## Big Data Analytics - CS7070

## **Programming Project #2**

- 3. Phase-3: Write a program for Spark (in PySpark, Scala, or Spark-JAVA) that takes as input a graph in the form of 2-hop projection (value) for each node (key) and produces a list of (key, value) pairs such that for each node (key), the value is all the triangles incident on that node.
- a. Items to be submitted: (i) yourprogram source code, (ii) Output of your program for the graph of TinyDataSet, and (ii) Output of your program for the graph of SmallDataSet.

## **Solution:**

#### (i) Source code:

```
1. %pyspark
2. phase2_tiny_rdd = sc.textFile("/user/maria_dev/Phase2_output_tinydataset.txt")
3. phase2_tiny_rdd.collect()
5. ['{1: [{3: [1, 2, 4, 6], 4: [1, 3, 5, 6, 7]}]}', '{2: [{3: [1, 2, 4, 6], 6: [2, 3, 4, 5
    , 10]}]}', '{3: [{1: [3, 4], 2: [3, 6], 4: [1, 3, 5, 6, 7], 6: [2, 3, 4, 5, 10]}]}', '{
   4: [{1: [3, 4], 3: [1, 2, 4, 6], 5: [4, 6, 8, 9], 6: [2, 3, 4, 5, 10], 7: [4, 8, 10]}]}
       '{5: [{4: [1, 3, 5, 6, 7], 6: [2, 3, 4, 5, 10], 8: [5, 7, 9], 9: [5, 8, 10]}]}',
    : [{2: [3, 6], 3: [1, 2, 4, 6], 4: [1, 3, 5, 6, 7], 5: [4, 6, 8, 9], 10: [6, 7, 9]}]}',
    '{7: [{4: [1, 3, 5, 6, 7], 8: [5, 7, 9], 10: [6, 7, 9]}]}', '{8: [{5: [4, 6, 8, 9], 7:
    [4, 8, 10], 9: [5, 8, 10]}]}', '{9: [{5: [4, 6, 8, 9], 8: [5, 7, 9], 10: [6, 7, 9]}]}'
    , '{10: [{6: [2, 3, 4, 5, 10], 7: [4, 8, 10], 9: [5, 8, 10]}]}']
6.
7. %pyspark
8. phase2_tiny_transformed = phase2_tiny_rdd.map(lambda x:eval(x))
9. phase2 tiny transformed.collect()
10.
11. [{1: [{3: [1, 2, 4, 6], 4: [1, 3, 5, 6, 7]}]}, {2: [{3: [1, 2, 4, 6], 6: [2, 3, 4, 5, 1
   0]}]}, {3: [{1: [3, 4], 2: [3, 6], 4: [1, 3, 5, 6, 7], 6: [2, 3, 4, 5, 10]}]}, {4: [{1:
    [3, 4], 3: [1, 2, 4, 6], 5: [4, 6, 8, 9], 6: [2, 3, 4, 5, 10], 7: [4, 8, 10]}]}, {5: [
    {4: [1, 3, 5, 6, 7], 6: [2, 3, 4, 5, 10], 8: [5, 7, 9], 9: [5, 8, 10]}]}, {6: [{2: [3,
   6], 3: [1, 2, 4, 6], 4: [1, 3, 5, 6, 7], 5: [4, 6, 8, 9], 10: [6, 7, 9]}]}, {7: [{4: [1
    , 3, 5, 6, 7], 8: [5, 7, 9], 10: [6, 7, 9]}]}, {8: [{5: [4, 6, 8, 9], 7: [4, 8, 10], 9:
    [5, 8, 10]}]}, {9: [{5: [4, 6, 8, 9], 8: [5, 7, 9], 10: [6, 7, 9]}]}, {10: [{6: [2, 3,
    4, 5, 10], 7: [4, 8, 10], 9: [5, 8, 10]}]}]
12.
13. %pyspark
14. phase2_intermediate = phase2_tiny_transformed.map(lambda x:{k:list(dicts.keys()) for k,
   v in x.items() for dicts in v })
15. phase2 intermediate.collect()
17. [{1: [3, 4]}, {2: [3, 6]}, {3: [1, 2, 4, 6]}, {4: [1, 3, 5, 6, 7]}, {5: [4, 6, 8, 9]},
    {6: [2, 3, 4, 5, 10]}, {7: [4, 8, 10]}, {8: [5, 7, 9]}, {9: [5, 8, 10]}, {10: [6, 7,
   }]
18.
19. %pyspark
20. from itertools import combinations
```

```
21. phase2_flattened = phase2_intermediate.map(lambda x:{k:sorted([k]+v) for k,v in x.items
       () })
22. phase2_flattened.collect()
23.
24. [{1: [1, 3, 4]}, {2: [2, 3, 6]}, {3: [1, 2, 3, 4, 6]}, {4: [1, 3, 4, 5, 6, 7]}, {5: [4,
         5, 6, 8, 9]}, {6: [2, 3, 4, 5, 6, 10]}, {7: [4, 7, 8, 10]}, {8: [5, 7, 8, 9]}, {9: [5,
        8, 9, 10]}, {10: [6, 7, 9, 10]}]
25.
26. %pyspark
27. phase2 rdd = phase2 flattened.map(lambda x:{k:list(combinations(v,3)) for k,v in x.item
28. phase2_rdd_new = phase2_rdd
30. phase2 rdd.collect()
31.
32. [{1: [(1, 3, 4)]}, {2: [(2, 3, 6)]}, {3: [(1, 2, 3), (1, 2, 4), (1, 2, 6), (1, 3, 4), (
       1, 3, 6), (1, 4, 6), (2, 3, 4), (2, 3, 6), (2, 4, 6), (3, 4, 6)]}, {4: [(1, 3, 4), (1,
       3, 5), (1, 3, 6), (1, 3, 7), (1, 4, 5), (1, 4, 6), (1, 4, 7), (1, 5, 6), (1, 5, 7), (1,
       (4, 5), (3, 4, 5), (3, 4, 6), (3, 4, 7), (3, 5, 6), (3, 5, 7), (3, 6, 7), (4, 5, 6), (4, 5, 6)
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       ), (2, 3, 6), (2, 3, 10), (2, 4, 5), (2, 4, 6), (2, 4, 10), (2, 5, 6), (2, 5, 10), (2,
       6, 10), (3, 4, 5), (3, 4, 6), (3, 4, 10), (3, 5, 6), (3, 5, 10), (3, 6, 10), (4, 5, 6),
       (4, 5, 10), (4, 6, 10), (5, 6, 10)]}, {7: [(4, 7, 8), (4, 7, 10), (4, 8, 10), (7, 8, 1
       0)]}, {8: [(5, 7, 8), (5, 7, 9), (5, 8, 9), (7, 8, 9)]}, {9: [(5, 8, 9), (5, 8, 10), (5
       , 9, 10), (8, 9, 10)]}, {10: [(6, 7, 9), (6, 7, 10), (6, 9, 10), (7, 9, 10)]}]
33.
34. %pyspark
35. def trans(x,y):
36.
              x.update(y)
37.
               return x
38.
39. phase2 flat rdd = phase2 rdd.reduce(lambda x,y:trans(x,y))
40. phase2 flat rdd
42. {1: [(1, 3, 4)], 2: [(2, 3, 6)], 3: [(1, 2, 3), (1, 2, 4), (1, 2, 6), (1, 3, 4), (1, 3,
         6), (1, 4, 6), (2, 3, 4), (2, 3, 6), (2, 4, 6), (3, 4, 6)], 4: [(1, 3, 4), (1, 3, 5),
       (1, 3, 6), (1, 3, 7), (1, 4, 5), (1, 4, 6), (1, 4, 7), (1, 5, 6), (1, 5, 7), (1, 6, 7),
        (3, 4, 5), (3, 4, 6), (3, 4, 7), (3, 5, 6), (3, 5, 7), (3, 6, 7), (4, 5, 6), (4, 5, 7)
       , (4, 6, 7), (5, 6, 7)], 5: [(4, 5, 6), (4, 5, 8), (4, 5, 9), (4, 6, 8), (4, 6, 9), (4,
        8, 9), (5, 6, 8), (5, 6, 9), (5, 8, 9), (6, 8, 9)], 6: [(2, 3, 4), (2, 3, 5), (2, 3, 6)]
       ), (2, 3, 10), (2, 4, 5), (2, 4, 6), (2, 4, 10), (2, 5, 6), (2, 5, 10), (2, 6, 10), (3,
       4, 5), (3, 4, 6), (3, 4, 10), (3, 5, 6), (3, 5, 10), (3, 6, 10), (4, 5, 6), (4, 5, 10)
       , (4, 6, 10), (5, 6, 10)], 7: [(4, 7, 8), (4, 7, 10), (4, 8, 10), (7, 8, 10)], 8: [(5,
       7, 8), (5, 7, 9), (5, 8, 9), (7, 8, 9)], 9: [(5, 8, 9), (5, 8, 10), (5, 9, 10), (8, 9,
       [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10], [10]
43.
44. %pyspark
45. rdd = sc.parallelize([phase2_flat_rdd])
46. rdd.collect()
48. \ [\{1: [(1, 3, 4)], 2: [(2, 3, 6)], 3: [(1, 2, 3), (1, 2, 4), (1, 2, 6), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 3, 4), (1, 4, 4), (1, 4, 4), (1, 4, 4), (1, 4, 4), (1, 4, 4), (1, 4, 4), (1, 4, 4)
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         (1, 3, 6), (1, 3, 7), (1, 4, 5), (1, 4, 6), (1, 4, 7), (1, 5, 6), (1, 5, 7), (1, 6, 7)
       , (3, 4, 5), (3, 4, 6), (3, 4, 7), (3, 5, 6), (3, 5, 7), (3, 6, 7), (4, 5, 6), (4, 5, 7
       ), (4, 6, 7), (5, 6, 7)], 5: [(4, 5, 6), (4, 5, 8), (4, 5, 9), (4, 6, 8), (4, 6, 9), (4
       , 8, 9), (5, 6, 8), (5, 6, 9), (5, 8, 9), (6, 8, 9)], 6: [(2, 3, 4), (2, 3, 5), (2, 3,
       6), (2, 3, 10), (2, 4, 5), (2, 4, 6), (2, 4, 10), (2, 5, 6), (2, 5, 10), (2, 6, 10), (3
       , 4, 5), (3, 4, 6), (3, 4, 10), (3, 5, 6), (3, 5, 10), (3, 6, 10), (4, 5, 6), (4, 5, 10
       ), (4, 6, 10), (5, 6, 10)], 7: [(4, 7, 8), (4, 7, 10), (4, 8, 10), (7, 8, 10)], 8: [(5,
```

```
7, 8), (5, 7, 9), (5, 8, 9), (7, 8, 9)], 9: [(5, 8, 9), (5, 8, 10), (5, 9, 10), (8, 9,
    10)], 10: [(6, 7, 9), (6, 7, 10), (6, 9, 10), (7, 9, 10)]}]
49.
50.
51. %pyspark
52. def filter_elements(x):
53.
        for k,v in x.items():
           b = []
54.
55.
            for i,item in enumerate(v):
56.
                if k in item:
57.
                    b.append(item)
58.
59.
       return {k:b}
60. phase2 rdd 2 = phase2 rdd new.map(lambda x: filter elements(x))
61. phase2 rdd 2.collect()
62.
63. [{1: [(1, 3, 4)]}, {2: [(2, 3, 6)]}, {3: [(1, 2, 3), (1, 3, 4), (1, 3, 6), (2, 3, 4), (
   2, 3, 6), (3, 4, 6)]}, {4: [(1, 3, 4), (1, 4, 5), (1, 4, 6), (1, 4, 7), (3, 4, 5), (3,
   4, 6), (3, 4, 7), (4, 5, 6), (4, 5, 7), (4, 6, 7)]}, {5: [(4, 5, 6), (4, 5, 8), (4, 5,
   9), (5, 6, 8), (5, 6, 9), (5, 8, 9)]}, {6: [(2, 3, 6), (2, 4, 6), (2, 5, 6), (2, 6, 10)
    , (3, 4, 6), (3, 5, 6), (3, 6, 10), (4, 5, 6), (4, 6, 10), (5, 6, 10)]}, {7: [(4, 7, 8)
    , (4, 7, 10), (7, 8, 10)]}, {8: [(5, 7, 8), (5, 8, 9), (7, 8, 9)]}, {9: [(5, 8, 9), (5,
    9, 10), (8, 9, 10)]}, {10: [(6, 7, 10), (6, 9, 10), (7, 9, 10)]}]
64.
65. %pyspark
66. #same as phase2_flattened. "rr" can be replaced with phase2_flattened
68. phase1_rdd = sc.textFile("/user/maria_dev/Phase1_output_tinydataset.txt")
69. phase1 op rdd = phase1 rdd.map(lambda x:eval(x)).map(lambda x:\{x[0]:x[1]\})
70. phase1 op rdd.collect()
71.
72. def trans_1(x,y):
73.
       x.update(y)
74.
       return x
75.
76. rr = phase1_op_rdd.reduce(lambda x,y : trans_1(x,y))
78. {1: [3, 4], 2: [3, 6], 3: [1, 2, 4, 6], 4: [1, 3, 5, 6, 7], 5: [4, 6, 8, 9], 6: [2, 3,
   4, 5, 10], 7: [4, 8, 10], 8: [5, 7, 9], 9: [5, 8, 10], 10: [6, 7, 9]}
79.
80.
81. %pyspark
82. def filter non triangles(x):
84.
       for k,v in x.items():
           b=[]
85.
86.
            1=[]
87.
            for tuples in v:
88.
                if k not in tuples:
89.
                    b.append("a")
90.
91.
                    l = list(tuples)
92.
                    1.remove(k)
93.
                    a = 1[0]
94.
                    if l[1] in rr.get(a):
95.
                        b.append(tuples)
96.
97.
       return {k:b}
98.
99.
100.
```

# (ii) Output for Tiny Dataset:

```
1. {1: [(1, 3, 4)]}
2. {2: [(2, 3, 6)]}
3. {3: [(1, 3, 4), (2, 3, 6), (3, 4, 6)]}
4. {4: [(1, 3, 4), (3, 4, 6), (4, 5, 6)]}
5. {5: [(4, 5, 6), (5, 8, 9)]}
6. {6: [(2, 3, 6), (3, 4, 6), (4, 5, 6)]}
7. {7: []}
8. {8: [(5, 8, 9)]}
9. {9: [(5, 8, 9)]}
10. {10: []}
```

## (iii) Output for Small Dataset:

```
1. {1: [(1, 2, 3), (1, 2, 6), (1, 6, 10)]}
 2. {2: [(1, 2, 3), (1, 2, 6), (2, 3, 7), (2, 6, 7), (2, 6, 11), (2, 7, 11)]}
 3. {3: [(1, 2, 3), (2, 3, 7), (3, 4, 5), (3, 4, 8), (3, 7, 8), (3, 7, 12), (3, 8, 12)]}
 4. {4: [(3, 4, 5), (3, 4, 8), (4, 5, 9), (4, 8, 9), (4, 8, 13), (4, 9, 13)]}
 5. {5: [(3, 4, 5), (4, 5, 9)]}
 6. {6: [(1, 2, 6), (1, 6, 10), (2, 6, 7), (2, 6, 11), (6, 7, 11), (6, 10, 11)]}
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