Problem 1.1

a)

$$\sigma = \frac{Q}{\pi R^2}$$

b)

$$dq = \sigma 2\pi r' dr' = \frac{2Qr'dr'}{R^2}$$

c)

$$dE = \frac{1}{4\pi\epsilon_0} d\frac{dQ}{(d^2+r^2)^{\frac{3}{2}}} = \frac{1}{4R^2\pi\epsilon_0} d\frac{2Qrdr}{(d^2+r^2)^{\frac{3}{2}}} = \frac{d}{2R^2\pi\epsilon_0}\frac{Qrdr}{(d^2+r^2)^{\frac{3}{2}}}$$

d)

$$\begin{aligned} E &= \int_0^R \frac{d}{2R^2\pi\epsilon_0}\frac{Qr}{(d^2+r^2)^{\frac{3}{2}}}dr \\ &= \frac{Qd}{2R^2\pi\epsilon_0}\int_0^R \frac{r}{(d^2+r^2)^{\frac{3}{2}}}dr \end{aligned}$$

e)

$$\begin{aligned} E &= \frac{Qd}{2R^2\pi\epsilon_0}\left[-\frac{1}{\sqrt{d^2+r^2}}\right]_0^R \\ &= \frac{Q}{2\pi R^2\epsilon_0}\left(1-\frac{d}{\sqrt{d^2+R^2}}\right) \end{aligned}$$