Mathematical Thinking - Week 2

September 27, 2023

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1 A Trip to Cantorsville

1. What are other ways in which the managaer of the Hilbert Hotel in Cantorsville could have accomodated the people coming from the infinitely many Hilbert Hotels?

2. What are other ways in which the managaer of the Hilbert Hotel in Cantorsville could have accomodated the people coming from the infinitely many Hilbert Hotels if it is alright to leave some of the rooms empty?
   Hint: There are infinitely many prime numbers.

2 Cantor's Diagonalization Argument

1. Each real number $r \in [0, 1)$ can be denoted as $r_i = 0.d_{i1}d_{i2}d_{i3}d_{i4}\ldots$ for $i \in \mathbb{N}$. Can you construct a bijection $f : \mathbb{N} \to [0, 1)$? If not, use Cantor's Diagonalization Argument to show that such a function would be surjective.

3 Towards the Real Numbers

1. Show that there does not exist any rational number x for which $x^2 = 3$.
   Is it too early to ask this question?

2. Verify each of the ten field axioms for rationals using the algebra of integers.