Problem 0.2

a)

$$
\begin{aligned}
a(t) &= v_0 \frac{d}{dt}\left(e^{-\gamma t}\sin\omega t\right) \\
&= v_0\left(-e^{-\gamma t}\sin\omega t + e^{-\gamma t}\cos\omega t\right)
\end{aligned}
$$

b)

$$
\begin{aligned}
y(t) &= v_0 \int e^{-\gamma t}\sin\omega t \; dt \\
&= -v_0\frac{1}{\gamma}\left(e^{-\gamma t}\sin\omega t - \omega\int e^{-\gamma t}\cos\omega t\; dt\right) \\
&= -v_0\frac{1}{\gamma}\left(e^{-\gamma t}\sin\omega t + \omega\frac{1}{\gamma}\left(e^{-\gamma t}\cos\omega t - \omega\int e^{-\gamma t}(-\sin\omega t)\; dt\right)\right) \\
&= -\frac{v_0}{\gamma}\left(e^{-\gamma t}\sin\omega t + \frac{\omega}{\gamma}e^{-\gamma t}\cos\omega t + \frac{\omega^2}{\gamma}\int e^{-\gamma t}(\sin\omega t)\; dt\right)
\end{aligned}
$$

It can be observed that the integral $\int e^{-\gamma t}\sin\omega t\; dt$ appears at the both side of the equations. We can rearrange the terms to solve for it

$$
\begin{aligned}
v_0\int e^{-\gamma t}\sin\omega t\; dt + v_0\frac{\omega^2}{\gamma^2}\int e^{-\gamma t}(\sin\omega t)\; dt &= -\frac{v_0}{\gamma}e^{-\gamma t}\left(\sin\omega t + \frac{\omega}{\gamma}\cos\omega t\right) \\
v_0\left(1+\frac{\omega^2}{\gamma^2}\right)\int e^{-\gamma t}\sin\omega t\; dt &= -\frac{v_0}{\gamma}e^{-\gamma t}\left(\sin\omega t + \frac{\omega}{\gamma}\cos\omega t\right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
y(t) &= v_0\int e^{-\gamma t}\sin\omega t\; dt \\
&= -\frac{v_0}{\gamma\left(1+\frac{\omega^2}{\gamma^2}\right)}e^{-\gamma t}\left(\sin\omega t + \frac{\omega}{\gamma}\cos\omega t\right) \\
&= -\frac{v_0\gamma}{\omega^2+\gamma^2}e^{-\gamma t}\left(\sin\omega t + \frac{\omega}{\gamma}\cos\omega t\right) \\
&= -\frac{v_0}{\omega^2+\gamma^2}e^{-\gamma t}\left(\gamma\sin\omega t + \omega\cos\omega t\right)
\end{aligned}
$$