# Problem 4.1

## a)

$$
\begin{aligned}
F &= \int I d\vec{l} \times \vec{B} \\
&= \int_{-\infty}^{\infty} I \begin{bmatrix} dx \\ 0 \\ 0 \end{bmatrix} \times B_0 \begin{bmatrix} e^{-\left(\frac{y}{b}\right)^2} \\ e^{-\left(\frac{x}{a}\right)^2} \\ 0 \end{bmatrix} \\
&= IB_0 \int_{-\infty}^{\infty} e^{-\left(\frac{x}{a}\right)^2} \hat{k}\ dx \\
&= IB_0 a\sqrt{\pi}\hat{k}
\end{aligned}
$$

## b)

$$
\begin{aligned}
F &= \int_{-\infty}^{\infty} I \begin{bmatrix} 0 \\ dy \\ 0 \end{bmatrix} \times B_0 \begin{bmatrix} e^{-\left(\frac{y}{b}\right)^2} \\ e^{-\left(\frac{x}{a}\right)^2} \\ 0 \end{bmatrix} \\
&= IB_0 \int_{-\infty}^{\infty} -e^{-\left(\frac{y}{b}\right)^2} \hat{k}\ dy \\
&= -IB_0 b\sqrt{\pi}\hat{k}
\end{aligned}
$$

## c)

$$
\begin{aligned}
F &= \int_{-\infty}^{\infty} I \begin{bmatrix} \cos(\arctan m)dx \\ \sin(\arctan m)dy \\ 0 \end{bmatrix} \times B_0 \begin{bmatrix} e^{-\left(\frac{y}{b}\right)^2} \\ e^{-\left(\frac{x}{a}\right)^2} \\ 0 \end{bmatrix} \\
&= IB_0 \int_{-\infty}^{\infty} \left( \frac{1}{\sqrt{1+m^2}} e^{-\left(\frac{x}{a}\right)^2} \right) \hat{k}\ dx - \int_{-\infty}^{\infty} \left( \frac{m}{\sqrt{1+m^2}} e^{-\left(\frac{y}{b}\right)^2} \right) \hat{k}\ dy \\
&= IB_0\sqrt{\pi} \left( \frac{a}{\sqrt{1+m^2}} - \frac{mb}{\sqrt{1+m^2}} \right) \hat{k}
\end{aligned}
$$

## d)

When m = 0

$$
IB_0\sqrt{\pi} \left( \frac{a}{\sqrt{1+0}} - \frac{0m}{\sqrt{1+0}} \right) \hat{k} = IB_0 a\sqrt{\pi}\hat{k}
$$

When m = $\infty$, $\lim_{m\to\infty} \frac{1}{\sqrt{1+m^2}} = 0$, $\lim_{m\to\infty} \frac{m}{\sqrt{1+m^2}} = 1$

$$
IB_0\sqrt{\pi} \left( \frac{a}{\sqrt{1+m^2}} - \frac{mb}{\sqrt{1+m^2}} \right) \hat{k} = -IB_0 b\sqrt{\pi}\hat{k}
$$