Mathematical Thinking - Week 3

October 5, 2023

Contents

1 Currency Game

1. Can you find 3 other ways of paying Rs 1, using just 17 and 10 Rupee coins?

2. If the shopkeeper has no change, then how can we pay Rs. 144 using only 17 and 10 rupee coins? Can we pay Rs 143?

2 Divisibility

1. Try your hand at proving the following:
   (a) If a divides b, then $2a$ divides $6b$.
   (b) If a divides b and c divides d, then ac divides bd.
   (c) If a divides b, then a^2 divides b^2
   (d) If a^2 divides b^2, then do you think a must divide b?

3 Greatest Common Divisor

1. If they exist, find all the possible solutions $m, n \in \mathbb{Z}$ for which,
   (a) $89m + 97n = 150$
   (b) $49m + 56n = 22$
   (c) $315m + 189n = 42$

2. Indian currency coins come as $1, 2, 5, 10$, and 20 Rupee(s) coins. Suppose the 1 Rupee coin is demonetised and you want to demonetise two more of the four remaining coins and keep only coins of two values in circulation without affecting transaction, which pair(s) of coins would you choose and why?

4 The Euclidean Algorithm

1. For $a, b \in \mathbb{Z}$, show that $\gcd(a, b) = \gcd(a - kb, b)$ for all $k \in \mathbb{N}$ using the principle of mathematical induction.

2. Try your hand at computing the gcd of 900 and 55 using the Euclidean algorithm.

3. If you double both numbers (i.e. take 1800 and 110), then what do you observe in the steps of the algorithm? How does the final gcd change?

5 Proof of the Euclidean Algorithm

1. The Euclidean algorithm eventually stops, as we showed. Can you make your own algorithm that takes two numbers as input and does something to them at each step, but with the property that the algorithm never stops.

2. Can you modify your algorithm so that it stops for some values of input, but goes on forever for other choices?

6 Test for Divisibility

1. Construct the test for divisibility by 13. Hint: Use the method used for the construction of test for divisibility by 7.