Problem 5.1

a)

$$
\begin{aligned}
d\vec{B} &= \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3} \\
&= \frac{\mu_0}{4\pi}\frac{I}{\sqrt{R^2+y^2}^3}\begin{bmatrix}0\\-dy\\0\end{bmatrix}\times\begin{bmatrix}-R\\-y\\0\end{bmatrix} \\
&= \frac{\mu_0}{4\pi}\frac{IR}{(R^2+y^2)^{\frac{3}{2}}}dy\ \hat{k}
\end{aligned}
$$

$$
\vec{B} = \frac{\mu_0}{4\pi}\int_0^\infty \frac{IR}{(R^2+y^2)^{\frac{3}{2}}}dy\ \hat{k} = \frac{I\mu_0}{4\pi R}\hat{k}
$$

b)

$$
\begin{aligned}
d\vec{B} &= \frac{\mu_0}{4\pi}\frac{I}{\sqrt{R^2}^3}\begin{bmatrix}\cos(\theta+\frac{\pi}{2})\\ \sin(\theta+\frac{\pi}{2})\\0\end{bmatrix}Rd\theta\times R\begin{bmatrix}-\cos\theta\\-\sin\theta\\0\end{bmatrix} \\
&= \frac{\mu_0}{4\pi}\frac{I}{R}d\theta\ \hat{k}
\end{aligned}
$$

$$
\vec{B} = \frac{\mu_0}{4\pi}\int_\pi^{\frac{3}{2}\pi}\frac{I}{R}d\theta\ \hat{k} = \frac{\mu_0 I}{8R}\ \hat{k}
$$

c)

$$
\begin{aligned}
d\vec{B} &= \frac{\mu_0}{4\pi}\frac{I}{\sqrt{R^2+x^2}^3}\begin{bmatrix}dx\\0\\0\end{bmatrix}\times\begin{bmatrix}-x\\R\\0\end{bmatrix} \\
&= \frac{\mu_0}{4\pi}\frac{IR}{(R^2+x^2)^{\frac{3}{2}}}dx\ \hat{k}
\end{aligned}
$$

$$
\vec{B} = \frac{\mu_0}{4\pi}\int_0^\infty \frac{IR}{(R^2+x^2)^{\frac{3}{2}}}dx\ \hat{k} = \frac{I\mu_0}{4\pi R}\hat{k}
$$

d)

$$
\vec{B}_{\text{total}} = 2\frac{I\mu_0}{4\pi R}\hat{k} + \frac{\mu_0 I}{8R}\ \hat{k} = 8.927\times10^{-6}T\ \hat{k}
$$

e)

They are already indicated by unit vectors