

Mathematical Thinking - Week 2

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Contents

1	A Trip to Cantorsville	1
2	Cantor's Diagonalization Argument	1

1 A Trip to Cantorsville

1. What are other ways in which the manager of the Hilbert Hotel in Cantorsville could have accommodated the people coming from the infinitely many Hilbert Hotels?
2. What are other ways in which the manager of the Hilbert Hotel in Cantorsville could have accommodated the people coming from the infinitely many Hilbert Hotels if it is alright to leave some of the rooms empty?

Hint: There are infinitely many prime numbers.

2 Cantor's Diagonalization Argument

1. Each real number $r \in [0, 1)$ can be denoted as $r = 0.d_{i1}d_{i2}d_{i3}d_{i4} \dots$ for $i \in \mathbb{N}$. Can you construct a bijection $f : \mathbb{N} \rightarrow [0, 1)$? If not, use Cantor's Diagonalization Argument to show that such a function would be surjective.