

- (1) (MCQ) Which is larger? Twice the sum of the first 1000 natural numbers, or the sum of the first 1000 odd natural numbers?
- (a) Twice the sum of the first 1000 natural numbers.
 - (b) the sum of the first 1000 odd natural numbers.
 - (c) They are equal.

Correct answer: (b).

- (2) (Short answer) Find a formula for $1 + 4 + 7 + \cdots + 3n - 2$ (the sum of first n numbers which leave remainder 1 when divided by 3).

Correct answer: $(3n - 1)n/2$.

- (3) (MSQ) Which following is the same as

$$\sum_{i=1}^n \sum_{j=1}^i (2i + j)$$

for all positive integers n ?

(a)

$$\sum_{i=1}^n \sum_{j=1}^i (2j + i)$$

(b)

$$\sum_{j=1}^n \sum_{i=j}^n (2i + j)$$

(c)

$$\sum_{j=1}^n \sum_{i=1}^j (2i + j)$$

(d)

$$\sum_{i=1}^n \sum_{j=i}^n (2j + i)$$

Correct answers: (b) and (d).

- (4) (Short Answer) The *Online Encyclopedia of Integer Sequences (OEIS)* (<https://oeis.org>) has a huge collection of integer sequences. Each sequence has a unique identifier. For example, the sequence of triangular numbers has identifier A000217 and can be found at <https://oeis.org/A000217>. Consider the sequence $\{C_n\}$ given by the rule:

- $C_0 = 1$.
- $C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}$ for $n \geq 1$.

What is the unique identifier of this sequence on OEIS?

Correct Answer: A000108

- (5) (Numerical, exact value) Use OEIS to find C_{30} in the previous problem

Correct answer: 3814986502092304

- (6) (MCQ) Let $S_n = \{x \in \mathbf{N} \mid x \text{ is divisible by } n\}$. Then $S_9 \cap S_{12}$ is equal to which of the following sets?

- (a) S_{72}
- (b) S_3
- (c) S_{36}
- (d) S_{24}

Correct answer: (c).

- (7) Let S_n be as in the previous problem. How many natural numbers less than 100 are elements of $S_9 \cup S_{12}$?

- (a) 3
- (b) 17
- (c) 19
- (d) 33