PHYS 2415 Formula Sheet

By Hanzhi Zhou

Electricity

$E = \frac{1}{4\pi\epsilon}\frac{Q}{r^2}$

$\Delta V = V_B - V_A = \frac{\Delta U}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$

$\Delta U = U_b - U_a = q(V_b - V_a)$

$V = k\frac{Q}{r} = k\int \frac{dq}{r}$

$\vec{\mathbf{E}} = -\vec{\nabla} V$

$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$

$U_{\text{system}} = k \sum_{\text{pairs i j}} \frac{q_i q_j}{r_{ij}}$

$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0}\iiint \rho dV$

$E_{\text{rod}} = k\frac{\lambda L}{x\sqrt{x^2 + L^2/4}} \approx \frac{\lambda}{2\pi\epsilon_0 x}$

$E_{\text{ring}} = k\frac{Qx}{(x^2 + R^2)^{\frac{3}{2}}}$

$E_{\text{disk}} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right] \approx \frac{\sigma}{2\epsilon_0}$

$E_{\text{conducting plane}} = \frac{\sigma}{\epsilon_0}$

dipole moment $p = Ql$

$\tau = \vec{\mathbf{p}} \times \vec{\mathbf{E}} = pE\sin\theta$

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = -pE\int_{\theta_1}^{\theta_2} \sin\theta d\theta$$
$$= pE(\cos\theta_2 - \cos\theta_1) = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$$

$C_{\text{pplate}} = K\epsilon_0\frac{A}{d}$

$U = \frac{1}{C}\int_0^Q q dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

$u = \frac{1}{2}\epsilon_0 \vec{\mathbf{E}}^2 (J/m^3)$

$R = \rho\frac{l}{A}$

$\rho = \rho_0[1 + \alpha(T - T_0)]$

$V = V_0 \sin 2\pi f t = V_0 \sin\omega t$

$I_0 \sin\omega t$

$\overline{P} = \frac{1}{2}I_0^2 R = \frac{1}{2}\frac{V_0^2}{R} = I_{\text{rms}}V_{\text{rms}}$

$I_{\text{rms}} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}}$

$V_{\text{rms}} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}}$

$j = \frac{I}{A} = -ne\vec{\mathbf{v}}_d$

Number of electrons over volume

$n = \frac{N}{V}$

$\varepsilon = IR + \frac{Q}{C}$

$Q = C\varepsilon\left(1 - e^{\frac{-t}{RC}}\right)$

$Q = Q_0 e^{\frac{-t}{RC}}$

Magnetism

$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \quad \vec{\mathbf{F}} = I\vec{\boldsymbol{l}} \times \vec{\mathbf{B}}$

$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$

$\vec{\boldsymbol{\tau}} = NI\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$

$U = \int \tau d\theta = -\mu B\cos\theta = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$

$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{\boldsymbol{l}} \times \hat{\boldsymbol{r}}}{r^2} = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{\frac{3}{2}}}$ Coil

$B_{\text{Solenoid}} = \frac{1}{2}\frac{\mu_0 NI}{l}$

$B_{\text{long wire}} = \frac{\mu_0 I}{2\pi r}$ (from Ampere's law)

$\mathcal{X}_\mu = \frac{\mu - \mu_0}{\mu_0}$ magnetic susceptibility

$\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{\partial}{\partial t}\oiint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$

$\varepsilon_1 = -M\frac{dI_2}{dt}, \ \varepsilon_2 = -M\frac{dI_1}{dt}$

$L = \frac{N\Phi_B}{I}, \ \varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$

$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al, \ \mu = \frac{1}{2}\frac{B^2}{\mu_0}$

$I = I_0 e^{-\frac{t}{\tau}}, \ \tau = \frac{L}{R}$

$Z_{RLC} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$\cos\phi = \frac{V_{R0}}{V_0} = \frac{R}{Z}$

$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 I_{encl} + \mu_0\epsilon_0\frac{\partial}{\partial t}\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$

$\oiint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

Waves

$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{E}{\rho}}, \mu = \frac{m}{l}$

$E = 2\pi^2 m f^2 A^2 = 2\pi^2 \rho S v t f^2 A^2$

$E = 2\pi^2 \mu v t f^2 A^2$

Intensity $I = \frac{\overline{P}}{S}$

$\frac{\partial^2 D}{\partial x^2} = \frac{\mu}{F_T}\frac{\partial^2 D}{\partial t^2}$

$E = E_0 \sin(kx - \omega t)$

$k = \frac{2\pi}{\lambda}, \ \omega = 2\pi f, \ f\lambda = \frac{\omega}{k} = v$

$E_0 = cB_0$

$\mu = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$

$\vec{\mathbf{S}} = \frac{1}{\mu_0}(\vec{\mathbf{E}} \times \vec{\mathbf{B}}) = \frac{1}{2}\epsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2$

$\overline{S} = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_0 B_0}{2\mu_0}$

$P_{\text{refl}} = \frac{2}{Ac}\frac{dU}{dt} = \frac{2\overline{S}}{c}$

$c = \frac{1}{\mu_0\epsilon_0}$

Geometric Optics

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$f = \frac{R}{2}$$

$$m = -\frac{d_i}{d_o}$$

Sign convention

1. d_o positive if object at side where the light comes from, negative otherwise
2. d_i positive if image at the opposite side where the light comes from, negative otherwise.
3. f positive for converging lens, negative for diverging lens, infinity for mirror

Lensmaker's equation. Note: if concave, R is negative.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$n = \frac{c}{\sqrt{\mu\epsilon}}$$

$$\lambda_n = \frac{v}{f} = \frac{c}{nf} = \frac{\lambda}{n}$$

where λ_n is the wavelength in the material with index of refraction n.

$P = \frac{1}{f}$ Units: dipoter. $1D = 1m^{-1}$

Diffraction and Interference

Single-slit

Position of **minima**. Note that D is the width of the slit and $m = 0$ corresponds to the central maximum.

$$D\sin\theta = m\lambda,\ m = \pm1, \pm2$$

$$I_\theta = I_0 \left[\frac{\sin\left(\frac{\pi D \sin\theta}{\lambda}\right)}{\left(\frac{\pi D \sin\theta}{\lambda}\right)}\right]^2$$

Double-slit

Constructive interference (bright)

$$d\sin\theta = m\lambda,\ m = 0, \pm1, \pm2...$$

Destructive interference (dark)

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda,\ m = 0, \pm1, \pm2...$$

$$I_\theta = I_0 \cos^2\left(\frac{\delta}{2}\right)$$

$$\delta = \frac{2\pi}{\lambda} d\sin\theta$$

Thin-film

If light reflects from the surface where $n_1 < n_2$, there will be 180° phase shift ($\frac{1}{2}\lambda$ wavelength).
Constructive interference

$$\frac{2tn}{\lambda_{\text{vacuum}}} = m + \frac{1}{2}$$

Destructive interference

$$\frac{2tn}{\lambda_{\text{vacuum}}} = m$$

where t is the thickness and n is the index of refraction of the material

Diffraction grating

Positions of principle **maxima**. d is the width between slits.

$$d\sin\theta = m\lambda$$

Other

Angular resolution (Rayleigh criterion). D: diameter.

$$\theta = \frac{1.22D}{\lambda}$$