Problem 3.2

a)

$$C = K\varepsilon_0 \frac{A}{d}$$

b)

$$r = \rho \frac{d}{A}$$

c)

$$\begin{aligned} V(t) &= \frac{Q(t)}{C} \\ I(t) &= -\frac{dQ}{dt} = -\frac{V(t)}{r} = -\frac{Q(t)}{Cr} \end{aligned}$$

Solve the separable differential equation:

$$\begin{aligned} \int \frac{1}{Q} dQ &= \int \frac{1}{Cr} dt \\ \ln|Q| &= -\frac{t}{Cr} + D \\ Q &= De^{\frac{-t}{Cr}} \end{aligned}$$

Given the initial value $Q(0) = Q_0$, $D = Q_0$.

$$\begin{aligned} \frac{1}{4} Q_0 &= Q_0 e^{\frac{-t}{Cr}} \\ \ln\frac{1}{4} &= \frac{-t}{Cr} \\ t &= Cr\ln 4 \end{aligned}$$

Therefore $Cr\ln 4$ amount of time is required for the charge to decrease from Q_0 to $\frac{1}{4}Q_0$

d)

$$\begin{aligned} dE = I(t)^2\ r\ dt &= \left(\frac{dQ}{dt}\right)^2\ r\ dt \\ &= \left(-\frac{Q_0}{Cr}e^{\frac{-t}{Cr}}\right)^2\ r\ dt \\ &= \frac{Q_0^2}{C^2 r} e^{\frac{-2t}{Cr}}\ dt \end{aligned}$$

e)

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\begin{aligned}
E &= \frac{Q_0^2}{C^2 r}\int_0^{\infty} e^{\frac{-2t}{Cr}}\,dt \\
&= \frac{Q_0^2}{C^2 r}\frac{C}{2r}\left(1 - e^{-\infty}\right) \\
&= \frac{1}{2}\frac{Q_0^2}{C}
\end{aligned}
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f)

The energy stored in a capacitor is $\frac{1}{2}\frac{Q_0^2}{C}$ when it is fully charged to Q_0. It is the same as the total energy dissipated through the resistor calculated in **e).**