Problem 5.2

a)

$$\varepsilon = \frac{d\Phi}{dt} = B\frac{dA}{dt} = BLv$$

b)

$$I = \frac{\varepsilon}{R} = \frac{BLv}{R}$$

c)

$$F = Id\vec{l} \times \vec{B} = -ILB = -\frac{(BL)^2 v}{R}$$

d)

$$F = ma = m\frac{dv}{dt} = -\frac{(BL)^2 v}{R}$$

$$\frac{1}{v}\frac{dv}{dt} = -\frac{(BL)^2}{Rm}$$

e)

$$\int \frac{1}{v} dv = \int -\frac{(BL)^2}{Rm} dt$$

$$\ln|v| = -\frac{(BL)^2}{Rm} t + C$$

$$v = Ce^{-\frac{(BL)^2}{Rm}t}$$

$$v(0) = C = v_0 \Rightarrow C = v_0$$

$$v = v_0 e^{-\frac{(BL)^2}{Rm}t}$$

f)

$$d = \int_0^\infty v_0 e^{-\frac{(BL)^2}{Rm}t} dt = -v_0 \lim_{a\to\infty} \frac{Rm}{(BL)^2} \left[e^{-\frac{(BL)^2}{Rm}t} \right]_0^a = v_0 \frac{Rm}{(BL)^2}$$

g)

$$P(t) = \frac{(BLv)^2}{R} = \frac{(BL)^2}{R} v_0^2 \left(e^{-2\frac{(BL)^2}{Rm}t} \right)$$

h)

$$\begin{aligned} W &= \int_0^\infty \frac{(BL)^2}{R} v_0^2 \left(e^{-2\frac{(BL)^2}{Rm}t} \right) dt \\ &= \frac{(BL)^2}{R} v_0^2 \lim_{a\to\infty} \left(-\frac{Rm}{2(BL)^2} \right) \left[e^{-2\frac{(BL)^2}{Rm}t} \right]_0^a \\ &= \frac{1}{2} m v_0^2 \end{aligned}$$