

Problem 6.1

a)

$$v = \omega l$$

b)

$$d\varepsilon = Bvdl = B\omega ldl$$

c)

$$\varepsilon = \int_0^L B\omega ldl = B\omega \left[\frac{1}{2}l^2 \right]_0^L = \frac{1}{2}B\omega L^2$$

d)

$$I = \frac{8\varepsilon}{R} = \frac{4B\omega L^2}{R}$$

e)

$$\begin{aligned} \vec{\tau} &= 8 \int \vec{l} \times (I d\vec{l} \times \vec{B}) = \frac{32B\omega L^2}{R} \int \vec{l} \times (d\vec{l} \times \vec{B}) \\ &= \frac{32B\omega L^2}{R} \int_0^L ldl B \hat{k} \\ &= \frac{16B^2\omega L^4}{R} \hat{k} \end{aligned}$$

f)

$$P_{\text{mech}} = \vec{\tau} \cdot \vec{\omega} = \frac{16B^2\omega^2 L^4}{R}$$

g)

$$P_{\text{elec}} = I^2 R = \left(\frac{4B\omega L^2}{R} \right)^2 R = \frac{16B^2\omega^2 L^4}{R} = P_{\text{mech}}$$