ASSIGNMENT

(1) Consider the set $D_n \subset \mathbb{N} \times \mathbb{N}$. The elements of D_n are (x, y) where $0 \leq x \leq y \leq n$. e.g., $D_2 = \{(0,0), (0,1), (0,2), (1,1), (1,2), (2,2)\}$. What is the cardinality of D_n?

(2) Let a be a positive integer. The Collatz sequence $C(a)$ is defined recursively as follows:

$$\begin{aligned} a_1 &= a \\ a_{n+1} &= a_n/2 \text{ if } a_n \text{ is even, and} \\ a_{n+1} &= 3a_n + 1 \text{ if } n \text{ is odd.} \end{aligned}$$

Find all the terms of the Collatz sequences $C(1)$ and $C(12)$. Can you find a formula for the nth term?

(3) Look up the sequences $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$ Can you find a formula for the nth term of this sequence?

(4) Find a formula for the nth positive odd number.

(5) Can you find a formula for the nth number with odd digits?

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