Problem 1.3

a)

i)

Upward if $z_0 > \frac{h}{2}$ since there're more charges at the bottom, downward if $z_0 < \frac{h}{2}$ since there're more charges at the top

ii)

When $z_0 = \frac{h}{2}$. This point is the center of the cylindrical shell, and the electrical forces should be zero by symmetry.

iii)

When $z_0 >> h$, the cylindrical shell is a ring with radius R. Therefore,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qz_0}{(z_0^2+R^2)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0}\frac{\sigma 2\pi Rhz_0}{(z_0^2+R^2)^{\frac{3}{2}}} = \frac{\sigma Rh}{2\epsilon_0}\frac{z_0}{(z_0^2+R^2)^{\frac{3}{2}}}$$

When $z_0 >> h$ and $z_0 >> R$, the cylindrical shell acts like a point charge.

$$E = \frac{1}{4\pi\epsilon_0}\frac{\sigma 2\pi Rh}{z_0^2} = \frac{\sigma Rh}{2\epsilon_0}\frac{1}{z_0^2}$$

b)

$dQ = \sigma 2\pi R dz$

c)

$d = |z_0 - z|$

d)

$$dE = \frac{1}{4\pi\epsilon_0}\frac{d(dQ)}{(d^2+R^2)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0}\frac{(z_0-z)\sigma 2\pi R}{((z_0-z)^2+R^2)^{\frac{3}{2}}}dz = \frac{\sigma R}{2\epsilon_0}\frac{z_0-z}{((z_0-z)^2+R^2)^{\frac{3}{2}}}dz$$

e)

$$E = \frac{\sigma R}{2\epsilon_0}\int_0^h \frac{z_0-z}{((z_0-z)^2+R^2)^{\frac{3}{2}}}dz$$

f)

By calculator,

$$E = \frac{\sigma R}{2\epsilon_0}\left(\frac{1}{\sqrt{R^2+(z_0-h)^2}} - \frac{1}{\sqrt{R^2+z_0^2}}\right)$$

The expectations **i)** and **ii)** are all met.

g)

Rewriting the formula in **f)**

$$E = \frac{\sigma R}{2\epsilon_0}\left[\frac{1}{z_0 - h}\left(1 + \frac{R^2}{(z_0 - h)^2}\right)^{-\frac{1}{2}} - \frac{1}{z_0}\left(1 + \frac{R^2}{z_0^2}\right)^{-\frac{1}{2}}\right]$$

By first order binomial approximation, $(1 + x)^n \approx 1 + nx$

$$E = \frac{\sigma R}{2\epsilon_0}\left[\frac{1}{z_0 - h}\left(1 - \frac{1}{2}\frac{R^2}{(z_0 - h)^2}\right) - \frac{1}{z_0}\left(1 - \frac{1}{2}\frac{R^2}{z_0^2}\right)\right]$$

$$= \frac{\sigma R}{2\epsilon_0}\left[\frac{1}{z_0 - h} - \frac{1}{z_0} - \frac{1}{2}\frac{R^2}{(z_0 - h)^3} + \frac{1}{2}\frac{R^2}{z_0^3}\right]$$

Because z_0 is large, we ignore the terms containing the cube of z_0 in the denominator.

$$E = \frac{\sigma R}{2\epsilon_0}\left[\frac{1}{z_0 - h} - \frac{1}{z_0}\right]$$

$$= \frac{\sigma R}{2\epsilon_0}\frac{h}{(z_0 - h)z_0}$$

$$\approx \frac{\sigma R}{2\epsilon_0}\frac{h}{z_0^2}$$