Take one Domain and draw the graph (Normal distribution) (Empirical rule)

NORMAL DISTRIBUTION – COMPLETE NOTES

1.Definition

Normal Distribution is a **continuous probability distribution** that is:

- Symmetrical
- Bell-shaped
- Mean = Median = Mode
- Defined by two parameters:
  - Mean (μ)
  - Standard Deviation (σ)

It is also called **Gaussian distribution**.

2. Important Properties

1. Bell-shaped curve
2. Symmetric about mean
3. Total area under curve = 1 (100%)
4. Tails approach x-axis but never touch
5. Spread controlled by standard deviation

3. Formula of Normal Distribution

[f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}]

Where:

- μ = Mean
- σ = Standard Deviation
- $\pi \approx 3.14$
- $e \approx 2.718$

4.Empirical Rule (68–95–99.7 Rule)

For any normal distribution:

Range Percentage of Data

Range Percentage of Data

$\mu \pm 1\sigma$ 68%

$\mu \pm 2\sigma$ 95%

$\mu \pm 3\sigma$ 99.7%

Percentage Breakdown

- 34% between μ and $+1\sigma$
- 34% between μ and -1σ
- 13.5% between 1σ and 2σ (each side)
- 2.35% between 2σ and 3σ (each side)
- 0.15% beyond 3σ

Domain Example 1: Student Marks Distribution

Mean score (μ) = 70
Standard deviation (σ) = 10

Using Empirical Rule:

- 68% students score between **60 and 80**
- 95% score between **50 and 90**
- 99.7% score between **40 and 100**

Interpretation:

- Very few students score below 40 or above 100
- Helps in grading system

Domain Example 2: Healthcare – Blood Pressure

Mean BP = 120 mmHg
$\sigma = 15$

- 68% between 105 and 135
- 95% between 90 and 150

Doctors use this to identify abnormal patients.

Domain Example 3: Banking – Customer Income

Mean income = ₹50,000
σ = ₹8,000

- 68% earn between ₹42,000–₹58,000
- 95% between ₹34,000–₹66,000

Used for:

- Loan eligibility
- Risk assessment

Effect of Standard Deviation

Small σ

- Narrow curve
- Data tightly packed
- High consistency

Large σ

- Wide curve
- Data spread out
- Less consistency

Standard Normal Distribution (Z-Distribution)

When:

- $\mu = 0$
- $\sigma = 1$

Formula for Z-score:

$$Z = \frac{X - \mu}{\sigma}$$

Used to:

- Find probabilities
- Compare different datasets

Numerical Example

Suppose:
$\mu = 100$
$\sigma = 20$
Find Z-score for $X = 140$

[Z = \frac{140 - 100}{20}]

[Z = 2]

Meaning:
140 is 2 standard deviations above mean.

According to Empirical Rule:
About 2.5% of values lie above this.

Graph Description (Exam Writing)

Draw bell-shaped curve.
Mark:

- μ at center
- $\mu \pm 1\sigma$
- $\mu \pm 2\sigma$
- $\mu \pm 3\sigma$
  Label percentages:
- 34%, 13.5%, 2.35%, 0.15%

Why Normal Distribution is Important?

1. Used in Machine Learning
2. Used in Hypothesis Testing
3. Used in Quality Control
4. Used in Risk Management
5. Used in Six Sigma

Business Importance

- Detect defects
- Improve product quality
- Predict demand
- Analyze performance
- Reduce risk

Final Summary

Normal Distribution:

- Bell-shaped
- Symmetric
- Mean = Median = Mode

- Controlled by μ and σ

Empirical Rule:

- 68% within 1σ
- 95% within 2σ
- 99.7% within 3

Normal Distribution Graph (Bell Curve)

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              *       *
           *             *
         *                  *
       *                       *
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    85    90    95   100   105   110   115


      μ-3σ  μ-2σ  μ-1σ   μ   μ+1σ  μ+2σ  μ+3σ
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Where:

- μ = 100
- σ = 5
- μ ± 1σ → 95 to 105

- $\mu \pm 2\sigma \rightarrow 90$ to 110
- $\mu \pm 3\sigma \rightarrow 85$ to 115