

<p style="text-align: center;">Mathematical thinking Questions Week 1</p>
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1. Let A and B be nonempty sets and $A \subset B$. Suppose set B has a least upper bound. Then
 - (i) Prove that set A also has a least upper bound.
 - (ii) Prove that the supremum of A is less than or equal to the supremum of B .
2. Let F be an ordered field and $x, y, z \in F$. Using only the definitions, prove that if $x > 0$ and $y < z$, then $xy < xz$.
3. Which of the following statements is/are correct?
 - (a)
4. Which of the following is/are irrational numbers?
 - (a) π
 - (b) $\sqrt{25}$
 - (c) $\sqrt{5}$
 - (d) $\sqrt{8}$
5. Let a and b be two real numbers such that $a < b$. Prove that there exist infinitely many real numbers α such that $a < \alpha < b$.
6. Which of the following options is/are true?
 - (a) If p is any prime number, then \sqrt{p} is an integer.
 - (b) If x is an irrational number and y is a rational number, then $x + y$ is an irrational number.
 - (c) If x is an irrational number and y is a rational number, then $x - y$ is an integer.
 - (d) If x is an irrational number and y is a rational number, then $\frac{x}{y}$ is a rational number.
7. Let S be a set of real numbers. Set S is said to be bounded if there exist two real numbers α and β such that $\alpha < x < \beta, \forall x \in S$. Prove that any subset of S has the supremum and the infimum in \mathbb{R} .
8. Give an example (with the explanation) of a set of real numbers:
 - (a) Which has the supremum but not the infimum.
 - (b) Which has the infimum but not the supremum.
- 9.