

## ASSIGNMENT

- (1) Consider the set  $D_n = \{(x, y) \mid 0 \leq x \leq y \leq n \text{ where } x, y \in \mathbb{N}\}$ . e.g.,  $D_2 = \{(0, 0), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$ . What is the cardinality of  $D_n$ ?
- (2) Let  $a$  be a positive integer. The Collatz sequence  $C(a)$  is defined recursively as follows:

$$\begin{aligned}a_1 &= a \\a_{n+1} &= a_n/2 \text{ if } a_n \text{ is even, and} \\a_{n+1} &= 3a_n + 1 \text{ if } n \text{ is odd.}\end{aligned}$$

Find all the terms of the Collatz sequences  $C(1)$  and  $C(12)$ .  
Can you find a formula for the  $n$ th term?

- (3) Look up the sequences 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ... Can you find a formula for the  $n$ th term of this sequence?
- (4) Find a formula for the  $n$ th positive odd number.
- (5) Can you find a formula for the  $n$ th number with odd digits?