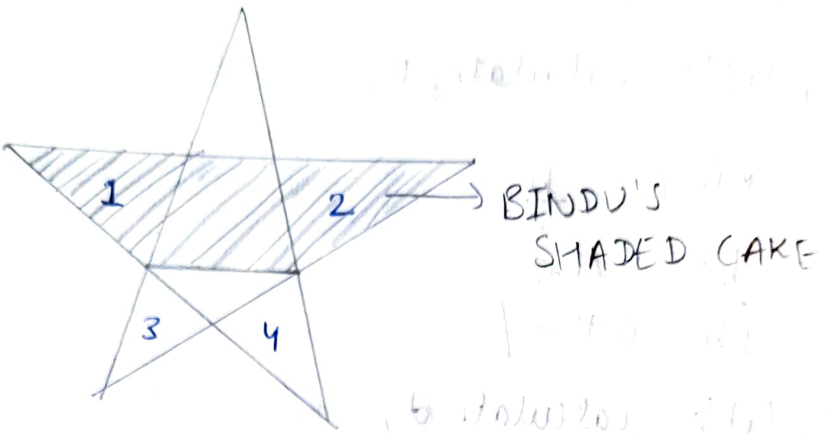
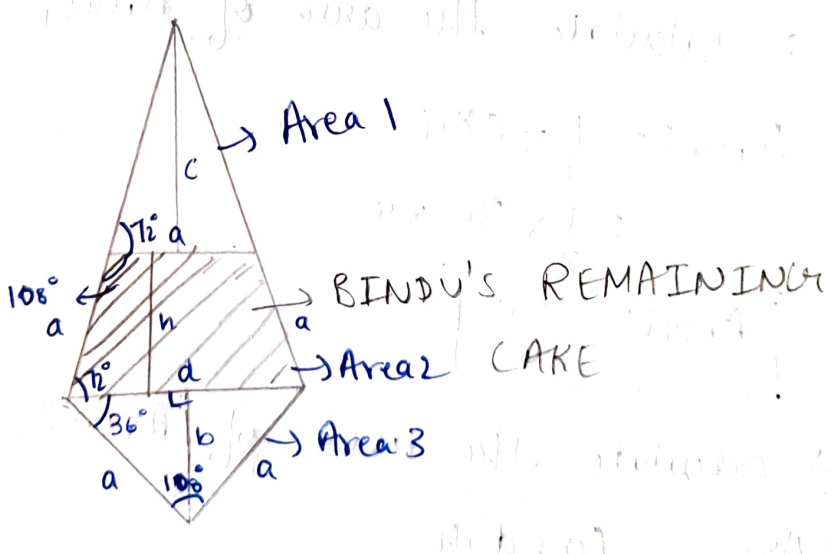
Monthly Maths Circle
India Challenge

Problem 1).



Since it is regular pentangonal star, so, all the ~~stars~~ triangles making the vertices of the star, are of same area.

So, for now lets cancel the equal components from both sides, Bindu and Anita's cakes, lets cancel 1 with 3 and 2 with 4, so the remaining ~~too~~ cake looks like this,



For the regular pentagon, the side will be a, let's assume. The angle will be 108°.

In Area 1, let's calculate c,

$$\tan 72^\circ = \frac{2c}{a}$$

$$\Rightarrow \boxed{c = \frac{3a}{2}}$$

In Area 2, let's calculate h,

$$\sin 72^\circ = \frac{h}{a}$$

$$\Rightarrow h = a \sin 72^\circ$$

$$\boxed{h = 0.95a}$$

In Area 3, let's calculate d,

$$\cos 36^\circ = \frac{d}{2a}$$

$$\Rightarrow d = 2a \cos 36^\circ$$

$$\boxed{d = 1.62a}$$

In Area 3, let's calculate b,

$$\sin 36^\circ = \frac{b}{a}$$

$$\Rightarrow \boxed{b = 0.59a}$$

Now, let's calculate the area of Area 1,

$$\text{Area 1} = \frac{1}{2} \times c \times a$$

$$= \frac{1}{2} \times \frac{3a}{2} \times a$$

$$\boxed{\text{Area 1} = \frac{3a^2}{4}}$$

Now, let's calculate the area of Area 2,

$$\text{Area 2} = \frac{(a+d)h}{2}$$

$$= \frac{(a+1.62a)0.95a}{2}$$

$$= 1.31a \times 0.95a$$

$$\boxed{\text{Area 2} = 1.2445a^2}$$

Now, let's calculate the area of Area 3,

$$\text{Area 3} = \frac{1}{2} \times d \times b$$

$$= \frac{1}{2} \times 1.62a \times 0.59a$$

$$= 0.81a \times 0.59a$$

$$\boxed{\text{Area 3} = 0.48a^2}$$

So, finally the total area of cake made by Anita is,

$$\text{Anita's cake area} = \text{Area 1} \times 3 + \text{Area 3}$$

$$= \frac{3a^2}{4} \times 3 + 0.48a^2$$

$$\boxed{\text{Anita's cake area} = 2.73a^2}$$

Similary, the total area of cake made by Bindu is,

$$\text{Bindu's cake area} = 2 \times \text{Area 1} + \text{Area 2}$$

$$= 2 \times \frac{3a^2}{4} + 1.24a^2$$

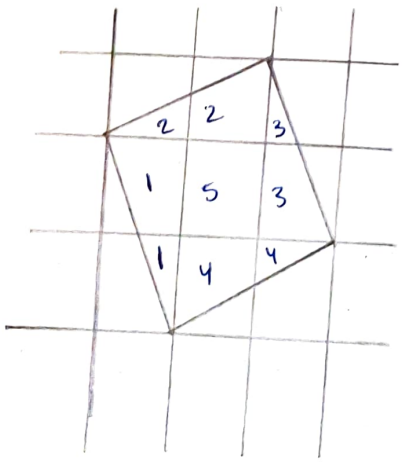
$$\boxed{\text{Bindu's cake area} = 2.74a^2}$$

So, approximately both the cakes made by Bindu and Anita is equal.

$$\boxed{\text{Anita's cake area} = \text{Bindu's cake area}}$$

So, Bindu's claim is right about their cake size being equal.

Problem 2)



The squares are numbered as follows.

So, the square with 5 times area is obtained.