VELOCITY DISTRIBUTION FUNTION OF DIFFERENT TYPES OF PLASMA THROUGH SIMULATION

Presented by,

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ABSTRACT

Velocity Distribution Function (VDF) is obtained from the combination of angular and energy distribution functions, Maxwell – Boltzmann Distribution. Particle simulations of bounded plasmas require assumptions on the distribution function of the incoming particles at the plasma–surface interface. In this project, types of velocity distribution functions of plasma particle emitted from plasma surface and their simulations through Monte-Carlo technique are discussed.

INTRODUCTION

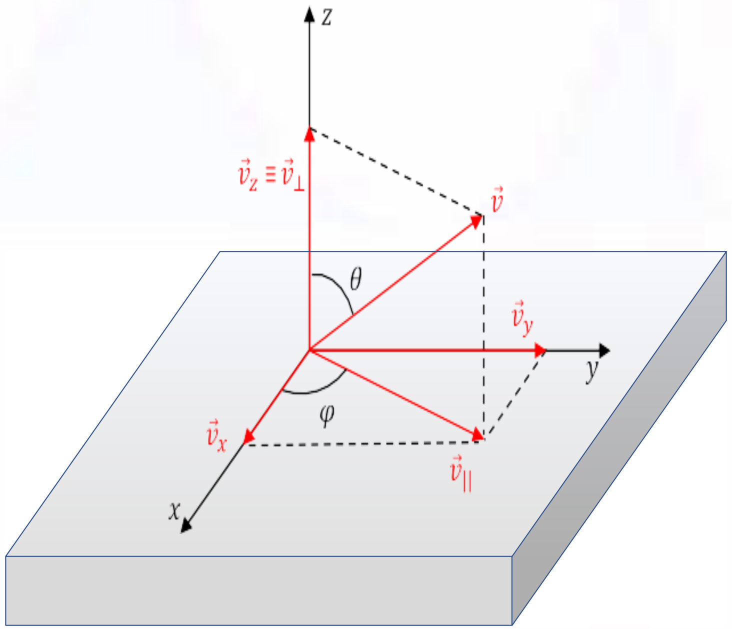
Numerical simulation is the process of mathematical model performed on a computer, which is designed to predict the behaviour of, or the outcome of, a real-world or physical system. The reliability of some mathematical models can be determined by comparing their results to the real-world outcomes they aim to predict. Computer simulations have become a useful tool for the mathematical modelling of many natural systems in physics , as well as human systems in Simulation of a system is represented as the running of the system's model. The emitted particles are characterized by a certain velocity (speed and direction) defined by a distribution function. The speed is inferred by an energy distribution function (EDF), while the direction is set according to an angular distribution function (ADF).

❑ ANGULAR DISTRIBUTION FUNCTION

- The ADF is defined as the number of particles emitted from the surface in a unit solid angle, divided by the total number of particles emitted from the surface.

- It indicates the solid-angle probability function (SAPF)

$$\boxed{g_{\Omega}(\theta, \varphi) = \frac{dN(\theta,\varnothing)}{N_0\, d\Omega}}$$



Where,
N_0 =total number of emitted particles
$dN(\theta,\varnothing)$ =number of particles emitted in a unit solid angle.
$d\Omega = \sin\theta d\theta d\varnothing$,
Solid angle defined by the polar angle θ & the azimuthal angle φ.
θ varies from 0 to $\pi/2$
φ varies from 0 to 2π.

ANGULAR DISTRIBUTION FUNCTION

- ISOTROPIC ANGULAR DISTRIBUTION FUNCTION

The solid-angle distribution is isotropic when the same number of particles is emitted in any unit solid angle, regardless of the direction of emission . The isotropic distribution is obtained for n=0. The isotropic distribution is also used to simulate the emission of secondary electrons in different types of plasmas . It characterizes the injection of thermalized particles through a boundary of the simulation domain.

- COSINE ANGULAR DISTRIBUTION FUNCTION

The distribution is cosine-type when the number of particles emitted in a unit solid angle is proportional to $\cos^n \theta$. The most common of the cosine-type distributions is pure cosine, with n= 1 . it is typical for the diffuse reflection of particles on a solid surface or for the secondary electron emission. sputtering under ion bombardment is described by cosine-type distributions of the sputtered atoms.

- Both isotropic and cosine-type solid-angle distributions can be defined by

$$\boxed{g_\Omega(\theta) = \frac{n+1}{2\pi} \cos^n \theta}$$

Both distributions are independent of the azimuthal angle φ.

❑ ENERGY DISTRIBUTION FUNCTION

- The Energy Distribution Function (EDF) of the emitted particles f(Ɛ), is defined as the number of particles emitted per unit energy with the kinetic energy around Ɛ (here expressed in eV). The corresponding Energy Probability function (EPF) is obtained by dividing the EDF by the no. the emitted particles Although it is a volume distribution rather than a surface one, the most known EPF is the Maxwell-Boltzmann distribution,

$$\boxed{f(\varepsilon) = \frac{dN(\varepsilon)}{N_0\, d\varepsilon} = C\sqrt{\varepsilon}\, \exp\left(-\frac{\varepsilon}{T}\right)}$$

C= Normalisation Constant

- For the isotropic distribution $C = \frac{2}{\pi}$ & for the cosine distribution C = 1.
- It characterizes a system of particles in thermal equilibrium at the temperature T (eV) .

MAXWELLIAN VELOCITY DISTRIBUTION FUNCTION

- The Velocity Distribution Function (VDF) is obtained from total distribution function F(Ɛ,θ) , Which is a combination of angular and energy distribution .
- It is not necessary to use the combined angular and energy distribution in order to obtain the VDF, Maxwell Boltzmann Distribution depends only on the speed of particle .

Analytically, the two distributions are multiplied when they are independent of each other.

$$\boxed{F(\mathcal{E},\theta) = \frac{dN(\mathcal{E},\theta)}{N_0\, d\mathcal{E}\, d\Omega} = f(\mathcal{E})\, g(\theta)}$$

Where, g (θ) = Angular Distribution Function
f (Ɛ) = Energy Distribution Function

- Therefore, VDF is given by

$$F(\mathcal{E},\theta) = C\sqrt{\mathcal{E}}\, \exp\left(-\frac{\mathcal{E}}{T}\right)$$

$$F(v) = C\sqrt{\mathcal{E}}\, \exp\left(-\frac{mv^2}{2eT}\right)$$

The particle speed v= $\sqrt{2e\mathcal{E}/m}$

❑ MAXWELLIAN VELOCITY DISTRIBUTION FUNCTION

- While the angular distribution serves compute velocity components ,

$$\mathbf{v_x} = \text{v} \sin\boldsymbol{\theta} \cos\varnothing$$

$$\mathbf{v_y} = \text{v} \sin\boldsymbol{\theta} \sin\varnothing$$

$$\mathbf{v_z} = \text{v} \cos\boldsymbol{\theta}$$

Velocity components $\mathbf{v_x}$ and $\mathbf{v_y}$ are parallel to the surface and $\mathbf{v_z}$ is perpendicular to the surface.

Where,

$$\boldsymbol{\varphi} = 2\pi\text{r}$$

$$\boldsymbol{\theta} = \text{arc cos}\left[(\mathbf{1}-\boldsymbol{r})^{\frac{\mathbf{1}}{\boldsymbol{n}+\mathbf{1}}}\right]$$

r = random number between 0 and **1.**

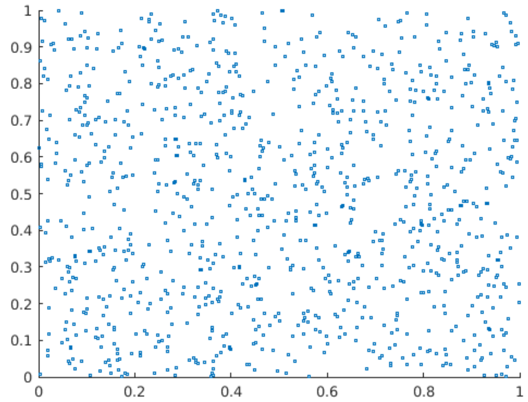
m = Mass of the electron = 9.11 x $\mathbf{10^{-31}}$ Kg.

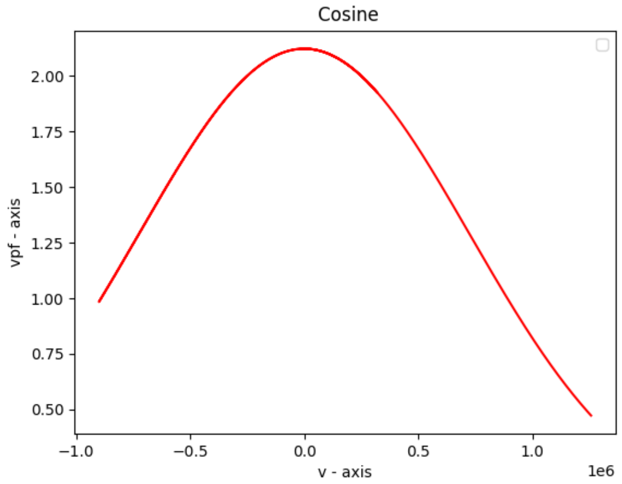
e = charge of electron = 1.602 x $\mathbf{10^{-19}}$ Columb.

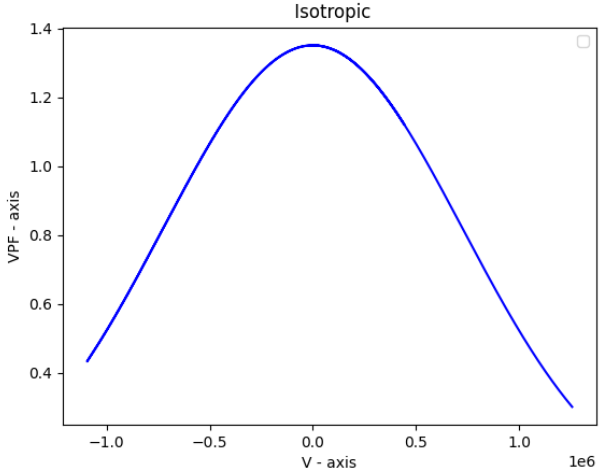
❑ MONTE –CARLO SIMULATION

- Monte Carlo method, or Monte Carlo experiment, is a broad class of computational algorithm that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle.
- In physics-related problems, Monte Carlo methods are useful for simulating systems with many coupled degrees of freedom , such as fluids, disordered materials, strongly coupled solids, and cellular structures
- Here, r is a random number between 0 and 1, so we generate **100000** random number between 0 to **1**.
- Here, we take $T = 3$ eV and $\mathcal{E} = \frac{3}{2}T$ to preserve the

  Debye length and Average kinetic energy.

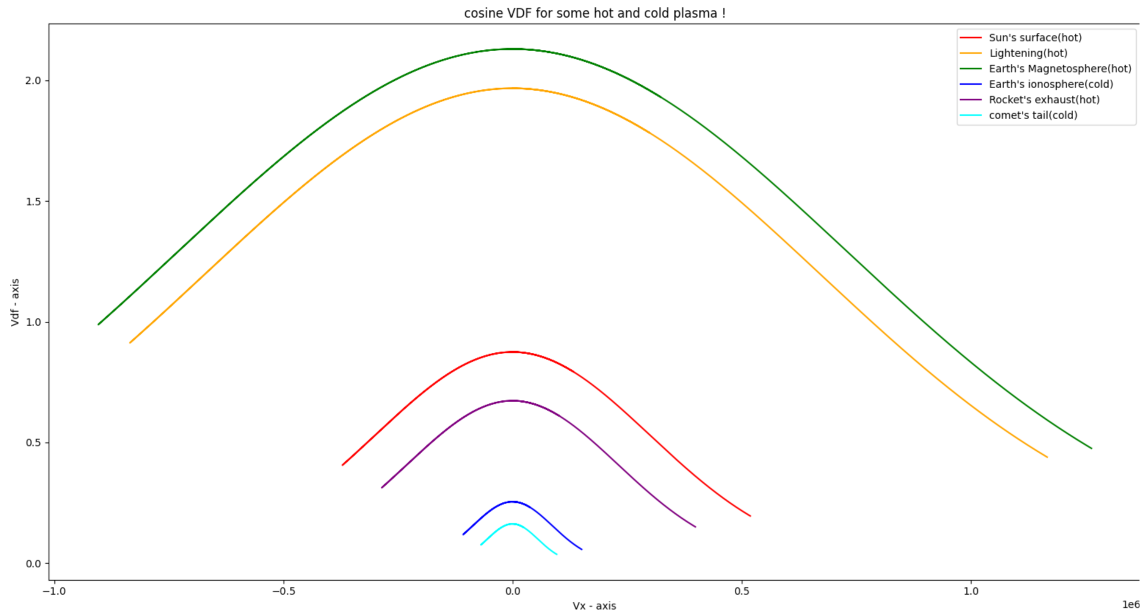


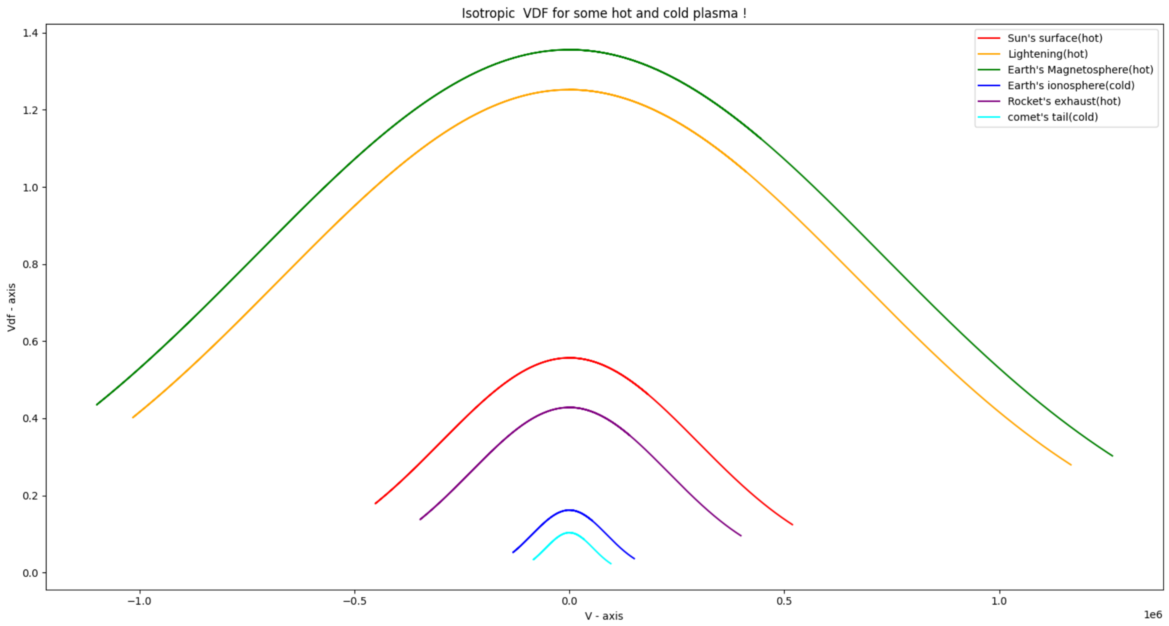




❑ SIMULATION FOR HOT & COLD PLASMAS

- In hot plasmas electrons and ions have nearly same energy i.e equal temperature. The term cold plasma refers to a condition where electrons have much higher energy than ions. A plasma is sometimes referred to as being "hot" if it is nearly fully ionized, or "cold" if only a small fraction, (for instance 1%) of the gas molecules are ionized.
- Here, we take some example of hot & cold plasmas like, Sun's surface(T=0.51 eV),Earth's magnetosphere(T=3.0248 eV),Lightening(T= 2.58 eV), Rocket exhaust (0.3016 eV),Earth's ionosphere(0.0431 eV), Comet's tail (T=0.0176 eV) to show its Cosine & Isotropic VDF graph through Simulation.
- It's very amazing and interesting fact, that, lightening and earth's magnetosphere are fully ionized and hotter than sun's surface. Lightening is 5 times hotter than sun's surface in kelvin's scale



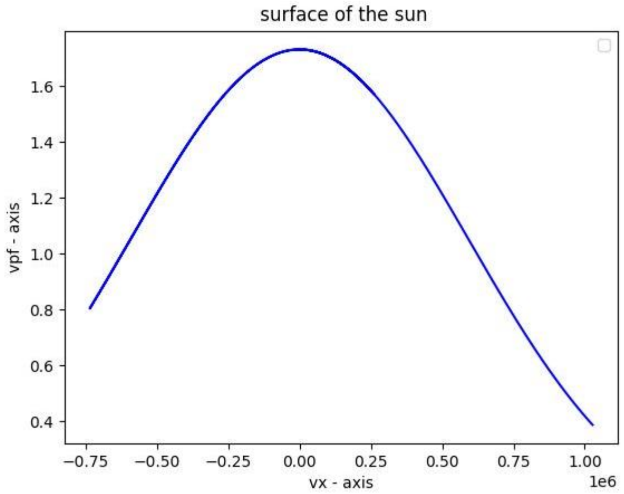


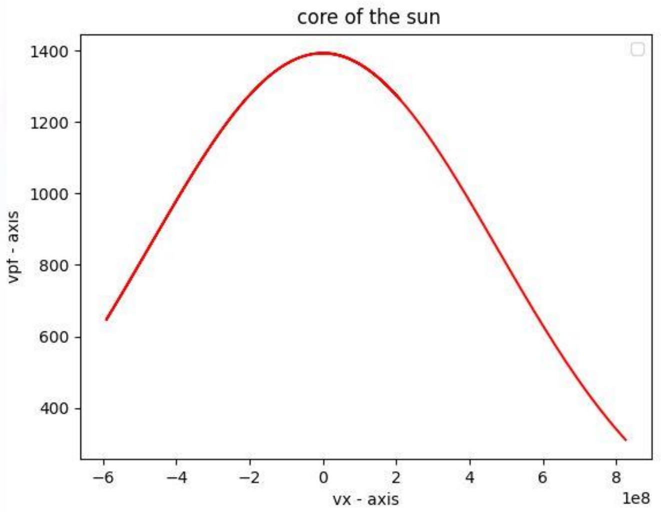
❑ SIMULATION FOR PLASMA AT SUN

- The core is made of hot, dense plasma (ions and electrons), at a pressure estimated at 265 billion bar at the centre. Due to fusion, the composition of the solar plasma drops from 68–70% hydrogen by mass at the outer core, to 34% hydrogen at the core/Sun centre.
The helium/hydrogen plasma in the core of the Sun is completely ionized. Core of the sun is more ionized than the surface of the sun. Through this comparison we get a brief idea about the VDF of nuclear fusion.

- Studying all the graphs we can say that, it is properly normalised so that the fact that the average value $< V_i >$ vanishes is physically evident by symmetry, each components of the velocity can be equally positive or negative.

- This is a normal distribution graph. They're all symmetric. The normal distribution cannot model skewed distributions. The mean, median, and mode are all equal.

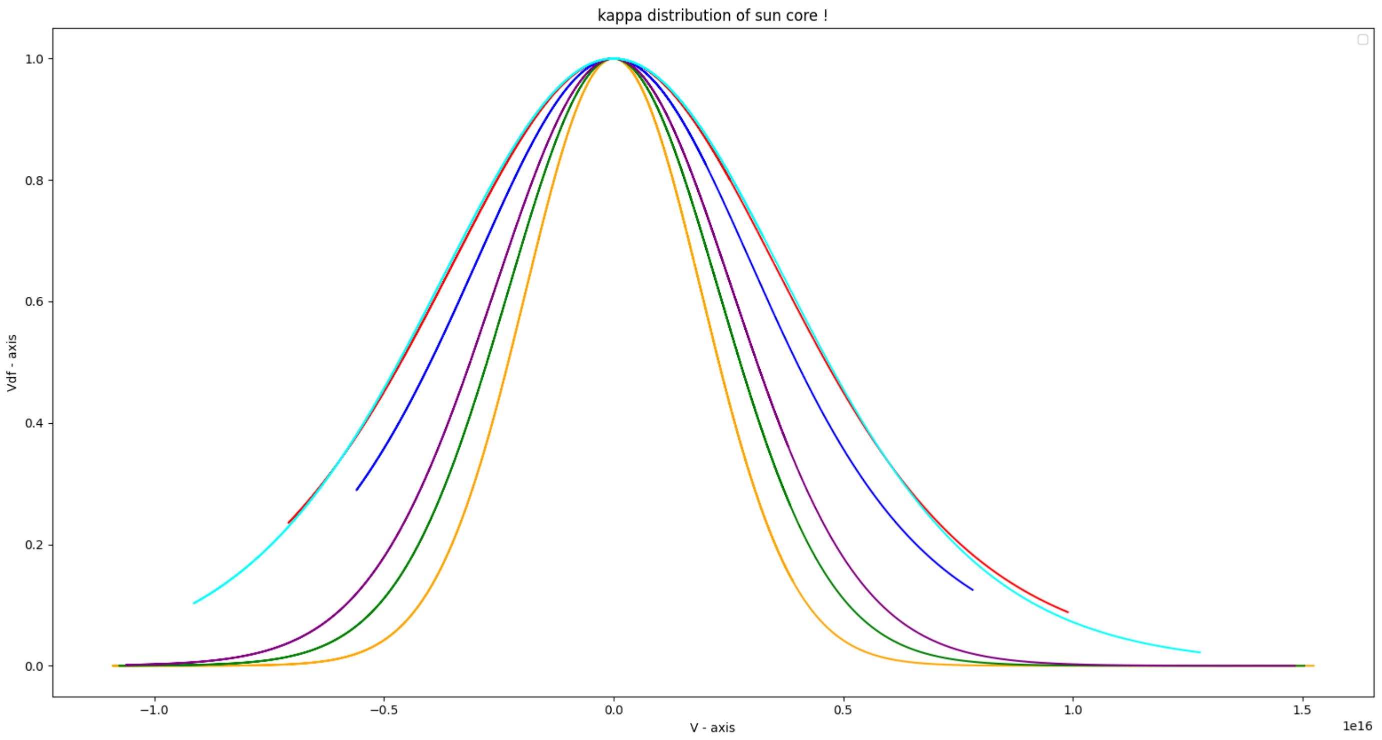




NON-MAXWELLIAN VELOCITY DISTRIBUTION FUNCTION

- Boltzmann-Gibbs (BG) Statistical Mechanics has withstood the test of time for describing classical systems in thermal equilibrium—a state where any flow of heat (e.g. thermal conduction, thermal radiation) is in balance. Any system in thermal equilibrium has its distribution function of velocities stabilized into a Maxwell distribution (in the absence of an external force). Maxwell distributions are quite rare in space and astrophysical plasmas instead, the vast majority of these plasmas reside in stationary states (i.e., their statistics are at least temporarily time invariant), that are typically not well described by Maxwell distributions, and thus not in thermal equilibrium.

- Particle velocity distribution functions (VDF) in space plasmas often show non Maxwellian super thermal tails decreasing as a power law of the velocity. Such distributions are well fitted by the so-called Kappa distribution. The presence of such distributions in different space plasmas suggests a universal mechanism for the creation of such super thermal tails.



❑ KAPPA VDF FOR DUSTY PLASMA

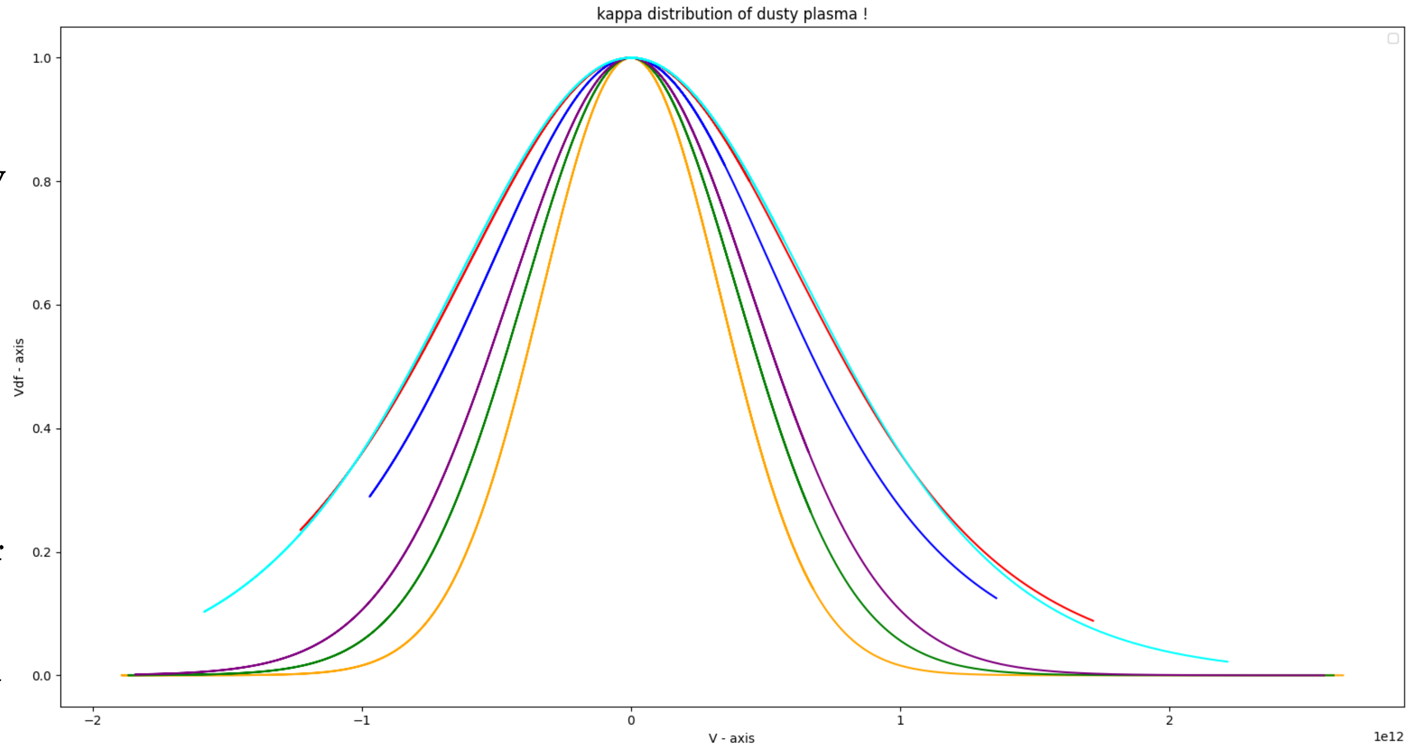
- A dusty plasma consists of electrons, ions, electrically-charged micron-size dust particles, and neutral gas atoms. Experiments with dusty plasmas are generally dissipative nonequilibrium systems driven by an external source, which results in electric fields and ion flows that can introduce energy into the collection of dust particles. In a two-dimensional dusty plasma levitated in a plasma sheath, a non-Gaussian high-energy tail was observed in the probability distribution function of particle displacement, and this feature was linked to super diffusion of particles.

- For the velocity distribution function, non-Gaussian distribution features, especially with energetic tails, are found in a wide range of plasmas, including low-collisional plasmas such as the solar wind . It is commonly found that the velocity distribution in the solar wind has high-energy tails, and it is often fit to the Kappa distribution.

$$\boxed{F(r,v) \propto \frac{1}{\left(1+\frac{v^2}{\kappa W^2}\right)^{1+\kappa}}}$$

where $W = [(2\boldsymbol{\kappa} - 3)k_B T_p / \boldsymbol{\kappa} m_p]^{1/2}$ is a characteristic speed, and T_p is the temperature of the particle with mass m_p . The parameter $\boldsymbol{\kappa}$ characterizes how far the system is from thermal equilibrium.

- When $\boldsymbol{\kappa} \rightarrow \infty$, the Kappa distribution approaches a Maxwellian, but when $\boldsymbol{\kappa}$ is finite, it has high-energy tails, with more high velocity particles than for a Maxwellian.



CONCLUSION

Maxwellian velocity distribution is frequently used for the simulation of incoming particles obtaining from a boundary surface. It' s very difficult for us to calculate all the VDFs and plot all the graphs. Monte Carlo simulation helps us to simulate all the VDFs graphically. Monte Carlo simulations are used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. It is a technique used to understand the impact of risk and uncertainty in prediction and forecasting models. In python, full process of simulation has been done. In this project, just few examples of plasma VDFs has been discussed. We can simulate other plasmas like nuclear fusion, nuclear fission, Aurora Boriolis etc. We can also calculate expectations, variance and mean root square of other plasma through Monte Carlo simulation. Thus, for better study in plasma physics as well as space physics, simulation is highly recommended.

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THANK YOU