Mathematical thinking
Practice assignment
Week 2

1. Negate the statement "Either X is true, or Y is true, but not both".

   (a) Both X and Y are false, or both X and Y are true.

   (b) Either X or Y are false or true.

   (c) X is false and Y is false.

   (d) Either X is true and Y is false, or X is false and Y is true.

   Answer: (a)

2. Which of the following options is/are true?

   (a) If p is any prime number, then $\sqrt{p}$ is an integer.

   (b) If x is an irrational number and y is a rational number, then $x + y$ is an irrational number.

   (c) If x is an irrational number and y is a rational number, then $x - y$ is an integer.

   (d) If x is an irrational number and y is a rational number, then $\frac{x}{y}$ is a rational number.

3. Let $S = \{\frac{(-1)^n}{2^n} | n \in \mathbb{N}\}$. Find the $Sup(S)$. It is enough to submit the final answer.
   Answer: 0.25

4. Which of the following statements is equivalent to the statement "not(For all real numbers satisfying $a < b$, there exists an $n \in \mathbb{N}$ such that $a + \frac{1}{n} < b$')"?

   (a) There exist real numbers satisfying $a < b$ where $a + \frac{1}{n} < b$ for all $n \in \mathbb{N}$.

   (b) For some real numbers $a < b$, there exists an $n \in \mathbb{N}$ such that $a + \frac{1}{n} < b$'.

   (c) For all real numbers satisfying $a < b$ where $a + \frac{1}{n} < b$ for all $n \in \mathbb{N}$.

   (d) For some real numbers $a < b$ where $a + \frac{1}{n} < b$ for some $n \in \mathbb{N}$.

   Answer: (a)

5. Give an example (with the explanation) of a set of real numbers that has an upper bound but not the supremum in the set.

6. Let S be a set of real numbers. Set S is said to be bounded if there exist two real numbers α and β such that $\alpha < x < \beta, \forall x \in S$. Prove that any subset of S has the supremum in $\mathbb{R}$.

7. Let F be an ordered field and $x, y, z \in F$. Using only the definitions, prove that if $x > 0$ and $y < z$, then $xy < xz$.

8. Let a and b be two real numbers such that $a < b$. Prove that there exist infinitely many real numbers α such that $a < \alpha < b$.