```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'cs231n/assignments/assignment1/'
FOLDERNAME = '682/assignment2/cs682'
assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
import sys
sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/My\ Drive/$FOLDERNAME/datasets/
!bash get_datasets.sh
%cd /content/drive/My\ Drive/$FOLDERNAME
```

```
Mounted at /content/drive
/content/drive/My Drive/682/assignment2/cs682/datasets
--2021-10-25 23:16:38--  https://www.cs.toronto.edu/~kriz/cifar-10-python.tar.gz
Resolving www.cs.toronto.edu (www.cs.toronto.edu)... 128.100.3.30
Connecting to www.cs.toronto.edu (www.cs.toronto.edu)|128.100.3.30|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 170498071 (163M) [application/x-gzip]
Saving to: ‘cifar-10-python.tar.gz’

cifar-10-python.tar 100%[===================>] 162.60M  52.8MB/s    in 3.2s

2021-10-25 23:16:42 (50.8 MB/s) - ‘cifar-10-python.tar.gz’ saved [170498071/170498071]

cifar-10-batches-py/
cifar-10-batches-py/data_batch_4
cifar-10-batches-py/readme.html
cifar-10-batches-py/test_batch
cifar-10-batches-py/data_batch_3
cifar-10-batches-py/batches.meta
cifar-10-batches-py/data_batch_2
cifar-10-batches-py/data_batch_5
cifar-10-batches-py/data_batch_1
/content/drive/My Drive/682/assignment2/cs682
```

Batch Normalization

One way to make deep networks easier to train is to use more sophisticated optimization procedures such as SGD+momentum, RMSProp, or Adam. Another strategy is to change the architecture of the network to make it easier to train. One idea along these lines is batch normalization which was proposed by [3] in 2015.

The idea is relatively straightforward. Machine learning methods tend to work better when their input data consists of uncorrelated features with zero mean and unit variance. When training a neural network, we can preprocess the data before feeding it to the network to explicitly decorrelate its features; this will ensure that the first layer of the network sees data that follows a nice distribution. However, even if we preprocess the input data, the activations at deeper layers of the network will likely no longer be decorrelated and will no longer have zero mean or unit variance since they are output from earlier layers in the network. Even worse, during the training process the distribution of features at each layer of the network will shift as the weights of each layer are updated.

The authors of [1] hypothesize that the shifting distribution of features inside deep neural networks may make training deep networks more difficult. To overcome this problem, [3] proposes to insert batch normalization layers into the network. At training time, a batch normalization layer uses a minibatch of data to estimate the mean and standard deviation of each feature. These estimated means and standard deviations are then used to center and normalize the features of the minibatch. A running average of these means and standard deviations is kept during training, and at test time these running averages are used to center and normalize features.

It is possible that this normalization strategy could reduce the representational power of the network, since it may sometimes be optimal for certain layers to have features that are not zero-mean or unit variance. To this end, the batch normalization layer includes learnable shift and scale parameters for each feature dimension.

[1] Sergey Ioffe and Christian Szegedy, "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift", ICML 2015.

```
%cd /content/drive/My Drive/682/assignment2
```

```
     /content/drive/My Drive/682/assignment2
```

```
# As usual, a bit of setup
import time
import numpy as np
import matplotlib.pyplot as plt
from cs682.classifiers.fc_net import *
from cs682.data_utils import get_CIFAR10_data
from cs682.gradient_check import eval_numerical_gradient, eval_numerical_gradient_array
from cs682.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

%load_ext autoreload
%autoreload 2

def rel_error(x, y):
    """ returns relative error """
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))

def print_mean_std(x,axis=0):
    print('  means: ', x.mean(axis=axis))
    print('  stds:  ', x.std(axis=axis))
    print()
```

```
# Load the (preprocessed) CIFAR10 data.
data = get_CIFAR10_data()
for k, v in data.items():
  print('%s: ' % k, v.shape)
```

```
X_train:  (49000, 3, 32, 32)
y_train:  (49000,)
X_val:  (1000, 3, 32, 32)
y_val:  (1000,)
X_test:  (1000, 3, 32, 32)
y_test:  (1000,)
```

Batch Normalization: Forward

In the file `cs682/layers.py`, implement the batch normalization forward pass in the function `batchnorm_forward`. Once you have done so, run the following to test your implementation.

Referencing the paper linked to above would be helpful!

```
# Check the training-time forward pass by checking means and variances
# of features both before and after batch normalization

# Simulate the forward pass for a two-layer network
np.random.seed(231)
N, D1, D2, D3 = 200, 50, 60, 3
X = np.random.randn(N, D1)
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)
a = np.maximum(0, X.dot(W1)).dot(W2)

print('Before batch normalization:')
print_mean_std(a,axis=0)
```

```
gamma = np.ones((D3,))
beta = np.zeros((D3,))
# Means should be close to zero and stds close to one
print('After batch normalization (gamma=1, beta=0)')
a_norm, _ = batchnorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=0)

gamma = np.asarray([1.0, 2.0, 3.0])
beta = np.asarray([11.0, 12.0, 13.0])
# Now means should be close to beta and stds close to gamma
print('After batch normalization (gamma=', gamma, ', beta=', beta, ')')
a_norm, _ = batchnorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=0)
```

```
     Before batch normalization:
       means:  [ -2.3814598  -13.18038246   1.91780462]
       stds:   [27.18502186 34.21455511 37.68611762]

     After batch normalization (gamma=1, beta=0)
       means:  [5.32907052e-17 7.04991621e-17 1.85962357e-17]
       stds:   [0.99999999 1.         1.        ]

     After batch normalization (gamma= [1. 2. 3.] , beta= [11. 12. 13.] )
       means:  [11. 12. 13.]
       stds:   [0.99999999 1.99999999 2.99999999]
```

```
# Check the test-time forward pass by running the training-time
# forward pass many times to warm up the running averages, and then
# checking the means and variances of activations after a test-time
# forward pass.

np.random.seed(231)
N, D1, D2, D3 = 200, 50, 60, 3
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)

bn_param = {'mode': 'train'}
gamma = np.ones(D3)
beta = np.zeros(D3)

for t in range(50):
  X = np.random.randn(N, D1)
  a = np.maximum(0, X.dot(W1)).dot(W2)
  batchnorm_forward(a, gamma, beta, bn_param)

bn_param['mode'] = 'test'
X = np.random.randn(N, D1)
a = np.maximum(0, X.dot(W1)).dot(W2)
a_norm, _ = batchnorm_forward(a, gamma, beta, bn_param)

# Means should be close to zero and stds close to one, but will be
# noisier than training-time forward passes.
print('After batch normalization (test-time):')
print_mean_std(a_norm,axis=0)
```

```
     After batch normalization (test-time):
       means:  [-0.03927354 -0.04349152 -0.10452688]
       stds:   [1.01531427 1.01238373 0.97819987]
```

Batch normalization: Backward Pass

Now implement the backward pass for batch normalization in the function `batchnorm_backward`.

To derive the backward pass you should write out the computation graph for batch normalization and backprop through each of the intermediate nodes. Some intermediates may have multiple outgoing branches; make sure to sum gradients across these branches in the backward pass.

Once you have finished, run the following to numerically check your backward pass.

```
# Gradient check batchnorm backward pass
np.random.seed(231)
N, D = 4, 5
x = 5 * np.random.randn(N, D) + 12
gamma = np.random.randn(D)
beta = np.random.randn(D)
dout = np.random.randn(N, D)

bn_param = {'mode': 'train'}
fx = lambda x: batchnorm_forward(x, gamma, beta, bn_param)[0]
fg = lambda a: batchnorm_forward(x, a, beta, bn_param)[0]
fb = lambda b: batchnorm_forward(x, gamma, b, bn_param)[0]

dx_num = eval_numerical_gradient_array(fx, x, dout)
da_num = eval_numerical_gradient_array(fg, gamma.copy(), dout)
db_num = eval_numerical_gradient_array(fb, beta.copy(), dout)

_ , cache = batchnorm_forward(x, gamma, beta, bn_param)
dx, dgamma, dbeta = batchnorm_backward(dout, cache)
#You should expect to see relative errors between 1e-13 and 1e-8
print('dx error: ', rel_error(dx_num, dx))
print('dgamma error: ', rel_error(da_num, dgamma))
print('dbeta error: ', rel_error(db_num, dbeta))
```

```
     dx error:  1.7029258328157158e-09
     dgamma error:  7.420414216247087e-13
     dbeta error:  2.8795057655839487e-12
```

Batch Normalization: Alternative Backward

In class we talked about two different implementations for the sigmoid backward pass. One strategy is to write out a computation graph composed of simple operations and backprop through all intermediate values. Another strategy is to work out the derivatives on paper. For example, you can derive a very simple formula for the sigmoid function's backward pass by simplifying gradients on paper.

Surprisingly, it turns out that you can do a similar simplification for the batch normalization backward pass too.

Given a set of inputs $X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_N \end{bmatrix}$, we first calculate the mean $\mu = \frac{1}{N}\sum_{k=1}^{N} x_k$ and variance $v = \frac{1}{N}\sum_{k=1}^{N}(x_k - \mu)^2.$
With μ and v calculated, we can calculate the standard deviation $\sigma = \sqrt{v + \epsilon}$ and normalized data Y with $y_i = \frac{x_i - \mu}{\sigma}$.

The meat of our problem is to get $\frac{\partial L}{\partial X}$ from the upstream gradient $\frac{\partial L}{\partial Y}$. It might be challenging to directly reason about the gradients over X and Y - try reasoning about it in terms of x_i and y_i first.

You will need to come up with the derivations for $\frac{\partial L}{\partial x_i}$, by relying on the Chain Rule to first calculate the intermediate $\frac{\partial \mu}{\partial x_i}, \frac{\partial v}{\partial x_i}, \frac{\partial \sigma}{\partial x_i}$, then assemble these pieces to calculate $\frac{\partial y_i}{\partial x_i}$. You should make sure each of the intermediary steps are all as simple as possible.

After doing so, implement the simplified batch normalization backward pass in the function `batchnorm_backward_alt` and compare the two implementations by running the following. Your two implementations should compute nearly identical results, but the alternative implementation should

```
np.random.seed(231)
N, D = 100, 500
x = 5 * np.random.randn(N, D) + 12
gamma = np.random.randn(D)
beta = np.random.randn(D)
dout = np.random.randn(N, D)

bn_param = {'mode': 'train'}
out, cache = batchnorm_forward(x, gamma, beta, bn_param)

t1 = time.time()
dx1, dgamma1, dbeta1 = batchnorm_backward(dout, cache)
t2 = time.time()
dx2, dgamma2, dbeta2 = batchnorm_backward_alt(dout, cache)
t3 = time.time()
```

```
print('dx difference: ', rel_error(dx1, dx2))
print('dgamma difference: ', rel_error(dgamma1, dgamma2))
print('dbeta difference: ', rel_error(dbeta1, dbeta2))
print('speedup: %.2fx' % ((t2 - t1) / (t3 - t2)))
```

```
dx difference:  6.284600172572596e-13
dgamma difference:  0.0
dbeta difference:  0.0
speedup: 1.88x
```

Fully Connected Networks with Batch Normalization

Now that you have a working implementation for batch normalization, go back to your `FullyConnectedNet` in the file `cs682/classifiers/fc_net.py`. Modify your implementation to add batch normalization.

Concretely, when the `normalization` flag is set to `"batchnorm"` in the constructor, you should insert a batch normalization layer before each ReLU nonlinearity. The outputs from the last layer of the network should not be normalized. Once you are done, run the following to gradient-check your implementation.

HINT: You might find it useful to define an additional helper layer similar to those in the file `cs682/layer_utils.py`.

```
np.random.seed(231)
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

# You should expect losses between 1e-4~1e-10 for W,
# losses between 1e-08~1e-10 for b,
# and losses between 1e-08~1e-09 for beta and gammas.
for reg in [0, 3.14]:
  print('Running check with reg = ', reg)
  model = FullyConnectedNet([H1, H2], input_dim=D, num_classes=C,
                            reg=reg, weight_scale=5e-2, dtype=np.float64,
                            normalization='batchnorm')

  loss, grads = model.loss(X, y)
  print('Initial loss: ', loss)

  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False, h=1e-5)
    print('%s relative error: %.2e' % (name, rel_error(grad_num, grads[name])))
  if reg == 0: print()
```

```
Running check with reg =  0
Initial loss:  2.2611955101340957
W1 relative error: 1.10e-04
W2 relative error: 2.85e-06
W3 relative error: 4.05e-10
b1 relative error: 2.22e-07
b2 relative error: 2.22e-08
b3 relative error: 1.01e-10
beta1 relative error: 7.33e-09
beta2 relative error: 1.89e-09
gamma1 relative error: 6.96e-09
gamma2 relative error: 1.96e-09

Running check with reg =  3.14
Initial loss:  6.996533220108303
W1 relative error: 1.98e-06
W2 relative error: 2.29e-06
W3 relative error: 2.79e-08
b1 relative error: 1.38e-08
b2 relative error: 7.99e-07
b3 relative error: 2.10e-10
beta1 relative error: 6.65e-09
beta2 relative error: 4.23e-09
gamma1 relative error: 6.27e-09
gamma2 relative error: 5.28e-09
```

Batch Normalization for Deep Networks

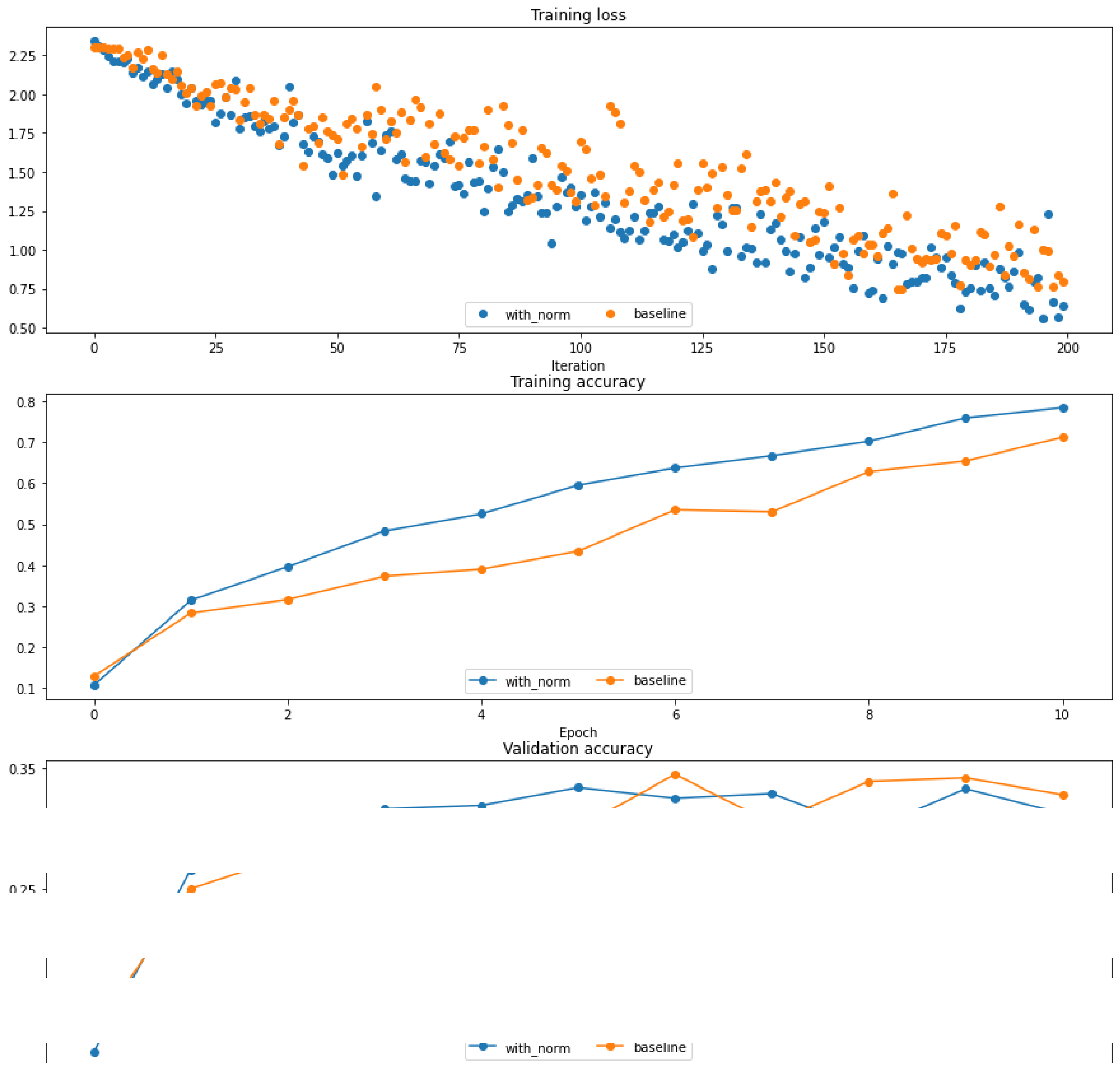
Run the following to train a six-layer network on a subset of 1000 training examples both with and without batch normalization.

```
np.random.seed(231)
# Try training a very deep net with batchnorm
hidden_dims = [100, 100, 100, 100, 100]

num_train = 1000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

weight_scale = 2e-2
bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, normalization='batchnorm
model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, normalization=None)

bn_solver = Solver(bn_model, small_data,
                num_epochs=10, batch_size=50,
                update_rule='adam',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True,print_every=20)
bn_solver.train()

solver = Solver(model, small_data,
                num_epochs=10, batch_size=50,
                update_rule='adam',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True, print_every=20)
solver.train()
```

```
(Iteration 1 / 200) loss: 2.340974
(Epoch 0 / 10) train acc: 0.107000; val_acc: 0.115000
(Epoch 1 / 10) train acc: 0.314000; val_acc: 0.266000
(Iteration 21 / 200) loss: 2.039345
(Epoch 2 / 10) train acc: 0.396000; val_acc: 0.280000
(Iteration 41 / 200) loss: 2.047471
(Epoch 3 / 10) train acc: 0.483000; val_acc: 0.316000
(Iteration 61 / 200) loss: 1.739554
(Epoch 4 / 10) train acc: 0.525000; val_acc: 0.319000
(Iteration 81 / 200) loss: 1.246974
(Epoch 5 / 10) train acc: 0.595000; val_acc: 0.334000
(Iteration 101 / 200) loss: 1.354827
(Epoch 6 / 10) train acc: 0.637000; val_acc: 0.325000
(Iteration 121 / 200) loss: 1.013708
(Epoch 7 / 10) train acc: 0.667000; val_acc: 0.329000
(Iteration 141 / 200) loss: 1.170422
(Epoch 8 / 10) train acc: 0.702000; val_acc: 0.298000
(Iteration 161 / 200) loss: 0.739973
(Epoch 9 / 10) train acc: 0.759000; val_acc: 0.333000
(Iteration 181 / 200) loss: 0.756491
(Epoch 10 / 10) train acc: 0.784000; val_acc: 0.312000
(Iteration 1 / 200) loss: 2.302332
(Epoch 0 / 10) train acc: 0.129000; val_acc: 0.131000
(Epoch 1 / 10) train acc: 0.283000; val_acc: 0.250000
(Iteration 21 / 200) loss: 2.041970
(Epoch 2 / 10) train acc: 0.316000; val_acc: 0.277000
(Iteration 41 / 200) loss: 1.900473
(Epoch 3 / 10) train acc: 0.373000; val_acc: 0.282000
(Iteration 61 / 200) loss: 1.713156
(Epoch 4 / 10) train acc: 0.390000; val_acc: 0.310000
(Iteration 81 / 200) loss: 1.662209
(Epoch 5 / 10) train acc: 0.434000; val_acc: 0.300000
(Iteration 101 / 200) loss: 1.696059
(Epoch 6 / 10) train acc: 0.535000; val_acc: 0.345000
(Iteration 121 / 200) loss: 1.557987
(Epoch 7 / 10) train acc: 0.530000; val_acc: 0.304000
(Iteration 141 / 200) loss: 1.432189
(Epoch 8 / 10) train acc: 0.628000; val_acc: 0.339000
(Iteration 161 / 200) loss: 1.034116
(Epoch 9 / 10) train acc: 0.654000; val_acc: 0.342000
(Iteration 181 / 200) loss: 0.905795
(Epoch 10 / 10) train acc: 0.712000; val_acc: 0.328000
```

Run the following to visualize the results from two networks trained above. You should find that using batch normalization helps the network to converge much faster.

```
def plot_training_history(title, label, baseline, bn_solvers, plot_fn, bl_marker='.', bn_mark
    """utility function for plotting training history"""
    plt.title(title)
    plt.xlabel(label)
    bn_plots = [plot_fn(bn_solver) for bn_solver in bn_solvers]
    bl_plot = plot_fn(baseline)
    num_bn = len(bn_plots)
    for i in range(num_bn):
        label='with_norm'
        if labels is not None:
            label += str(labels[i])
        plt.plot(bn_plots[i], bn_marker, label=label)
    label='baseline'
    if labels is not None:
        label += str(labels[0])
    plt.plot(bl_plot, bl_marker, label=label)
    plt.legend(loc='lower center', ncol=num_bn+1)


plt.subplot(3, 1, 1)
plot_training_history('Training loss','Iteration', solver, [bn_solver], \
                      lambda x: x.loss_history, bl_marker='o', bn_marker='o')
plt.subplot(3, 1, 2)
plot_training_history('Training accuracy','Epoch', solver, [bn_solver], \
                      lambda x: x.train_acc_history, bl_marker='-o', bn_marker='-o')
plt.subplot(3, 1, 3)
plot_training_history('Validation accuracy','Epoch', solver, [bn_solver], \
                      lambda x: x.val_acc_history, bl_marker='-o', bn_marker='-o')

plt.gcf().set_size_inches(15, 15)
plt.show()
```



Batch Normalization and Initialization

We will now run a small experiment to study the interaction of batch normalization and weight initialization.

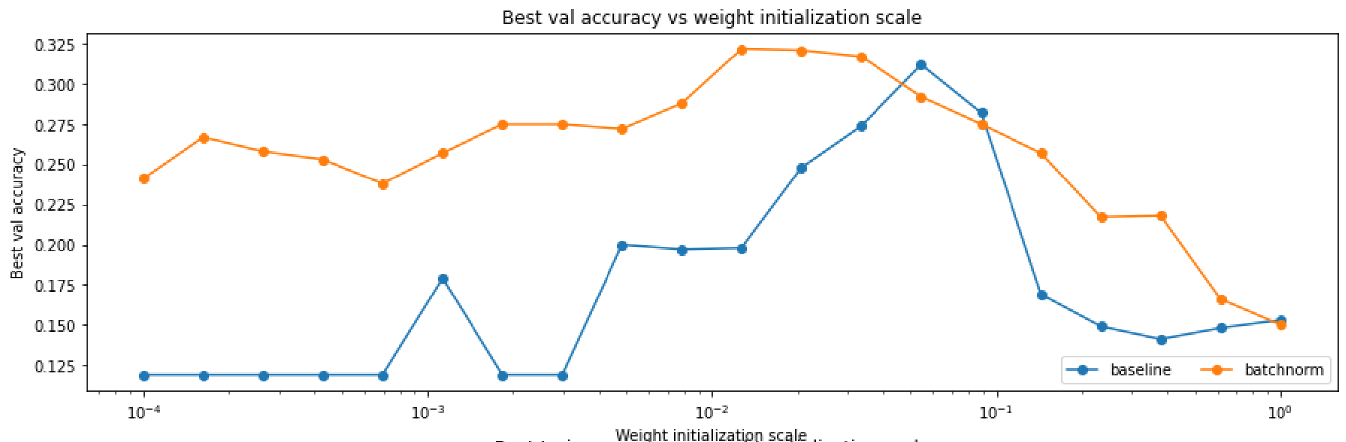
The first cell will train 8-layer networks both with and without batch normalization using different scales for weight initialization. The second layer will plot training accuracy, validation set accuracy, and training loss as a function of the weight initialization scale.

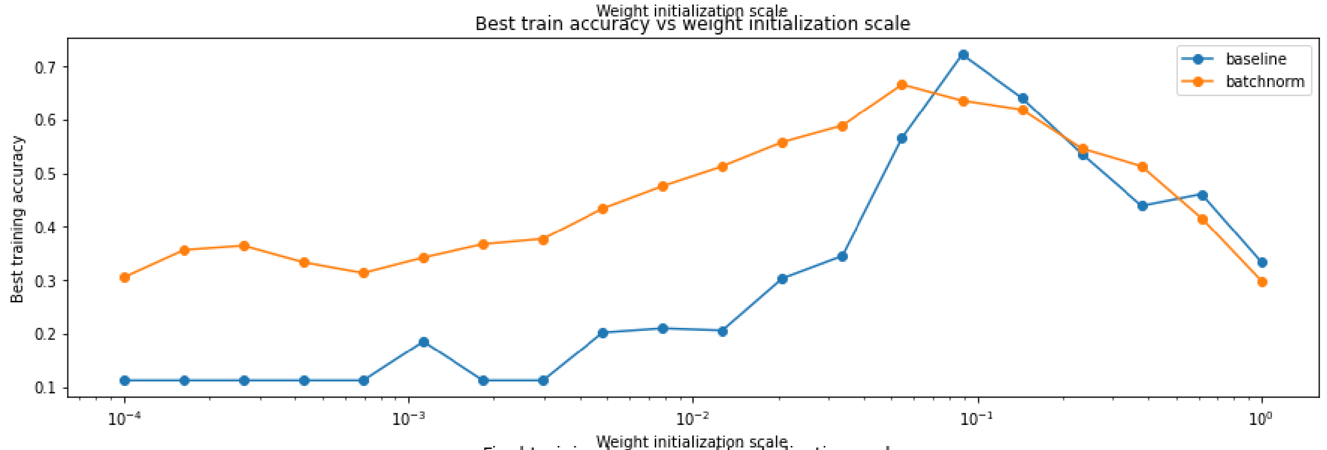
```
np.random.seed(231)
# Try training a very deep net with batchnorm
```

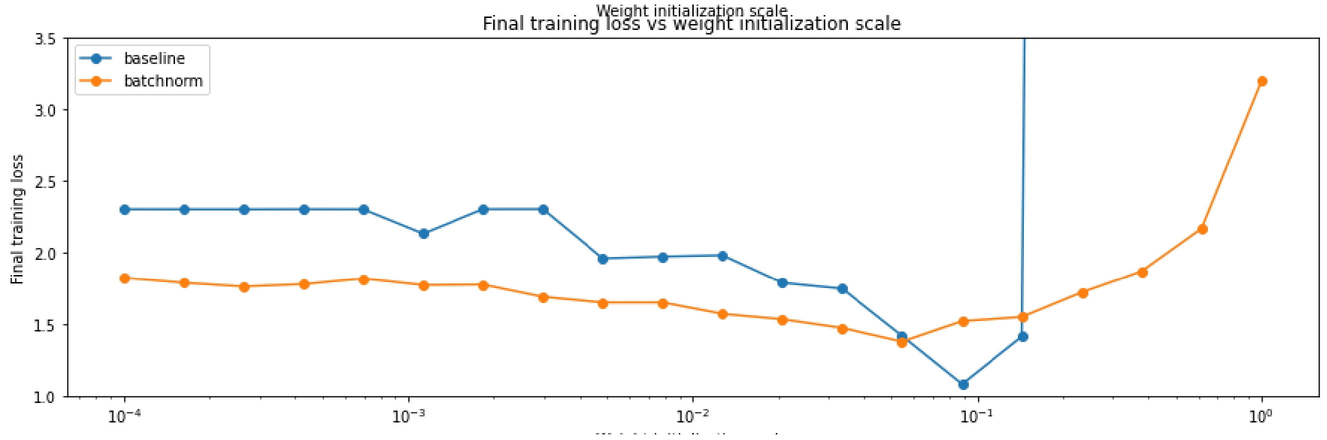
```
hidden_dims = [50, 50, 50, 50, 50, 50, 50]
num_train = 1000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

bn_solvers_ws = {}
solvers_ws = {}
weight_scales = np.logspace(-4, 0, num=20)
for i, weight_scale in enumerate(weight_scales):
  print('Running weight scale %d / %d' % (i + 1, len(weight_scales)))
  bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, normalization='batchno
  model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, normalization=None)

  bn_solver = Solver(bn_model, small_data,
                  num_epochs=10, batch_size=50,
                  update_rule='adam',
                  optim_config={
                    'learning_rate': 1e-3,
                  },
                  verbose=False, print_every=200)
  bn_solver.train()
  bn_solvers_ws[weight_scale] = bn_solver

  solver = Solver(model, small_data,
                  num_epochs=10, batch_size=50,
                  update_rule='adam',
                  optim_config={
                    'learning_rate': 1e-3,
                  },
                  verbose=False, print_every=200)
  solver.train()
  solvers_ws[weight_scale] = solver
```

```
Running weight scale 1 / 20
Running weight scale 2 / 20
Running weight scale 3 / 20
Running weight scale 4 / 20
Running weight scale 5 / 20
Running weight scale 6 / 20
Running weight scale 7 / 20
Running weight scale 8 / 20
Running weight scale 9 / 20
Running weight scale 10 / 20
Running weight scale 11 / 20
Running weight scale 12 / 20
Running weight scale 13 / 20
Running weight scale 14 / 20
Running weight scale 15 / 20
Running weight scale 16 / 20
Running weight scale 17 / 20
Running weight scale 18 / 20
Running weight scale 19 / 20
Running weight scale 20 / 20
```

```
# Plot results of weight scale experiment
best_train_accs, bn_best_train_accs = [], []
best_val_accs, bn_best_val_accs = [], []
final_train_loss, bn_final_train_loss = [], []

for ws in weight_scales:
  best_train_accs.append(max(solvers_ws[ws].train_acc_history))
  bn_best_train_accs.append(max(bn_solvers_ws[ws].train_acc_history))

  best_val_accs.append(max(solvers_ws[ws].val_acc_history))
  bn_best_val_accs.append(max(bn_solvers_ws[ws].val_acc_history))

  final_train_loss.append(np.mean(solvers_ws[ws].loss_history[-100:]))
  bn_final_train_loss.append(np.mean(bn_solvers_ws[ws].loss_history[-100:]))

plt.subplot(3, 1, 1)
plt.title('Best val accuracy vs weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Best val accuracy')
plt.semilogx(weight_scales, best_val_accs, '-o', label='baseline')
plt.semilogx(weight_scales, bn_best_val_accs, '-o', label='batchnorm')
plt.legend(ncol=2, loc='lower right')

plt.subplot(3, 1, 2)
plt.title('Best train accuracy vs weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Best training accuracy')
plt.semilogx(weight_scales, best_train_accs, '-o', label='baseline')
plt.semilogx(weight_scales, bn_best_train_accs, '-o', label='batchnorm')
plt.legend()

plt.subplot(3, 1, 3)
plt.title('Final training loss vs weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Final training loss')
plt.semilogx(weight_scales, final_train_loss, '-o', label='baseline')
plt.semilogx(weight_scales, bn_final_train_loss, '-o', label='batchnorm')
plt.legend()
plt.gca().set_ylim(1.0, 3.5)

plt.gcf().set_size_inches(15, 15)
plt.show()
```







Inline Question 1:

Describe the results of this experiment. How does the scale of weight initialization affect models with/without batch normalization differently, and why?

Answer:

1: We can witness the problem of vanishing gradients in the first and the second plots. Accuracy for baseline starts very low whereas that of the batchnorm starts high mostly because it is not so sensitive to weight initialization.
2: I think in the second plot, we are overfitting and getting training accuracies so high which is prevented by batchnorm.

3: When the weight scale is extremely high, loss is high (both ends) in the third plot. Batchnorm prevents this exact problem by standardizing the outputs at the end of each affine layer.

Batch normalization and batch size

We will now run a small experiment to study the interaction of batch normalization and batch size.

The first cell will train 6-layer networks both with and without batch normalization using different batch sizes. The second layer will plot training accuracy and validation set accuracy over time.

```
def run_batchsize_experiments(normalization_mode):
    np.random.seed(231)
    # Try training a very deep net with batchnorm
    hidden_dims = [100, 100, 100, 100, 100]
    num_train = 1000
    small_data = {
      'X_train': data['X_train'][:num_train],
      'y_train': data['y_train'][:num_train],
      'X_val': data['X_val'],
      'y_val': data['y_val'],
    }
    n_epochs=10
    weight_scale = 2e-2
    batch_sizes = [5,10,50]
    lr = 10**(-3.5)
    solver_bsize = batch_sizes[0]

    print('No normalization: batch size = ',solver_bsize)
    model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, normalization=None)
    solver = Solver(model, small_data,
                    num_epochs=n_epochs, batch_size=solver_bsize,
                    update_rule='adam',
                    optim_config={
                      'learning_rate': lr,
                    },
                    verbose=False)
    solver.train()

    bn_solvers = []
    for i in range(len(batch_sizes)):
        b_size=batch_sizes[i]
        print('Normalization: batch size = ',b_size)
        bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, normalization=no
        bn_solver = Solver(bn_model, small_data,
                        num_epochs=n_epochs, batch_size=b_size,
                        update_rule='adam',
                        optim_config={
                          'learning_rate': lr,
                        },
                        verbose=False)
        bn_solver.train()
        bn_solvers.append(bn_solver)

    return bn_solvers, solver, batch_sizes

batch_sizes = [5,10,50]
bn_solvers_bsize, solver_bsize, batch_sizes = run_batchsize_experiments('batchnorm')
```

```
No normalization: batch size =  5
Normalization: batch size =  5
Normalization: batch size =  10
Normalization: batch size =  50
```

```
plt.subplot(2, 1, 1)
plot_training_history('Training accuracy (Batch Normalization)','Epoch', solver_bsize, bn_sol
                      lambda x: x.train_acc_history, bl_marker='-^', bn_marker='-o', labels=b
plt.subplot(2, 1, 2)
plot_training_history('Validation accuracy (Batch Normalization)','Epoch', solver_bsize, bn_s
                      lambda x: x.val_acc_history, bl_marker='-^', bn_marker='-o', labels=bat

plt.gcf().set_size_inches(15, 10)
plt.show()
```

Inline Question 2:

Describe the results of this experiment. What does this imply about the relationship between batch normalization and batch size? Why is this relationship observed?

Answer:

We can see that the performance is postively correlated to the batch size. So worst performance occurs for smaller batch sizes. This happens as when we standardize data using mean and variance of a smaller sample, it actually transforms our data by not keeping its characteristics intact. The means of the samples can be very different and we are no longer can treat batch normalization as a transformation only method. In case of higher batch size, mean represents more of data , variance is for more of data . So, when we scale and shift, we are essentially only transforming, not affecting data's characteristics.

Layer Normalization

Batch normalization has proved to be effective in making networks easier to train, but the dependency on batch size makes it less useful in complex networks which have a cap on the input batch size due to hardware limitations.

Several alternatives to batch normalization have been proposed to mitigate this problem; one such technique is Layer Normalization [2]. Instead of normalizing over the batch, we normalize over the features. In other words, when using Layer Normalization, each feature vector corresponding to a single datapoint is normalized based on the sum of all terms within that feature vector.

[2] Ba, Jimmy Lei, Jamie Ryan Kiros, and Geoffrey E. Hinton. "Layer Normalization." stat 1050 (2016): 21.

Inline Question 3:

Which of these data preprocessing steps is analogous to batch normalization, and which is analogous to layer normalization?

1. Scaling each image in the dataset, so that the RGB channels for each row of pixels within an image sums up to 1.
2. Scaling each image in the dataset, so that the RGB channels for all pixels within an image sums up to 1.

3. Subtracting the mean image of the dataset from each image in the dataset.
4. Setting all RGB values to either 0 or 1 depending on a given threshold.

Answer:

2. Layer norm (size fo feature vector=3)
3. Batch norm (size = number of images in dataset)

Layer Normalization: Implementation

Now you'll implement layer normalization. This step should be relatively straightforward, as conceptually the implementation is almost identical to that of batch normalization. One significant difference though is that for layer normalization, we do not keep track of the moving moments, and the testing phase is identical to the training phase, where the mean and variance are directly calculated per datapoint.

Here's what you need to do:

- In `cs682/layers.py`, implement the forward pass for layer normalization in the function `layernorm_backward`.

Run the cell below to check your results.

- In `cs682/layers.py`, implement the backward pass for layer normalization in the function `layernorm_backward`.

Run the second cell below to check your results.

- Modify `cs682/classifiers/fc_net.py` to add layer normalization to the `FullyConnectedNet`. When the `normalization` flag is set to `"layernorm"` in the constructor, you should insert a layer normalization layer before each ReLU nonlinearity.

Run the third cell below to run the batch size experiment on layer normalization.

```
# Check the training-time forward pass by checking means and variances
# of features both before and after layer normalization

# Simulate the forward pass for a two-layer network
np.random.seed(231)
N, D1, D2, D3 =4, 50, 60, 3
X = np.random.randn(N, D1)
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)
a = np.maximum(0, X.dot(W1)).dot(W2)

print('Before layer normalization:')
```

```
print_mean_std(a,axis=1)

gamma = np.ones(D3)
beta = np.zeros(D3)
# Means should be close to zero and stds close to one
print('After layer normalization (gamma=1, beta=0)')
a_norm, _ = layernorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=1)

gamma = np.asarray([3.0,3.0,3.0])
beta = np.asarray([5.0,5.0,5.0])
# Now means should be close to beta and stds close to gamma
print('After layer normalization (gamma=', gamma, ', beta=', beta, ')')
a_norm, _ = layernorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=1)
```

```
Before layer normalization:
  means:  [-59.06673243 -47.60782686 -43.31137368 -26.40991744]
  stds:   [10.07429373 28.39478981 35.28360729  4.01831507]

After layer normalization (gamma=1, beta=0)
  means:  [ 4.81096644e-16 -7.40148683e-17  2.22044605e-16 -5.92118946e-16]
  stds:   [0.99999995 0.99999999 1.         0.99999969]

After layer normalization (gamma= [3. 3. 3.] , beta= [5. 5. 5.] )
  means:  [5. 5. 5. 5.]
  stds:   [2.99999985 2.99999998 2.99999999 2.99999907]
```

```
# Gradient check layernorm backward pass
np.random.seed(231)
N, D = 4, 5
x = 5 * np.random.randn(N, D) + 12
gamma = np.random.randn(D)
beta = np.random.randn(D)
dout = np.random.randn(N, D)

ln_param = {}
fx = lambda x: layernorm_forward(x, gamma, beta, ln_param)[0]
fg = lambda a: layernorm_forward(x, a, beta, ln_param)[0]
fb = lambda b: layernorm_forward(x, gamma, b, ln_param)[0]

dx_num = eval_numerical_gradient_array(fx, x, dout)
da_num = eval_numerical_gradient_array(fg, gamma.copy(), dout)
db_num = eval_numerical_gradient_array(fb, beta.copy(), dout)

_, cache = layernorm_forward(x, gamma, beta, ln_param)
dx, dgamma, dbeta = layernorm_backward(dout, cache)

#You should expect to see relative errors between 1e-12 and 1e-8
print('dx error: ', rel_error(dx_num, dx))
print('dgamma error: ', rel_error(da_num, dgamma))
print('dbeta error: ', rel_error(db_num, dbeta))
```

```
    dx error:  1.4336158494902849e-09
    dgamma error:  4.519489546032799e-12
    dbeta error:  2.276445013433725e-12
```

Layer Normalization and batch size

We will now run the previous batch size experiment with layer normalization instead of batch normalization. Compared to the previous experiment, you should see a markedly smaller influence of batch size on the training history!

```
ln_solvers_bsize, solver_bsize, batch_sizes = run_batchsize_experiments('layernorm')

plt.subplot(2, 1, 1)
plot_training_history('Training accuracy (Layer Normalization)','Epoch', solver_bsize, ln_sol
                      lambda x: x.train_acc_history, bl_marker='-^', bn_marker='-o', labels=b
plt.subplot(2, 1, 2)
plot_training_history('Validation accuracy (Layer Normalization)','Epoch', solver_bsize, ln_s
                      lambda x: x.val_acc_history, bl_marker='-^', bn_marker='-o', labels=bat

plt.gcf().set_size_inches(15, 10)
plt.show()
```

```
No normalization: batch size =  5
Normalization: batch size =  5
Normalization: batch size =  10
Normalization: batch size =  50
```

Inline Question 4:

When is layer normalization likely to not work well, and why?

1. Using it in a very deep network
2. Having a very small dimension of features
3. Having a high regularization term

Answer: 2,3

1. Normalization was created to be able to increase the capacity of the network and so that we can learn more deep networks and that stays true for layer norm.
2. As was proven when we took a small batch size, the representative power of the mean and variance reduces in case of small number of sample, so data point A will have a very different mean and variance across features than data point B, if there are only a few features. Therefore, the noisy layernorm outputs will cause the network to not work well.
3. Regularization is supposed to enforce smoothening of the predictor in question, so high regularization means the less sensitivity to data and therefore underfitting will happen even with layernorm.