Problem 2.2

a)

i)

First calculate the charge as a function of radius for $r \leq r_0$

$$Q(r) = \int \rho \, dV = \int_0^r \frac{C}{r'} 4\pi r'^2 dr' = 4C\pi \int_0^r r' dr' = 4\pi C \frac{1}{2} r^2 = 2\pi C r^2$$

For $r \leq r_0$

$$\Phi(r) = \frac{Q}{\epsilon_0} = \frac{2\pi C r^2}{\epsilon_0} = E(r) \times 4\pi r^2$$
$$E(r) = \frac{C}{2\epsilon_0}$$

For $r > r_0$

$$\Phi(r) = \frac{Q}{\epsilon_0} = \frac{2\pi C r_0^2}{\epsilon_0} = E(r) \times 4\pi r^2$$
$$E(r) = \frac{C r_0^2}{2\epsilon_0 r^2}$$

ii)

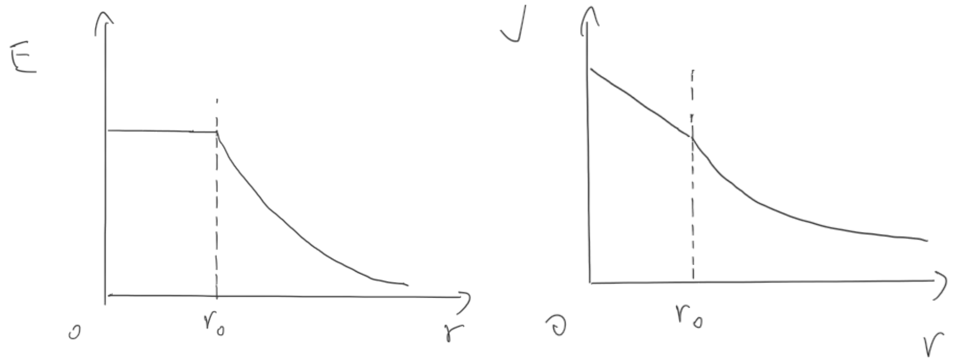
For $r > r_0$

$$V(r) = -\frac{C r_0^2}{2\epsilon_0} \int_\infty^r \frac{1}{r'^2} dr' = \frac{C r_0^2}{2\epsilon_0} \frac{1}{r}$$

For $r \leq r_0$

$$\begin{aligned} V(r) &= V(r_0) - \int_{r_0}^r \frac{C}{2\epsilon_0} dr' \\ &= \frac{C r_0}{2\epsilon_0} - \frac{C}{2\epsilon_0} [r - r_0] \\ &= \frac{C}{\epsilon_0} \left(r_0 - \frac{r}{2} \right) \end{aligned}$$

iii)



b)

i)

First find the charge enclosed as a function of radius.

For $r < a$, $Q(r) = 0$, because charges reside on the surface of a conductor.
For $a < r < b$, $Q(r) = +2Q$.

For $b < r < c$, $Q(r) = 0$, because the $+2Q$ inside attracts $-2Q$ charges on the inner surface of the conducting shell, and the net charge enclosed is zero.

For $r > c$, $Q(r) = +Q$.

Then, use Gauss's law to find the electric field

$$\Phi(r) = \frac{Q(r)}{\epsilon_0} = E(r)\ 4\pi r^2$$
$$E(r) = \frac{Q(r)}{4\pi\epsilon_0 r^2}$$

For $r < a$,

$$E(r) = \frac{0}{4\pi\epsilon_0 r^2} = 0$$

For $a < r < b$,

$$E(r) = \frac{2Q}{4\pi\epsilon_0 r^2} = \frac{Q}{2\pi\epsilon_0 r^2}$$

For $b < r < c$,

$$E(r) = \frac{0}{4\pi\epsilon_0 r^2} = 0$$

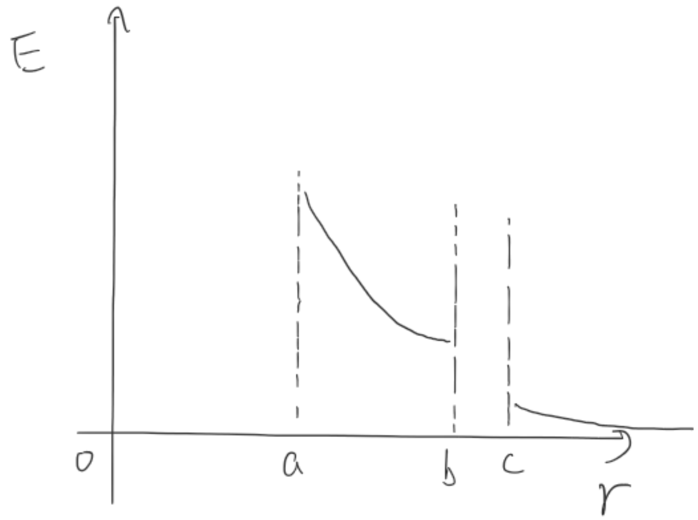
For $r > c$,

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

ii)

-2Q amount of charges appear on the inner surface of the shell. +Q amount of charges appear on the outer surface of the shell.

iii)


E
0
a
b
c
r