

Math Assignment

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Q1. Answer A - 1

Inputting this logical equation into MATLAB, the value of $X = 1$.

This is so, $X=(1 \sim 0)$ evaluates to true because 1 does not equal 0. The second and third statements are false as they claim 2 is greater than 2 and also 7 is less than 4. This gives us true given false and false.

This evaluates to true, leaving $X=1$.

Q2. Answer E - none of the above

$$r = [-7.7417, -0.2583]$$

Q3. Answer E - none of these

$$e = 17.9876$$

Q5. Answer C - 1.15630, 1.10167

In order to find the parameters, we use the following formula with :

$$x = T_k, y = S_k$$

$$a = \frac{nS_{xy} - S_x S_y}{nS_{xx} - (S_x)^2}$$

$$b = \frac{S_{xx} S_y - S_{xy} S_x}{nS_{xx} - (S_x)^2}$$

Next, we evaluate the variables with the data :

$$\begin{aligned} S_{xy} &= (0)(1.15) + (1)(2.32) + (2)(3.32) + (3)(4.53) + (4)(5.65) + (5)(6.97) + (6)(8.02) + (7)(9.23) \\ &= 192.73 \end{aligned}$$

$$S_x = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$

$$S_y = 1.15 + 2.32 + 3.32 + 4.53 + 5.65 + 6.97 + 8.02 + 9.23 = 41.19$$

$$S_x^2 = \sum(x^2) = 140$$

$$a = 1.1563$$

$$b = 1.10167$$

Q6. Answer E - none of the above

In order to find the parameters in a least squares sense for the function

$f(x) = \alpha e^{\beta x}$, we must take the natural log of both sides to change it to linear form.

we get:

$$\ln(y) = \ln(\alpha e^{\beta x}) = \ln(\alpha) + \ln(e^{\beta x}) = \beta x + \ln(\alpha)$$

let $\ln(y) = Y$, $x = X$, $\ln(\alpha) = a_0$ and $\beta = a_1$.

same formula as for a normal linear least squares regression :

$$\beta = \frac{nS_{xy} - S_x S_y}{nS_{xx} - (S_x)^2}$$

$$\alpha = \frac{S_{xx} S_y - S_{xy} S_x}{nS_{xx} - (S_x)^2}$$

We then evaluate the variables to be used in the formulas :

$$S_x = 1 + 3 + 6 + 9 + 15 = 34$$

$$S_y = \ln(5.12) + \ln(3) + \ln(2.48) + \ln(2.34) + \ln(2.18) = 5.2695$$

$$S_{xy} = (1)(\ln(5.12)) + (3)(\ln(3)) + (6)(\ln(2.48)) + (9)(\ln(2.34)) + (15)(\ln(2.18)) = 29.7198$$

$$S_{xx} = \sum(x^2) = 352$$

Substituting into the formulae we get :

$$a_1 = \frac{(5)(29.7198) - (34)(5.2695)}{(5)(352) - (34)^2} = -0.050603$$

$$a_0 = (352)(5.2695) - (29.7198)(34) / (5)(352) - (34)^2 = 1.397998$$

But since $\ln(\alpha) = a_0$, then $a = e^{a_0}$ which gives us :

$$\alpha = e^{1.397998} = 4.047089$$

Q7. Answer A - 1.9681, 3.1468

let $Y = y$, $X = 1/x$, $a_1 = \beta$, and $a_0 = \alpha$.

substituting values into the formula :

$$SX = \sum X = 1.6778$$

$$SY = \sum Y = 15.12$$

$$SXY = \sum XY = 6.9386$$

$$SXX = \sum (X)^2 = 1.55679$$

Then filling into the formulae:

$$a_1 = (5)(6.9386) - (1.6778)(15.12) / (5)(1.556789) - (1.6778)^2 = 3.1468 = \beta$$

$$a_0 = (1.55679)(15.12) - (6.9386)(1.6778) / (5)(1.155679) - (1.6778)^2 = 1.9681 = \alpha$$

Q8. Answer D - -1.19004

given -

$$f(x) = \log_4(\cos(x))$$

$$x_0 = 0.5, x_1 = 1, x_2 = 1.5$$

calculating the outputs of the initial values to the function :

$$F[x_0] = \log_4(\cos(0.5)) = -0.094197$$

$$F[x_1] = \log_4(\cos(1)) = -0.44408$$

$$F[x_2] = \log_4(\cos(1.5)) = -1.91069$$

calculating the divided differences :

$$f[x_i, x_{i+1}] = f[x_{i+1}] - f[x_i] / x_{i+1} - x_i$$

$$f[x_i, x_{i+1}, x_{i+2}] = f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}] / x_{i+2} - x_i$$

$$F[X_0, X_1] = F[X_1] - F[X_0] / X_1 - X_0 = -0.44408 - (-0.094197) / 1 - 0.5 = -0.699766$$

$$F[X_1, X_2] = F[X_2] - F[X_1] / X_2 - X_1 = -1.91064 - (-0.44408) / 1.5 - 1 = -2.93312$$

$$F[X_0, X_1, X_2] = F[X_1, X_2] - F[X_0, X_1] / X_2 - X_0 = -2.93312 - (-0.699766) / 1.5 - 0.5 = -2.233354$$

interpolate for the point $X=1.3$:

$$P_2(X) = F[X_0] + F[X_0, X_1](X - X_0) + F[X_0, X_1, X_2](X - X_0)(X - X_1)$$

$$= -0.094197 + (-0.699766)(X - 0.5) + (-2.233354)(X - 0.5)(X - 1)$$

$$\therefore P_2(X = 1.3)$$

$$= -0.094197 + (-0.699766)(X - 0.5) + (-2.233354)(1.3 - 0.5)(1.3 - 1)$$

$$= -1.19001476 \approx -1.19004$$

Q9. Answer E - none of the above

using the Lagrange method :

The given function : $f(x) = x^3 \log_2(x)$

initial values: $x_0 = 2$, $x_1 = 3$, $x_2 = 7$

using the formula:

$$f(x) = (y_1)(x - x_1)(x - x_2) / (x_0 - x_1)(x_0 - x_2) + (y_2)(x - x_0)(x - x_2) / (x_1 - x_0)(x_1 - x_2) +$$

$$(y_3) (x - x_0)(x - x_1) / (x_2 - x_0)(x_1 - x_2)$$

To get y, we substitute the following -

$$y_1 = (2)^{3\log_2(2)} = 8$$

$$y_2 = (3)^{3\log_2(3)} = 42.79398752$$

$$y_3 = (7)^{3\log_2(7)} = 962.9227383$$

after substitution -

$$f(x) = (8) (x - 3)(x - 7) / (2 - 3)(2 - 7) + (42.79 \dots) (x - 2)(x - 7) / (3 - 2)(3 - 7) + (962.922 \dots) (x - 2)(x - 3) / (7 - 2)(7 - 3)$$

evaluating at x=5 :

$$f(5) = (8) (5 - 2)(5 - 7) / (-1)(-5) + (42.7 \dots) (5 - 2)(5 - 7) / (1)(-4) + (962.9 \dots) (5 - 2)(5 - 3) / (5)(4)$$

$$= 346.6678$$