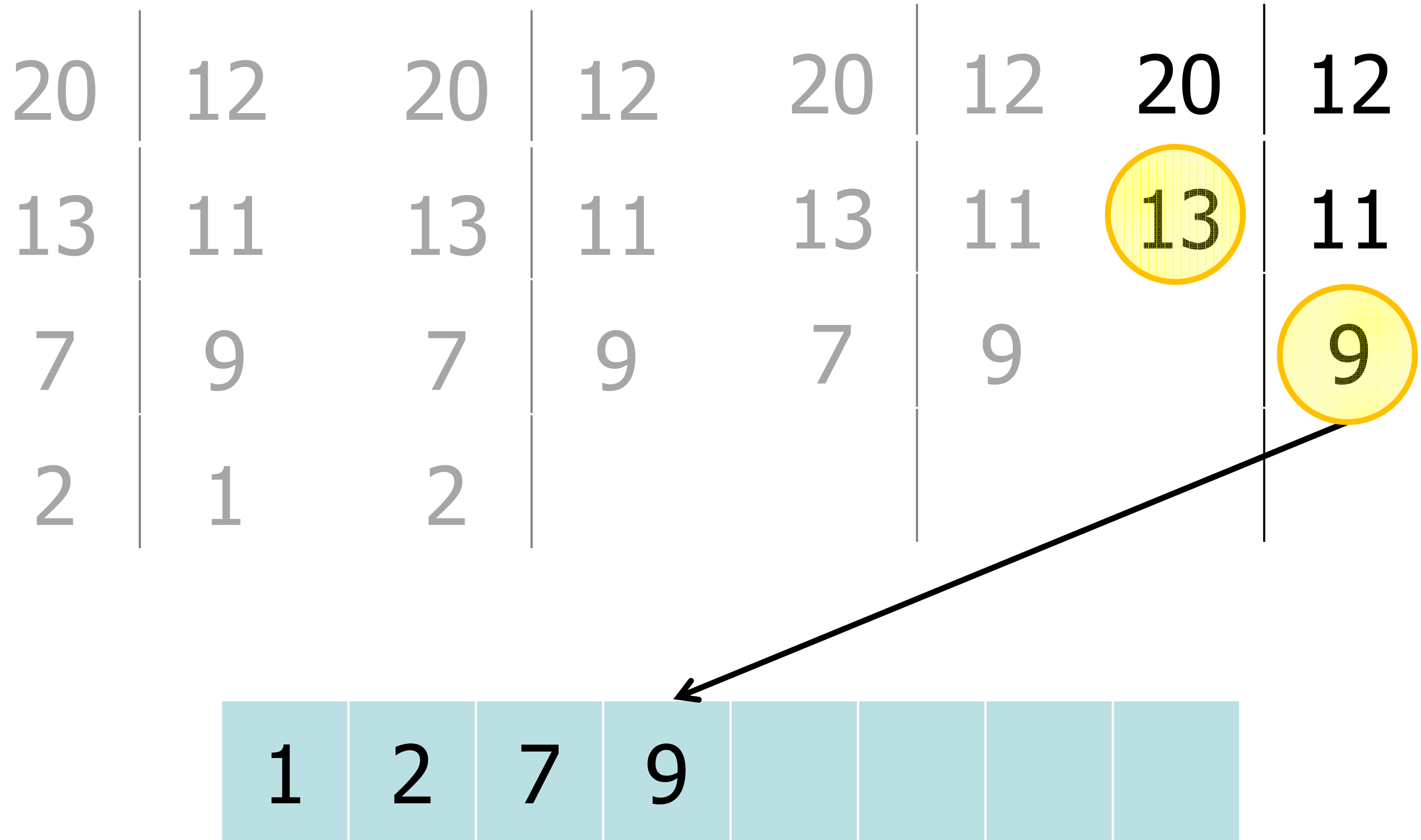
Merging Two Sorted Lists



Merging Two Sorted Lists



Merging Two Sorted Lists



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Merging Two Sorted Lists

20	12	20	12	20	12	20	12
13	11	13	11	13	11	13	11
7	9	7	9	7	9		
2	1	2					

1	2	7	9	11	12	13	20

Merge: Running Time

Given two lists:

- A of size n/2
- B of size n/2

Total running time:??

Merge: Running Time

Given two lists:

- A of size n/2
- B of size n/2

Total running time: $O(n) = cn$

- In each iteration, move one element to final list

Merge-Sort Analysis

Let $T(n)$ be the worst-case running time for an array of n elements.

MergeSort(A, n)

if $(n=1)$ **then return;** ←------------------- $\theta(1)$

else:

$X \leftarrow$ Merge-Sort**(**…**);** ←------------ $T(n/2)$

$Y \leftarrow$ Merge-Sort**(**…**);** ←------------ $T(n/2)$

return Merge **(**$X, Y, n/2$**);** ←------------ $\theta(n)$

MergeSortAnalysis

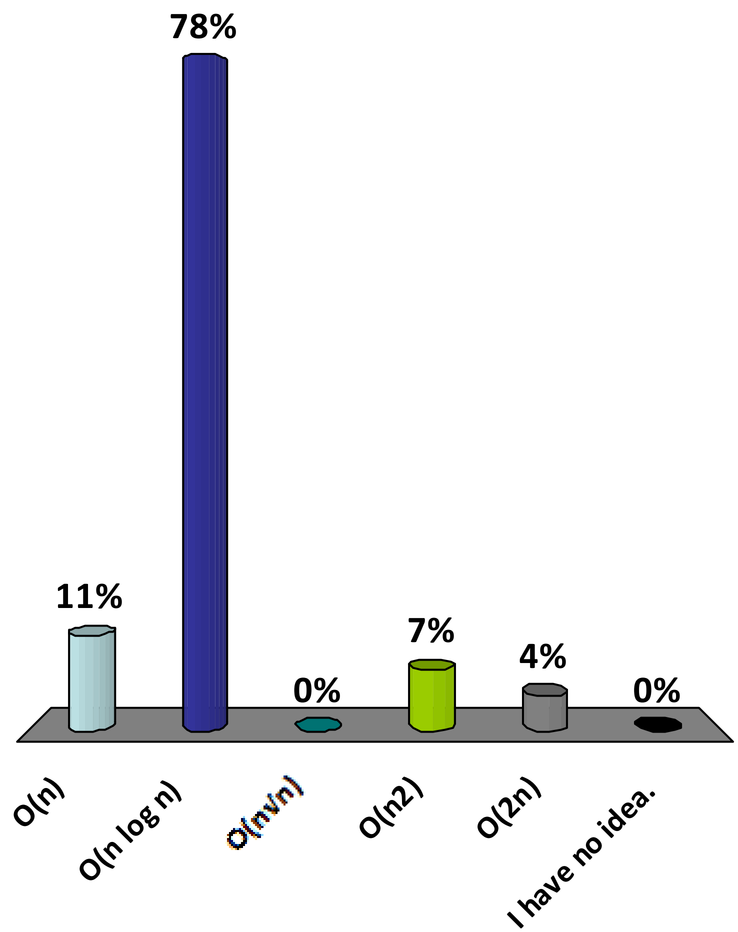
Let $T(n)$ be the worst-case running time for an array of n elements.

$$T(n) = \theta(1) \quad \textbf{if } (n=1)$$

$$= 2T(n/2) + cn \quad \textbf{if } (n>1)$$

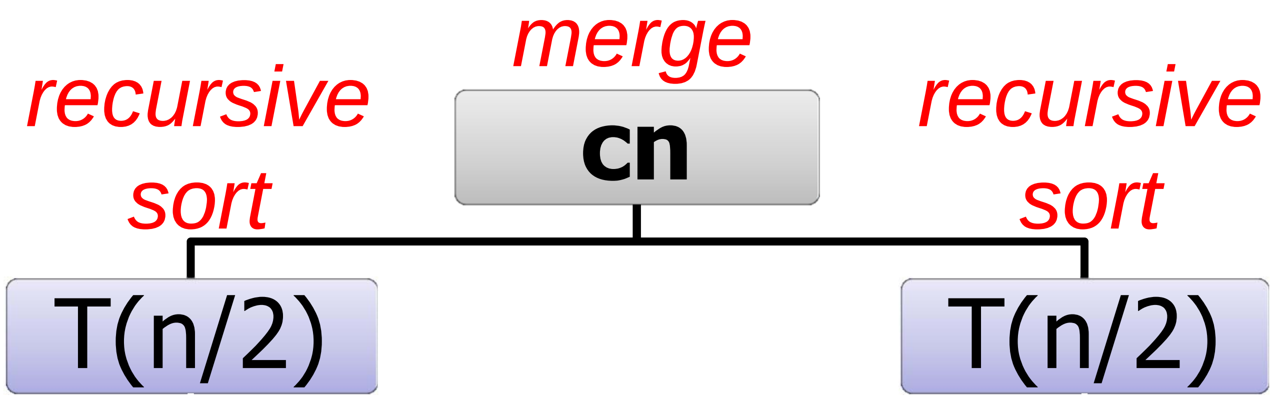
What is the (worst-case) running time of MergeSort?

A. O(n)
B. O(n log n)
C. O(n√n)
D. O(n^2)
E. O(2^n)
F. I have no idea.



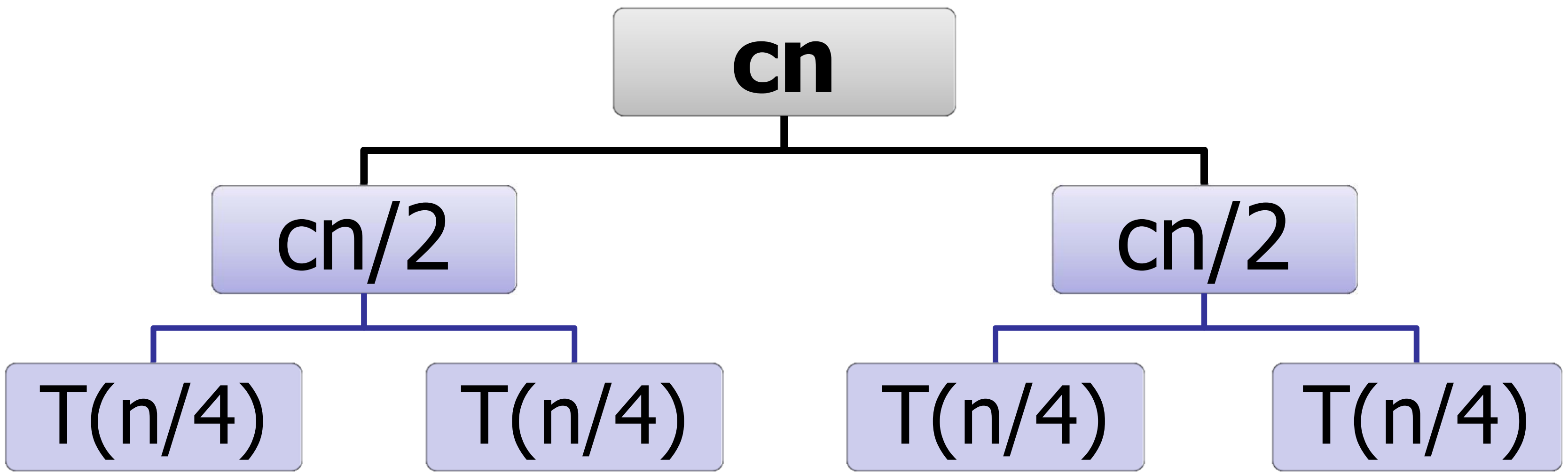
MergeSortAnalysis

$T(n) = 2T(n/2) + cn$



MergeSortAnalysis

$T(n) = 2T(n/2) + cn$



MergeSort Analysis

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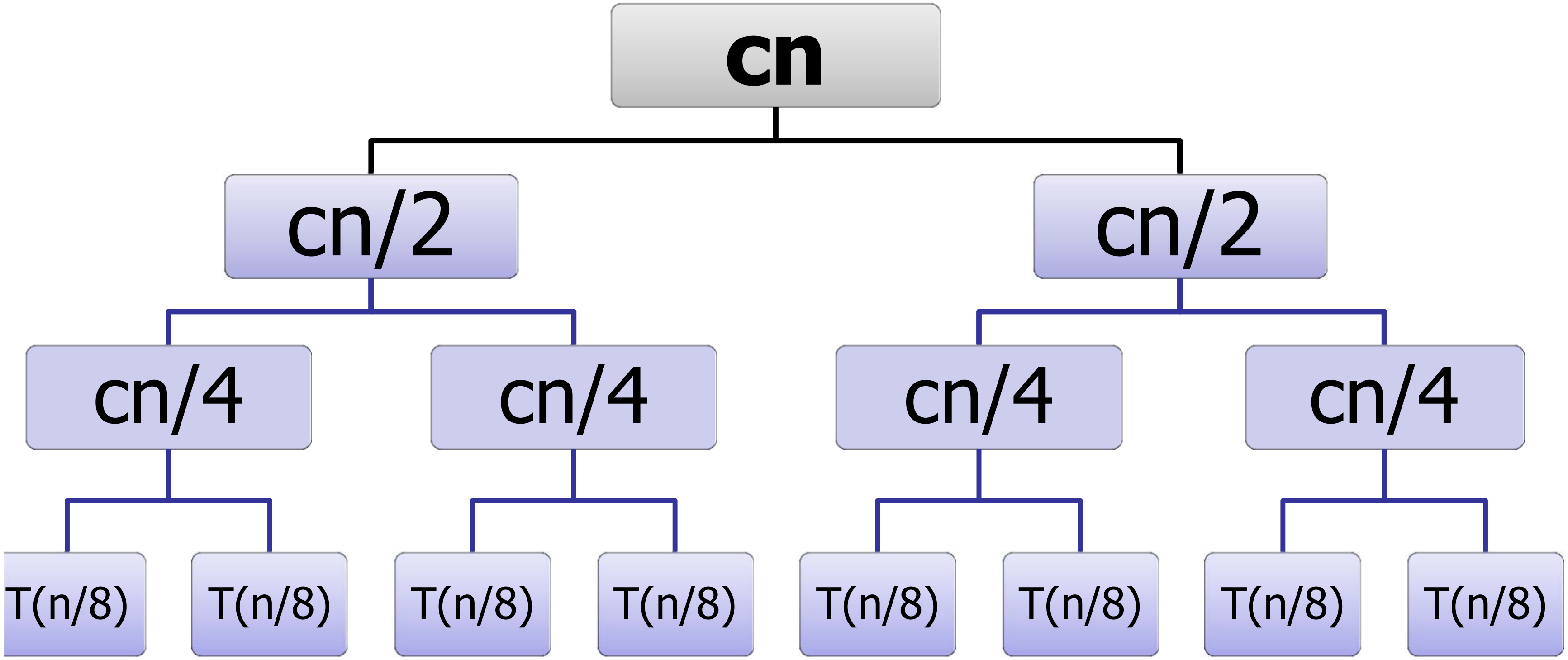
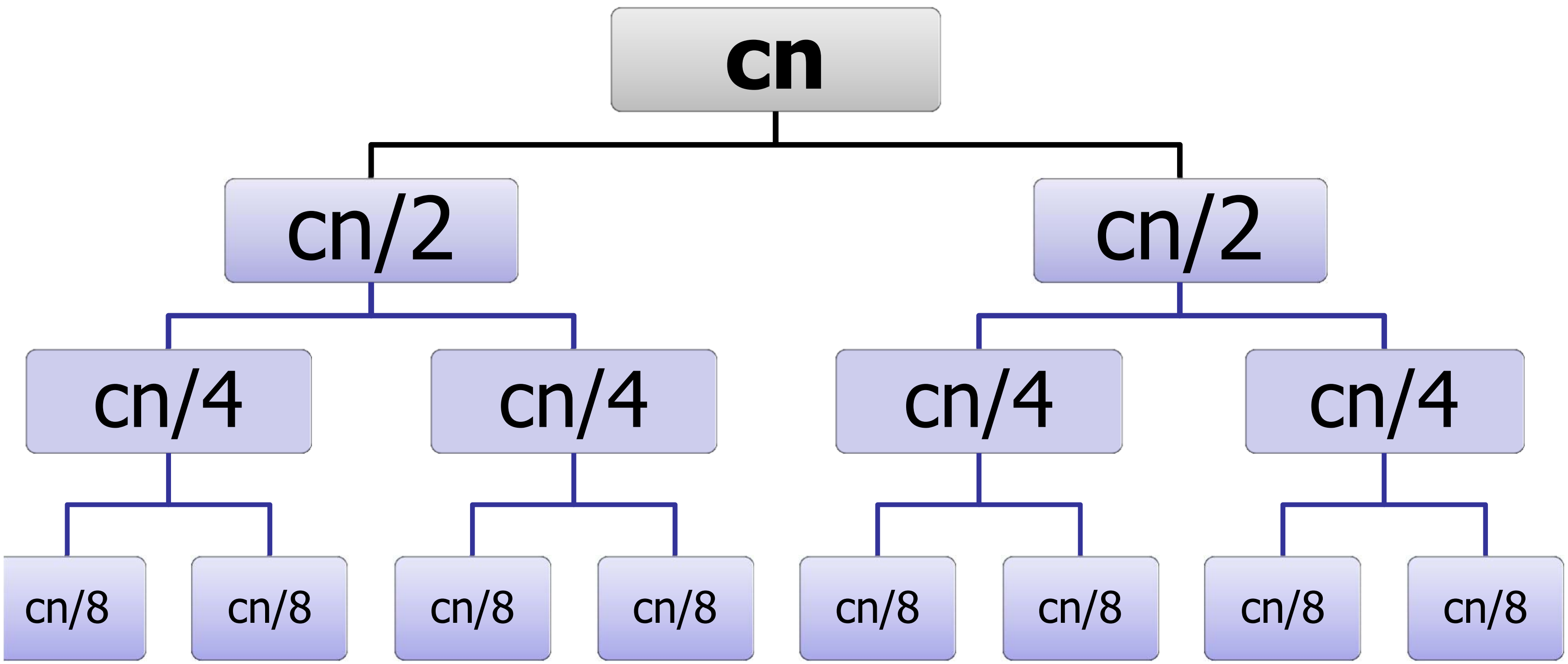


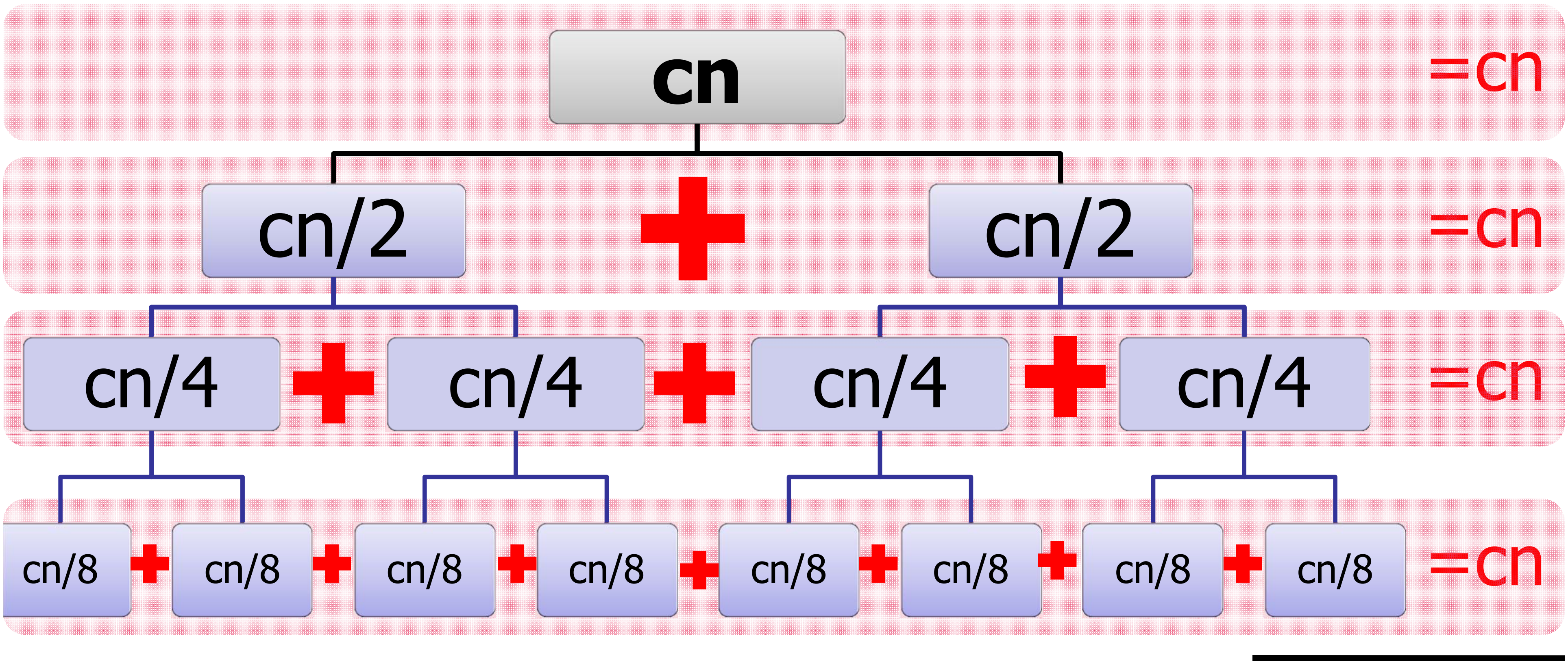
MergeSort Analysis

$T(n) = 2T(n/2) + cn$



MergeSort Analysis

T(n) = 2T(n/2) + cn



MergeSort Analysis

$T(n) = 2T(n/2) + cn$

Level	Number
0	1
1	2
2	4
3	8
4	16
…	…
h	??

$$\text{Number} = 2^{\text{Level}}$$

MergeSort Analysis

T(n) = 2T(n/2) + cn

Level	Number
0	1
1	2
2	4
3	8
4	16
…	…
h	n

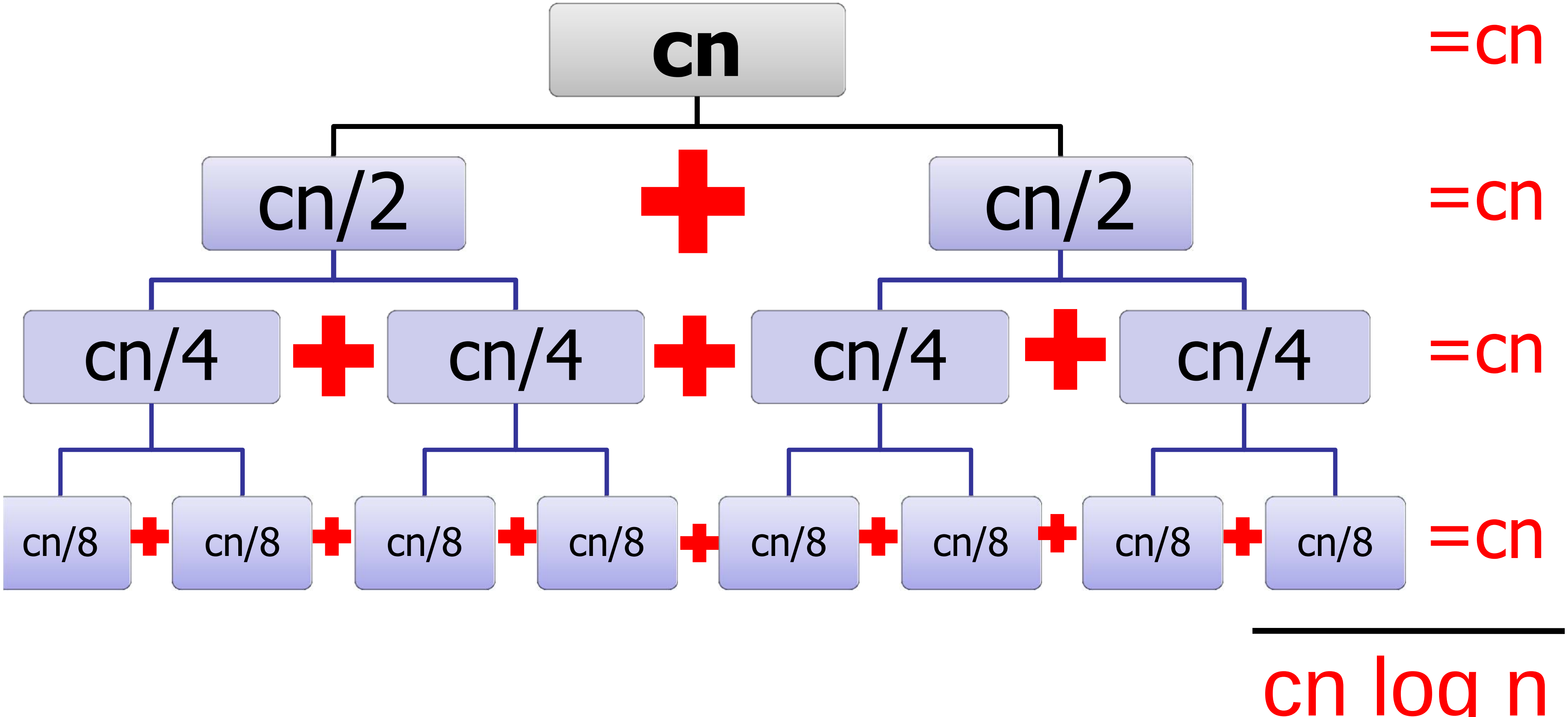
$$\text{Number} = 2^{\text{Level}}$$

$$n = 2^h$$

$$\log n = h$$

MergeSort Analysis

$T(n) = 2T(n/2) + cn$



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MergeSortAnalysis

$T(n) = O(n \log n)$

MergeSort(A, n)

if (n=1) **then return;**

else:

X ← MergeSort**(**…**);**

Y ← MergeSort**(**…**);**

return Merge **(**X,Y, n/2**);**

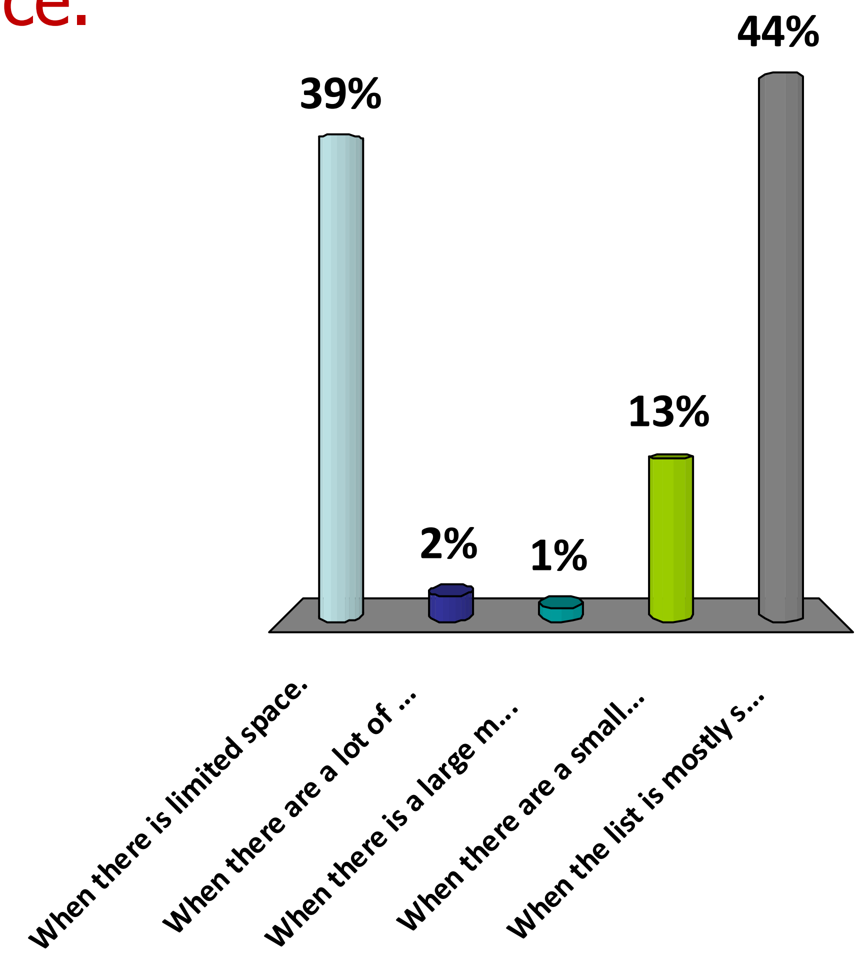
Performance Profiling

(*Dracula* vs. *Lewis & Clark*)

Version	Change	Running Time
Version 1		4,311.00s
Version 2	Better file handling	676.50s
Version 3	Faster sorting	6.59s
Version 4	No sorting!	2.35s

When is it better to use InsertionSort instead of MergeSort?

A. When there is limited space.
B. When there are a lot of items to sort.
C. When there is a large memory cache.
D. When there are a small number of items.
E. When the list is mostly sorted.



MergeSort

When the list is mostly sorted:

- InsertionSort is fast!
- MergeSort is O(n log n)

How "close to sorted" should a list be for InsertionSort to be faster?

MergeSort

Small number of items to sort:

- MergeSort is slow!
- Caching performance, branch prediction, etc.
- User InsertionSort for $n < 1024$, say.

Base case of recursion:

- Use slower sort.

MergeSort

Space usage:

- Need extra space to do merge.
- Merge copies data to new array.
- How much extra space??

NB Challenge of the Day 2:

How much space does MergeSort need to sort n items?

MergeSort

Stability:

- MergeSort is stable if “merge” is stable.
- Merge is stable if carefully implemented.

Sorting Analysis

Summary:

- BubbleSort: $O(n^2)$
- SelectionSort: $O(n^2)$
- InsertionSort: $O(n^2)$
- MergeSort: $O(n \log n)$

Properties: time, space, stability

For next time…

Friday lecture:

– More sorting

Problem Set 3:

– Released today. Due next week.