

Problem 7.2

a)

$$\mathbf{I}_1 = \frac{V_0}{\frac{1}{j\omega C}} = V_0 j\omega C$$

$$|\mathbf{I}_1| = V_0 \omega C$$

$$\phi = \arctan \frac{\omega C}{0} = \frac{\pi}{2}$$

$$I_1(t) = V_0 \omega C \sin\left(\omega t + \frac{\pi}{2}\right)$$

b)

$$\mathbf{I}_2 = \frac{V_0}{R + j\omega L} = V_0 \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$I_2 = |\mathbf{I}_2| = \frac{V_0}{R^2 + \omega^2 L^2} \sqrt{R^2 + \omega^2 L^2} = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\phi = \arctan \frac{-\omega L}{R} = -\arctan \frac{\omega L}{R}$$

$$\cos \phi = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\sin \phi = \frac{\frac{-\omega L}{R}}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} = \frac{-\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

c)

$$\begin{aligned} \mathbf{I}_0 &= \mathbf{I}_1 + \mathbf{I}_2 = V_0 j\omega C + V_0 \frac{R - j\omega L}{R^2 + \omega^2 L^2} \\ &= V_0 \frac{j\omega C(R^2 + \omega^2 L^2) + R - j\omega L}{R^2 + \omega^2 L^2} \\ &= V_0 \frac{R + j\omega(CR^2 + \omega^2 CL^2 - L)}{R^2 + \omega^2 L^2} \end{aligned}$$

$$|\mathbf{I}_0| = \frac{V_0}{R^2 + \omega^2 L^2} \sqrt{R^2 + \omega^2 (CR^2 + \omega^2 CL^2 - L)^2}$$

$$\phi = \arctan \left(\frac{\omega(CR^2 + \omega^2 CL^2 - L)}{R} \right)$$

d)

$$\cos \phi = \frac{1}{\sqrt{1 + \left(\frac{\omega(CR^2 + \omega^2 CL^2 - L)}{R} \right)^2}} = \frac{R}{\sqrt{R^2 + \omega^2(CR^2 + \omega^2 CL^2 - L)^2}}$$