Mathematical thinking
Graded assignment
Week 2
Total marks: 20

1. A function f in 2 variables is said to be *good* if "For every real number x, there exists a real number y such that $f(x, y) = 0$". What does it mean to say that a function f is not good? [2 marks]

   (a) For every real number x, there exists a real number y such that $f(x, y) \neq 0$

   (b) There does not exist any real number x such that $f(x, y) = 0$ for some real number y.

   (c) There exists a real number x such that $f(x, y) \neq 0$ for any real number y.

   (d) For any real number x, there are no real number y such that $f(x, y) = 0$

   Answer: (c)

2. Which of the following sets has an upper bound but does not contain the supremum?[2 marks]

   (a) $D = \{x \in \mathbb{Q} \mid \sin(\pi x) = 0\}$

   (b) $S = \{x \in \mathbb{Q}' \mid 0 < x < \pi\}$, where $\mathbb{Q}'$ is the set of all irrational numbers.

   (c) $T = \{1/n \mid n \in \mathbb{N}\}$

   (d) $U = \{x \in \mathbb{R} \mid x^2 < 4\}$

   Answer: (b),(d)

3. Find the supremum of the set $\left\{1 - \frac{(-1)^n}{n} \mid n \in \mathbb{N}\right\}$. It is enough to submit the final answer. [2 marks]
   Answer: 1

4. Which of the following is/are irrational numbers? [1 mark]

   (a) π

   (b) $\sqrt{25}$

   (c) $\sqrt{5}$

   (d) $\sqrt{8}$

5. Prove that for any $\epsilon > 0$ there exist a integer m such that $\frac{1}{m} < \epsilon$. [3 marks]

6. Consider the set $S = \mathbb{Q}^+ \cup \{0\}$, where $\mathbb{Q}^+$ is the collection of all positive rational numbers. Prove or disprove that S is a field. [3 marks]

7. Let A and B be nonempty sets and $A \subset B$. Suppose set B has a least upper bound. Then

   (i) Prove that set A also has a least upper bound. [2 marks]

   (ii) Prove that the supremum of A is less than or equal to the supremum of B. [2 marks]

8. Consider the set $S = \{5^n \mid n \in \mathbb{N}\}$. Prove that the set S has no upper bound. [3 marks]