Exploring Modern Mathematics Through the Art of Tiling

Disha Kuzhively
disha.jk@icts.res.in

International Centre for Theoretical Sciences - TIFR, Bangalore

Science Gallery Bangalore

Outline

What is Tiling?

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Tiling is the covering of a surface using geometric shapes while ensuring there are no overlaps and no gaps.

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Tiling is the covering of a surface using geometric shapes while ensuring there are no overlaps and no gaps.

Picture of tiled floor/wall

Picture of checked fabric

Picture of folded origami tesselation

Tiling

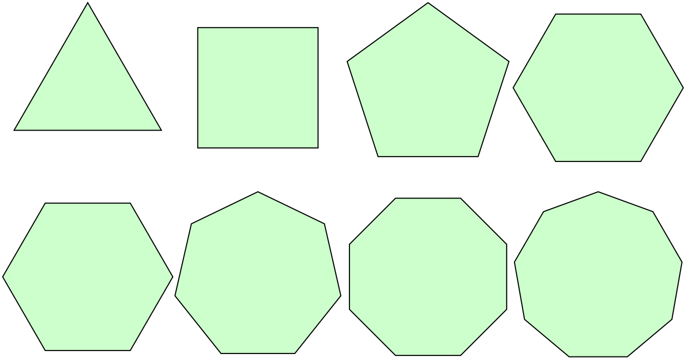
What is Tiling?

Tiling is the covering of a surface using geometric shapes while ensuring there are no overlaps and no gaps.

Tiles with curved edges

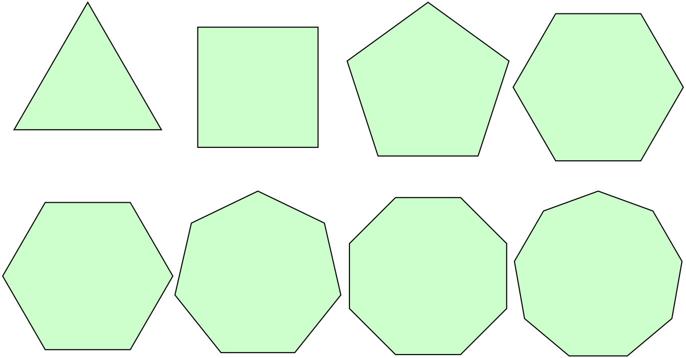
Tiling with Convex Regular Polygons

A convex regular polygon has n straight edges and n equal internal angles. It is convex because any line segment joining two vertices passes entirely through the polygon's interior.



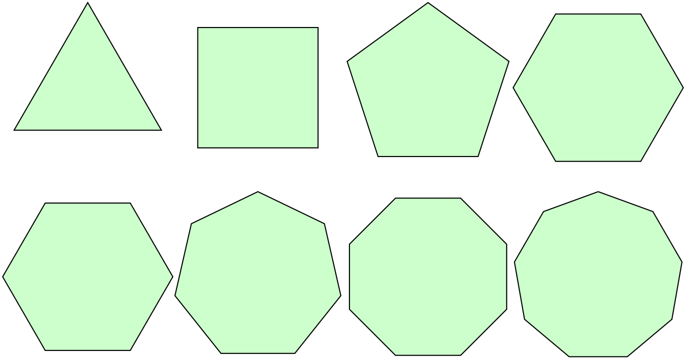
Tiling with Convex Regular Polygons

Can you tile with convex regular polygons?



Tiling with Convex Regular Polygons

How many edge to edge tilings are possible using only one type of tile?



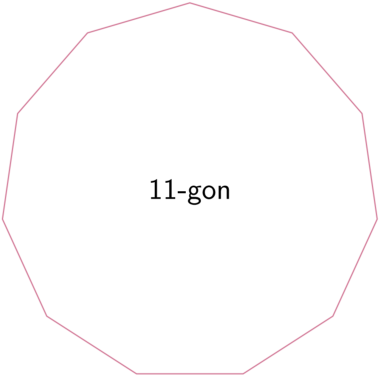
Tiling with Convex Regular Polygons

How many edge to edge tilings are possible using only one type of tile?

edge to edge tiling example	not edge to edge tiling example

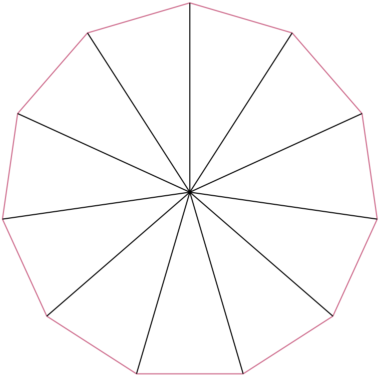
Tiling with Convex Regular Polygons

Internal angle of an n-sided polygon



Tiling with Convex Regular Polygons

Internal angle of an n-sided polygon



for $n = 11$, we have 11 triangles.

Tiling with Convex Regular Polygons

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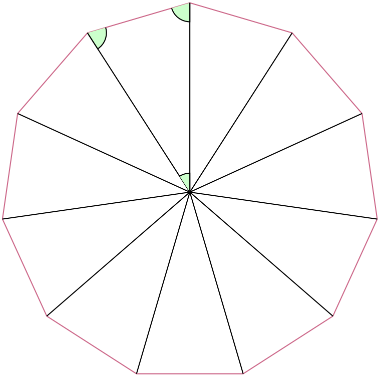

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Sum of angles of one triangle $= \pi$
$\pi = 180^\circ$

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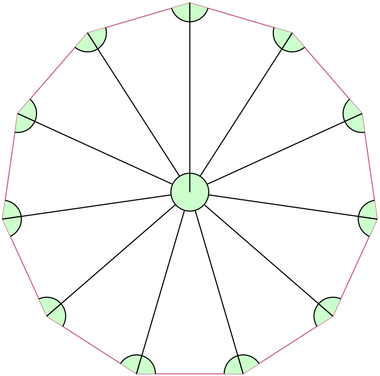

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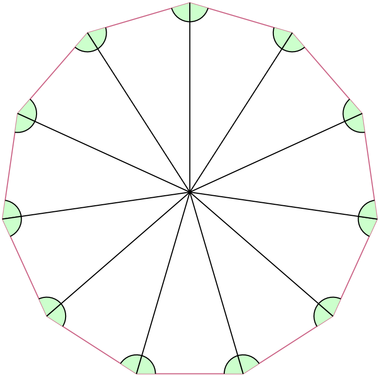

for $n = 11$, we have 11 triangles.
Sum of angles of one triangle $= \pi$
$\pi = 180^\circ$
Sum of angles of 11 triangles $= 11\pi$
Sum of internal angles $= 11\pi - 2\pi$
Each internal angle $= (11\pi - 2\pi)/11$

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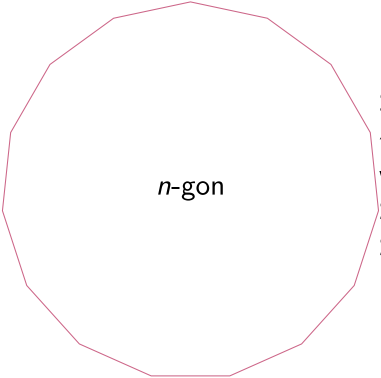
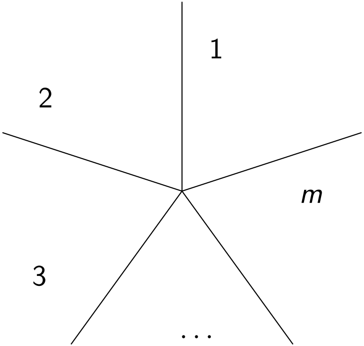
Internal angle of an n-sided polygon



Sum of angles of one triangle $= \pi$
$\pi = 180^\circ$
when we have n triangles,
Sum of angles of n triangles $= n\pi$
Sum of internal angles $= n\pi - 2\pi$
Internal angle $= (n-2)\pi/n = A_n$

Tiling with Convex Regular Polygons

How many edge to edge tilings are possible using only one type of tile?



Internal angle $= (n-2)\pi/n = A_n$
$m \times A_n = 360^\circ = 2\pi$

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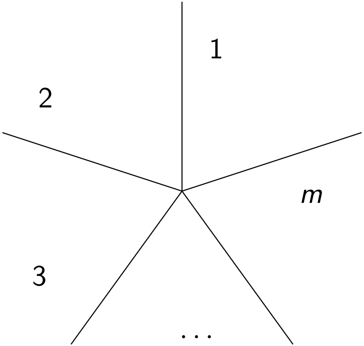

Internal angle $= (n-2)\pi/n = A_n$
$m \times A_n = 360^\circ = 2\pi$

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{2}$$

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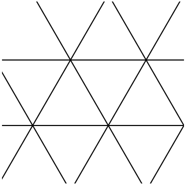

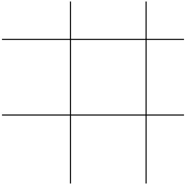

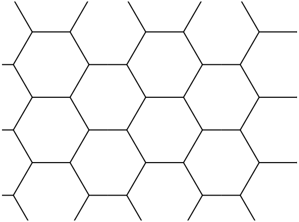

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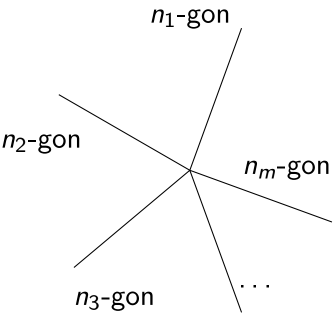

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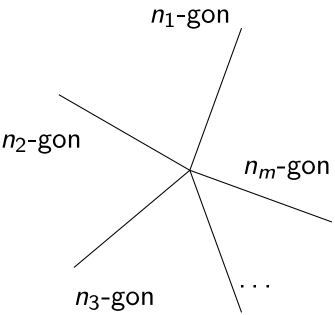
How many edge to edge tilings are possible using more than one type of tile?



Internal angle $= (n-2)\pi/n = A_n$

$$A_{n_1} + A_{n_2} + A_{n_3} + \ldots + A_{n_m} = 2\pi$$

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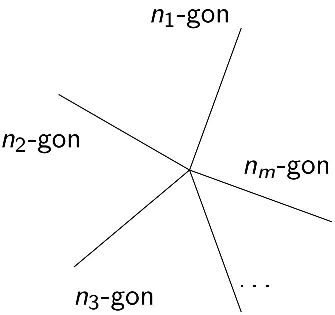

Internal angle $= (n-2)\pi/n = A_n$

$$A_{n_1} + A_{n_2} + A_{n_3} + \ldots + A_{n_m} = 2\pi$$

$$\frac{m}{2} - \left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \ldots + \frac{1}{n_m} \right) = 1$$

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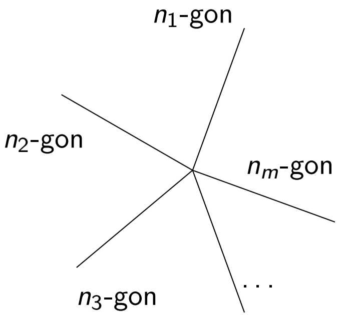

$$\text{Internal angle} = (n-2)\pi/n = A_n$$

$$A_{n_1} + A_{n_2} + A_{n_3} + \ldots + A_{n_m} = 2\pi$$

$$\frac{m}{2} - \left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \ldots + \frac{1}{n_m}\right) = 1$$

Tiling with Convex Regular Polygons

How many edge to edge tilings are possible using more than one type of tile?
Consider $m = 3$

$$\frac{m}{2} - \left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \ldots + \frac{1}{n_m}\right) = 1$$

$$\frac{3}{2} - \left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}\right) = 1$$

Escher

Take from sswp

Wang's Conjecture

Are there sets of tiles that tile only periodically?

Escher's ascending descending - penrose staircases

Ratio of number of darts and kites if the ratio was rational? - periodic Tiling place the forced pieces first when you have choices - one can lead to a point where no more pieces can be legally added number of penrose tilings? - 'uncountable' uncountable meaning? - e.g. rational numbers are countable, real numbers are uncountable

Reference

Vigyan Pratibha Learning Unit - `https://vigyanpratibha.in/`