# Mathematical Thinking

## Assignment Questions

## Week 3

Total Marks: 20

1. Let $b, n \in \mathbb{N}$ be such that $n \mid b$ and $n \mid (b+1)$. Find the value of $n$.

   **Answer:** 1

2. Let $k = \gcd(1092, 5005)$. Use the Euclidean algorithm to compute $k$, then express $k$ as a linear combination of 1092 and 5005.

   **Answer:** $k = 91$, $(23)1092 + (-5)(5005) = 91$

3. Suppose $\gcd(a+b, a-b) = k$, where $a, b, k \in \mathbb{N}$. Which of the following options is (are) correct?

   a. $\gcd(2a, 2b) = k$

   b. $\gcd(2a, a-b) = k$

   c. $\gcd(a, b) = k$

   d. $\gcd(a+b, 2b) = k$

   **Answer:** (b), (d)

4. Every odd integer is of the form ____________.

   a. either $1 + (n-1)^2$ or $2 + (2n-1)^2, n \in \mathbb{N}$

   b. either $6n - 3$ or $6n - 5, n \in \mathbb{N}$

   c. $3n - 2, n \in \mathbb{N}$

   d. either $4n - 1$ or $4n - 3, n \in \mathbb{N}$

**Answer:** (d)

5. Suppose $\gcd(a, b) = 1$. If $c \mid a$ and $d \mid b$, then prove that $\gcd(c, d) = 1$. (2 marks)
6. If $a \mid bc$ and $\gcd(a, b) = 1$, then prove that $a \mid c$. (2 marks)
7. Using the induction method prove that for any $n \in \mathbb{N}$, $2 \mid n(n+1)$ and $3 \mid n(n+1)(n+2)$.
8. Consider two integers $x$ and $y$ such that $6x + 12y = 3$. Prove or disprove that such integers exist or not. If it exists then write some values of $x$ and $y$.

# graded

1. Find the remainder when $2^{20} + 3^{30} + 4^{40} + 5^{50}$ is divided by 7. (2 marks)

   **Answer:** 6

2. Let $a, b \in \mathbb{N}$. Which of the following options is (are) always true? (2 marks)

   a. If $a^3 \mid b^3$, then $a \mid b$.
   b. If $a^a \mid b^b$, then $a \mid b$.
   c. If $a^b \mid b^a$, then $a \mid b$.
   d. If $a^2 \mid 2b^2$, then $a \mid b$.

   **Answer:** (a),(b),(d)

3. Let $m, n \in \mathbb{N}$ and $\gcd(m, n) = 1$. Which of the following options is (are) always true? (1 mark)

   a. $\exists\, x, y \in \mathbb{Z}$ such that $mx - ny = 1$.
   b. $\exists\, x, y \in \mathbb{Z}$ such that $mx + ny = mn$.
   c. $\gcd(xm, xn) = 1 \;\forall\, x \in \mathbb{Z}$.
   d. $\gcd(xm, yn) = 1 \;\forall\, x, y \in \mathbb{Z}$ and $\gcd(x, y) = 1$.

   **Answer:** (a), (b)

4. Which of the following equations have solutions $a, b \in \mathbb{Z}$? (2 marks)

   a. $12a + 20b = 42$
   b. $152a + 102b = 3$
   c. $23a + 11b = 120$
   d. $21a + 91b = 50$

**Answer:** (c)

5. Consider a number $a = a_1\ a_2\ a_3 \ldots a_n$, where $a_1, a_2, a_3, \ldots, a_n$ are the digits for integers $a$. If $a$ is divisible by 8, then prove that the number which is formed by the last 3 digits $(a_{n-2}\ a_{n-1}\ a_n)$ of $a$ is divisible by 8. (3 marks)

6. Using the division algorithm show that if $n \mid m$ and $n \mid k$, then $n^2 \mid mk$. (2 marks)

7. Prove that $\gcd(\gcd(m, n), k) = \gcd(m, \gcd(n, k))$. (4 marks)

8. Let $r$ be the remainder obtained when $m$ is divided by $n$. Let $g = \gcd(m, n)$ and $g' = \gcd(n, r)$. Prove that $g' \mid g$. (4 marks)