

## Math assignment 1

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Question 1 answer

option D - 1 1

Explaination -

```
>> x=[7:9; 6 12 19;-2:0];  
>> y=x(3,:);  
>> w=y(1,3);  
>> size(w)
```

ans =

1 1

```
>>
```

Question 2 answer

option C - [6 0 12 12]+[0 6 12 0]

Explaination -

We epresent in the following format -  $[x^3 \ x^2 \ x \ C]$

```
>> [6 0 12 12]+[0 6 12 0]
```

ans =

6 6 24 12

```
>>
```

Question 3 answer

option D - 1 1

Explaination -

```
>> x=[7:9; 6 12 19;-2:0];  
>> y=x(3,:);  
>> w=y(1,3);  
>> size(w)
```

ans =

1 1

```
>>
```

Question 4 answer

option A - 5

Explaination -

Given a function  $f(x) = \cos x$

exact value of  $\cos 1 = 0.5403$

By using Taylor's series expansion at  $a=0$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots$$

for  $x=1$

$$\cos 1 = P_n(1) = 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} - \frac{1}{10!} + \dots$$

for  $t = 1.0 \times 10^{-5} = 0.00001$  such that  $|\cos 1 - P_n(1)| < t$

now,

$$P_3(1) = 1 - 1/2! + 1/4! = 0.5416$$

$$P_4(1) = 1 - 1/2! + 1/4! + 1/6! = 0.5402$$

$$P_5(1) = 1 - 1/2! + 1/4! + 1/6! + 1/8! = 0.5403$$

$$[\cos 1 - P_5(1)] = [0.5403023 - 0.5403025]$$

$$= 0.00000027 < \epsilon$$

hence minimum degree is 5

Question 5 answer

option C - 11

The number of iterations it takes to find a root depends on the acceptable error. At each iteration in the bisection method the absolute error becomes half of the previous iteration, Therefore

$$E = (b-a)/2^n = \log_2(b-a/e)$$

bisection method states

$$n \geq \log(b-a) - \log E / \log 2$$

So,

$$n \geq \log(2-0.5) - \log(0.001)/\log 2$$

$$n \geq 0.17609 - 3 / 0.30102$$

$$n \geq 10.55$$

Hence  $n=11$  iterations

Question 6 answer

option E - none of the above

Explanation-

$$\begin{array}{ccc} -1 & 1 & -4 \\ -2 & 2 & 1 \\ 3 & 3 & 2 \end{array}$$

we ll find the inverse first

$$\begin{array}{ccc|ccc} -1 & 1 & -4 & 1 & 0 & 0 \\ -2 & 2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 2 & 0 & 0 & 1 \end{array}$$

$R1 \leftarrow R1 \text{ divided by } (-1)$

$$\begin{array}{ccc|ccc} -1 & -1 & -4 & -1 & 0 & 0 \\ -2 & 2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 2 & 0 & 0 & 1 \end{array}$$

$R2 \leftarrow R2 + 2 \times R1$

$$\begin{array}{ccc|ccc} -1 & -1 & -4 & -1 & 0 & 0 \\ 0 & 0 & 9 & -2 & 1 & 0 \\ 3 & 3 & 2 & 0 & 0 & 1 \end{array}$$

$R3 \leftarrow R3 - 3R1$

|    |    |     |    |   |   |
|----|----|-----|----|---|---|
| -1 | -1 | -4  | -1 | 0 | 0 |
| 0  | 0  | 9   | -2 | 1 | 0 |
| 0  | 6  | -10 | 3  | 0 | 1 |

interchanging rowa  $R2 \leftrightarrow R3$

|    |    |     |    |   |   |
|----|----|-----|----|---|---|
| -1 | -1 | -4  | -1 | 0 | 0 |
| 0  | 6  | -10 | 3  | 0 | 1 |
| 0  | 0  | 9   | -2 | 1 | 0 |

$R2 \leftarrow R2$  divided by 6

|    |    |      |     |   |     |
|----|----|------|-----|---|-----|
| -1 | -1 | -4   | -1  | 0 | 0   |
| 0  | 1  | -5/3 | 1/2 | 0 | 1/6 |
| 0  | 0  | 9    | -2  | 1 | 0   |

$R1 \leftarrow R1 + R2$

|   |   |      |      |   |     |
|---|---|------|------|---|-----|
| 1 | 0 | 7/3  | -1/2 | 0 | 1/6 |
| 0 | 1 | -5/3 | 1/2  | 0 | 1/6 |
| 0 | 0 | 9    | -2   | 1 | 0   |

$R3 \leftarrow R3$  divided by 9

|   |   |      |      |     |     |
|---|---|------|------|-----|-----|
| 1 | 0 | 7/3  | -1/2 | 0   | 1/6 |
| 0 | 1 | -5/3 | 1/2  | 0   | 1/6 |
| 0 | 0 | 1    | -2/9 | 1/9 | 0   |

$R1 \leftarrow R1 - 7/3R3$

$$\begin{array}{ccc|ccc}
 1 & 0 & 0 & 1/54 & -7/27 & 1/6 \\
 0 & 1 & -5/3 & 1/2 & 0 & 1/6 \\
 0 & 0 & 1 & -2/9 & 1/9 & 0
 \end{array}$$

$$R2 \leftarrow R2 + 5/3 R3$$

$$\begin{array}{ccc|ccc}
 1 & 0 & 0 & 1/54 & -7/27 & 1/6 \\
 0 & 1 & 0 & 7/54 & 5/27 & 1/6 \\
 0 & 0 & 1 & -2/9 & 1/9 & 0
 \end{array}$$

Solution:

$$\begin{array}{ccc}
 1/54 & -7/27 & 1/6 \\
 7/54 & 5/27 & 1/6 \\
 -2/9 & 1/9 & 0
 \end{array}$$

Question 7 answer

option D -  $x = (1, 2, 1)^T$

Explanation-

$$\begin{array}{ccc|c}
 0 & 4 & 1 & 9 \\
 1 & 1 & 3 & 6 \\
 2 & -2 & 1 & -1
 \end{array}$$

$$R2 \leftrightarrow R1$$

$$\begin{array}{ccc|c} 1 & 1 & 3 & 6 \\ 0 & 4 & 1 & 9 \\ 2 & -2 & 1 & -1 \end{array}$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\begin{array}{ccc|c} 1 & 1 & 3 & 6 \\ 0 & 4 & 1 & 9 \\ 0 & -4 & -5 & -13 \end{array}$$

$$R_1 \rightarrow 4R_1 - R_2, \quad R_3 \rightarrow R_3 + R_2$$

$$\begin{array}{ccc|c} 4 & 0 & 11 & 15 \\ 0 & 4 & 1 & 9 \\ 0 & 0 & -4 & -4 \end{array}$$

$$R_2 \rightarrow 4R_2 + R_3, \quad R_1 \rightarrow 4R_1 + 11R_3$$

$$\begin{array}{ccc|c} 16 & 0 & 0 & 16 \\ 0 & 4 & 1 & 9 \\ 0 & 0 & -4 & -4 \end{array}$$

$$R_2 \rightarrow R_2/16, \quad R_1 \rightarrow R_1/16, \quad R_3 \rightarrow R_3/-4$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}$$

$$(X_1, X_2, X_3) = (1, 2, 1)^T$$

Question 8 answer

option E - none of the above

$$\begin{pmatrix} 1 & 8 & 6 \end{pmatrix} \cdot \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = 6$$

$$\begin{pmatrix} 7 & 3 & 1 \end{pmatrix} \cdot \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = -8$$

$$\begin{pmatrix} 6 & 7 & 12 \end{pmatrix} \cdot \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = 2$$

$$x_1 + x_2 + 6x_3 = 6$$

$$7x_1 + 3x_2 + x_3 = -8$$

$$6x_1 + 7x_2 + 12x_3 = 2$$

we put the equations in suitable order to use gauss-sided method

$$7x_1 + 3x_2 + x_3 = -8 \quad \{1\}$$

$$x_1 + x_2 + 6x_3 = 6 \quad \{2\}$$

$$6x_1 + 7x_2 + 12x_3 = 2 \quad \{3\}$$

from  $\{1\}$

$$x_1 = 1/7[-8-3x_2-x_3] \quad \{4\}$$

$$x_2 = 1/8[6-x_1-6x_3] \quad \{5\}$$

$$x_3 = 1/12[2-6x_1-7x_2] \quad \{6\}$$

initial approximation  $\rightarrow x_1=2, x_2=4, x_3=5$

$$x_1 = 1/7[-8-3x_2-x_3] = 1/7[-8-3(4)-5] = -25/7 = -3.5714$$

$$x_2 = 1/8[6-x_1-6x_3] = 1/8[6-3.5714-6(5)] = -2.5535$$

$$x_3 = 1/12[2-6x_1-7x_2] = 1/12[2-6(-3.5714)-7(-2.5535)] = 3.4419$$



upon doing all the 2nd iteration we get,

$$x_1 = -0.5402$$

$$x_2 = -1.7639$$

$$x_3 = 1.4657$$

upon doing all the 3rd iteration we get,

$$x_1 = -0.5962$$

$$x_2 = 0.2747$$

$$x_3 = 0.6021$$

so option E is correct