Mixed-effect Modelling: Experiment II (Centroid)

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This document presents statistical analysis of the user responses in an immersive sound localization test in which users attempted to localize the edges and centroid of a 2D spatially sonified shape in an image. Users' responses were recorded for five different shapes. Images were presented to them in a random order with a random horizontal-offset for each image.

The users performed this test in two scenarios: 'Stationary' and 'Moving'. In the stationary scenario, the users stood at the center of the room, where they were allowed to turn their heads and move around their position, but not walk. In the moving scenario, they were encouraged to walk around in the room. There was no time constraint for this test. Based on the results from Experiment I, the combined modulation technique was used for sonifying the 2D images.

The miss-distance indicates the angular distance between the shape's mathematical centroid and the perceived center pointed at by the users. The images are named for convenience based on their shapes as: 'Amoeba', 'Bow', 'Gradient', 'Ring' and 'Snake'.

(a.) Reading the data file

In this experiment, we used a 2×5 within-subjects design with two independent variables: two scenarios (stationary, and moving), and five shapes ('Bow', 'Ring', 'Amoeba', 'Snake', 'Gradient'), yielding ten total responses per participant for the perceived center. In total, the study resulted in 220 responses (10 × 22) for the comparison of the perceived spatial data properties in terms of the shapes' centroid.

```
image_data <- read_csv('../data/miss-distances-4.csv')
```

```
## Warning: Missing column names filled in: 'X1' [1]
```

```
## Parsed with column specification:
## cols(
##   X1 = col_double(),
##   Scenario = col_character(),
##   User_id = col_character(),
##   Shapes = col_character(),
##   value = col_double()
## )
```

(b.) Hypotheses

Null hypothesis for Experiment II is that there is no difference in the user responses depending on their physical motion and shapes of the aural images. Alternate hypotheses, with respect to the centroid of the shapes, are:

- $H_{\alpha-scenario}$: There is a difference in the average miss-distances between the perceived center and the mathematical centroid of an aural image in stationary and moving scenarios, and
- $H_{\alpha-shape}$: There is a difference in the average miss-distance between the perceived center and the mathematical centroid among user responses for at least one of the shapes when compared to other shapes.

(c.) Mixed-effect Modelling and Levene's Test

We used mixed-effect modelling with Satterthwaite approximation to analyze the angular miss-distances between the centroid and the user-responses. We used Levene's test to test the homogeneity of variance condition and found out that our data (miss-distances) do not satisfy the equal variance condition. We did the log transformation of our data and fitted them to the model. Since the log transform is monotonic, this would still give us the interpretation that a significant p-value means participants were better or worse at locating the center of each shape. After taking the log transform, our data satisfied the equal variance condition.

The normality of the data was not a big concern for our model, as the categorical predictors only require the mean of the group to be normally distributed, which is not an issue with the amount of data collected.

We used Tukey's method for p-value adjustment while doing the pairwise comparison to control the error rate. Tukey's method is the most common correcting approach as it is not overly conservative with respect to Type I error and also provides good power for detecting differences.

We used an α level of 0.05 in all significance tests and report the significance values for $p < .05$, $p < .01$, and $p < .001$, abbreviated by $(*)$, $(**)$ and $(***)$ respectively.

```
image_data$Shapes = factor(image_data$Shapes,
                           levels = c("Snake", "Amoeba", "Ring", "Gradient", "Bow"))

# Levene's test
leveneTest(log(value) ~ Shapes*Scenario, data = image_data)
```

```
## Levene's Test for Homogeneity of Variance (center = median)
##        Df F value Pr(>F)
## group   9    1.61 0.1139
##       210
```

```
mixed.lmer <- lmer(log(value) ~ Shapes*Scenario + (1 | User_id),
                   data = image_data)
#tidy(mixed.lmer)
summary(mixed.lmer, ddf = "Satterthwaite")
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: log(value) ~ Shapes * Scenario + (1 | User_id)
##    Data: image_data
##
## REML criterion at convergence: 372.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.6141 -0.5220  0.1469  0.5875  2.2475
##
```

```
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  User_id  (Intercept) 0.1144   0.3382
##  Residual             0.2506   0.5006
## Number of obs: 220, groups:  User_id, 22
##
## Fixed effects:
##                                    Estimate Std. Error         df t value
## (Intercept)                         3.575071   0.128804 111.457709  27.756
## ShapesAmoeba                       -0.284460   0.150934 189.000001  -1.885
## ShapesRing                         -0.649743   0.150934 189.000001  -4.305
## ShapesGradient                     -0.714105   0.150934 189.000001  -4.731
## ShapesBow                          -0.322904   0.150934 189.000001  -2.139
## ScenarioStationary                  0.007626   0.150934 189.000001   0.051
## ShapesAmoeba:ScenarioStationary     0.037582   0.213453 189.000001   0.176
## ShapesRing:ScenarioStationary      -0.061358   0.213453 189.000001  -0.287
## ShapesGradient:ScenarioStationary  -0.156151   0.213453 189.000001  -0.732
## ShapesBow:ScenarioStationary       -0.093482   0.213453 189.000001  -0.438
##                                   Pr(>|t|)
## (Intercept)                        < 2e-16 ***
## ShapesAmoeba                         0.0610 .
## ShapesRing                         2.68e-05 ***
## ShapesGradient                     4.36e-06 ***
## ShapesBow                            0.0337 *
## ScenarioStationary                   0.9598
## ShapesAmoeba:ScenarioStationary      0.8604
## ShapesRing:ScenarioStationary        0.7741
## ShapesGradient:ScenarioStationary    0.4653
## ShapesBow:ScenarioStationary         0.6619
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##             (Intr) ShpsAm ShpsRn ShpsGr ShpsBw ScnrSt ShA:SS ShR:SS ShG:SS
## ShapesAmoeb -0.586
## ShapesRing  -0.586  0.500
## ShapesGrdnt -0.586  0.500  0.500
## ShapesBow   -0.586  0.500  0.500  0.500
## ScenrSttnry -0.586  0.500  0.500  0.500  0.500
## ShpsAmb:ScS  0.414 -0.707 -0.354 -0.354 -0.354 -0.707
## ShpsRng:ScS  0.414 -0.354 -0.707 -0.354 -0.354 -0.707  0.500
## ShpsGrdn:SS  0.414 -0.354 -0.354 -0.707 -0.354 -0.707  0.500  0.500
## ShpsBw:ScnS  0.414 -0.354 -0.354 -0.354 -0.707 -0.707  0.500  0.500  0.500
```

```
#anova(mixed.lmer)

#difflsmeans(mixed.lmer)
```

```
emmeans(mixed.lmer, list(pairwise~Shapes), adjust="tukey")
```

```
## NOTE: Results may be misleading due to involvement in interactions

## $`emmeans of Shapes`
```

```
##  Shapes   emmean    SE   df lower.CL upper.CL
##  Snake      3.58 0.104 58.7     3.30     3.86
##  Amoeba     3.31 0.104 58.7     3.04     3.59
##  Ring       2.90 0.104 58.7     2.62     3.18
##  Gradient   2.79 0.104 58.7     2.51     3.06
##  Bow        3.21 0.104 58.7     2.93     3.49
##
## Results are averaged over the levels of: Scenario
## Degrees-of-freedom method: kenward-roger
## Results are given on the log (not the response) scale.
## Confidence level used: 0.95
## Conf-level adjustment: sidak method for 5 estimates
##
## $`pairwise differences of Shapes`
##  contrast          estimate    SE  df t.ratio p.value
##  Snake - Amoeba       0.266 0.107 189  2.489  0.0974
##  Snake - Ring         0.680 0.107 189  6.375  <.0001
##  Snake - Gradient     0.792 0.107 189  7.423  <.0001
##  Snake - Bow          0.370 0.107 189  3.463  0.0059
##  Amoeba - Ring        0.415 0.107 189  3.886  0.0013
##  Amoeba - Gradient    0.527 0.107 189  4.933  <.0001
##  Amoeba - Bow         0.104 0.107 189  0.974  0.8665
##  Ring - Gradient      0.112 0.107 189  1.047  0.8330
##  Ring - Bow          -0.311 0.107 189 -2.912  0.0325
##  Gradient - Bow      -0.423 0.107 189 -3.959  0.0010
##
## Results are averaged over the levels of: Scenario
## Degrees-of-freedom method: kenward-roger
## Results are given on the log (not the response) scale.
## P value adjustment: tukey method for comparing a family of 5 estimates
```

```
emmeans(mixed.lmer, list(pairwise~Scenario), adjust="tukey")
```

```
## NOTE: Results may be misleading due to involvement in interactions

## $`emmeans of Scenario`
##  Scenario   emmean     SE   df lower.CL upper.CL
##  Moving       3.18 0.0865 29.1     2.98     3.38
##  Stationary   3.13 0.0865 29.1     2.93     3.34
##
## Results are averaged over the levels of: Shapes
## Degrees-of-freedom method: kenward-roger
## Results are given on the log (not the response) scale.
## Confidence level used: 0.95
## Conf-level adjustment: sidak method for 2 estimates
##
## $`pairwise differences of Scenario`
##  contrast            estimate     SE  df t.ratio p.value
##  Moving - Stationary   0.0471 0.0675 189 0.697   0.4866
##
## Results are averaged over the levels of: Shapes
## Degrees-of-freedom method: kenward-roger
## Results are given on the log (not the response) scale.
```

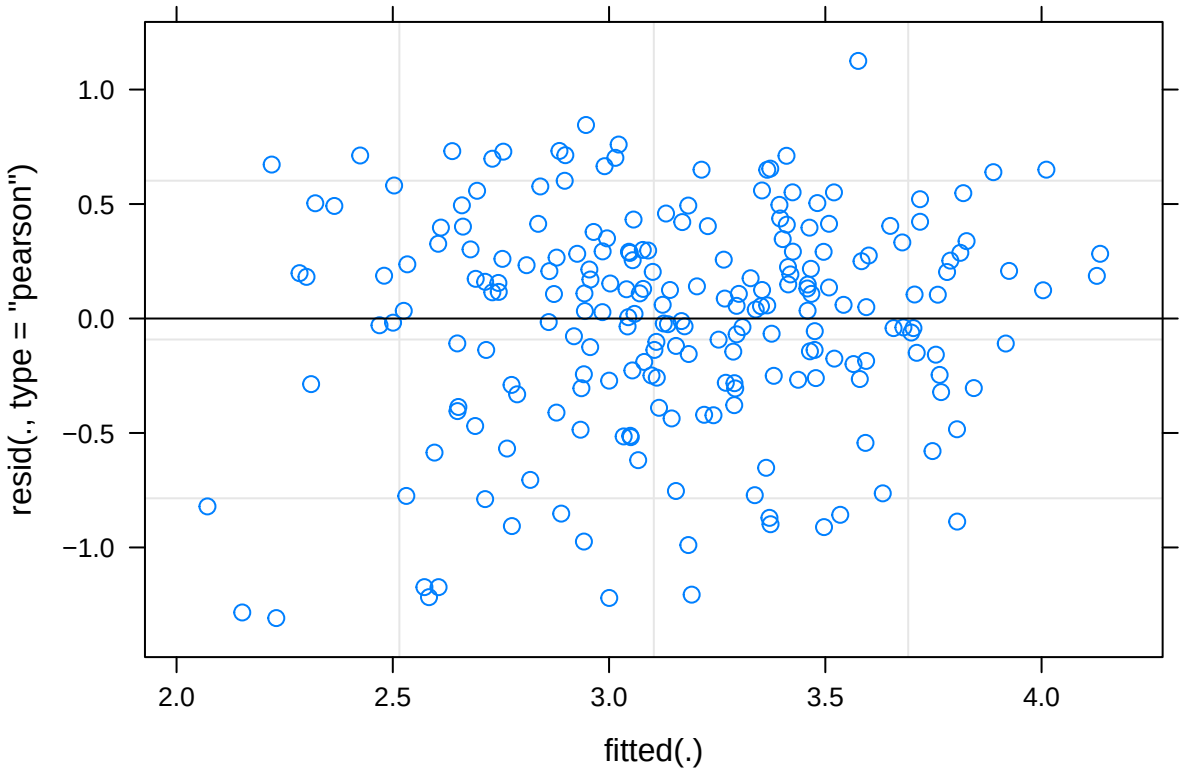
(d.) Checking the equal variance assumption

```
Levene.Model.F <- lm(abs(residuals(mixed.lmer))^2 ~ Shapes*Scenario, data = image_data)

#ANOVA of the squared residuals

anova(Levene.Model.F)
```

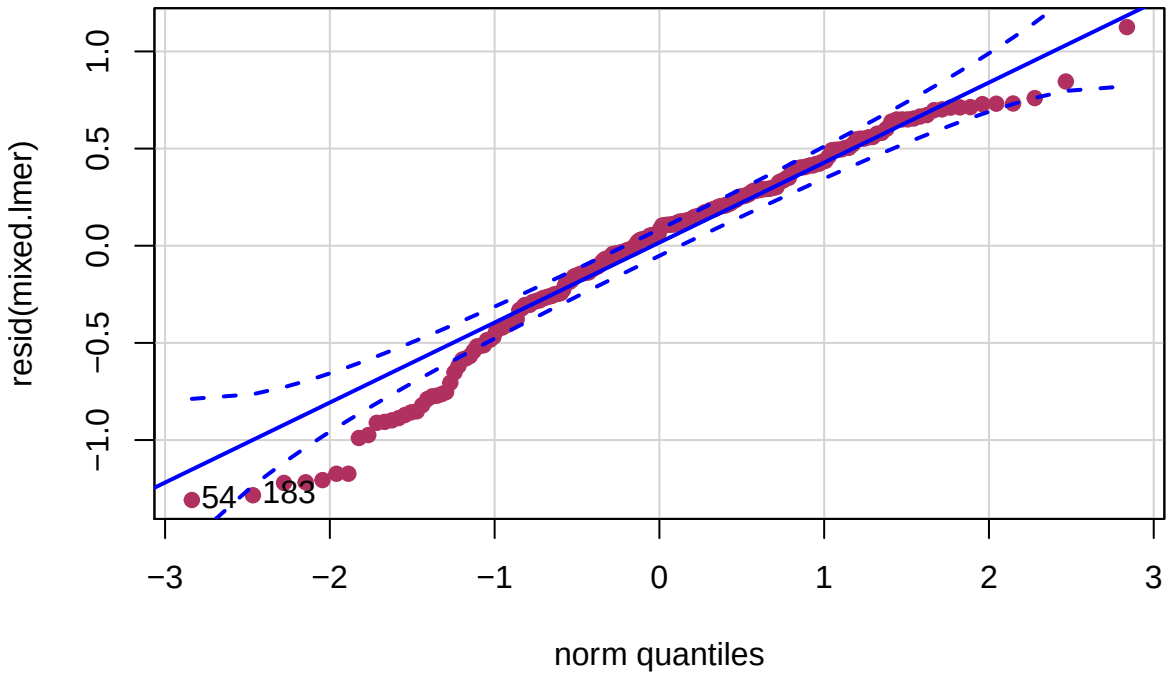
```
## Analysis of Variance Table
##
## Response: abs(residuals(mixed.lmer))^2
##                   Df  Sum Sq Mean Sq F value  Pr(>F)
## Shapes             4  0.5867 0.14667  1.4136 0.23057
## Scenario           1  0.3792 0.37919  3.6545 0.05728 .
## Shapes:Scenario    4  0.4123 0.10307  0.9933 0.41216
## Residuals        210 21.7898 0.10376
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(e.) Checking the normality assumption

```
plot(mixed.lmer)
```



```
qqPlot(resid(mixed.lmer), pch=19, col = "maroon")
```




```
## [1]  54 183
```

(f.) Interpretation of the results

With respect to the centroid, we conclude from our analysis that we fail to reject our first null hypothesis $H_{0-scenario}$. The p-value from the 'pairwise differences of scenario' in the average miss-distances is 0.4866. Therefore, we do not have sufficient evidence to conclude that there exists a difference in user's responses of the perceived center of the aural images between the stationary and moving scenario.

For pairwise differences in shapes, we found out that the 'Snake' shape differs from the 'Ring', 'Bow' and 'Gradient' shape at $(***)$ significance level, p-value = <.0001, 0.0059 and <0.0001 respectively. The 'Amoeba' shape differs from the 'Ring' and the 'Gradient' shape at $(***)$ significance level, p-value = 0.0013 and <.0001 respectively. The 'Bow' shape differs from the 'Gradient' shape at $(***)$ significance level, p-value = 0.0010 and from the 'Ring' shape at $(*)$ significance level, p-value = 0.0325.

Therefore, we reject the null hypothesis $H_{0-shape}$ and conclude that there exists sufficient evidence that user-responses for the perceived center for the shapes differ from each other.