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Trajectory Planning of Redundant Manipulators Moving along Constrained Path and Avoiding Obstacles

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Abstract

An optimum trajectory planning for a planar redundant manipulators is presented by minimizing the power consumption, when its end effector is commanded to move in its prescribed path. The Equations of motion of the robot manipulator are determined using Lagrange formulation. In order to accomplish a desired trajectory, the displacements of the revolute joints are interpolated as polynomial functions of time. The proposed approach generates the trajectory for the manipulator joints and end-effector by minimizing total power consumption by taking the kinematic, dynamic constraints and the obstacles in the workspace in to account. The methodology is illustrated using a planar 3-Degrees of Freedom (DOF) robot using two types of motion planning simulations; point to point motion and continuous path motion planning. Simulation results have been reported for the joint trajectories, end-effector motion for the above two cases. Results of joint trajectories are smooth and efficient.

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1. Introduction

The primary task of the manipulator is to maneuver the end-effector to follow a desired trajectory. If the number of DOF of manipulator are excess than the desired DOF to execute a given task is mentioned as a redundant robots [1]. The degree of redundancy is defined as excess number of DOF that the manipulator possess. Redundant manipulators offers flexibility by exploiting its degree of redundancy while performing a desired task. The flexibility in the redundant manipulators helps to achieve secondary criterion like obstacle avoidance [2], singularity avoidance

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[3] and joint limit avoidance [4] in addition to the primary task of the end-effector. For a given end-effector trajectory, there are an infinite number of robot configurations.

Therefore, inverse kinematics and trajectory planning of the redundant manipulators gives the choice of selecting the best configuration from the set of infinite configurations, this problem is referred as redundancy resolution. Redundancy resolution has been performed by formulating it as an optimization problem and finding the best configuration which minimizes the objective. Popular approach for solving redundancy resolution is done with the utilization of pseudo inverse of the manipulator's Jacobian, pseudo inverse approach has been carried out with the use of null space term for optimizing the secondary criterion such as obstacle, singularity and joint-limit avoidance.

Trajectory planning of redundant manipulators is another important aspect that is considered with the concern of obtaining smooth trajectories, it is important to have smooth trajectories because they avoids mechanical vibrations of the robot. Menasri et al. [5] proposed trajectory planning of redundant manipulators by formulating it as a bi-level optimization problem and solved with the help of bi-genetic algorithm.

Another class of approaches for solving trajectory planning is with the use of B-spline interpolation [6, 7] techniques. In these approaches trajectory planning is carried out without using inverse of Jacobian matrix, it is with the idea of using an optimal motion planning with optimal time as criterion for path planning, which was suggested under B-Spline assumption in the task space [8]. A novel method based on the variational approach [9] was planned, where, the trajectories of the robot joint space was modelled as B-spline curve and the measure of performance is directly integrated through the trajectory of the end-effector. Xidias [10] proposed an optimal time trajectory planning for hyper-redundant manipulator in 3-Dimensional workspaces in presence of obstacles. This approach evaluates an optimization problem to determine the minimum time trajectory while performing required tasks.

Many studies have been carried out on optimization of robot trajectories in terms of consumption of minimum energy [11] and torque minimization [12]. Devendra and manish [13] proposed an approach to optimize the torque applied at the joints and they used genetic algorithm to interpret the problem. Hirakawa and Kawamura [14] proposed a new scheme for accomplishing trajectory of a redundant robotic arm that minimize the total energy consumption of robot. Saramago and Stefen [15] developed trajectory of a robotic arm with the target of optimizing energy consumption and time of travelling. Ding et al. [16] used neural networks for optimized dynamics of redundant robots. Zhang et al. [17] proposed an approach for minimized applied torque of redundant robot by using a unified quadratic programming based dynamical system. Most of the literature in trajectory planning is done in joint space for point to point applications where, initial and final point of the end-effector is known.

Trajectory planning by avoiding obstacles in the workspace is the crucial requirement for redundant manipulators. Baillieul [18] introduced an extended Jacobian method, used to adopt obstacle avoidance scheme by optimizing a distance criteria. The equations of joint velocities have replaced by an extra set of equations same as unknown joint velocities. Nakamura [19] developed an obstacle avoidance scheme by representing the obstacles as a potential functions, which were used to determine the motion of the joint variable. The inverse kinematic solution and redundancy resolution of manipulators were obtained with the usage of pseudo-inverse of the manipulator Jacobian by taking order of priority of tasks in to account. Khatib [20] proposed an artificial potential field method, which is a widely utilized method for obstacle avoidance. In this approach, robot maneuvers in the field of forces which pulls the robot's end-effector towards the goal and drive away the parts of the robot from the obstacles. Maciejewski and Klein [21] developed an obstacle avoidance technique by incorporating a secondary task, which controls the point on the robot adjacent to the obstacle, by maximizing its distance from the obstacle. Most of the collision avoidance techniques have been performed by pseudo inverse approach by exploiting the null space term. This approach is highly expensive in computing pseudo-inverse and also yields kinematically singular configurations.

From the literature it was observed that most of the work has been carried out in trajectory planning is on point to point motion applications without obstacles in the workspace. This paper illustrates trajectory planning of 3-DOF planar redundant manipulator by optimizing power consumption of the robot joints for point to point and continuous path applications by considering obstacles in the work space. Trajectory planning had performed at joint level and in the Cartesian space. For collision avoidance, obstacles are modelled effectively and collision check has performed. Modelling of obstacles have done by a computational geometric technique called ray casting algorithm. The use of classical optimization algorithm, and effective modeling of obstacles makes the approach effective and computationally efficient.

2. Problem Formulation

In this paper, 3R planar manipulator is considered for optimum trajectory planning is shown in fig. 1.

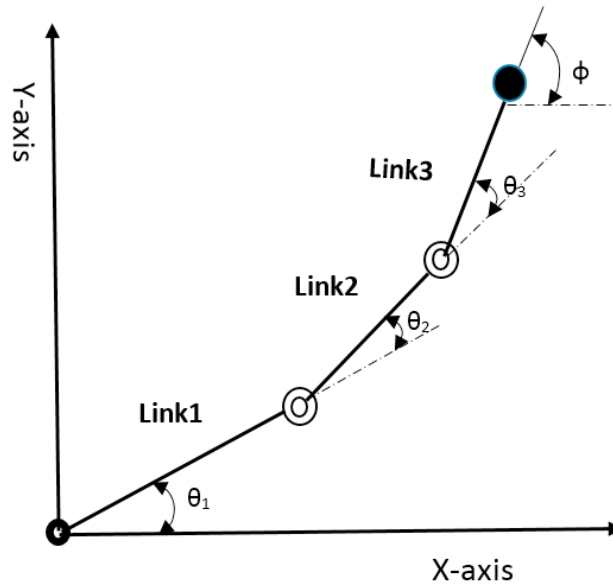


Fig. 1. Schematic representation of 3-dof serial manipulator

The end-effector of the robot is assumed to move in a desired trajectory and corresponding smooth joint displacements, joint velocities and joint torques has been evaluated. Trajectory planning problem has formulated as an optimization problem where, the objective function was to minimize the power consumption while the end-effector of robot is moving along a given trajectory. The revolute joint displacements are approximated as polynomial functions of the time. Since the objective of the optimization problem is power consumption which is related with torque applied at joints and angular velocities of joints the dynamic parameters are also considered. The physical properties of i^{th} link of the manipulator are represented as length l_i , mass m_i moment of inertia about its center of rotation is I_i . Torque at each joint has been evaluated from the dynamic model of 3-DOF manipulator. The equations of motion of the robot arm are computed by using Lagrange equation in the form given below

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \left(\frac{\partial L}{\partial q_i} \right) = \tau_i \quad (1)$$

Where, L represents the Lagrangian, which equals to the difference between total kinetic energy (K.E.) and total potential energy (P.E.) of the manipulator. The torque applied at the manipulator joint is represented as τ_i .

The total kinetic and potential energies of the manipulator are given by

$$K.E = \frac{1}{2} (I_1 \dot{\theta}_1^2) + \frac{1}{2} I_2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} I_3 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)^2 + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 \quad (2)$$

$$P.E = \frac{1}{2} (m_1 g L_1 \sin \theta_1) + m_2 g \left(L_1 \sin \theta_1 + \frac{1}{2} (L_2 \sin (\theta_1 + \theta_2)) \right) + m_3 g (L_1 \sin \theta_1 + L_2 \sin \theta_2 + L_3 \sin (\theta_1 + \theta_2 + \theta_3)) \quad (3)$$

After applying Lagrangian equation of motion to all the robot joints and the total torque is put in the matrix form as

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \\ \dot{\theta}_3^2 \end{bmatrix} + \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_3 \\ \dot{\theta}_2 \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} G_{11} \\ G_{22} \\ G_{33} \end{bmatrix} \quad (4)$$

where, M , C , H , and G represents the inertia matrix, centrifugal matrix, Coriolis matrix, and gravity matrix.

The trajectory of each joint of the three degree-of-freedom planar redundant manipulator can be designed as a quintic polynomial as follows:

$$\theta_i(t) = a_i t^5 + b_i t^4 + c_i t^3 + d_i t^2 + e_i t + f_i \quad (5)$$

where, $a_i, b_i, c_i, d_i, e_i, f_i$ are the coefficients to be determined based on end conditions and objective function to be minimized. The constant f_i represents initial joint position while e_i can be determined from end velocity conditions.

3. Methodology

The objective of the current problem is to move the end-effector of the planar redundant manipulator along a given trajectory to a specified location with minimum consumed power. The objective of optimization problem is formulated as

$$f = \sum \left| (\omega_i T_i)^2 \right| = \sum \left| (\dot{\theta}_i T_i)^2 \right| \quad (6)$$

Subjected to constraints

$$g_1 = (x_{te} - x_{tp})^2 + (y_{te} - y_{tp})^2 \quad (7)$$

$$g_2 = \ddot{\theta}_i \quad (8)$$

where, ω_i is the angular velocity of the i^{th} joint, T_i is the torque applied at the i^{th} joint, x_{te} , y_{te} are the coordinates of end-effector x_{tp} , y_{tp} are the coordinates of task space and $\ddot{\theta}_i$ represents angular acceleration at the i^{th} joint.

The classical optimization method has been adopted in order to evaluate the optimal trajectory planning of planar redundant manipulator. The optimization problem is solved by utilizing a gradient based classical optimization algorithm, which takes care the constraints of equality by sequential quadratic programming (SQP). Here, trajectory planning has been done and simulated for point to point motion robots with and without obstacles in the workspace and continuous motion robot in the prescribed path.

3.1. Obstacle avoidance

Redundant manipulators should be under control while tracing the required trajectory of the end-effector, simultaneously making sure that no part of the manipulator hits the obstacles in the task space. Collision avoidance needs collision detection, for which links of the robot are casted as line segments and the obstacles in the workspace are casted as polygons. The problem of identifying collisions of links and the obstacles is boils down to the intersection of line segment and polygons. These intersections are determined with the use of the **inpolygon** function of Matlab, based on ray casting algorithm. Once, the robot configurations responsible for the collision are identified, then those configurations are to be avoided, which was done by giving more displacements at the joints so that the links colliding the obstacles move away from the obstacle and hence collisions are avoided.

4. Results and Discussion

This section illustrates how the trajectory planning has been done for planar redundant manipulator for a given initial and goal location i.e. (point to point motion) by avoiding obstacles and continuous path motion while optimizing the power consumption of robot. The kinematic control of point to point motion and continuous path motion of robots is desirable for pick and place and continuous arc welding applications. Hence, for these two cases inverse kinematics and trajectory planning is crucial. A gradient based classical optimization approach has been

used for simulating these two cases and the results are shown.

4.1. Trajectory Planning for Continuous Path

A 3-degrees of freedom planar redundant manipulator has considered for optimal trajectory planning of continuous motion robot. In this case, end effector is constrained to move in straight line path while optimizing power consumption, which is shown in Fig. 2. A non-linear constrained optimization algorithm has been used for achieving required objective of minimum power consumption, given in Eq. 6. The constraints of optimization problem are reaching the end-effector to the task space and accelerations of joints which are shown in Eq. 7&8. In the objective formulation, joint motions are interpolated as fifth order polynomials. Coefficients of the polynomials are determined as the output of the optimization algorithm. Once the coefficients of the joints has evaluated at minimum objective requirement, corresponding joint trajectories are evaluated for a sequence of time interval of 10 seconds. For simulation of this case the parameters of manipulator considered are mass ($m_1=1$ kg $m_2=2$ kg $m_3=1$ kg), link length ($l_1=l_2=l_3=0.01$ m), inertia of links. Fig. 3 depicts the variation of joint displacement for a given time interval. It was observed that variation of joint angle 1 is more compared to the other joints. Fig. 4 shows the variation of angular velocity against the time, it is clearly understood that joint velocities are satisfying end conditions and also ensuring smooth variation. Fig. 5 depicts the variation of angular accelerations at respective joints. Fig. 6 shows the variation of joint torque. Since the objective is minimization of power consumption which leads to minimization torque applied at joints and it was clearly noticed from fig. 6 the variation of torque at corresponding joints are very less

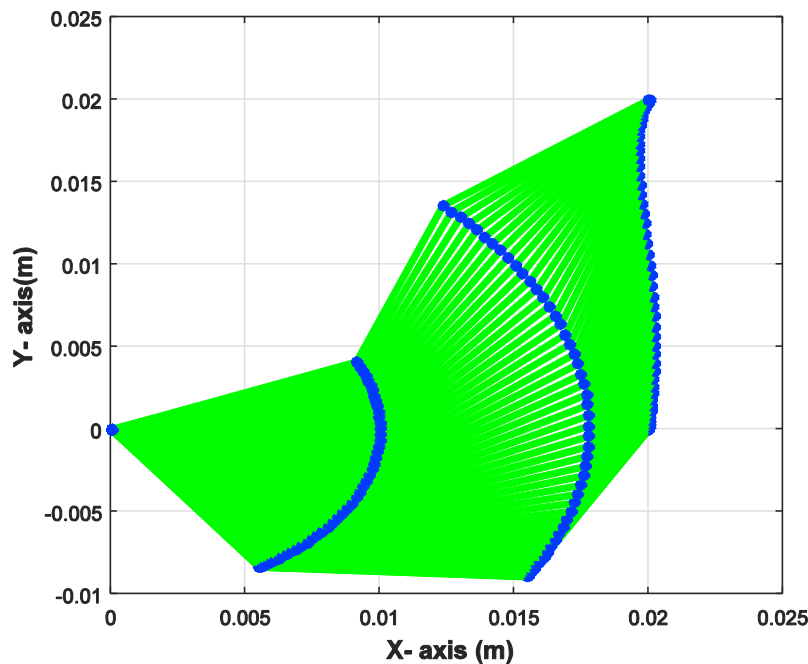


Fig. 2. Manipulator configurations while traversing continuous path in Cartesian space.

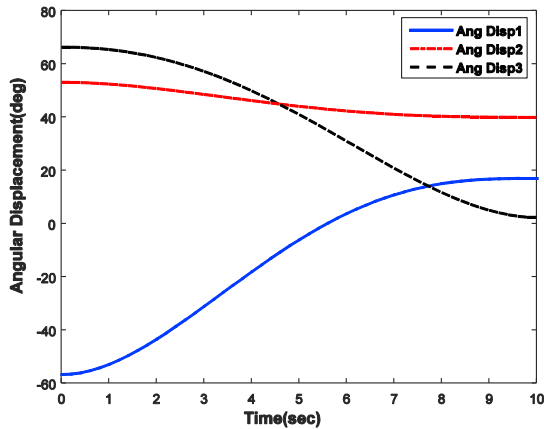


Fig. 3. Angular displacement of the joints for a given trajectory

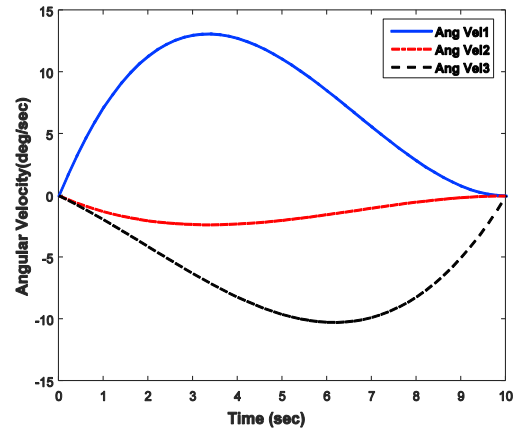


Fig. 4. Angular velocity of the joints for a given trajectory

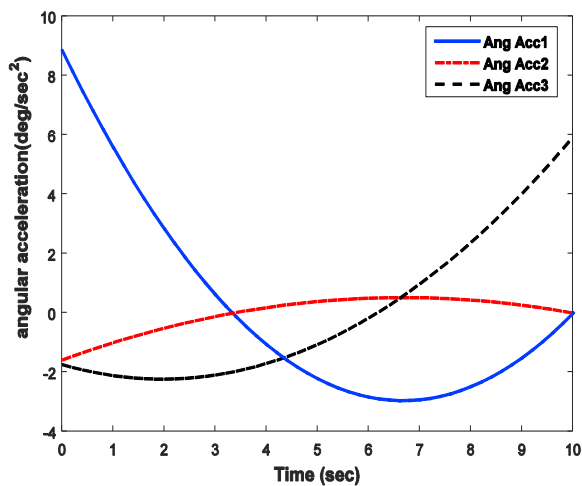


Fig. 5. Angular acceleration of the joints for a given trajectory

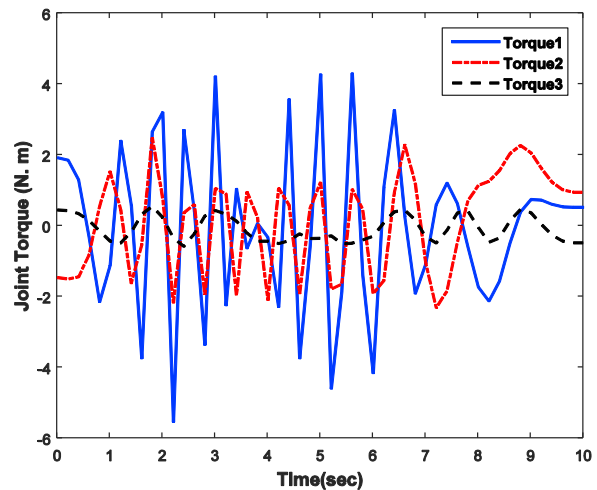


Fig. 6. Applied torques at the joints for a given trajectory

4.2. Trajectory Planning for Point to Point Motion

In this case, Trajectory planning of 3-DOF manipulator for point to point motion has been simulated with and without obstacles in the workspace. The optimal trajectories has been evaluated by posing it as a non-linear constrained optimization problem with an objective shown in Eq. 6. The constraints of an optimization problem is the end-conditions of the manipulator such as starting point, goal point, start velocities and end velocities. Fig. 7 shows the path traced by end-effector by minimizing the objective while satisfying the constraints. In this case, collision avoidance is also considered, for which collision detection is required. Collision detection is carried out by finding the intersecting points of links and obstacles and then by increasing the joint displacements such that the

configurations leading to collision move away from obstacle, which was explained in sec. 3. 1. Fig. 8 shows the path traced by end-effector without considering collision avoidance, whereas in the result of Fig. 9 shows the path traced by end-effector with obstacles in the workspace. It was observed from the result, that the end-effector of the robot was reaching the goal point without colliding the obstacle.

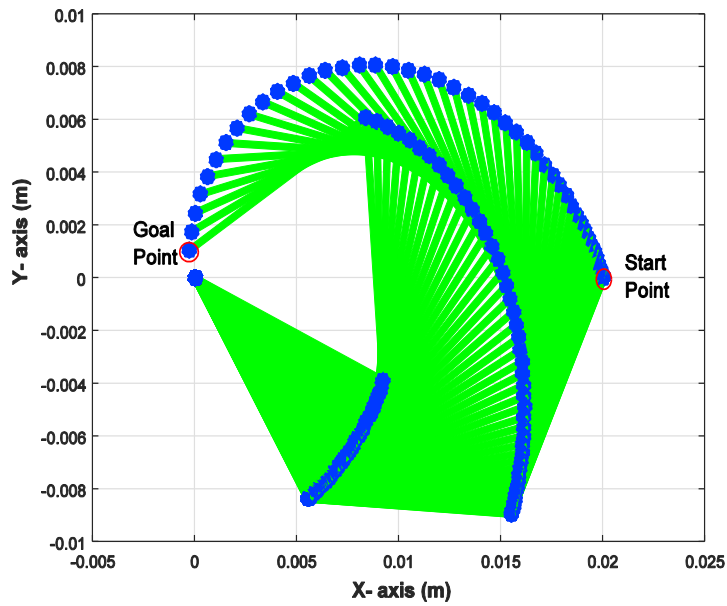


Fig. 7. Manipulator configurations while traversing point to point motion in Cartesian space.

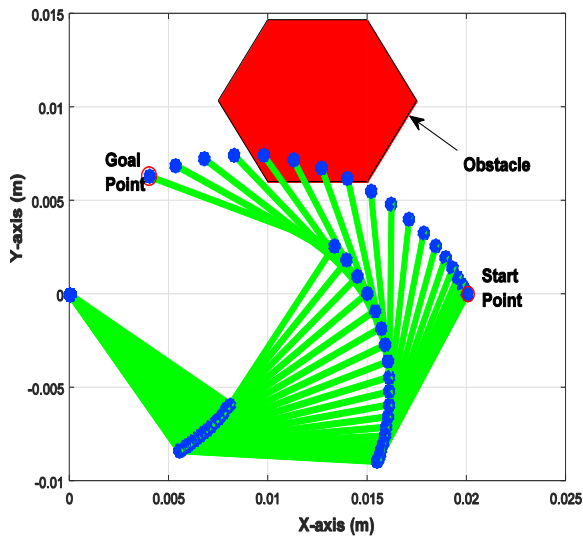


Fig. 8. Manipulator configurations without collision avoidance.

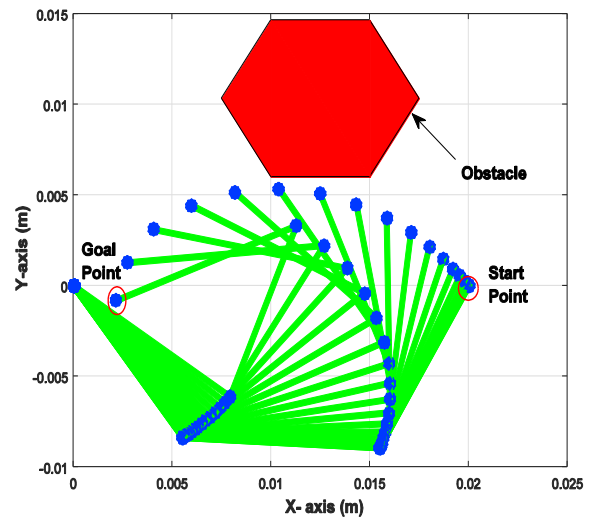


Fig. 9. Manipulator configurations with collision avoidance

5. Conclusions

Trajectory planning of planar redundant manipulator is performed with the objective of minimizing power

consumption by considering kinematic and dynamic constraints. Joint trajectories are evaluated by interpolating them using fifth order polynomial function. For simulations, two types of robot motions have been considered; continuous motion and point to point motion. Optimal trajectories have obtained by considering obstacles in the workspace. Since, there is no inverse of Jacobian the approach is computationally efficient. As the method involves the dynamic modelling for obtaining joint torques this can be applied in design of serial manipulators. Further, this approach can be easily extended to more DOF manipulators and more number of obstacles in the workspace. Effective modelling of obstacles and efficient collision avoidance technique makes the approach applicable for convex and non-convex obstacles.

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