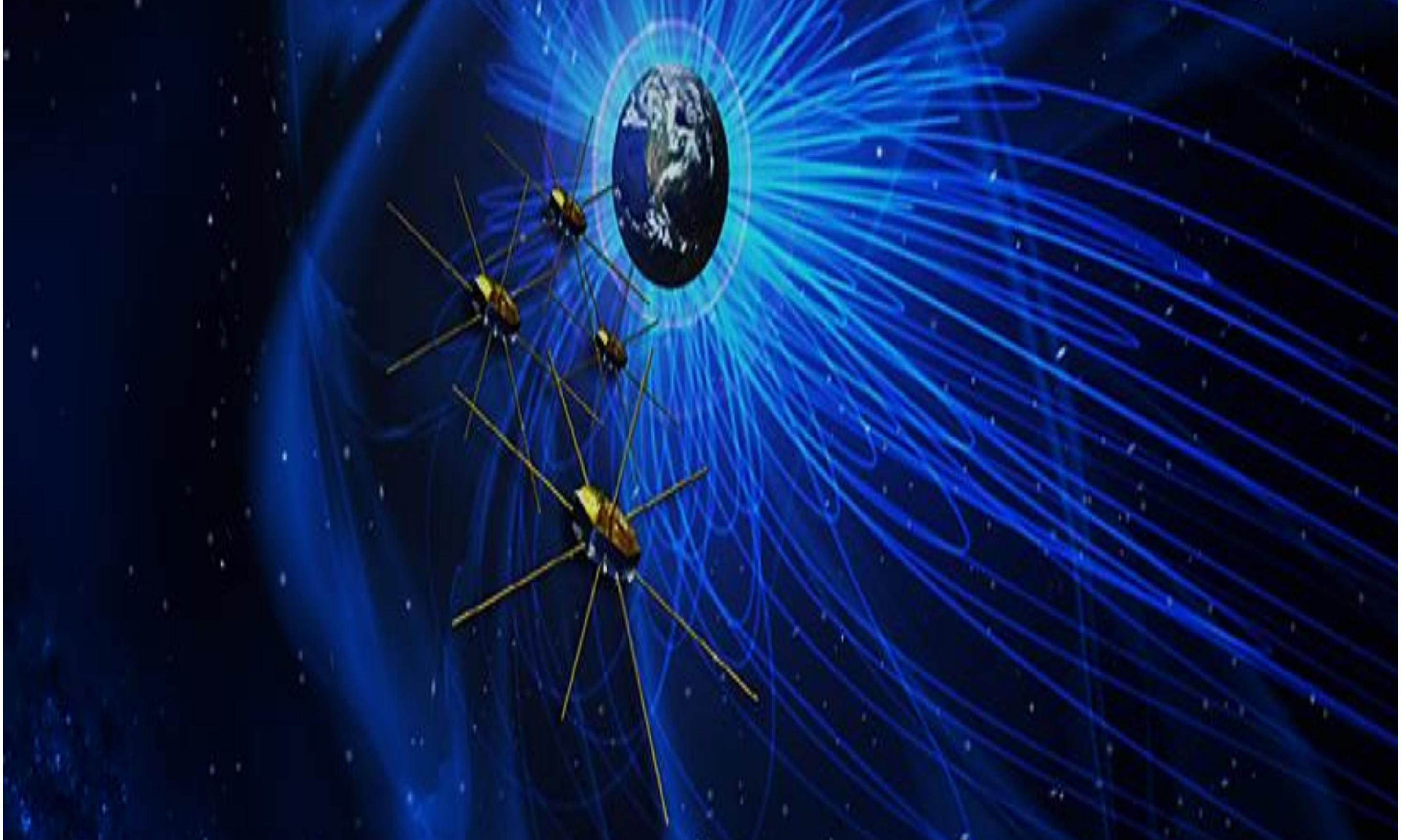
A PROJECT ON

VELOCITY DISTRIBUTION FUNCTION OF DIFFERENT TYPES OF PLASMAS THROUGH SIMULATION



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CERTIFICATE

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ABSTRACT

Velocity Distribution Function (VDF) is obtained from the combination of angular and energy distribution functions, Maxwell – Boltzmann Distribution. Particle simulations of bounded plasmas require assumptions on the distribution function of the incoming particles at the plasma–surface interface. In this project, types of velocity distribution functions of plasma particle emitted from plasma surface and their simulations through Monte Carlo technique are discussed.

INTRODUCTION

NUMERICAL SIMULATION

Numerical simulation is the process of mathematical modelling, performed on a computer, which is designed to predict the behaviour of, or the outcome of, a real-world or physical system. The reliability of some mathematical models can be determined by comparing their results to the real-world outcomes they aim to predict. Computer simulations have become a useful tool for the mathematical modelling of many natural systems in physics , as well as human systems in Simulation of a system is represented as the running of the system's model. It can be used to explore and gain new insights into new technology and to estimate the performance of systems too complex for analytical solutions.

NUMERICAL SIMULATION IN PLASMA PHYSICS

In the case of particle simulations with boundary conditions, such as 2D or 3D simulations of fluids, including, here, gases and plasmas bounded by solid, liquid, or virtual surfaces, an important assumption concerns the particles entering the simulation domain at boundaries, regardless of their generation process: injection, surface reflection, or surface emission. All particles originating from the boundary surface will be further referred to as surface emitted particles, regardless of their generation method.**The emitted particles are characterized by a certain velocity (speed and direction) defined by a distribution function. The speed is inferred by an energy distribution function (EDF), while the direction is set according to an angular distribution function (ADF).**

VELOCITY DISTRIBUTION FUNCTION(VDF)

The **Velocity Distribution Function (VDF)** is obtained from total distribution function F(Ɛ,θ) , Which is a combination of **angular and energy distribution .** Analytically, the two distributions are **multiplied** when they are **independent of each other**.

$$F(\mathcal{E},\theta) = \frac{dN(\mathcal{E},\theta)}{N_0\ d\mathcal{E}d\Omega} = f(\mathcal{E})\ g(\theta) \qquad \text{..........(1)}$$

Where, g (θ) = Angular Distribution Function
f (Ɛ) = Energy Distribution Function

ANGUALR DISTRIBUTION FUNCTION(ADF)

Two types of distributions can be defined with respect to the emission angle: **solid-angle and polar-angle distribution functions.** The solid-angle distribution function (SADF) is usually referred to as the angular distribution function (ADF). The **SADF is defined as the number of particles emitted from the surface in a unit solid angle. Divided by the total number of particles,** it indicates the solid-angle probability function (SAPF)

$$g_\Omega(\theta,\varphi) = \frac{dN(\theta,\varnothing)}{N_0\ d\Omega} \qquad \text{.................. (2)}$$

Where N_0 **is the total number of emitted particles and dN(θ,⌀) is the number of particles emitted in the solid angle dΩ = sinθdθd⌀ defined by the polar angle** θ **and the azimuthal angle** φ**,** schematically shown in (Fig. 1) The **polar angle** θ is measured with respect to the surface normal. When particles are emitted on a single side ofa planar surface, i.e., in a total solid angle of 2π, the polar angle θ **varies from zero to** $\pi/2$ and the **azimuthal angle** φ **varies from zero to** 2π**. The distribution function is normalized to the total number of particles, while the probability function is normalized to 1. The use of the probability function is recommended because it ensures independence from**

the number of particles.

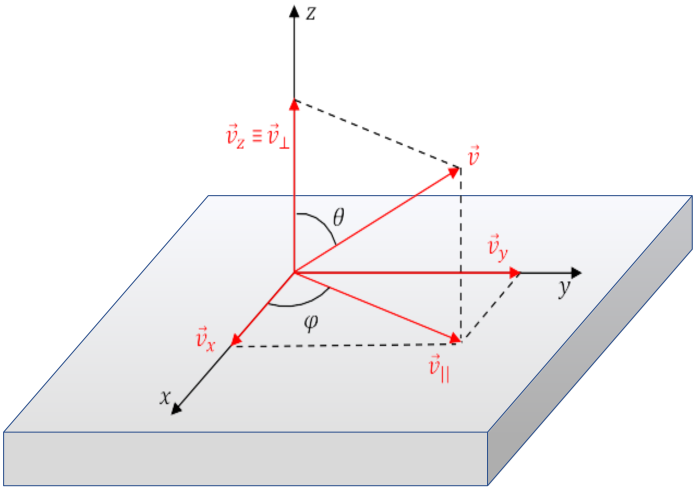


Fig: 1

The most common angular distributions are isotropic and cosine-type.

ISOTROPIC ANGULAR DISTRIBUTION FUNCTION

The solid-angle distribution is **isotropic** when **the same number of particles is emitted in any unit solid angle, regardless of the direction of emission.** It characterizes the injection of thermalized particles through a boundary of the simulation domain. Particular problems do not require the simulation of the entire (bulk) system but only of a small part of it. In such a case, the simulation domain has one or more virtual boundaries in contact with the bulk system. Matching the neighboring subsystems requires, among other parameters, matching the particle transfer. If particles in the bulk system are in thermal equilibrium, the particles injected into the simulation domain obey an isotropic angular distribution The isotropic distribution was also used to simulate the emission of secondary electrons in different types of plasmas **The isotropic distribution is obtained for n=0.** Both distributions are independent of the azimuthal angle φ.

COSINE ANGULAR DISTRIBUTION FUNCTION

The distribution is **cosine-type** when **the number of particles emitted in a unit solid angle is proportional to $\cos^n \theta$.** Target sputtering under ion bombardment is described by cosine-type distributions of the sputtered atoms. The most common of the **cosine-type distributions is pure cosine, with n= 1**. Besides the sputtering process, it is typical for the diffuse reflection of particles on a solid surface or for the secondary electron emission. In terms of probability, **both isotropic and cosine-type solid-angle distributions can be defined by**

$$g_\Omega(\theta) = \frac{n+1}{2\pi} \cos^n \theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

ENERGY DISTRIBUTION FUNCTION

The Energy Distribution Function (EDF) of the emitted particles f(Ɛ), is defined as the number of particles emitted per unit energy with the kinetic energy around Ɛ (here expressed in eV). The corresponding Energy Probability function (EPF) is obtained by dividing the EDF by the no the emitted particles Although it is a volume distribution rather than a surface one, **the most known EPF is the Maxwell-Boltzmann distribution,**

$$f(\mathcal{E}) = \frac{dN(\mathcal{E})}{N_0\, d\mathcal{E}} = C\sqrt{\mathcal{E}} \exp\left(-\frac{\mathcal{E}}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

with **C being a normalization constant**. The total Maxwell– Boltzmann distribution is defined by Eq. ((4) in energy and by an isotropic angular distribution. It will be hereafter referred to as isotropic Maxwell–Boltzmann distribution. It characterizes a system of particles in thermal equilibrium at the temperature T (also expressed in units of energy eV). For particles leaving a surface, Eq. (4) is specific for the electron EDF in the case of thermionic emission or for the EDF of gas particles reflected on the vessel's walls, when gas and walls are in thermal equilibrium It is also the case of thermalized particles injection through a boundary of the simulation domain, such as electrons or ions entering from plasma into the space charge sheath. **For the isotropic distribution $C = \frac{2}{\pi}$ and for the cosine distribution $C = 1$.**

NUMERICAL CALCULATION OF VDF

Numerically, **the particle speed** v= $\sqrt{2e\mathcal{E}/m}$ ....(5)is sampled according to the energy distribution, while the angular distribution serves compute **velocity components**

$$v_x = \text{v} \sin \theta \cos \varnothing$$
$$v_y = \text{v} \sin \theta \sin \varnothing \qquad (6)$$
$$v_y = \text{v} \cos \theta$$

Velocity components $\mathbf{v_x}$ **and** $\mathbf{v_y}$ **are parallel to the surface and** $\mathbf{v_y}$ **is perpendicular to the surface** (Fig. 1).

In practice, the solid-angle distribution is non-operable, serving to define the type of angular distribution, while the polar-angle distribution is used to numerically generate the distribution of particles **The polar angle probability function** corresponding to Eq. (3) are

$$g_\theta(\theta) = (n+1) \cos^n \theta \ \ \sin \theta \ \ldots\ldots\ldots\ldots\ldots\ldots \ (7)$$

All velocity distribution functions were obtained by the use of random numbers (Monte–Carlo technique). The **azimuthal angle** was generated as **φ= 2πr..............(8)**, with **r being a random number between 0 and 1.** The **polar angle θ was generated using the inverse of the cumulative distribution function algorithm.** For **isotropic and cosine angular distributions**, the algorithm was applied to Eq. (7)

$$\Theta = \text{arc cos}\,[(1-r)^{\frac{1}{n+1}}] \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (9)$$

Note that, since **r is a random number between 0 and 1**, Eq. (8) gives the **same angular distributions if the argument (1- r) is replaced by r .**

Note that, in particular cases, it is not necessary to use the combined angular and energy distribution in order to obtain the VDF, Maxwell Boltzmann Distribution depends only on the speed of particle and it can be treated as the product of three Independent normal distributions, each corresponding to one velocity component.

Therefore, **VDF is given by** (from Eq. (1), (4) and (5))

$$F(\mathcal{E},\theta) = C\sqrt{\mathcal{E}}\ \exp\left(-\frac{\mathcal{E}}{T}\right)$$

$$F(v) = C\sqrt{\mathcal{E}}\ \exp\left(-\frac{mv^2}{2eT}\right) \quad \ldots\ldots\ldots\ldots\ldots(10)$$

m = Mass of the electron = 9.11 x 10^{-31} Kg
e = charge of electron = 1.602 x 10^{-19} Coloumb

DIFFERENT TYPES OF VDF & THEIR SIMULATIONS

Monte Carlo method, or Monte Carlo experiment, is a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle.

They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. Monte Carlo methods are mainly used in three problem classes:[1] optimization, numerical integration, and generating draws from a probability distribution.

In physics-related problems, Monte Carlo methods are useful for simulating systems with many coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structures (see cellular Potts model, interacting particle systems, McKean–Vlasov processes, kinetic models of gases).

Here, r is a random number between 0 and 1, so we generate 100000 random number between 0 to 1(Fig:2).

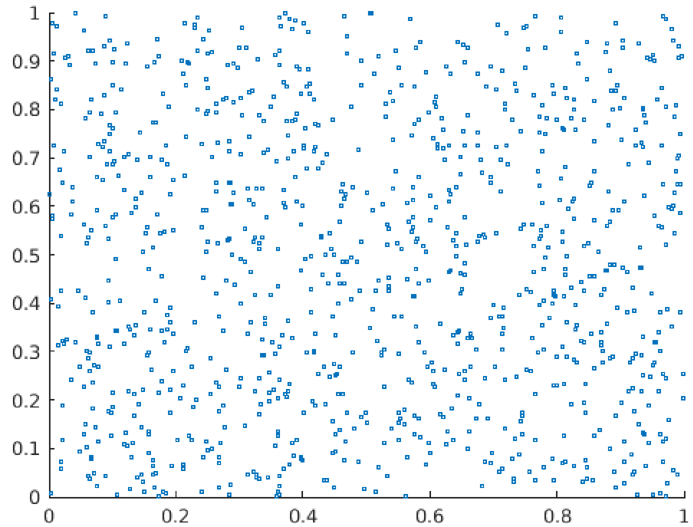
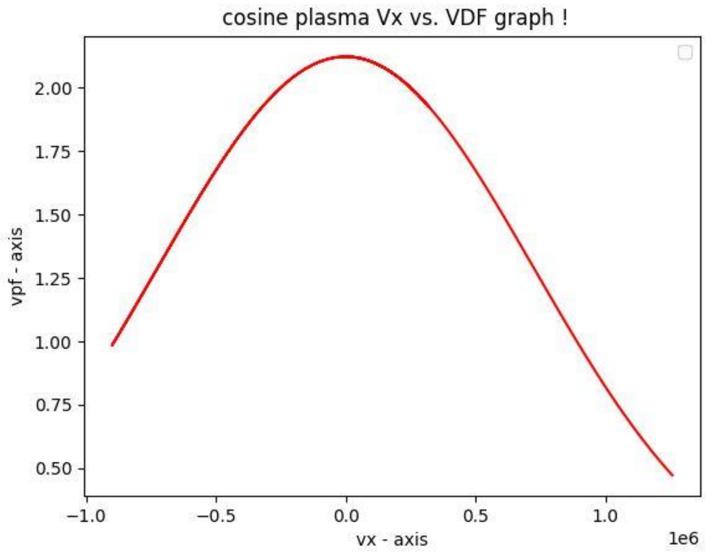
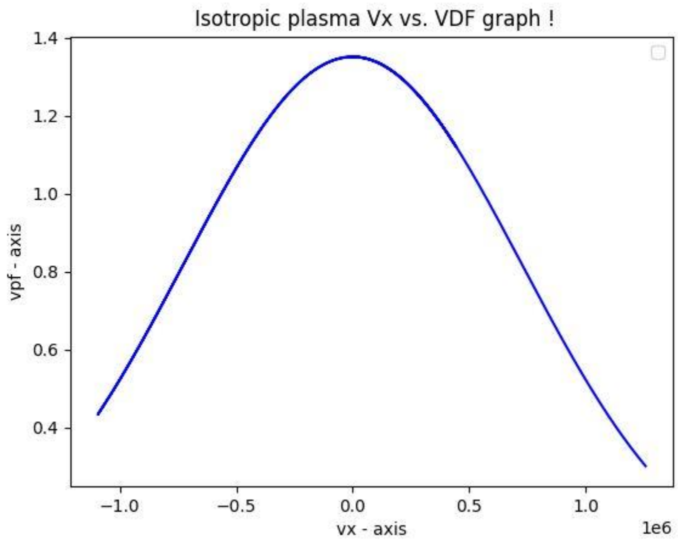


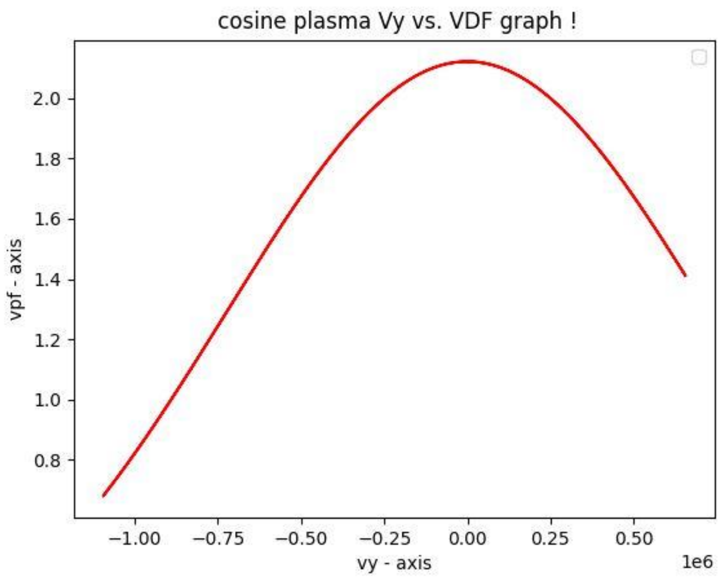
Fig:2

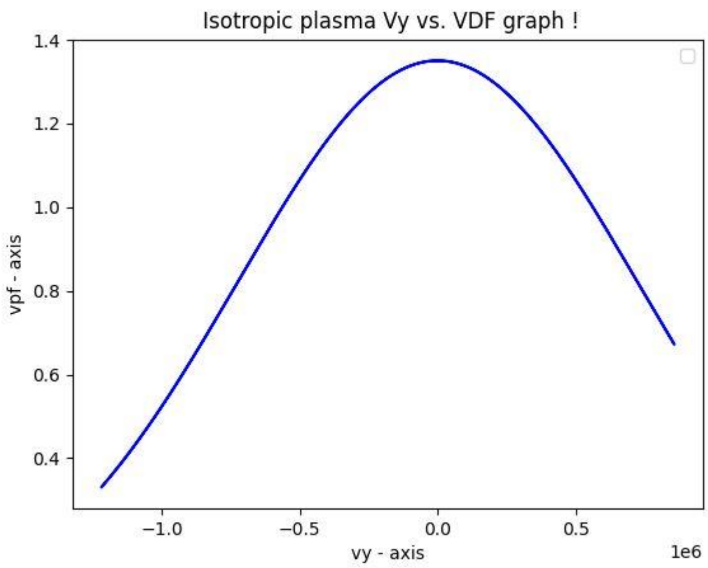
Here, taking T = 3 eV and $\mathcal{E} = \frac{3}{2}T$ to preserve the Debye length and Average kinetic energy, we get from Eq. (6), (8), (9) & (10)

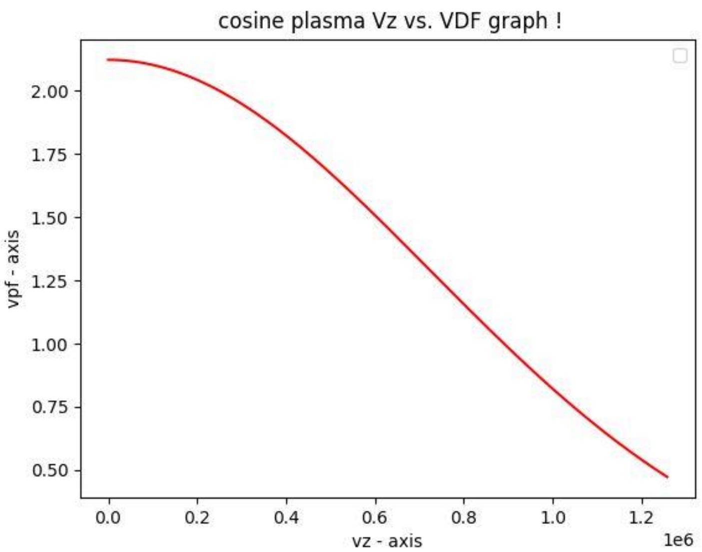
COSINE & ISOTROPIC VDF

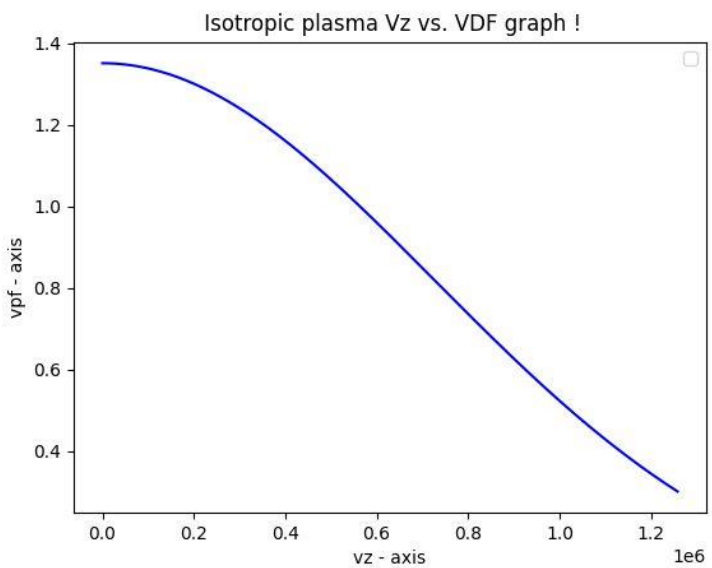












COSINE & ISOTROPIC VDF FOR SOME HOT AND COLD PLASMAS

In hot plasmas electrons and ions have nearly same energy i.e equal temperature. The term cold plasma refers to a condition where electrons have much higher energy than ions. A plasma is sometimes referred to as being “hot” if it is nearly fully ionized, or “cold” if only a small fraction, (for instance 1%) of the gas molecules are ionized.

No, we are going to talk about same example of hot and cold plasmas.

HOT PLASMA

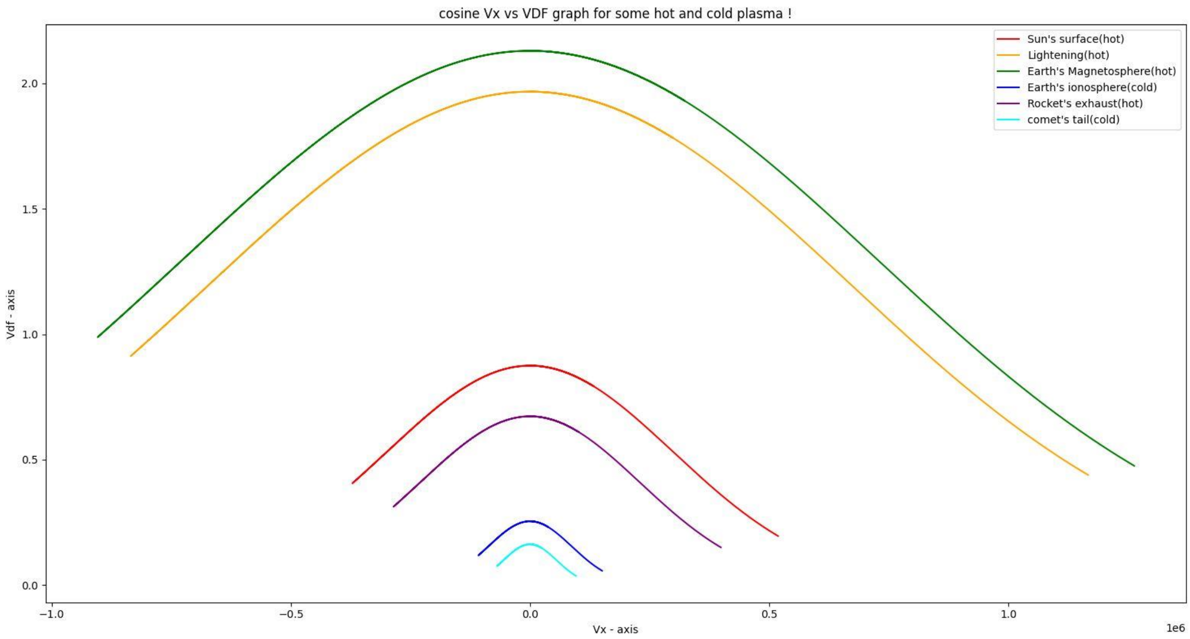
I. Sun’s surface, T=0.51 eV
II. Earth’s magnetosphere, T=3.0248 eV
III. Lightening, T= 2.58 eV
IV. Solar wind, T=689 eV
V. Sun’s core, 1292657.04 eV
VI. Rocket exhaust ,0.3016 eV

[**It’s very amazing and interesting fact, that, lightening and earth’s magnetosphere are fully ionized and hotter than sun’s surface. Lightening is 5 times hotter than sun’s surface in kelvin’s scale**]

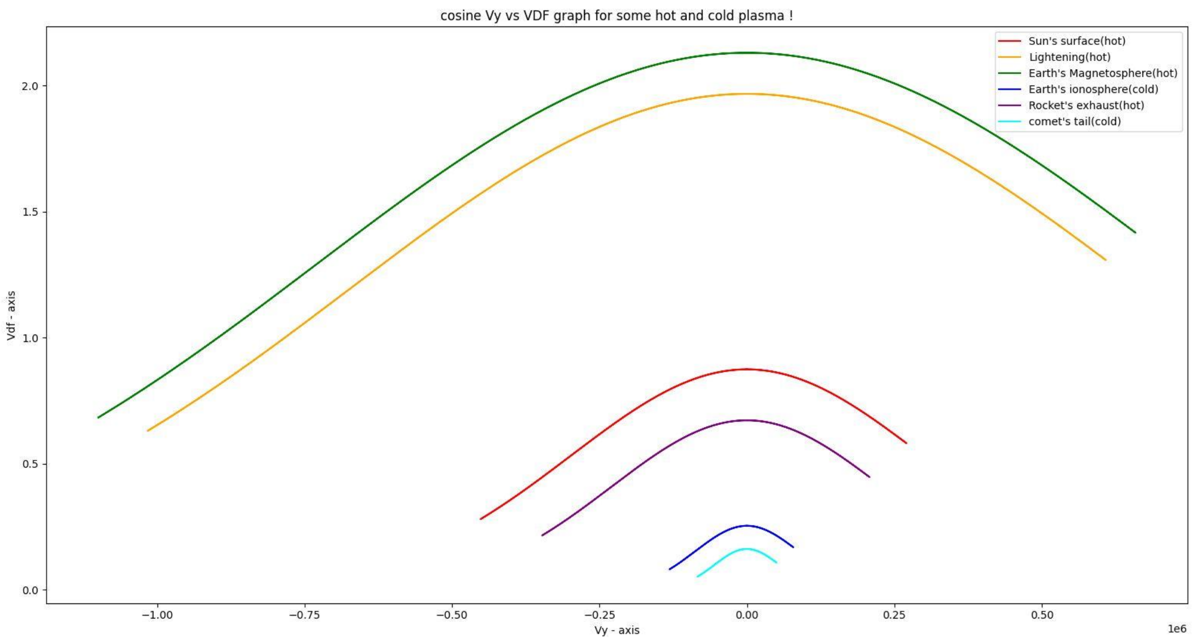
COLD PLASMA

I) Aurora, T=0.0259 eV
II) Comet’s tail, T=0.0176 eV
III) Interstellar gas cloudy, T=0.00861 eV
IV) TV. Plasma, T=0.027 eV

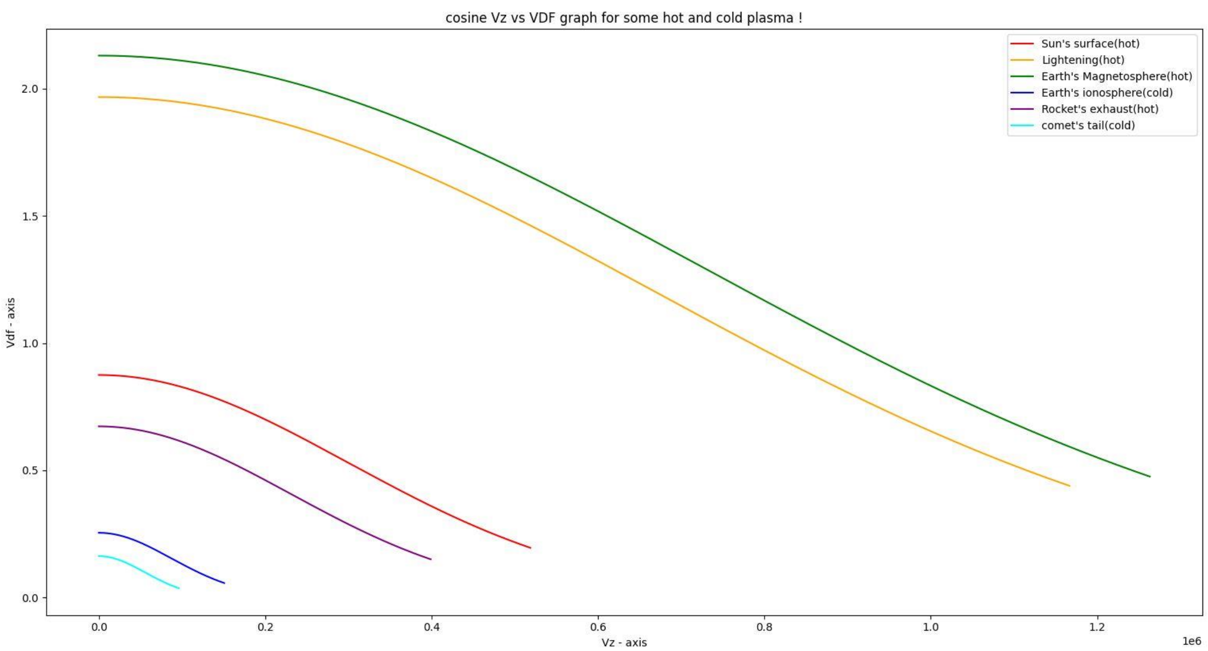
Here we have compared VDF graph among sun’s surface, lightening, earth’s magnetosphere, earth’s ionosphere, rocket’s exhaust and comet’s tail.



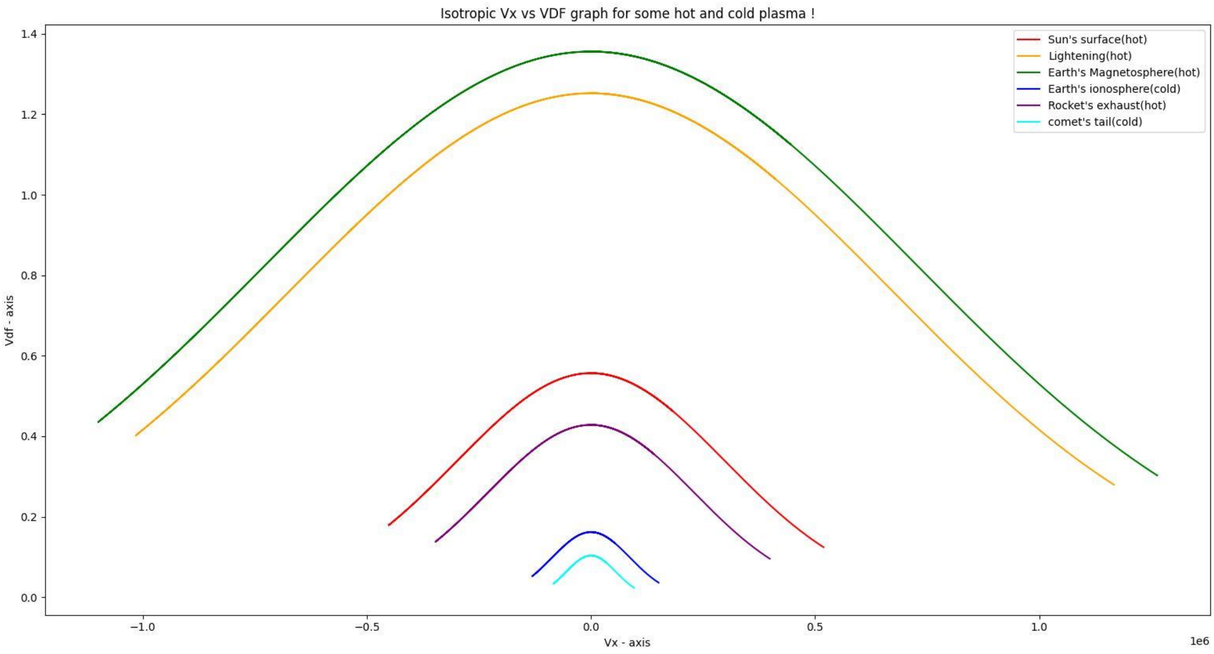
cosine Vx vs VDF graph for some hot and cold plasma !
Sun's surface(hot)
Lightening(hot)
Earth's Magnetosphere(hot)
Earth's ionosphere(cold)
Rocket's exhaust(hot)
comet's tail(cold)
Vdf - axis
Vx - axis
1e6

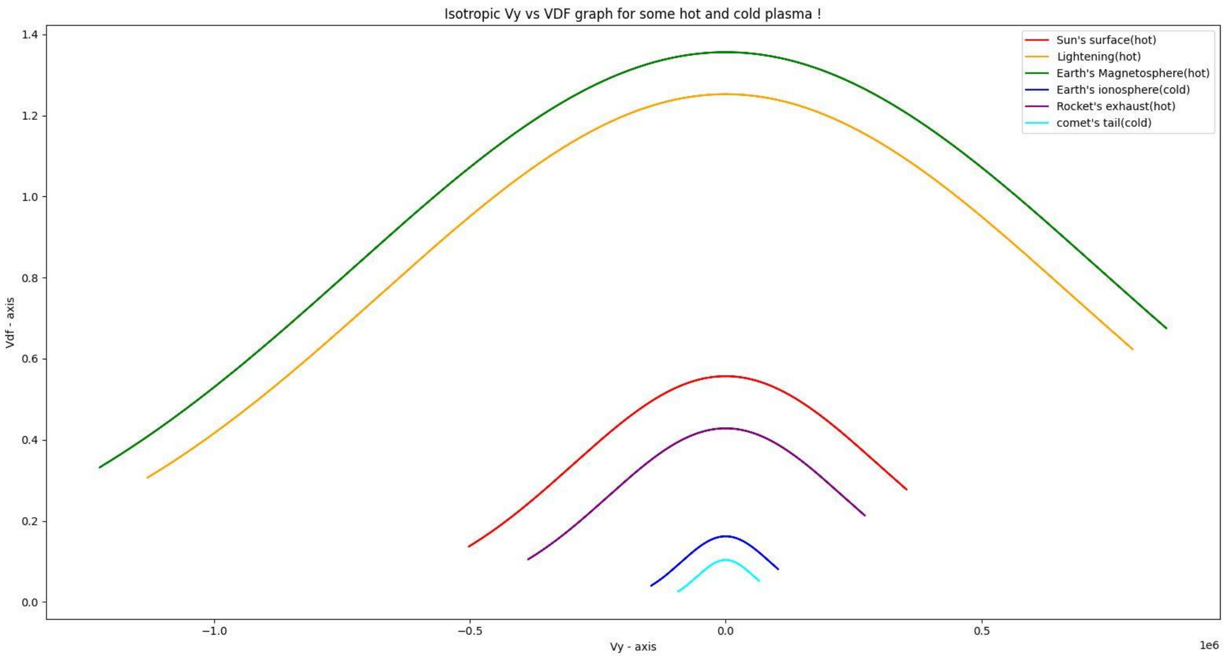


cosine Vy vs VDF graph for some hot and cold plasma !
Sun's surface(hot)
Lightening(hot)
Earth's Magnetosphere(hot)
Earth's ionosphere(cold)
Rocket's exhaust(hot)
comet's tail(cold)
Vdf - axis
Vy - axis
1e6



cosine Vz vs VDF graph for some hot and cold plasma !
Sun's surface(hot)
Lightening(hot)
Earth's Magnetosphere(hot)
Earth's ionosphere(cold)
Rocket's exhaust(hot)
comet's tail(cold)
Vdf - axis
Vz - axis
1e6


Isotropic Vx vs VDF graph for some hot and cold plasma !
Sun's surface(hot)
Lightening(hot)
Earth's Magnetosphere(hot)
Earth's ionosphere(cold)
Rocket's exhaust(hot)
comet's tail(cold)
Vdf - axis
Vx - axis
1e6


Isotropic Vy vs VDF graph for some hot and cold plasma !
Sun's surface(hot)
Lightening(hot)
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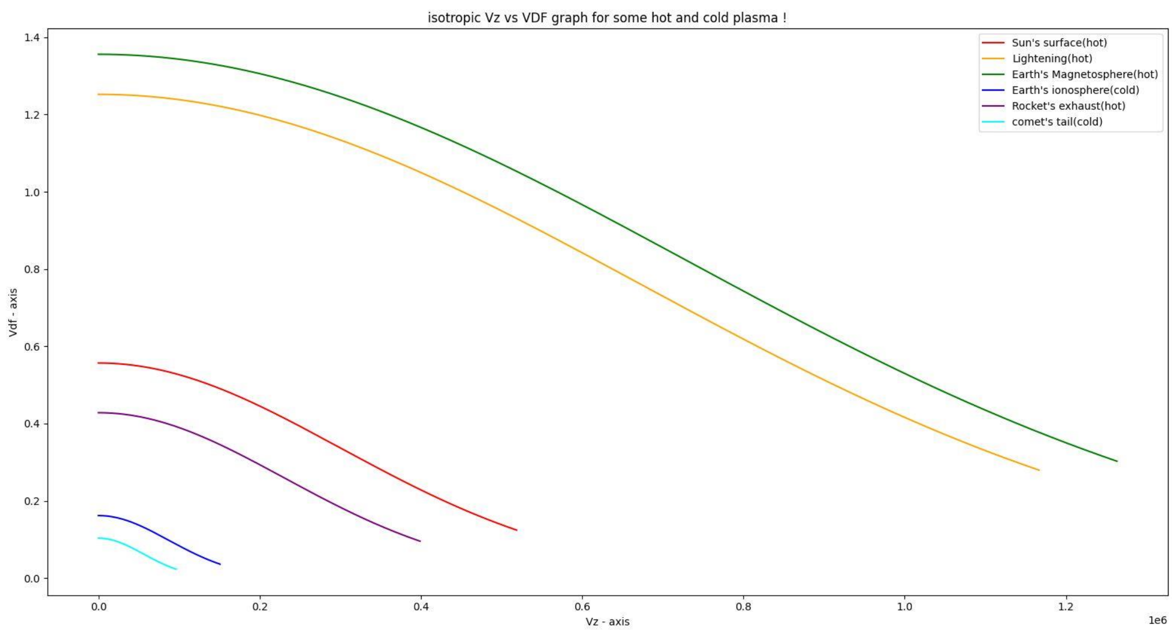
Earth's ionosphere(cold)
Rocket's exhaust(hot)
comet's tail(cold)
Vdf - axis
Vy - axis
1e6


isotropic Vz vs VDF graph for some hot and cold plasma !
Sun's surface(hot)
Lightening(hot)
Earth's Magnetosphere(hot)
Earth's ionosphere(cold)
Rocket's exhaust(hot)
comet's tail(cold)
Vdf - axis
Vz - axis
1e6

Studying all the graphs we can say that, it is properly normalised so that the fact that the average value $< V_i >$ vanishes is physically evident by symmetry, each components of the velocity can be equally positive or negative. Mathematically, we have,

$$< V_i > = \int_{-\infty}^{\infty} (F(V_i)\ V_i\, dv_i$$

$$= 0 \qquad\qquad \mathrm{i} = \mathrm{x, y, z}$$

The expectation of $< V_i > = 0 \qquad\qquad \mathrm{i} = \mathrm{x, y, z}$

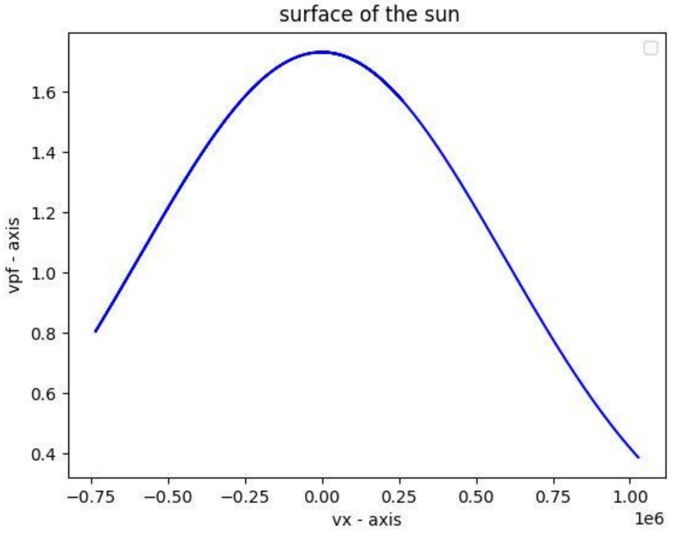
Variance of $V_i = < V_i^{\,2} > = \frac{1}{n} \int_{-\infty}^{\infty} F\,(V_i) V_i^{\,2} dv_i$

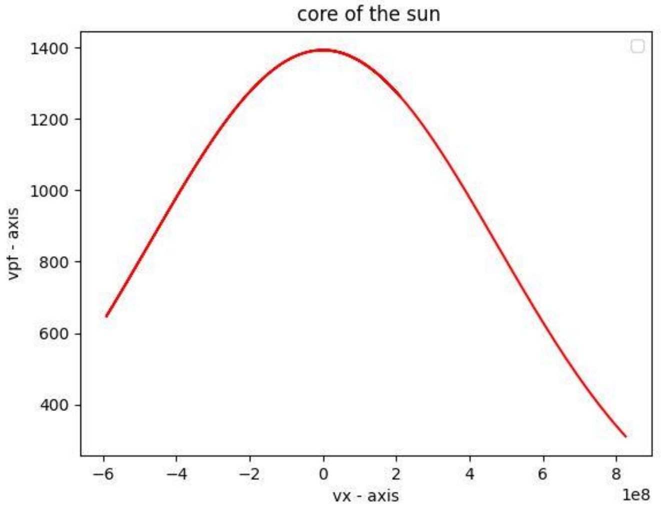
$$= \frac{kT}{m}$$

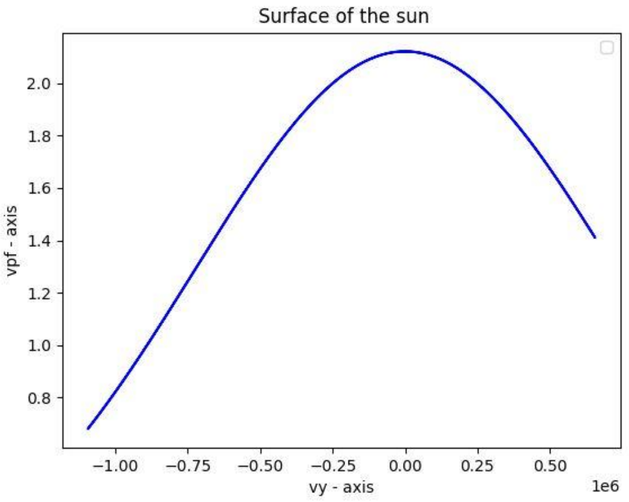
Root mean square is given by $(\frac{kT}{m})^{1/2}$

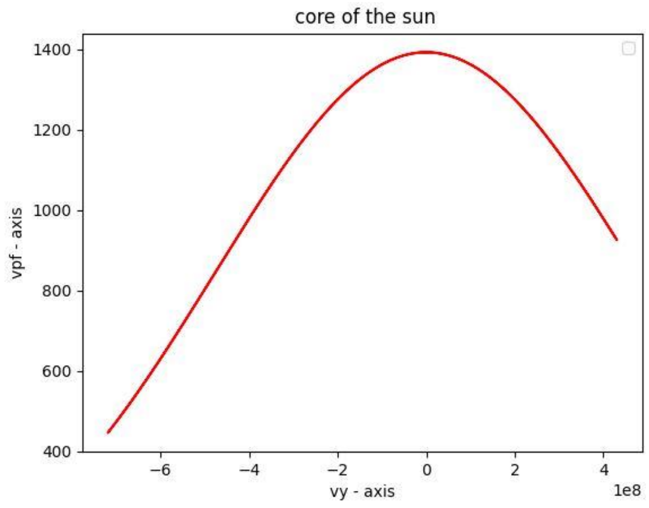
This is a normal distribution graph. They're all symmetric. The normal distribution cannot model skewed distributions. The mean, median, and mode are all equal.

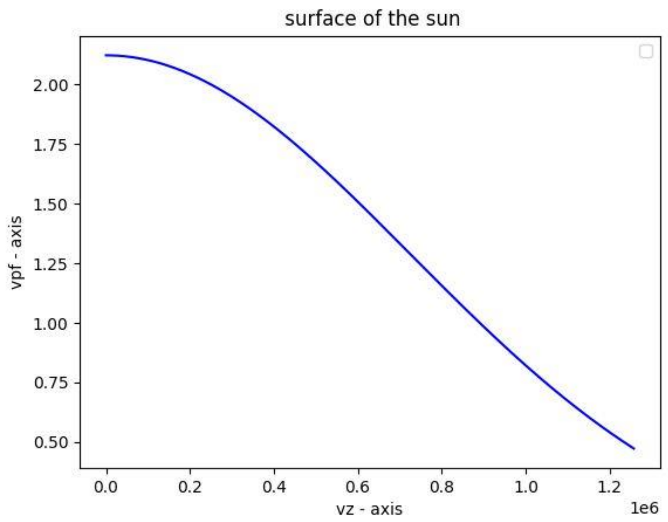
COMPARISON OF VDF BETWEEN SURFACE & CORE OF THE SUN

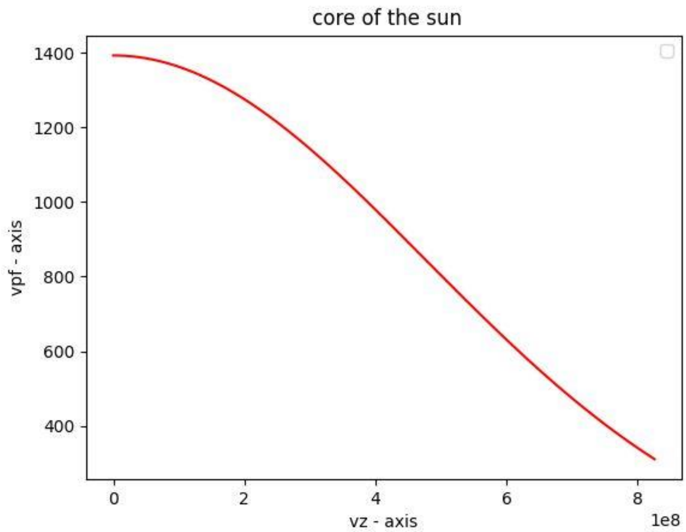












The core is made of hot, dense plasma (ions and electrons), at a pressure estimated at 265 billion bar at the centre. Due to fusion, the composition of the solar plasma drops from 68–70% hydrogen by mass at the outer core, to 34% hydrogen at the core/Sun centre. The helium/hydrogen plasma in the core of the Sun is completely ionized. Core of the sun is more ionized than the surface of the sun. Through this comparison we get a brief idea about the VDF of nuclear fusion.

SIMULATION OF NON-MAXWELLIAN VDF

Boltzmann-Gibbs (BG) Statistical Mechanics has withstood the test of time for describing classical systems in thermal equilibrium—a state where any flow of heat (e.g. thermal conduction, thermal radiation) is in balance. Any system in thermal equilibrium has its distribution function of velocities stabilized into a Maxwell distribution (in the absence of an external force). **Maxwell distributions are quite rare in space and astrophysical plasmas instead, the vast majority of these plasmas reside in stationary states (i.e., their statistics are at least temporarily time invariant),** that are typically not well described by Maxwell distributions, and thus not in thermal equilibrium.

Particle velocity distribution functions (VDF) in space plasmas often show non Maxwellian super thermal tails decreasing as a power law of the velocity. Such distributions are well fitted by the so-called Kappa distribution. The presence of such distributions in different space plasmas suggests a universal mechanism for the creation of such super thermal tails.

KAPPA VDF FOR DUSTY PLASMA

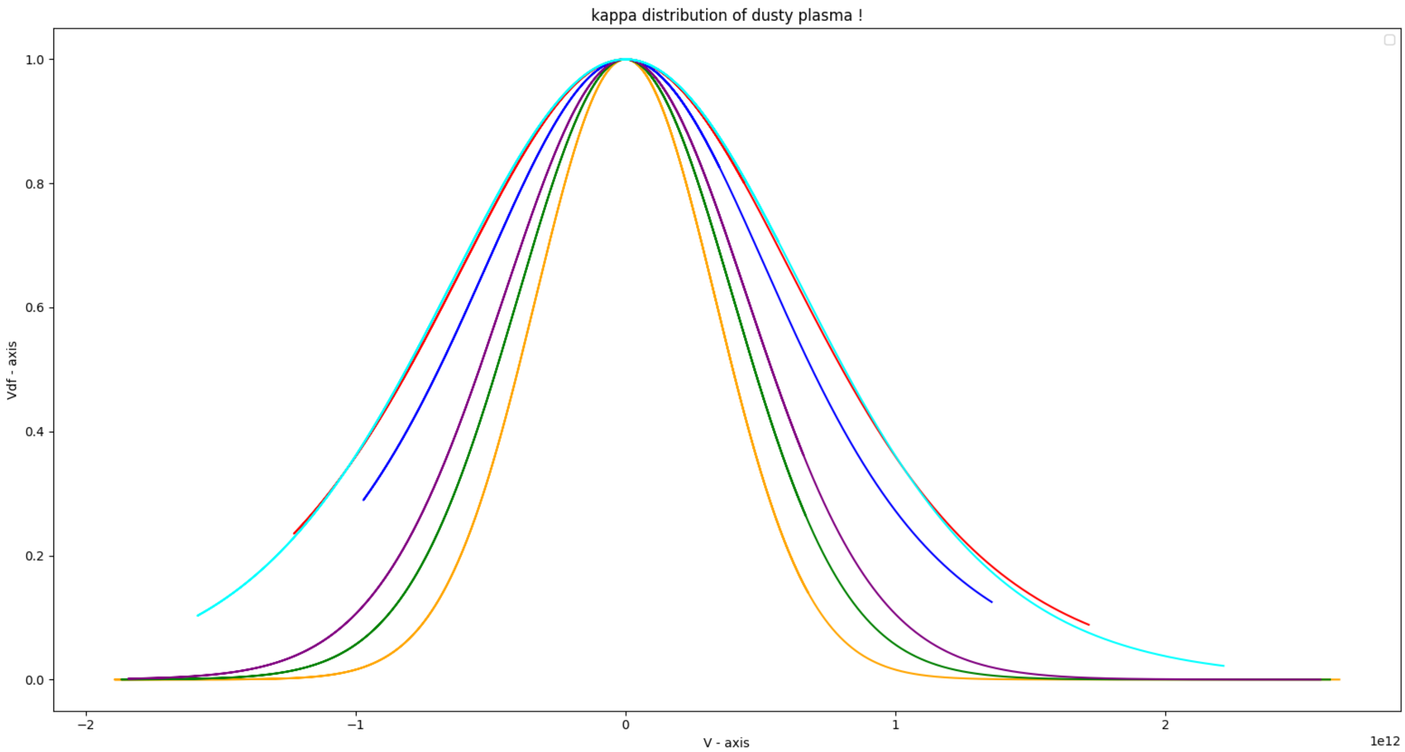
A dusty plasma consists of electrons, ions, electrically-charged micron-size dust particles, and neutral gas atoms. Experiments with dusty plasmas are generally dissipative nonequilibrium systems driven by an external source, which results in electric fields and ion flows that can introduce energy into the collection of dust particles. In a two-dimensional dusty plasma levitated in a plasma sheath, a non-Gaussian high-energy tail was observed in the probability distribution function of particle displacement, and this feature was linked to super diffusion of particles.

For the velocity distribution function, non-Gaussian distribution features, especially with energetic tails, are found in a wide range of plasmas, including low-collisionality plasmas such as the solar wind . It is commonly found that the **velocity distribution in the solar wind has high-energy tails, and it is often fit to the Kappa distribution.**

$$F(r,v) \propto \frac{1}{\left(1+\frac{v^2}{\kappa v_0{}^2}\right)^{1+\kappa}} \quad \text{...........}(11)$$

where $v_0 = [(2\kappa - 3)k_B T_p / \kappa m_p]^{1/2}$ is a characteristic speed, and T_p is the temperature of the particle with mass m_p. The parameter κ characterizes how far the system is from thermal equilibrium.

Now, Using Eq. (10) & (11) we get,



When κ → ∞, the Kappa distribution approaches a Maxwellian, but when κ is finite, it has high-energy tails, with more high velocity particles than for a Maxwellian.

CONCLUSION

Maxwellian velocity distribution is frequently used for the simulation of incoming particles obtaining from a boundary surface. It' s very difficult for us to calculate all the VDFs and plot all the graphs. Monte Carlo simulation helps us to simulate all the VDFs graphically. Monte Carlo simulations are used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. It is a technique used to understand the impact of risk and uncertainty in prediction and forecasting models. In python, full process of simulation has been done. In this project, just few examples of plasma VDFs has been discussed. We can simulate other plasmas like nuclear fusion, nuclear fission, Aurora Boriolis etc. We can also calculate expectations, variance and mean root square of other plasma through Monte Carlo simulation. Thus, for better study in plasma physics as well as space physics, simulation is highly recommended.

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