

## ACTIVITY 1.5

### Bijection and Cardinality

- (1) What are the terms in the set  $S_3 - S_2$ ?
- (2) The function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined as  $f(x) = x^3 - x$  is not injective. In what ways can the domain be restricted to make the function injective?
- (3) Consider the function  $g : \mathbb{N} \rightarrow \mathbb{Z}$  defined as

$$g(n) = \begin{cases} -n/2 & \text{if } n \text{ is even} \\ (n+1)/2 & \text{if } n \text{ is odd} \end{cases}$$

Is  $g$  an injection, a surjection, or both - a bijection? If  $g$  is a bijection, what does that say about the cardinality of  $\mathbb{N}$  and  $\mathbb{Z}$ . Note that  $\mathbb{N} \subset \mathbb{Z}$ .

- (4) Try to construct a bijective function from  $\mathbb{Z} \rightarrow \mathbb{N}$ .  
Hint: Next Lecture - Hilbert Hotel