AML5103 | Applied Probability and Statistics | Sessional-2

1. [10 points] [CO 1, BT 3] The APS dining hall offers four choices for lunch:

   (1) South Indian (2) North Indian (3) Chinese (4) Mexican.

   Each customer will order one lunch choice with respective probabilities 0.4, 0.25, 0.15, and 0.2 independently of other customers. For the scenario below, identify the correct random variable with the associated parameters clearly shown. Using the parameters, calculate the expected value and variance of that random variable. For example,

   $$X \sim \text{Bin}(n = 12, p = 0.2) \ \Rightarrow E[X] = np = 12 \times 0.2 \text{ and } \text{Var}[X] = np(1-p) = 12 \times 0.2 \times (1 - 0.2).$$

   The number of customers who will *not order* the Chinese among the next one hundred customers.

Solution:

$$X \sim \text{Bin}\left(n = 100, p = \underbrace{0.4 + 0.25 + 0.2}_{\text{south Indian + north Indian + Mexican}}\right)$$

$$\Rightarrow E[X] = np = 100 \times 0.85,$$
$$\text{Var}[X] = np(1-p) = 100 \times 0.85 \times (1-0.85).$$

2. [10 points] [CO 1, BT 3] The APS dining hall offers four choices for lunch:

(1) South Indian (2) North Indian (3) Chinese (4) Mexican.

Each customer will order one lunch choice with respective probabilities 0.4, 0.25, 0.15, and 0.2 independently of other customers. Calculate the probability that the tenth customer will be the sixth person to order an Indian lunch.

Solution:

$$X \sim \text{NegBin}\left(r = 6, p = \underbrace{0.4 + 0.25}_{\text{south Indian + north Indian}}\right)$$
$$\Rightarrow P(X = 10) = \binom{10-1}{6-1}(0.65)^6(1-0.65)^{10-6}.$$

3. [10 points] [CO 1, BT 3] The APS dining hall offers four choices for lunch:

(1) South Indian (2) North Indian (3) Chinese (4) Mexican.

Each customer will order one lunch choice with respective probabilities 0.4, 0.25, 0.15, and 0.2 independently of other customers. Given that the last three customers ordered an Indian lunch, calculate the probability that the next customer who will *not order* an Indian lunch will be the tenth person.

Solution: The orders are independent, so what the last three customers ordered does not matter.

$$X \sim \text{NegBin}\left(r = 1, p = \underbrace{0.15 + 0.2}_{\text{Chinese + Mexican}}\right)$$

$$\Rightarrow X \sim \text{Geom}\left(p = \underbrace{0.15 + 0.2}_{\text{Chinese + Mexican}}\right)$$

$$\Rightarrow P(X = 10) = (1 - 0.35)^{10-1} \times 0.35.$$

4. [10 points] [CO 1, BT 3] The APS dining hall offers four choices for lunch:

(1) South Indian (2) North Indian (3) Chinese (4) Mexican.

Each customer will order one lunch choice with respective probabilities 0.4, 0.25, 0.15, and 0.2 independently of other customers. Suppose that 40 of the previous 100 orders were for Indian lunches. You randomly choose 25 of those 100 orders and would like to know the number of non-Indian orders in them. What random variable models this scenario? Clearly state the parameters of that random variable, and calculate its expected value and variance.

Solution: The orders are independent, so what the last three customers ordered does not matter.

$$X \sim \text{HypGeom}\left(n_{success} = 60, n_{failure} = 40, n = 25\right)$$

$$\Rightarrow E[X] = n \times \frac{n_{success}}{n_{success} + n_{failure}} = 25 \times \frac{60}{60+40},$$

$$\text{Var}[X] = \frac{n \times \frac{n_{success}}{n_{success}+n_{failure}} \times \left(1 - \frac{n_{success}}{n_{success}+n_{failure}}\right)\left(n_{success} + n_{failure} - n\right)}{n_{success} + n_{failure} - 1}$$

$$= \frac{25 \times \frac{60}{60+40} \times \left(1 - \frac{60}{60+40}\right)(60 + 40 - 25)}{60 + 40 - 1}.$$

5. [10 points] [CO 2, BT 4] The number of times that a person contracts a cold in a given year is a Poisson random variable with parameter $\lambda = 6$. Suppose that a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 2$ for 75 percent of the population. For the remaining population, the drug has no appreciable effect on

colds. If an individual tries the drug for a year and has 1 cold in that time, we are interested in knowing how likely is it that the drug is beneficial for him or her.

(a) Is the quantity we are trying to calculate:

$$P(\text{Drug beneficial} \mid 1 \text{ cold})$$
or
$$P(1 \text{ cold} \mid \text{Drug beneficial})$$
or
$$P(1 \text{ cold AND Drug beneficial})?$$

Solution: $P(\text{Drug beneficial} \mid 1 \text{ cold})$.

(b) What is the probability that the drug is beneficial?

Solution: $P(\text{Drug beneficial}) = 0.75$.

(c) What is the probability that the drug is not beneficial?

Solution: $P(\text{Drug not beneficial}) = 1 - 0.75 = 0.25$.

(d) Given that the drug is beneficial, the number of colds $X \sim \text{Poi}(?)$

Solution: $X \,|\, \text{Drug beneficial} \sim \text{Poi}\left(\lambda = 2\,\frac{\text{colds}}{\text{year}}\right).$

(e) Given that the drug is not beneficial, the number of colds $X \sim \text{Poi}(?)$

Solution: $X \,|\, \text{Drug not beneficial} \sim \text{Poi}\left(\lambda = 6\,\frac{\text{colds}}{\text{year}}\right).$

(f) Apply Bayes' formula to calculate the probability in part (a).

Solution:

$$\begin{aligned}
P(\text{Drug beneficial} \mid 1 \text{ cold}) &= \frac{P(1 \text{ cold} \mid \text{Drug beneficial}) \times P(\text{Drug beneficial})}{P(1 \text{ cold})}\\
&= \frac{P(1 \text{ cold} \mid \text{DB}) \times P(\text{DB})}{P(1 \text{ cold} \mid \text{DB}) \times P(\text{DB}) + P(1 \text{ cold} \mid \text{DNB}) \times P(\text{DNB})}\\
&= \frac{\frac{e^{-2}2^1}{1!} \times 0.75}{\frac{e^{-2}2^1}{1!} \times 0.75 + \frac{e^{-6}6^1}{1!} \times 0.25}.
\end{aligned}$$