Problem 2.1

a)

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{d^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{z_0^2 + r^2}$$

b)

$$E_\perp = E_r \frac{z_0}{\sqrt{z_0^2 + r^2}} = \frac{1}{4\pi\epsilon_0}\frac{Qz_0}{(z_0^2 + r^2)^{\frac{3}{2}}}$$

c)

$$d\Phi = E_\perp dA = \frac{1}{4\pi\epsilon_0}\frac{Qz_0}{(z_0^2 + r^2)^{\frac{3}{2}}} 2\pi r dr = \frac{1}{2\epsilon_0}\frac{Qz_0}{(z_0^2 + r^2)^{\frac{3}{2}}} r dr$$

d)

$$\begin{aligned}
\Phi &= \int d\Phi \\
&= \int_0^{r_0} \frac{1}{2\epsilon_0}\frac{Qz_0}{(z_0^2 + r^2)^{\frac{3}{2}}} r dr \\
&= \frac{Qz_0}{2\epsilon_0}\int_0^{r_0} \frac{r}{(z_0^2 + r^2)^{\frac{3}{2}}} dr \\
&= \frac{Qz_0}{2\epsilon_0}\left(\frac{1}{z_0} - \frac{1}{\sqrt{r_0^2 + z_0^2}}\right) \\
&= \frac{Q}{2\epsilon_0}\left(1 - \frac{z_0}{\sqrt{r_0^2 + z_0^2}}\right)
\end{aligned}$$

e)

$$\Phi = \frac{Q}{2\epsilon_0}(1 - \cos\theta_0)$$