

If $\mu = 55$, $\sigma_a = 4$, $\sigma_b = 10$, $\sigma_c = 15$, In this which is better

Step 1: Understand the Meaning

- **Mean ($\mu = 55$)** → Average value is 55 in all three cases
- **Standard Deviation (σ)** → Measures spread or variability

Standard deviation tells us:

- How far values are from the mean
- How consistent the data is

Smaller σ = More consistent

Larger σ = More spread

Step 2: Compare the Three Cases

Case	Mean (μ)	Standard Deviation (σ)	Spread	Quality
A	55	4	Very small	✓ Best
B	55	10	Moderate	□ Average
C	55	15	Very large	✗ Poor

Step 3: Graphical Explanation (Concept)

If we draw three normal curves with mean = 55:

$\sigma = 4$

- Narrow and tall curve

- Data tightly clustered near 55
- High consistency

$\sigma = 10$

- Medium width curve
- Moderate variation

$\sigma = 15$

- Wide and flat curve
- Large variation
- Unstable system

Smaller σ gives better control.

Step 4: Business Interpretation (Manufacturing Example)

Suppose a factory produces medicine tablets with:

Target weight = 55 mg

Case A: $\sigma = 4$

- Most tablets between 51–59 mg
- Almost all tablets acceptable
- Low wastage
- High quality control

Case B: $\sigma = 10$

- Tablets between 45–65 mg
- Some tablets too heavy/light
- Moderate wastage

Case C: $\sigma = 15$

- Tablets between 40–70 mg
- Many defective
- Customer complaints
- High financial loss

So $\sigma = 4$ is best.

Step 5: Using Empirical Rule

For Normal Distribution:

When $\sigma = 4$

- 68% between 51 and 59
- 95% between 47 and 63
- Very controlled range

When $\sigma = 15$

- 68% between 40 and 70
- 95% between 25 and 85
- Very wide range

Wide range = Poor consistency.

Step 6: When Would Larger σ Be Better?

Only in rare cases like:

- Measuring diversity
- Studying income inequality
- Risk analysis

But in quality control and performance measurement:

Smaller σ is always better.

Final Exam Answer

Since all three distributions have the same mean ($\mu = 55$), the one with the smallest standard deviation is better.

$\boxed{\sigma = 4 \text{ is best}}$

Because:

- It has lowest variability
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- Values are closely concentrated around mean
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- It ensures better consistency and quality

Comparison of Standard Deviations ($\mu = 55$)

Height

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| $\sigma_a = 4$

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| $\sigma_b = 10$

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| $\sigma_c = 15$

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+-----→ X (Value)

55- σ 55 55+ σ