Problem 2.3

a)

$$dV = \frac{dQ}{4\pi\epsilon_0}\frac{1}{r} = \frac{\lambda dz}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+z^2}}$$

b)

$$\begin{aligned} V(x) &= \int dV \\ &= \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\lambda}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+z^2}}dz \\ &= \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{\sqrt{4x^2+L^2}+L}{2|x|}\right) \end{aligned}$$

c)

$$\begin{aligned} dV &= \frac{\lambda dz}{4\pi\epsilon_0}\frac{1}{z-z'} \\ V(z) &= \frac{\lambda}{4\pi\epsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{z-z'}dz' \\ &= -\frac{\lambda}{4\pi\epsilon_0}\ln\left|\frac{2z-L}{2z+L}\right| \end{aligned}$$

d)

$$W_P = q_e V_x(0.25) = -1.60\times10^{-19}C \times -51923.6V = 8.308\times10^{-15}J$$

e)

$V_z(0.75) = -28943.5V > V_x(0.25)$.

Therefore, point Q has a higher potential.

f)

$$W = q_e(V_z(0.75) - V_x(0.25)) = -1.60\times10^{-19}C(-28943.5V - -51923.6V) = -22980\ eV$$

Therefore, the potential energy of the electron decreases by 22980 eV