

Playing with Pascal's Triangle

Disha Kuzhively

ICTS-TIFR

1 June 2025

Outline

Warm-up

Outline

Warm-up

Pascal's Triangle
History

Outline

Warm-up

Pascal's Triangle
History

Sierpinski Gasket

Outline

Warm-up

Pascal's Triangle
History

Sierpinski Gasket

Dividing a circle into areas

Number of chocolates

Problem

Maya brought home chocolates for her 6 children Asha, Bindu, Chaitra, Divya, Esha, and Farah. She gave them to the youngest, Farah and asked her to distribute them among her siblings. Farah ate half of the chocolates and gave the rest to Esha and asked her to distribute them among her younger siblings. Esha did the same, ate half and gave the rest to Divya and so on. In the end, the oldest child Asha got only one chocolate. How many chocolates did Maya bring home?

Number of chocolates

Problem

Maya brought home chocolates for her 6 children Asha, Bindu, Chaitra, Divya, Esha, and Farah. She gave them to the youngest, Farah and asked her to distribute them among her siblings. Farah ate half of the chocolates and gave the rest to Esha and asked her to distribute them among her younger siblings. Esha did the same, ate half and gave the rest to Divya and so on. In the end, the oldest child Asha got only one chocolate. How many chocolates did Maya bring home?

What if Maya had 7 children instead of 6, what would be the number of chocolates Maya would have brought home then?

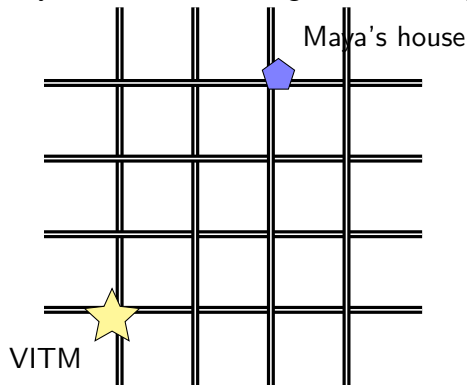
Sum of Numbers

Problem

Find the sum of $1 + 2 + 4 + 8 + \dots + 1024$.

Number of Routes

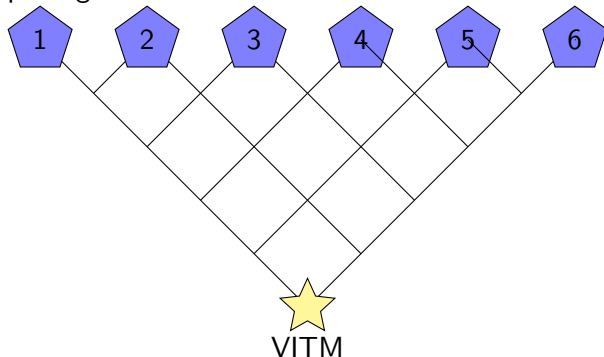
Below is the map of the city showing Maya's house and VITM. Maya visits VITM taking the shortest possible route.



Maya visits VITM taking the shortest possible route.
How many such routes are there?

Highest number of routes

The houses of 6 students who came to VITM today are marked in pentagons.

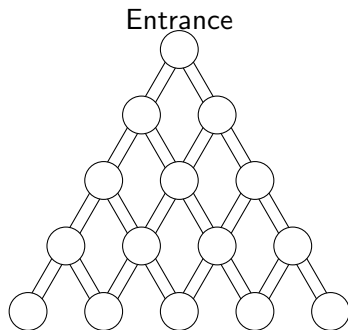


Which of these students have the highest number of routes to return home?

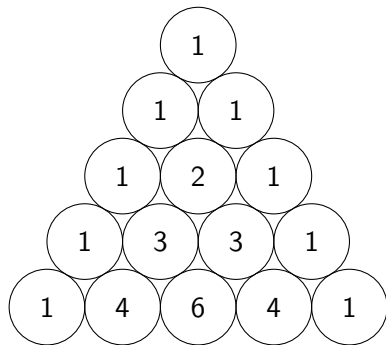
Keep in mind these students try to take the shortest possible routes.

Number of routes

Write in every circle the number of ways that lead to it from the entrance.



Pascal's Triangle



History

French mathematician Blair Pascal in 1665.

History

Italian algebraist Nicolo Tartaglia in 1556.

French mathematician Blair Pascal in 1665.

History

Chinese mathematician Yang Hui in 1303 based on Jia Xian's description from 11th century.

Italian algebraist Nicolo Tartaglia in 1556.

French mathematician Blair Pascal in 1665.

History

Persian mathematician Omar Khaayyam(1048-1131) citing book by Al-Karaji (953-1029).

Chinese mathematician Yang Hui in 1303 based on Jia Xian's description from 11th century.

Italian algebraist Nicolo Tartaglia in 1556.

French mathematician Blair Pascal in 1665.

History

Meru prastara, Halayudha in 10th century based on a commentary on Pingala's Chandahsastra (2nd or 3rd century BC).

Persian mathematician Omar Khaayyam (1048-1131) citing book by Al-Karaji (953-1029).

Chinese mathematician Yang Hui in 1303 based on Jia Xian's description from 11th century.

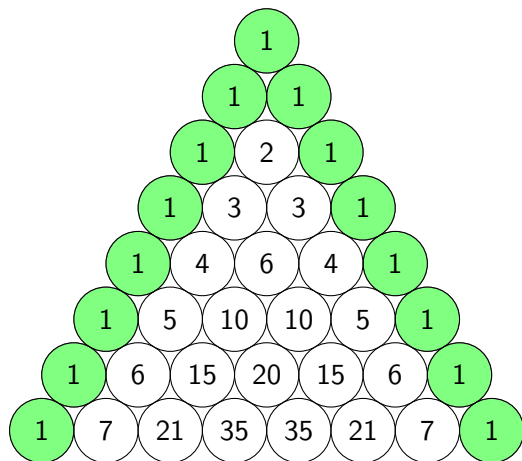
Italian algebraist Nicolo Tartaglia in 1556.

French mathematician Blaise Pascal in 1665.

Look for **patterns** in your Pascal's Triangle.

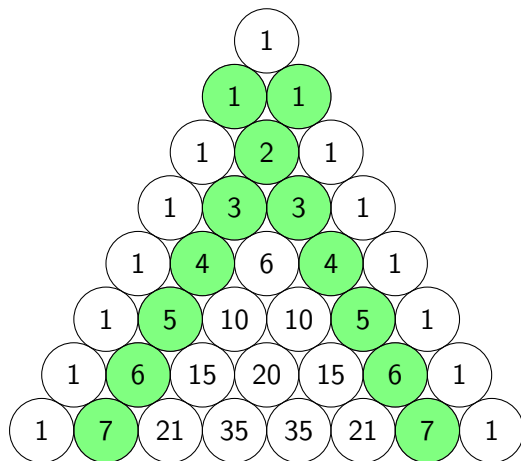
Take a new sheet of paper and list all the patterns you notice.

Patterns in Pascal's Triangle



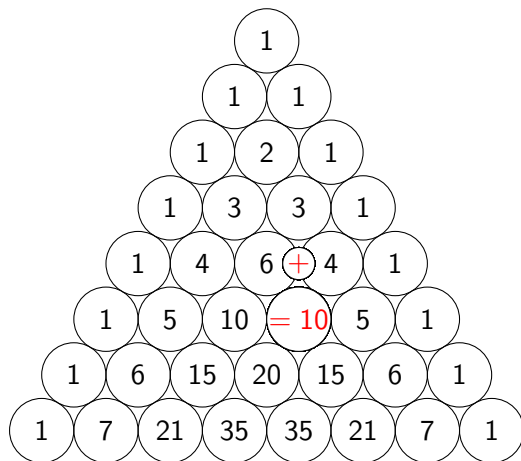
This triangle consists of "1"s on the left and right sides

Patterns in Pascal's Triangle



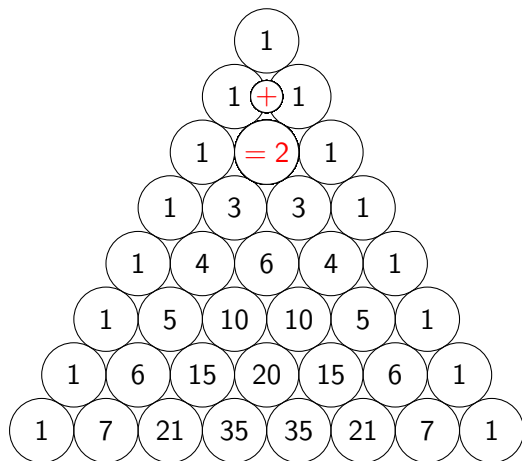
The second diagonal
contains consecutive
numbers

Patterns in Pascal's Triangle



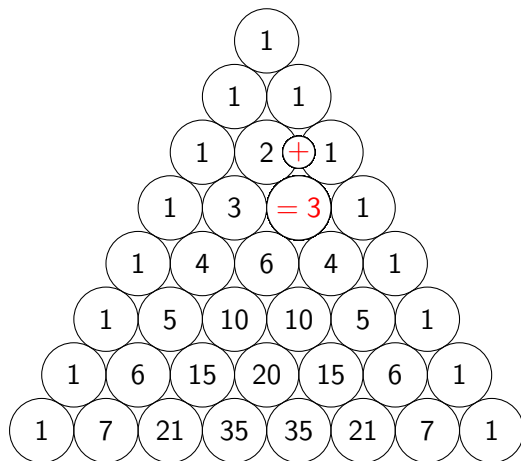
Every number in a row is the sum of the two numbers (to its' left and right) in the row above it.

Patterns in Pascal's Triangle



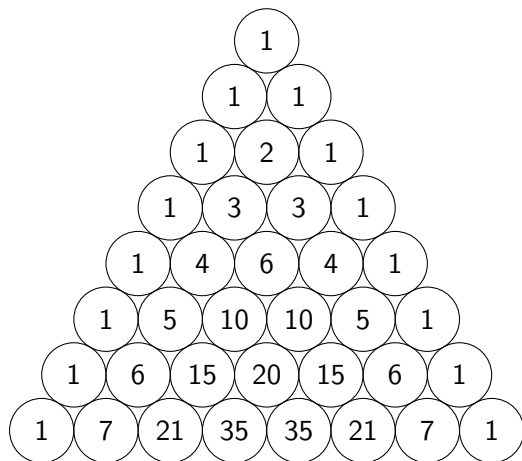
Every number in a row is the sum of the two numbers (to its' left and right) in the row above it.

Patterns in Pascal's Triangle



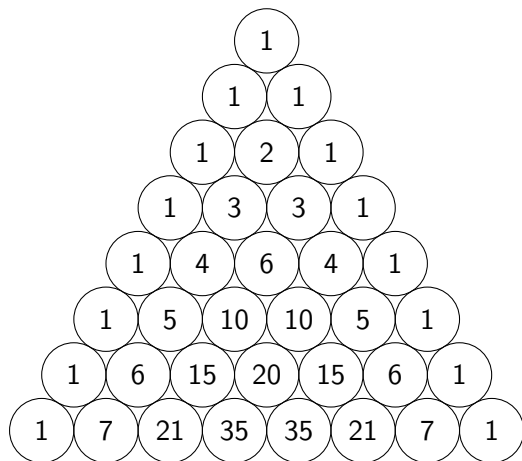
Every number in a row is the sum of the two numbers (to its' left and right) in the row above it.

Patterns in Pascal's Triangle



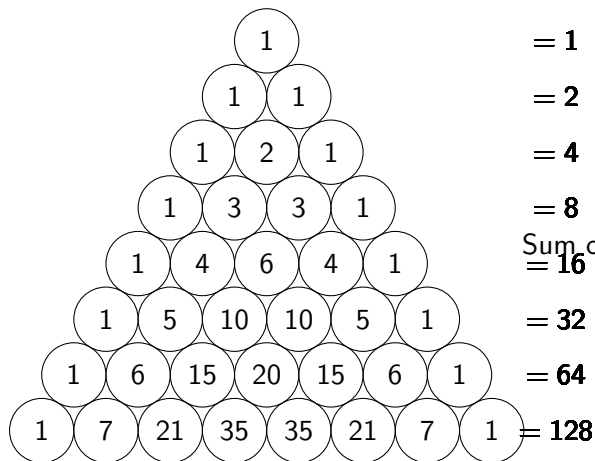
Sum of numbers in a row

Patterns in Pascal's Triangle

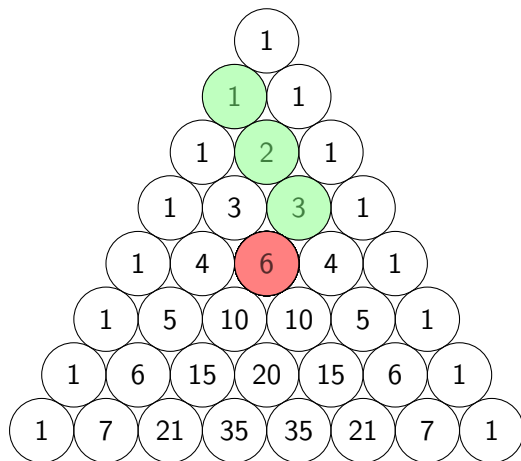


Sum of numbers in a row

Patterns in Pascal's Triangle

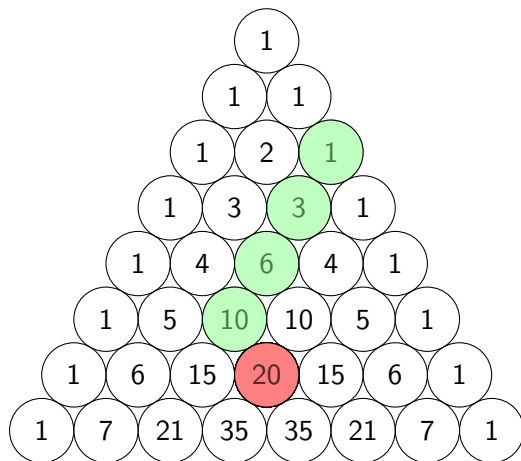


Patterns in Pascal's Triangle



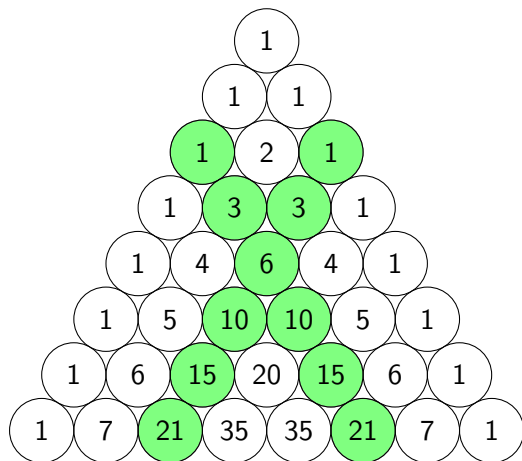
Sum of numbers in a
diagonal

Patterns in Pascal's Triangle

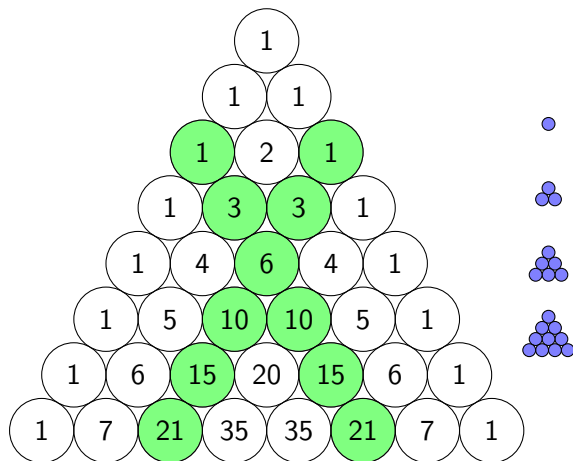


Sum of numbers in a diagonal

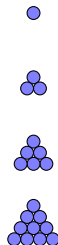
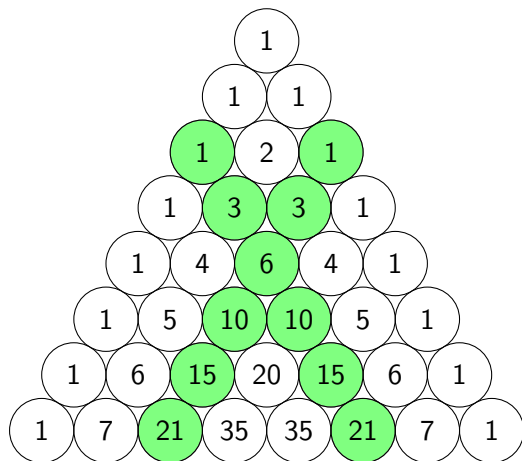
Patterns in Pascal's Triangle



Patterns in Pascal's Triangle

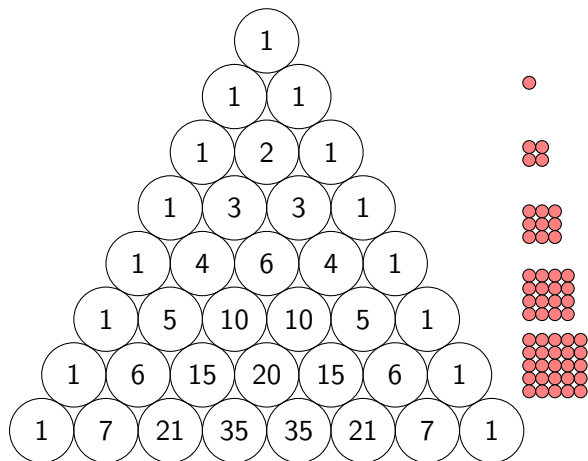


Patterns in Pascal's Triangle

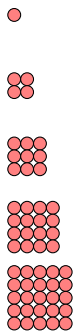
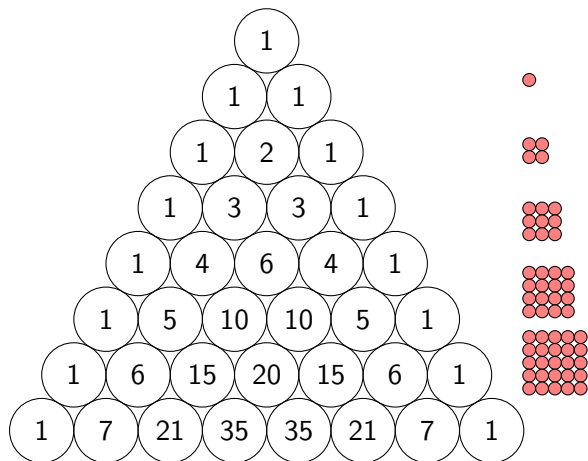


Triangular numbers

Patterns in Pascal's Triangle

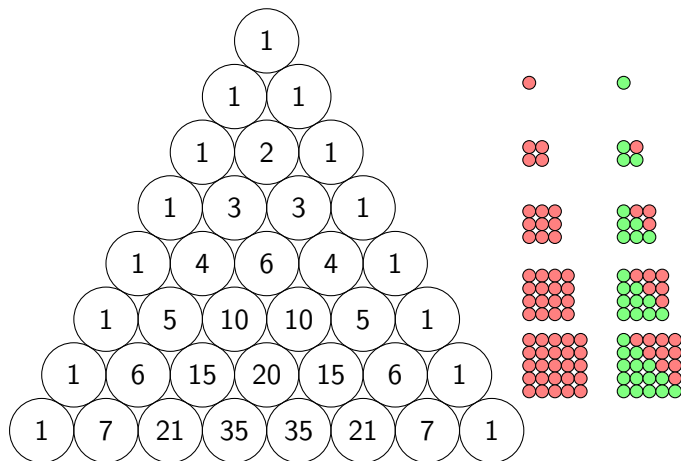


Patterns in Pascal's Triangle



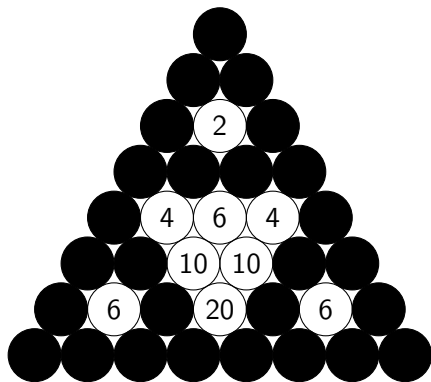
Sum of Squares

Patterns in Pascal's Triangle

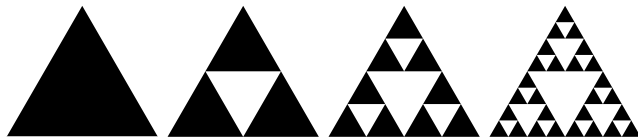


Color all the odd numbers in Pascal's triangle.

Sierpinski Gasket



Sierpinski Gasket



How will the triangle look if you color mod 3?

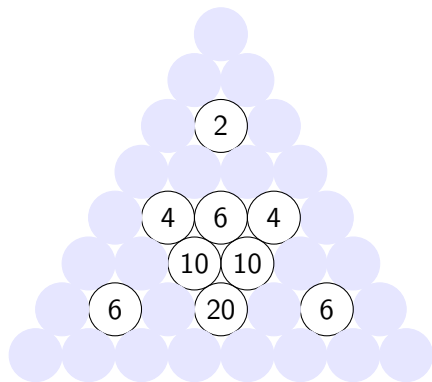
Sierpinski Gasket and Tower of Hanoi

The object of the game is to move all the disks over to Tower 3. You cannot place a larger disk onto a smaller disk.

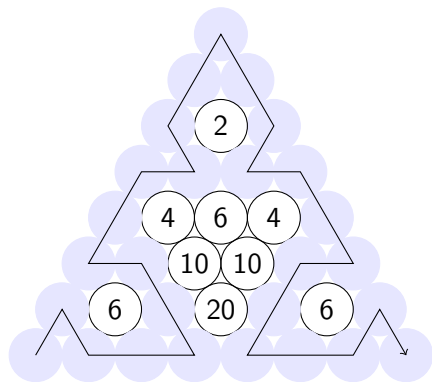
Sierpinski Gasket and Tower of Hanoi

List all the legal configurations.

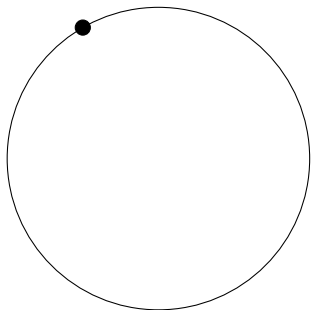
Sierpinski Gasket and Tower of Hanoi



Sierpinski Gasket and Tower of Hanoi

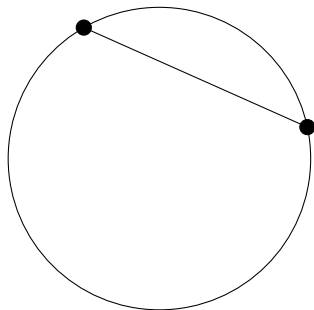


Dividing a circle into regions



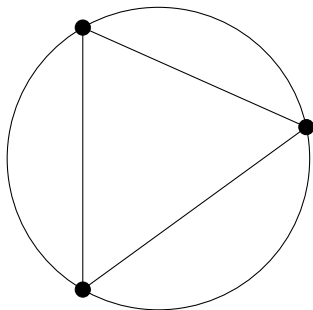
# points	# lines	# regions
1	0	1

Dividing a circle into regions



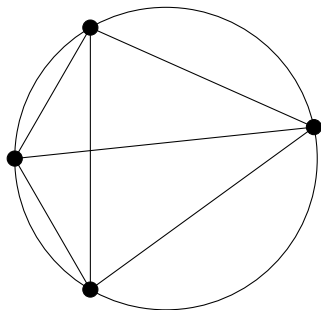
# points	# lines	# regions
1	0	1
2	1	2

Dividing a circle into regions



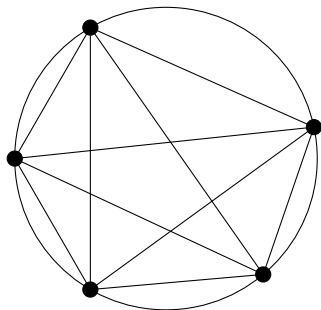
# points	# lines	# regions
1	0	1
2	1	2
3	3	4

Dividing a circle into regions



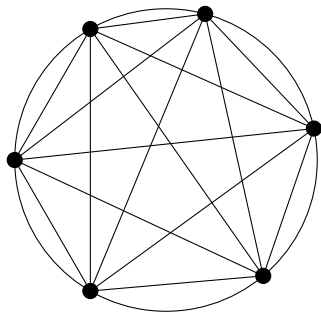
# points	# lines	# regions
1	0	1
2	1	2
3	3	4
4	6	8

Dividing a circle into regions



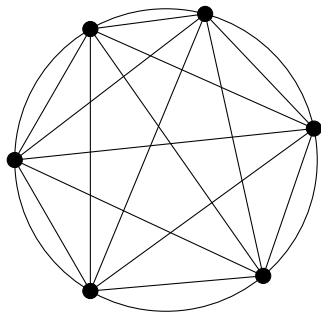
# points	# lines	# regions
1	0	1
2	1	2
3	3	4
4	6	8
5	10	16

Dividing a circle into regions



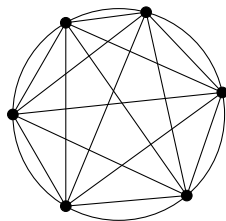
# points	# lines	# regions
1	0	1
2	1	2
3	3	4
4	6	8
5	10	16
6	15	?

Dividing a circle into regions



# points	# lines	# regions
1	0	1
2	1	2
3	3	4
4	6	8
5	10	16
6	15	31

Dividing a circle into regions



# points	# lines	# regions	# intersection
1	0	1	0
2	1	2	0
3	3	4	0
4	6	8	1
5	10	16	5
6	15	31	15