Chirp Mass estimation from LIGO Open Data

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1 Introduction

Einstein's Theory of General Relativity predicted gravitational waves, in 1916. On September 14, 2015, the Advanced LIGO (Laser Interferometer Gravitational-wave Observatory) detectors became successful in directly detecting the gravitational waves. Meticulous analysis by the LIGO Scientific team revealed that the source (in the first detected event labelled as GW150914) was a binary black hole merger (BBH) system [1]. The Nobel Prize in Physics for the year 2017 was awarded to Rainer Weiss, Barry C. Barish and Kip S. Thorne for decisive contributions to the LIGO detector and observation of gravitational waves. Till date, there have been six confirmed events, of which, one was a binary neutron star merger (BNS) system and the rest were BBH systems. The data recorded by the two aligned detectors, LIGO Hanford, WA and LIGO Livingston, LA, has been made openly available, which can be used for educational purposes. The data and Tutorials provided by the LIGO Open Science Center (LOSC) [2] was made use of, to get a basic understanding of the analysis and extraction of signal hidden in the noise, employing the technique called matched filtering.

In a binary black hole system, two black holes orbiting each other lose energy by emitting gravitational waves and come close to each other, this is known as the in-spiral phase. It is followed by the merger phase, where the black holes meet and combine to form a single large black hole. The distorted large black hole oscillating in shape, damps to its stable spherical shape in the process called ring-down, emitting gravitational waves throughout.

The python language based codes given in the tutorials uses pre-determined 'templates'. Waveform models that are constructed using various post-Newtonian theories are called as templates. As an overview, the in-spiral phase of the BBH system, has been described using post-Newtonian Dynamics and Kepler's Laws, merger phase requires Numerical Relativity and the ring-down phase uses Black Hole Perturbation Theory. The templates are used as 'filters' which extract the signal hidden in noise. The strain data obtained from the two detectors was whitened and band-passed, suppressing the noise and to make the weak signal visible. From the spectrogram the frequency versus time of the gravitational wave can be obtained, and from that an important quantity known as the *chirp mass* can be determined.

2 Gravitational waves

Gravitational waves, as described in General Theory of Relativity (GR), is a disturbance (perturbation) in the space-time fabric which travels at the speed of light. The 'strain' caused by a gravitational wave, denoted by h, is a ratio denoting the change in length to the length $\Delta L/L$, caused when a gravitational wave passes by. Gravitational waves are produced only if the second time derivative of the quadrupole moment of the mass distribution, is non-vanishing. h is given by,

$$h = \frac{2G}{rc^4}\frac{\mathrm{d}^2Q}{\mathrm{d}t^2},$$

where, r is the distance to the source, $G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$ is the gravitational constant, $c = 2.997 \times 10^8 ms^{-1}$ is the speed of light and Q is the quadrupole moment tensor. Two polarisations of gravitational waves are predicted by GR, plus polarised h_+ and cross polarised $h_\times$. The luminosity of a gravitational wave is given by,

$$L_{GW} \approx \frac{G}{c^5}\left(\frac{\mathrm{d}^3 Q}{\mathrm{d}t^3}\right)^2 .$$

Orbiting Binary System [3] Considering a circular orbiting binary system of masses m_1 and m_2 about the centre of mass frame, total mass $M = m_1 + m_2$, orbital separation a, orbital velocity $v = a\omega$, reduced mass $\mu = m_1 m_2/(m_1 + m_2)$. The orbital phase $\phi = \omega t$, is expressed as,

$$\phi = \left(\frac{v}{c}\right)^3 \frac{c^3 t}{GM}.$$

The time until coalescence τ_c, is given by the equation,

$$\begin{aligned} \tau_c &= \frac{5}{256\eta}\frac{GM}{c^3}\left(\frac{v_0}{c}\right)^{-8} \\ &= \frac{5}{256\eta}\frac{GM}{c^3}\left(\frac{\pi G M f_0}{c^3}\right)^{-8/3} \end{aligned} \tag{1}$$

since $v_0 = (\pi G M f_0)^{1/3}$ from Kepler's Laws, where v_0 and f_0 are initial velocity of the masses and frequency of the gravitational wave. The frequency of the gravitational wave f at any time, is twice the orbital frequency. The rate of frequency evolution with time is given by,

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3}, \tag{2}$$

where $\mathcal{M} = \mu^{3/5} M^{2/5} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{-1/5}$ is known as the *chirp mass*. For the binary system revolving in the x-y plane, the quadrupole moment tensor Q, was calculated to be,

$$Q = \frac{1}{2}\mu a^2 \begin{bmatrix} 1+\cos 2\phi & \sin 2\phi & 0 \\ \sin 2\phi & 1-\cos 2\phi & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The two polarisations of the wave h_+ and $h_\times$, at an angle of inclination ι from the line of sight, the phase of coalescence ϕ_c, is obtained as,

$$h_+(t) = -\frac{G\mathcal{M}}{c^2 r}\frac{1+\cos^2\iota}{2}\left(\frac{c^3(\tau_c - t)}{5G\mathcal{M}}\right)^{-1/4} \cos\left(2\phi_c - 2\left(\frac{c^3(\tau_c - t)}{5G\mathcal{M}}\right)^{5/8}\right), \tag{3}$$

$$h_\times(t) = -\frac{G\mathcal{M}}{c^2 r}\cos\iota\left(\frac{c^3(\tau_c - t)}{5G\mathcal{M}}\right)^{-1/4} \cos\left(2\phi_c - 2\left(\frac{c^3(\tau_c - t)}{5G\mathcal{M}}\right)^{5/8}\right). \tag{4}$$

3 Data Analysis

The gravitational wave event properties and the strain data are contained in downloadable json and hdf5 format files respectively, can be obtained from `https://losc.ligo.org/data/`. The raw strain data was plotted and is shown in the figure 1.

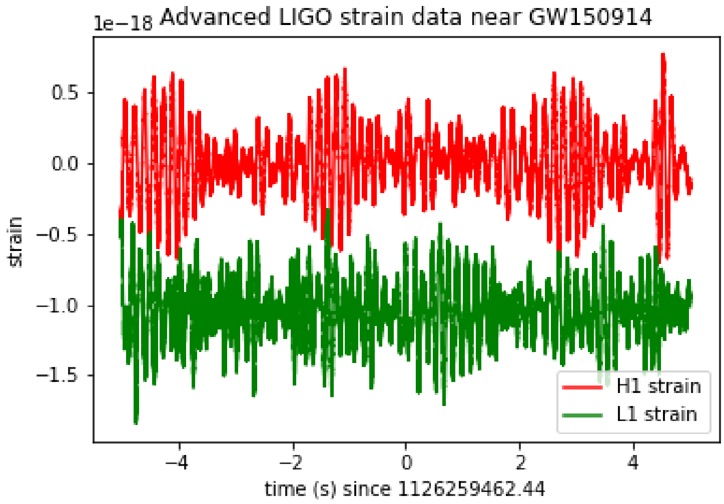


Figure 1: Raw data in a 5s neighbourhood of the event. The signal is not visible here, as it is very weak and dominated by noise.

3.1 Amplitude Spectral Density (ASD)

ASD is the square root of the power spectral density (PSD). The power spectral density gives the power of the strain data as a function of frequency. PSD is calculated using Welch's averaged periodogram method. In this method, time domain data is divided into certain overlapping segments, then the square of the FFT (Fast Fourier Transform) of each segment, is averaged over all the segments to give the power in the frequency domain (PSD).

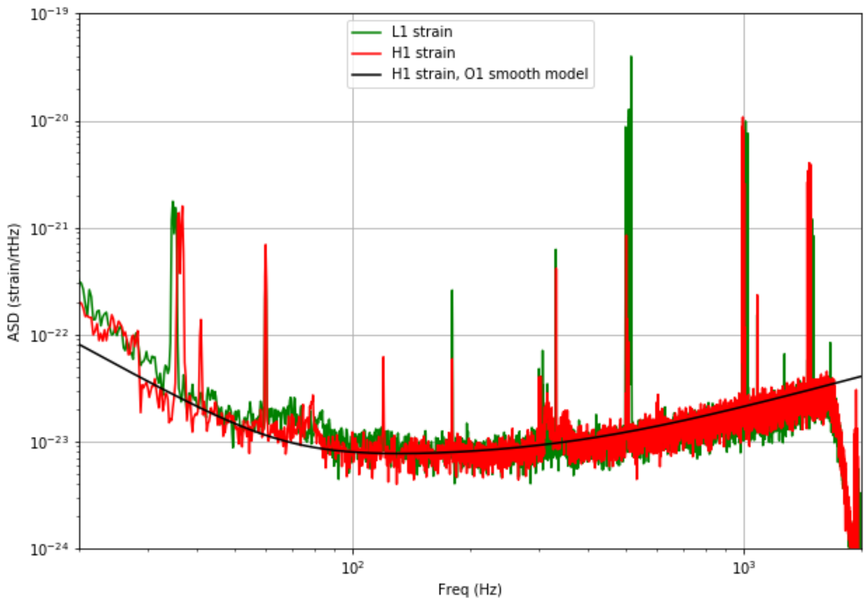


Figure 2: ASD plot between 20Hz and 2000Hz. Some spectral lines correspond to instrumental noise (mirror suspension resonances at 500Hz and harmonics, etc.)

The ASD of the two detectors is shown in the figure 2. The sampling rate of data is $f_s = 4096Hz$ and the Nyquist frequency follows to be $f_s/2 = 2048Hz$. The minimum frequency is at $20Hz$, below which LIGO is unable to capture due to very high noise. A signal with power (or amplitude) higher than base of the curve can be detected by LIGO, any signal lower than this curve would be weaker than the instrument's measuring capability.

3.2 Whitening

Noise which has power varying as a function of frequency is called as 'coloured' noise, it can be characterised as a curve in the ASD plot. If the power of the noise remains constant with varying frequency it is called as 'white' noise, characterised by a flat line in the ASD plot. In figure 2, it can be seen that noise in the detector is coloured. The strain data in the frequency domain can be 'whitened' by dividing it with the amplitude spectrum of the noise. Hence the noise would become constant and if any signal is present, will be visible. The data is also band-passed to remove high frequency noise.

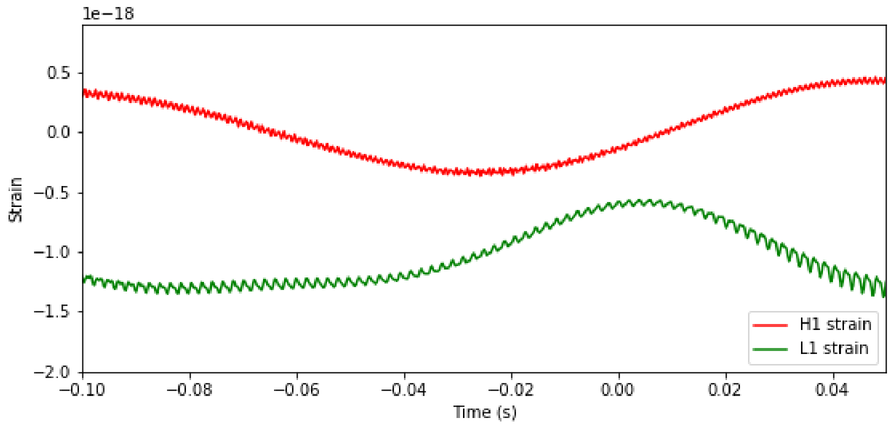


Figure 3: Strain data around the event, before whitening and band-passing.

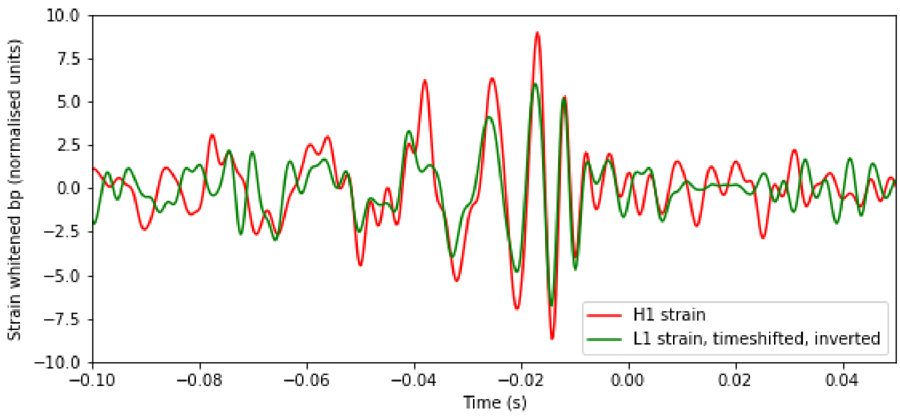
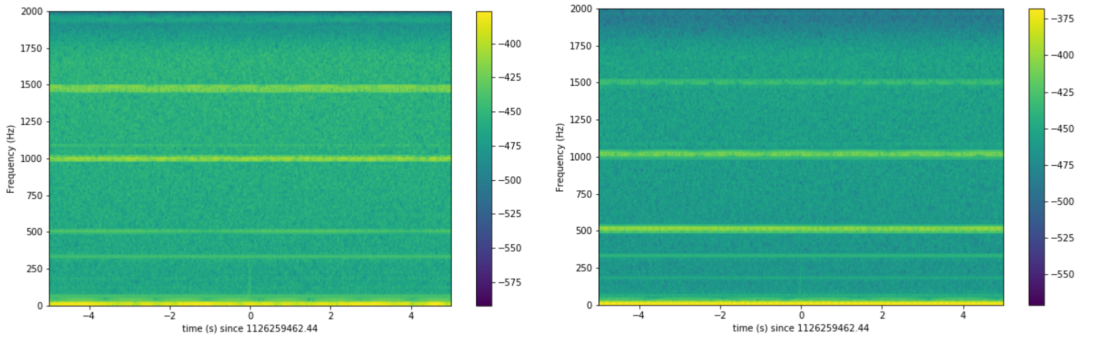


Figure 4: Strain data around the event, after whitening and band-passing.

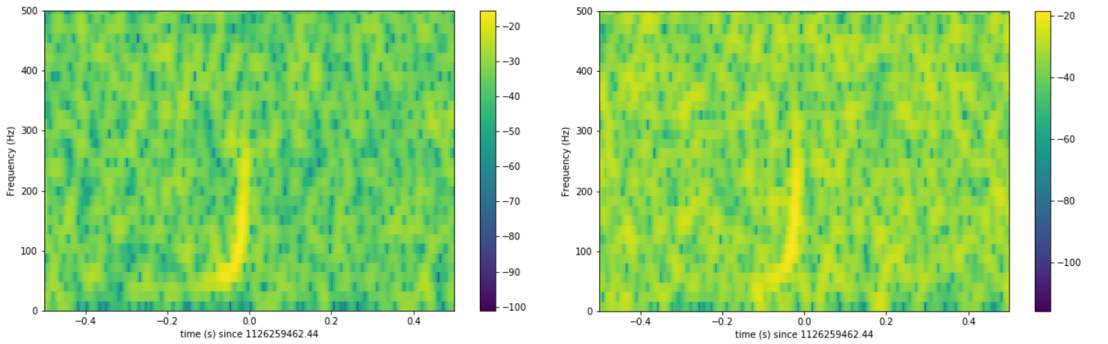
3.3 Spectrogram

A spectrogram is a 3D time-frequency representation of the data, with the third dimension (colour) indicating the normalised power of the strain data. A spectrogram is generated by breaking the data into several overlapping segments, fast fourier transforming (FFT) and periodogram for each segment calculated for the power. In the whitened and band-passed strain the signal is visible, the frequency rises sharply with time giving it a ‘chirp’ characteristic as shown in figure 6.



(a) LIGO Hanford (b) LIGO Livingstone

Figure 5: Spectrograms of strain data of the two detectors, before whitening. The colour indicates normalised power of the strain.



(a) LIGO Hanford (b) LIGO Livingstone

Figure 6: Spectrograms of strain data of the two detectors, after whitening. Colour indicates normalised power of the strain. The ‘chirp’ signal is visible.

3.4 Matched Filtering

Matched filtering is a technique used to find a known signal buried in noise (waveforms known) in an optimal way. If $s(t)$ is the signal we observe, it may have only noise $n(t)$, $s(t) = n(t)$, or it may also contain a gravitational wave signal $h(t)$, $s(t) = n(t) + h(t)$. $q(f) = h^*_{template}(f)/S_n(f)$ is defined, also known as the ‘matched filter’. The function $z(t)$ is the output for the matched filter [4] and is given by,

$$z(t) = 4\Re \int_0^\infty \frac{\tilde{s}(f)\tilde{h}^*_{template}(f)}{S_n(f)} e^{2\pi i f t}\,\mathrm{d}f\,,$$

where $\tilde{s}(f)$, $\tilde{h}^*_{template}(f)$ are in the frequency domain after Fourier Transform and (*) indicates complex conjugate, $S_n(f)$ is the power spectrum of noise.

The signal-to-noise ratio (SNR) $\rho(t)$, is defined as,

$$\rho(t) = \frac{|z(t)|}{\sigma},$$

where σ is the normalisation constant, σ^2 given by,

$$\sigma^2 = 4 \int_0^{\infty} \frac{|\tilde{h}^*_{template}(f)^2|}{S_n(f)} \, \mathrm{d}f \, .$$

A peak in the SNR $\rho(t)$ gives the value of arrival time of a signal, t_0.

Waveform Model The function $h_{template}(t)$ is the 'template' or 'waveform model' which is constructed using numerical relativity, black hole perturbation theory, etc. A template is shown in figure 7, for the merger of two black holes of masses $m_1 = 41.74 M_{\odot}$ and $m_2 = 29.24 M_{\odot}$.

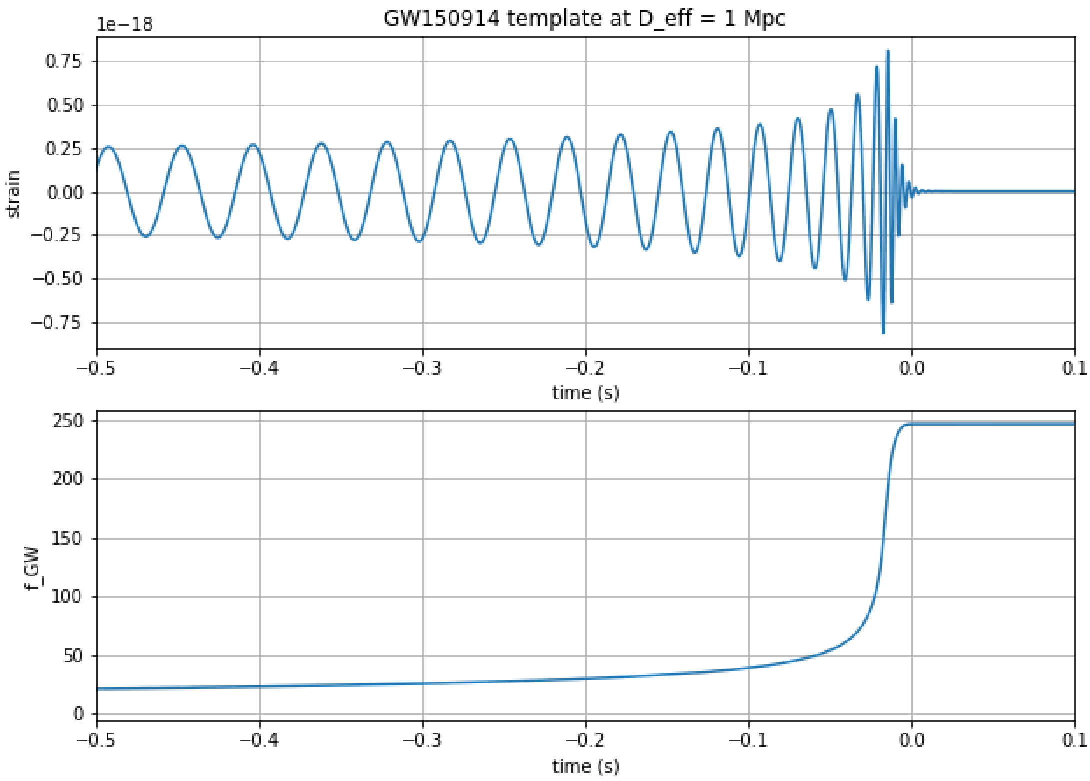


Figure 7: Waveform model, calculated at distance $D_{eff} = 1Mpc$ and frequency evolution with time of the gravitational wave.

Signal-to-Noise Ratio (SNR) The SNR peak is moved to occur at the end of the template to account for the time offset of the template. The template was also whitened, band-passed and plotted along with the signal. A peak occurred in the SNR, at time of arrival t_0 as shown in the figure 8. The estimated template is plotted along with the whitened and band-passed data in figure 9.

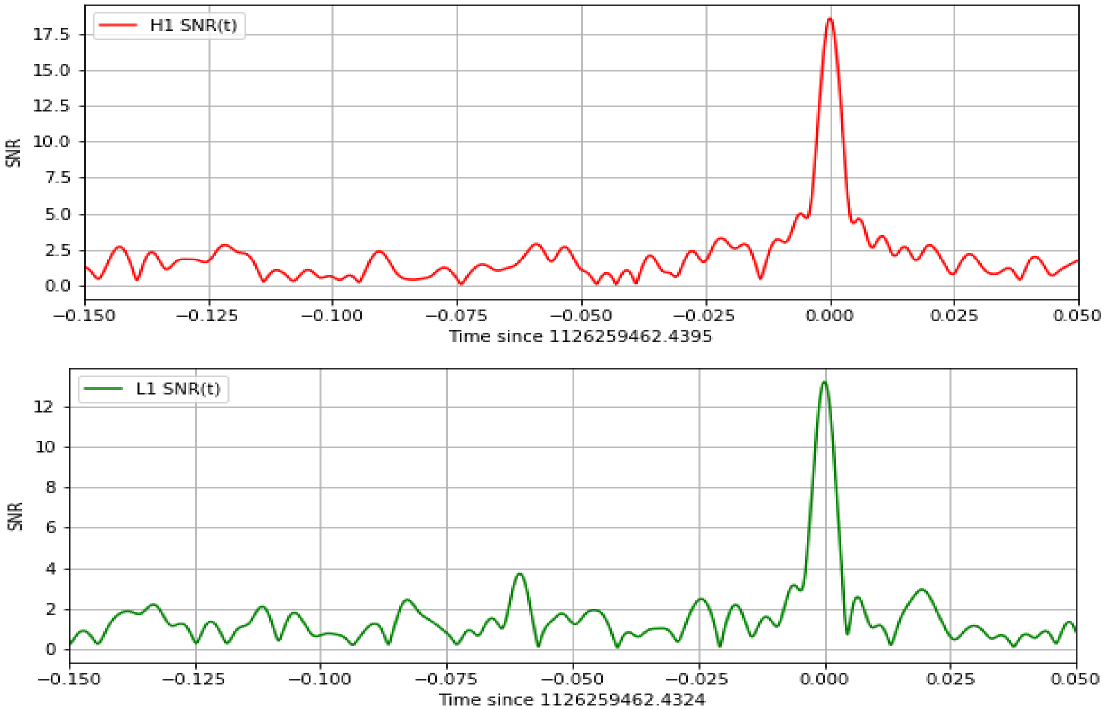


Figure 8: SNR versus time for both the detectors, red for LIGO Hanford, green for LIGO Livingstone.

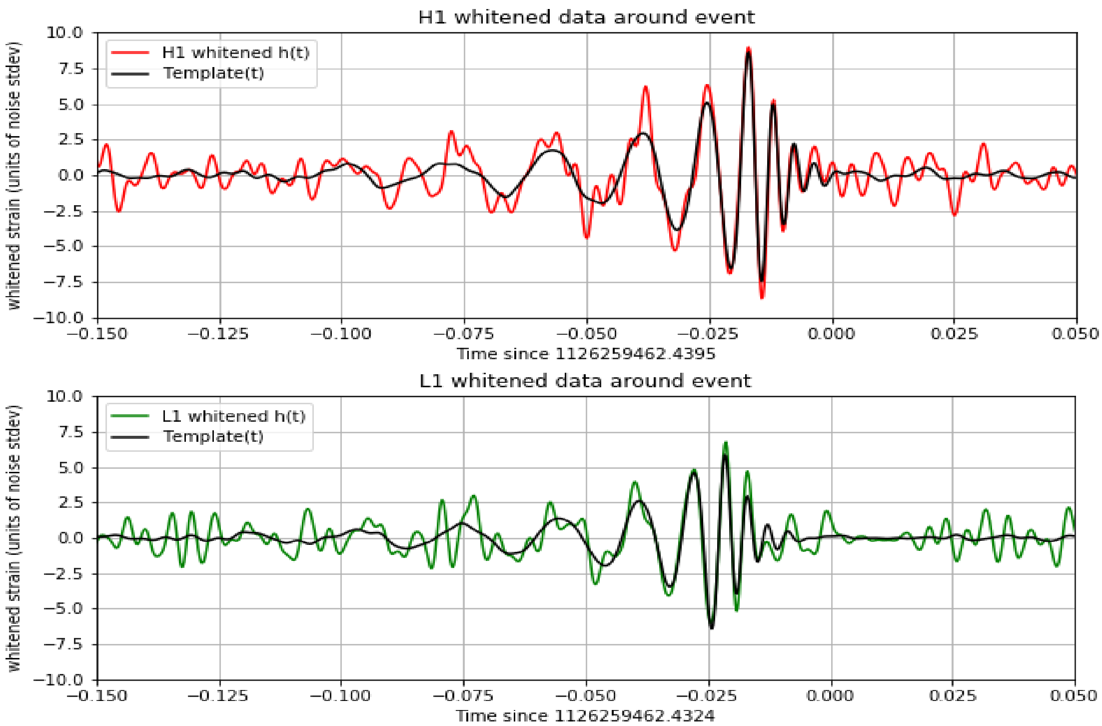


Figure 9: The matched template (in black) along with the received signal, red for LIGO Hanford, green for LIGO Livingstone.

4 Chirp Mass from Frequency

By integrating equation (2) the following equation is obtained,

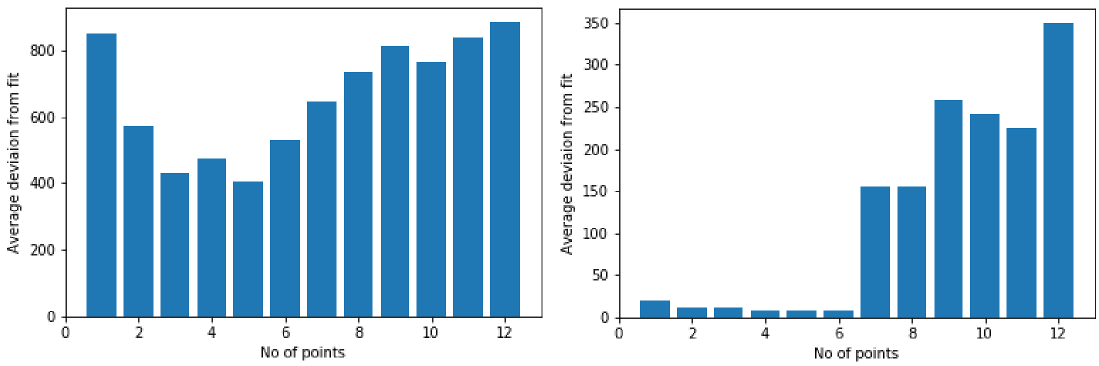
$$f^{-8/3}(t) = f_0^{-8/3} - \frac{256}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t - t_0)$$

where f_0 is the initial gravitational wave frequency received at time t_0 when it enters the band. From the spectrogram after whitening, frequency of the signal varying with time can be obtained in powerful events like GW150914. From the obtained frequency points a 'curve-fit' can be made by using the above equation as a model equation and the *chirp mass* $\mathcal{M}$ as the parameter to be determined. This equation was derived using Newtonian Dynamics and Kepler's laws and it can be approximated only in the in-spiral phase, it does not hold true for merger and ring-down phases. Hence only the frequencies up to the maximum strain are considered [5].

The data points that had to be considered for the curve-fit, was unknown without visual confirmation. To make it independent of visual confirmation, a loop was run to consider increasing number of points less than the frequency corresponding to the maximum strain(left neighbourhood). In each iteration of the loop the average deviation d of the data points with the curve-fit was calculated as,

$$d = \frac{\sum_{i=0}^{N} |f_{obs,i} - f_{fit,i}|}{N}$$

The left neighbourhood corresponding to the minimum average deviation d_{min} was selected as the best curve-fit and plotted.



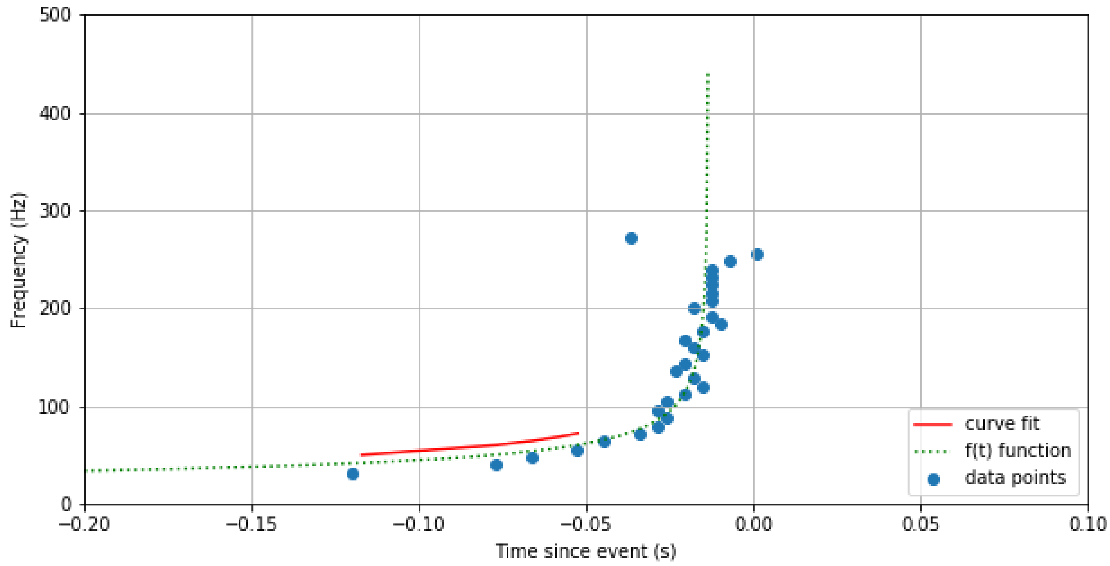
(a) LIGO Hanford

(b) LIGO Livingstone

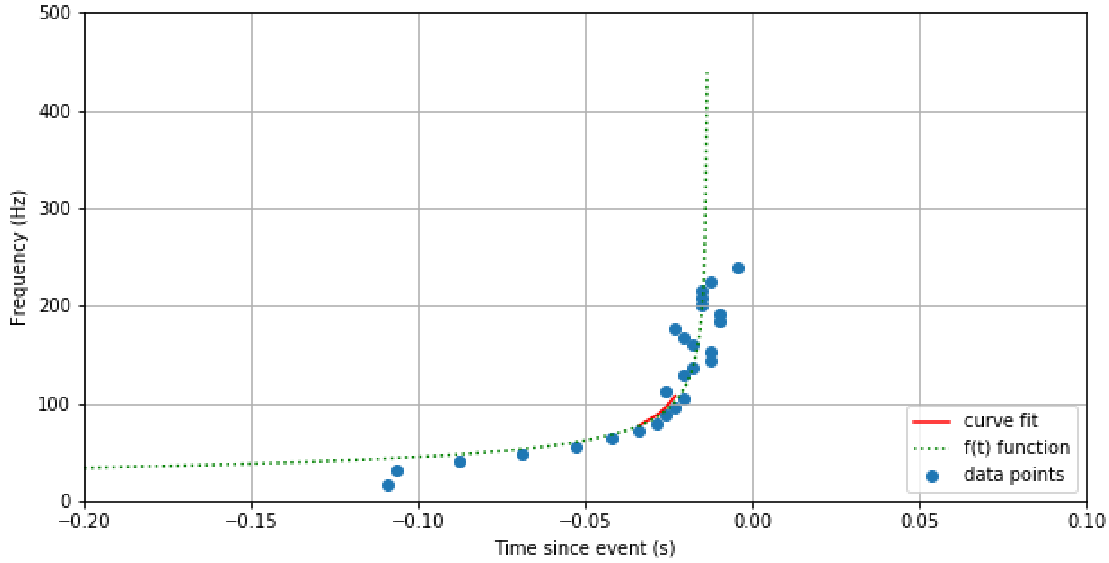
Figure 10: Average deviation calculated for different neighbourhood of points for both the detectors.

4.1 Results

From the best curve-fit, the chirp masses obtained were $29.59 M_{\odot}$ from LIGO Hanford and $30.74 M_{\odot}$ from LIGO Livingstone. The chirp mass as obtained from the template constructed using numerical relativity was $30.3168 M_{\odot}$ [1]. The curve-fit plots are shown in figure 11.



(a) LIGO Hanford



(b) LIGO Livingstone

Figure 11: Frequency versus time evolution plot from both the detectors. Curve-fit from the data points(blue) is plotted in red. The model function plotted in green (with chirp mass $\mathcal{M} = 30.3168 M_{\odot}$ from template, for reference).

[1] template used in the tutorials is not used in the actual results, since effects like combining multiple templates(Bayesian Posterior results), cosmological effects like red-shifting, were ignored for simplicity

4.2 Other plots

Newtonian Chirp Waveform Equation (3) obtained for gravitational wave from Newtonian dynamics was plotted.

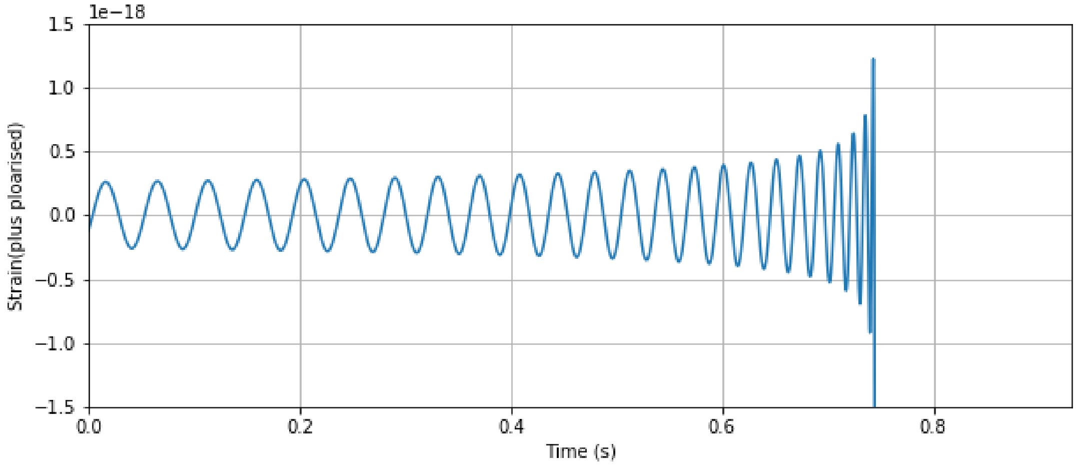


Figure 12: The strain waveform with distance $r = 1Mpc$, chirp mass $\mathcal{M} = 30.3168 M_{\odot}$ and angle of inclination $\iota = 0°$.

This shows that Newtonian dynamics is only valid for the in-spiral phase. To include the merger and ring-down phase, numerical relativity has to be used and waveforms stitched to get a complete waveform like in figure 7.

5 Conclusion

The simplified version of data analysis performed in LIGO, to extract the signal using matched-filtering and SNR estimation was understood and worked with. The estimation of chirp mass from the frequency data was limited by, expectation of a signal with enough power to be visible in the spectrogram. The results for chirp mass obtained agreed to the results with an error of 5-10%.

The discovery of gravitational waves proves the validation of Einstein's prediction in General Theory of relativity. Gravitational wave astronomy can be considered as a new window to explore the wonders of the cosmos and understand them better. The LIGO detector is one among the most advanced and requires pioneering physics and engineering. This project has given me considerable insight and knowledge about the science that goes into discovering gravitational waves and analysis of the data.

References

[1] B. P. Abbott, R. Abbott, T. Abbott, M. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. Adhikari, *et al.*, "Observation of gravitational waves from a binary black hole merger," *Physical review letters*, vol. 116, no. 6, p. 061102, 2016.

[2] "Ligo open science centre." `https://losc.ligo.org/about/`.

[3] J. D. Creighton and W. G. Anderson, *Gravitational-wave physics and astronomy: an introduction to theory, experiment and data analysis.* John Wiley & Sons, 2012.

[4] B. Allen, W. G. Anderson, P. R. Brady, D. A. Brown, and J. D. Creighton, "Findchirp: An algorithm for detection of gravitational waves from inspiraling compact binaries," *Physical Review D*, vol. 85, no. 12, p. 122006, 2012.

[5] B. P. Abbott, R. Abbott, T. Abbott, M. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. Adhikari, *et al.*, "The basic physics of the binary black hole merger gw150914," *arXiv preprint arXiv:1608.01940*, 2016.