ACTIVITY 1.5

Bijection and Cardinality

(1) What are the terms in the set $S_3 - S_2$?
(2) The function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^3 - x$ is not injective. In what ways can the domain be restricted to make the function injective?
(3) Consider the function $g : \mathbb{N} \to \mathbb{Z}$ defined as

$$g(n) = \begin{cases} -n/2 & \text{if } n \text{ is even} \\ (n+1)/2 & \text{if } n \text{ is odd} \end{cases}$$

Is g an injection, a surjection, or both - a bijection? If g is a bijection, what does that say about the cardinality of $\mathbb{N}$ and $\mathbb{Z}$. Note that $\mathbb{N} \subset \mathbb{Z}$.
(4) Try to construct a bijective function from $\mathbb{Z} \to \mathbb{N}$.
Hint: Hilbert Hotel

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