1 Derivation of Capacitance

We have the potential energy of series and parallel capacitors given as follows.

$$U_1 = \frac{1}{2}(C_1 + C_2)V^2 \tag{1}$$

$$U_2 = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)V^2 \tag{2}$$

Using Equation 2, we express C_1 using known values and C_2.

$$\frac{2U_1}{V^2} = C_1 + C_2 \Rightarrow C_1 = \frac{2U_1}{V^2} - C_2 \tag{3}$$

Now we substitute Equation 3 into Equation 2 to solve for C_2.

$$\begin{aligned} U_2 &= \frac{1}{2}\left(\frac{(\frac{2U_1}{V^2} - C_2)C_2}{\frac{2U_1}{V^2}}\right)V^2 \\ &= \frac{\frac{2U_1}{V^2}C_2 - C_2^2}{4U_1} \end{aligned}$$

Rearrange the equation

$$4U_1U_2 = \frac{2U_1}{V^2}C_2 - C_2^2$$

$$C_2^2 - \frac{2U_1}{V^2}C_2 + 4U_1U_2 = 0 \tag{4}$$

Equation 4 is a quadratic equation with C_2 being the unknown. Solving it using the root formula:

$$\begin{aligned} C_2 &= \frac{\frac{2U_1}{V^2} \pm \sqrt{\left(\frac{2U_1}{V^2}\right)^2 - 16U_1U_2}}{2} \\ &= \frac{U_1}{V^2} \pm \frac{1}{V^2}\sqrt{U_1^2 - 4U_1U_2} \end{aligned}$$

Now substitute C_2 into Equation 3 to solve for C_1.

$$\begin{aligned} C_1 &= \frac{2U_1}{V^2} - \left(\frac{U_1}{V^2} \pm \frac{1}{V^2}\sqrt{U_1^2 - 4U_1U_2}\right) \\ &= \frac{U_1}{V^2} \mp \frac{1}{V^2}\sqrt{U_1^2 - 4U_1U_2} \end{aligned}$$

Therefore,

$$\begin{cases} C_1 &= \frac{U_1}{V^2} + \frac{1}{V^2}\sqrt{U_1^2 - 4U_1U_2} \\ C_2 &= \frac{U_1}{V^2} - \frac{1}{V^2}\sqrt{U_1^2 - 4U_1U_2} \end{cases}$$

2 Error Propagation

$$C_1 = 0.0197702681531304 \pm 0.000327629137653663$$

$$C_2 = 0.0361213768295607 \pm 0.000327804658695168$$

$$\sigma_{C_1}, \sigma_{C_2} = \frac{\sqrt{\frac{V^2\left(U_1^2\sigma_{U1}^2 + 4U_1^2\sigma_{U2}^2 + 4U_2^2\sigma_{U1}^2 + \sigma_{U1}^2\right) + 4\sigma_V^2\left(U_1 - \sqrt{U_1(U_1-4U_2)}\right)^2}{V^2\left(U_1 - \sqrt{U_1(U_1-4U_2)}\right)^2}}\left(U_1 - \sqrt{U_1\left(U_1 - 4U_2\right)}\right)}{V^2}$$