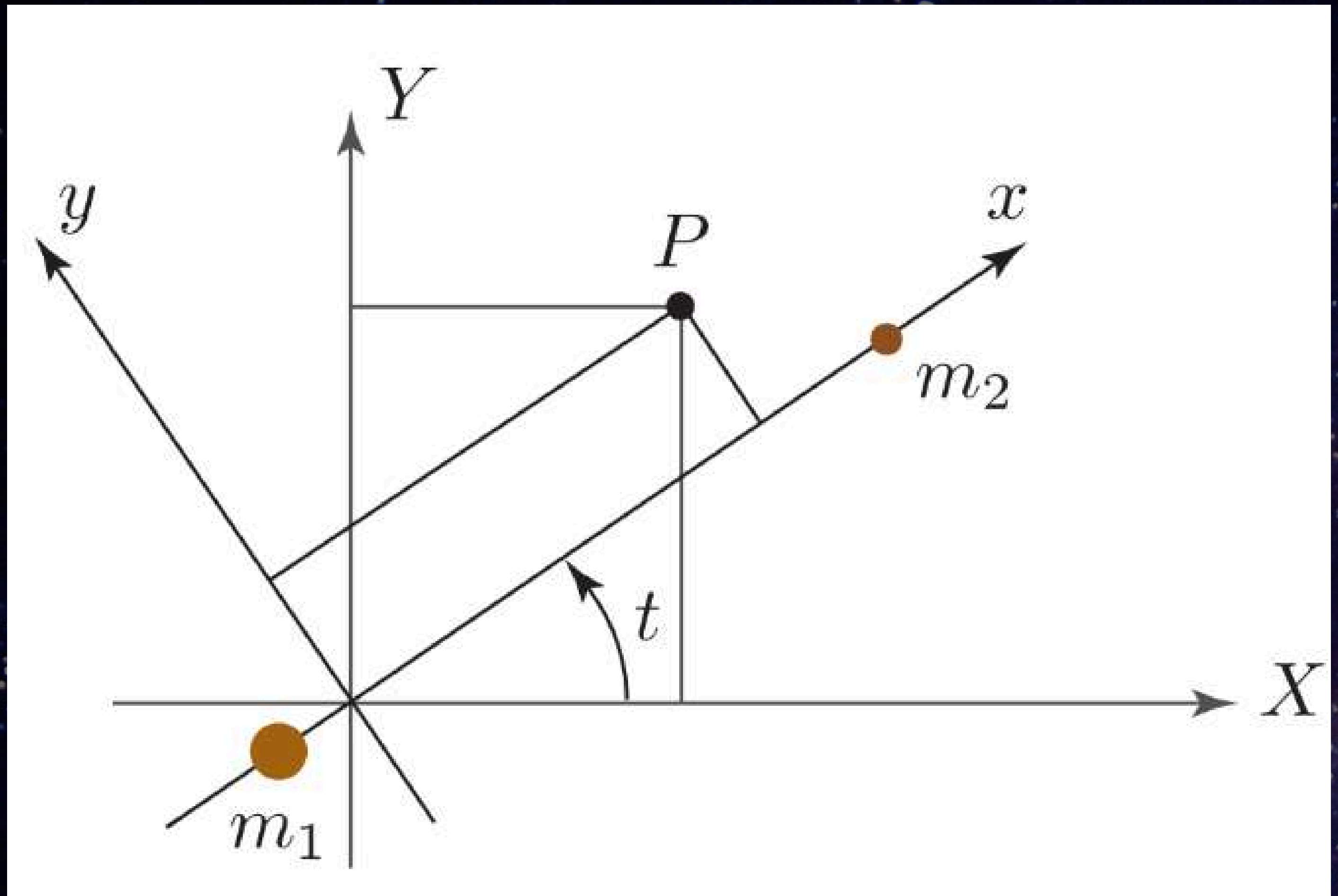
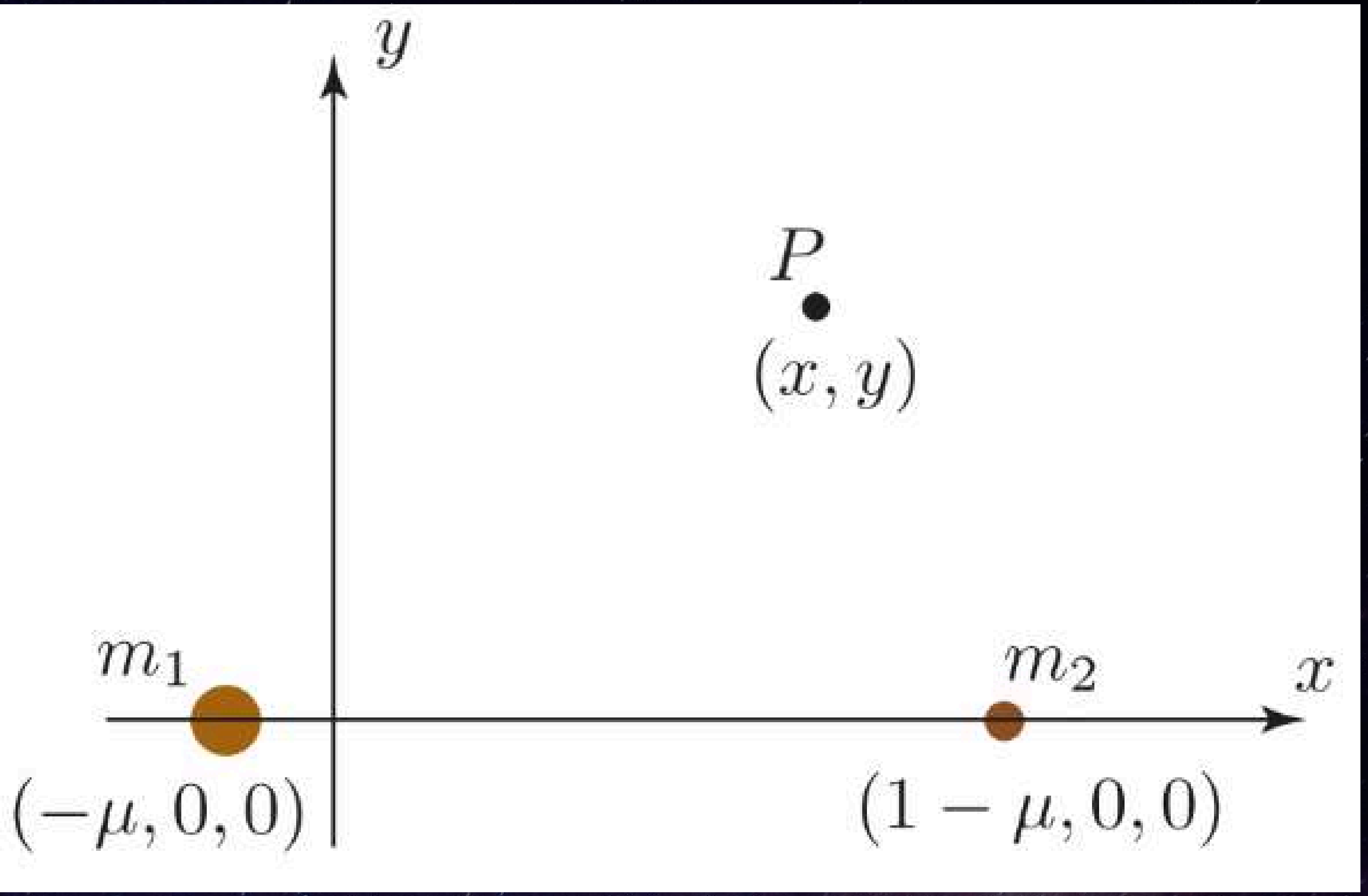
SPACE MISSION DESIGN

THREE BODY PROBLEM

Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D. (2011) Dynamical Systems, the Three-Body Problem and Space Mission Design. Marsden Books, ISBN 978-0-615-24095-4.

SOLUTION





Inertial and rotating frames x-y frame rotates wrt X-Y inertial frame

Rotating frame State space used to solve the planar constrained 3 body problem

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SOLUTION

Lagrangian : $L(x,y,z,\dot{x},\dot{y},\dot{z}) = \frac{1}{2}\left((\dot{x}-y)^2 + (\dot{y}+x)^2 + \dot{z}^2\right) - U(x,y,z)$

Equations of motion:

$$
\begin{aligned}
\dot{x} &= v_x, \\
\dot{y} &= v_y, \\
\dot{v}_x &= 2v_y - \frac{\partial \bar{U}}{\partial x}, \\
\dot{v}_y &= -2v_x - \frac{\partial \bar{U}}{\partial y},
\end{aligned}
$$

where

$$\bar{U}(x,y) = -\frac{1}{2}(x^2+y^2) + U(x,y,z),$$

$$U(x,y,z) = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{1}{2}\mu_1\mu_2$$

$$\mu_1 = 1-\mu \quad \text{and} \quad \mu_2 = \mu,$$

The energy integral give by $E(x,y,\dot{x},\dot{y}) = \frac{1}{2}(\dot{x}^2+\dot{y}^2) + \bar{U}(x,y)$ is conserved since the system is Hamiltonian

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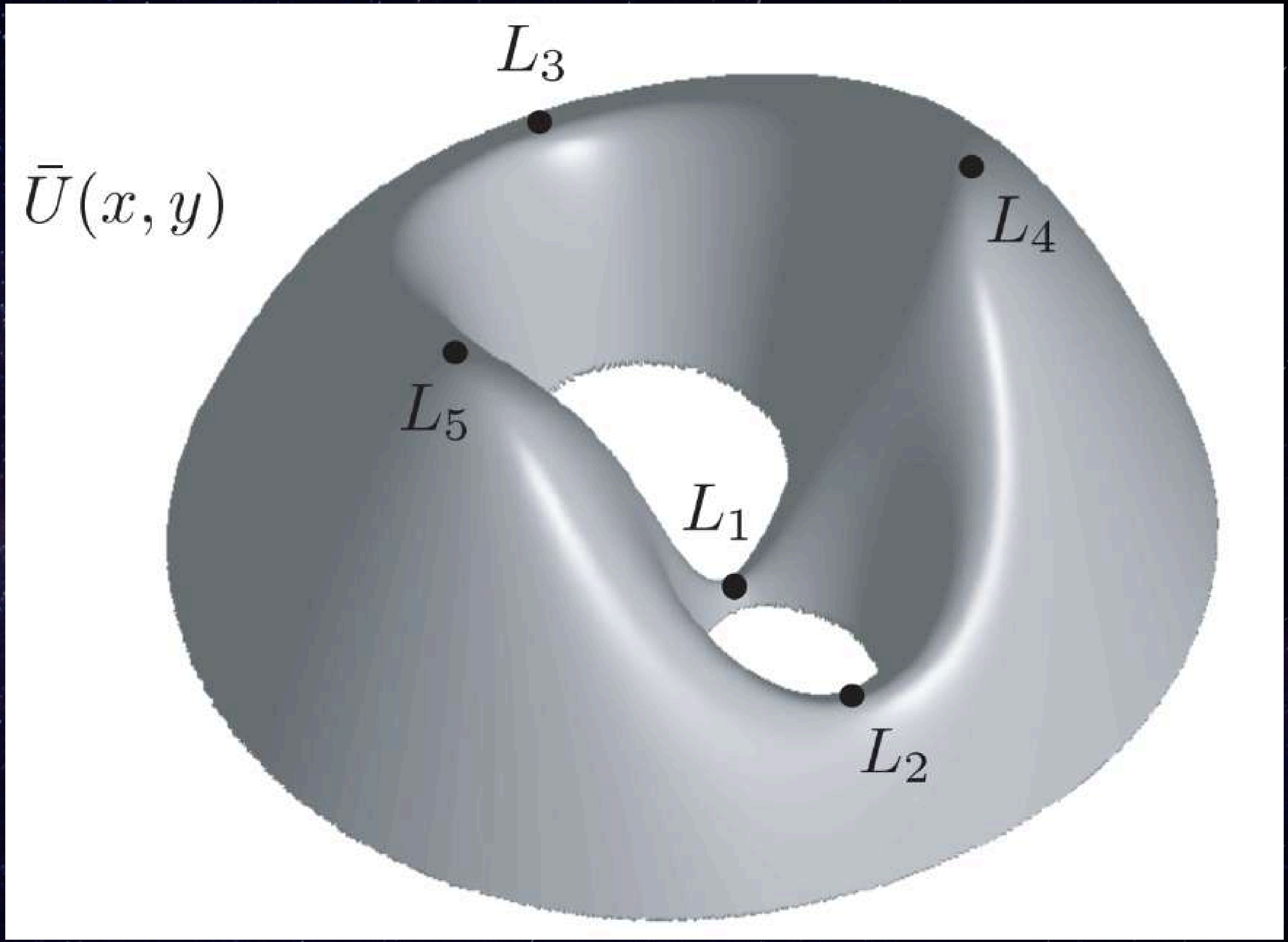
SOLUTION

$$
\begin{aligned}
\dot{x} &= v_x, \\
\dot{y} &= v_y, \\
\dot{v}_x &= 2v_y + ax, \\
\dot{v}_y &= -2v_x - by,
\end{aligned}
$$



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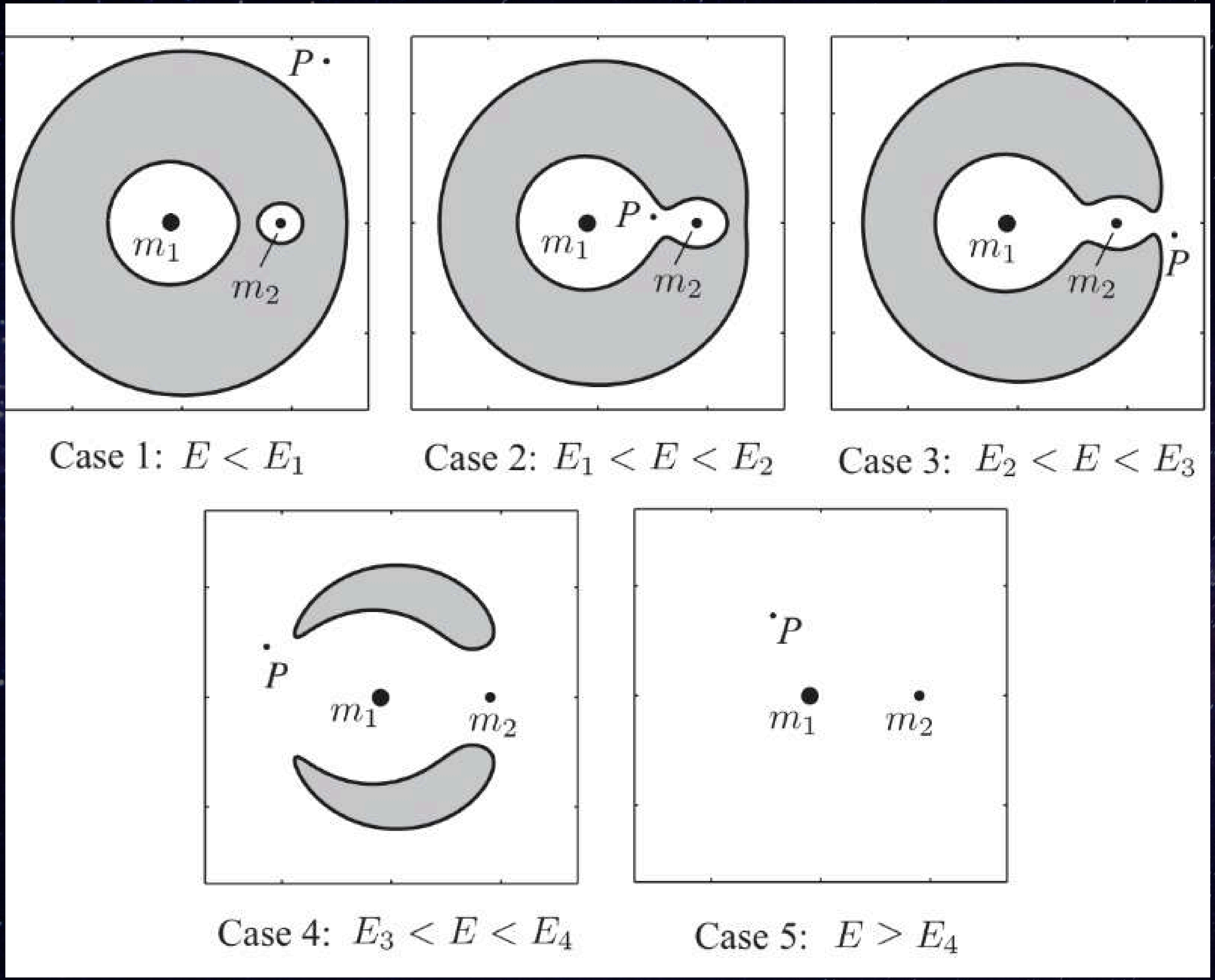
REALMS OF POSSIBLE MOTION



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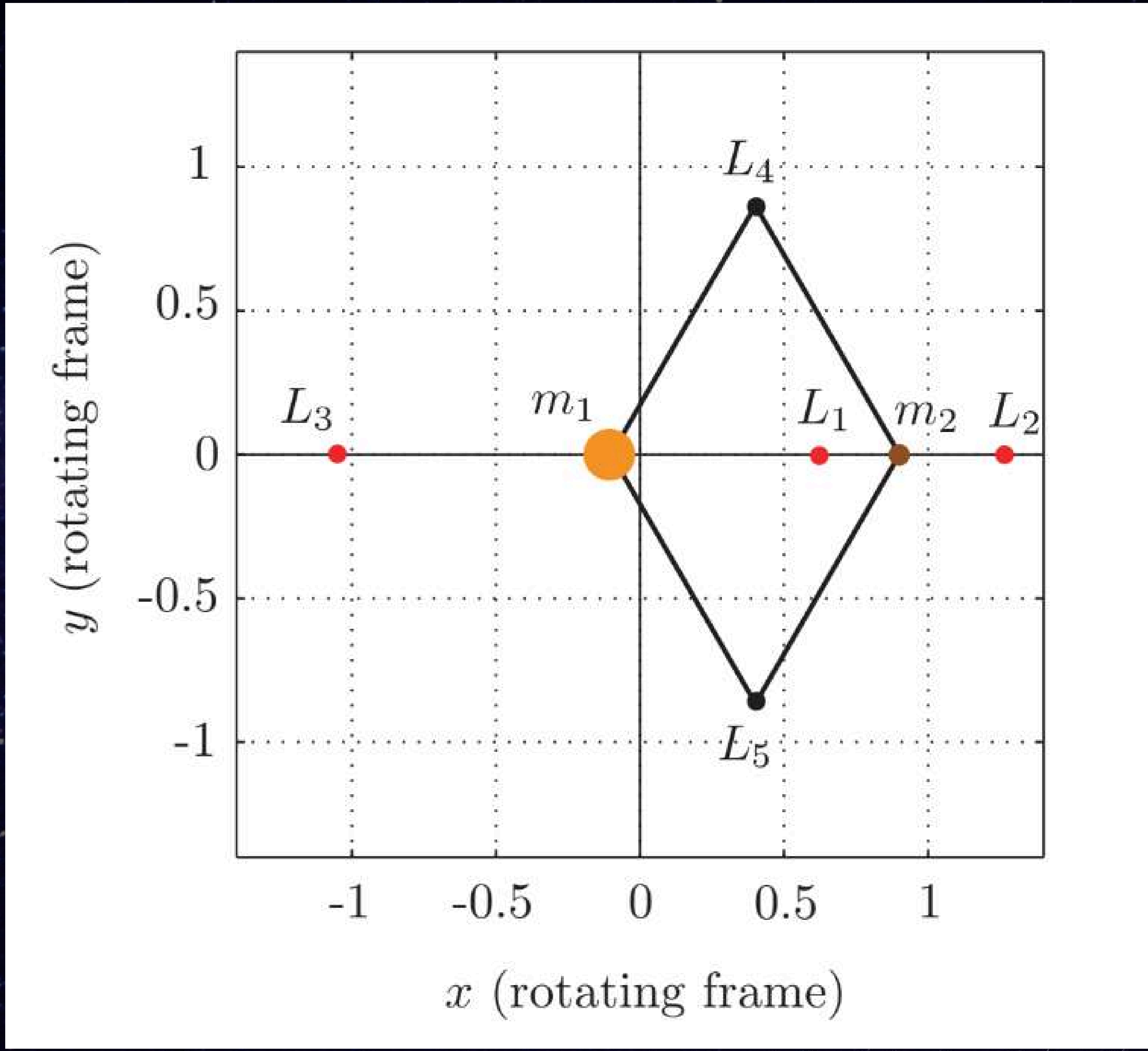
EQUILIBRIUM POINTS



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EQUILIBRIUM POINTS

$y \neq 0$.

$$0 = -\bar{U}_{r_1} = \mu r_2 - \frac{\mu}{r_2^2},$$

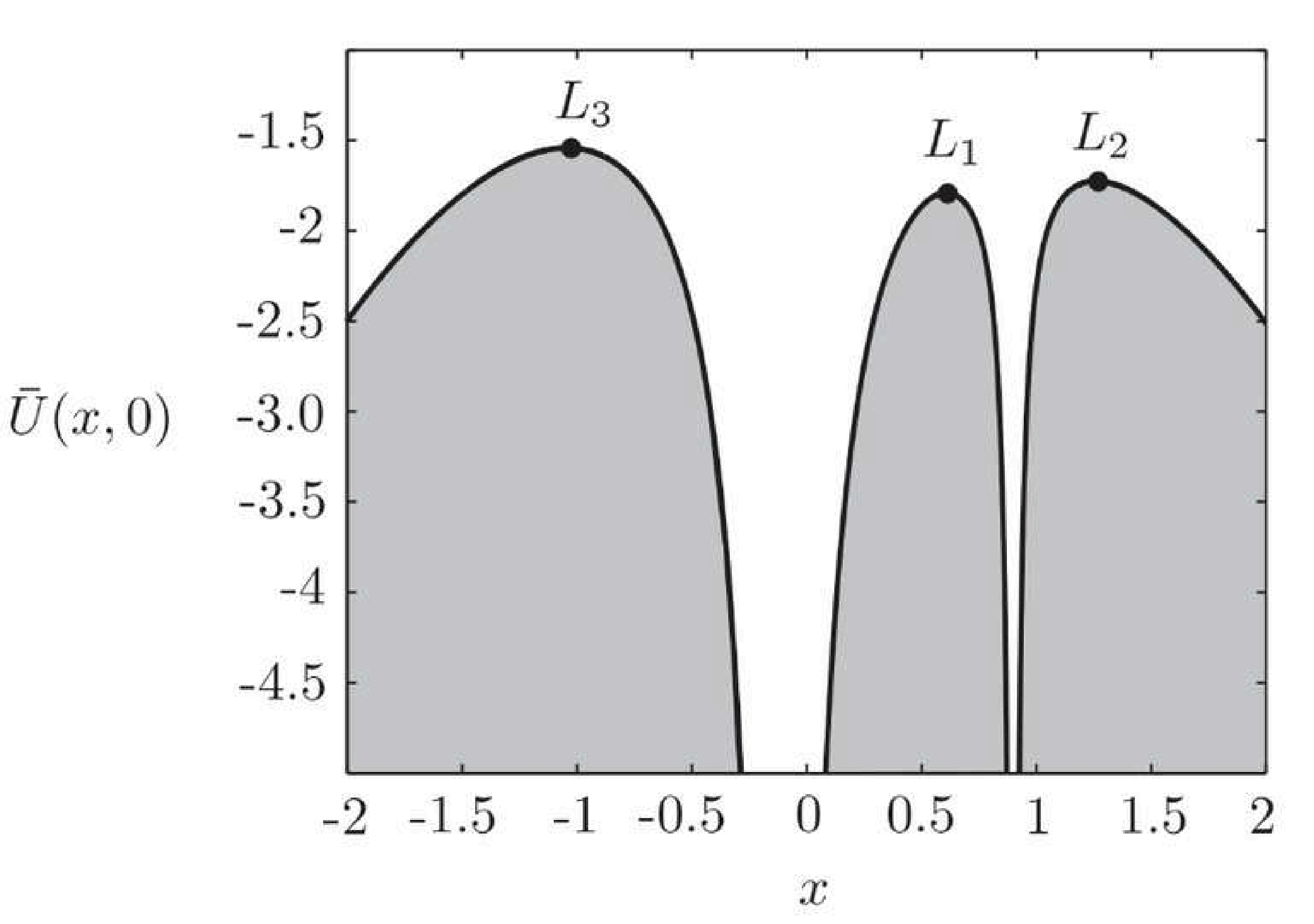
$$0 = -\bar{U}_{r_2} = (1-\mu) r_1 - \frac{(1-\mu)}{r_2^2}$$

$r_1 = r_2 = 1$.

$y = 0$.

$$\bar{U}(x,0) = -\frac{1}{2}x^2 - \frac{1-\mu}{|x+\mu|} - \frac{\mu}{|x-1+\mu|}$$

$$\frac{d}{dx}\bar{U}(x,0) = 0$$



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TRANSFER TUBES

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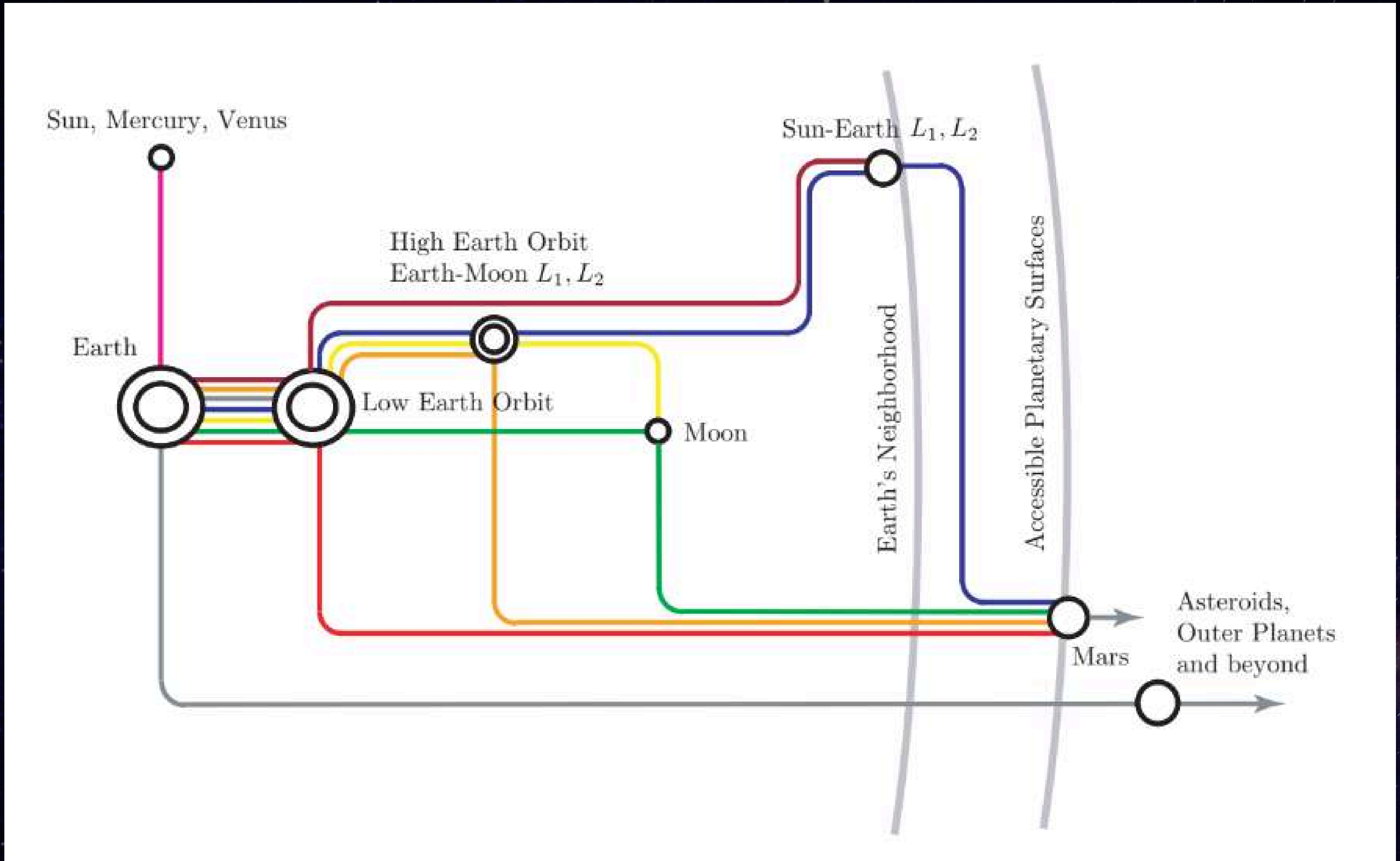

Metro Map

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STABILITY ANALYSIS IN EIGEN BASIS

$$\dot{\xi} = \lambda \xi,$$

$$\dot{\eta} = -\lambda \eta,$$

$$\dot{\zeta}_1 = \nu \zeta_2,$$

$$\dot{\zeta}_2 = -\nu \zeta_1,$$

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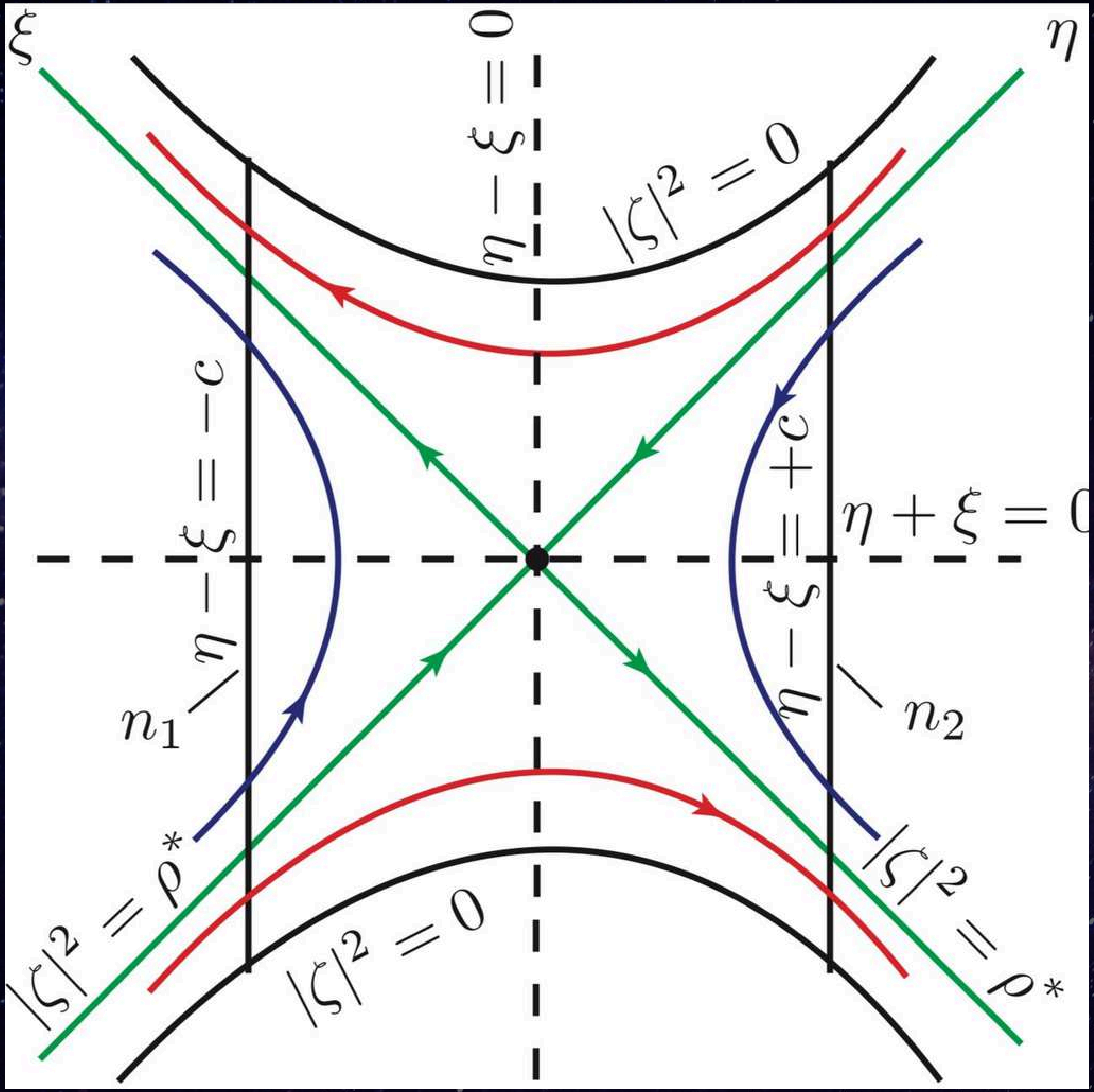

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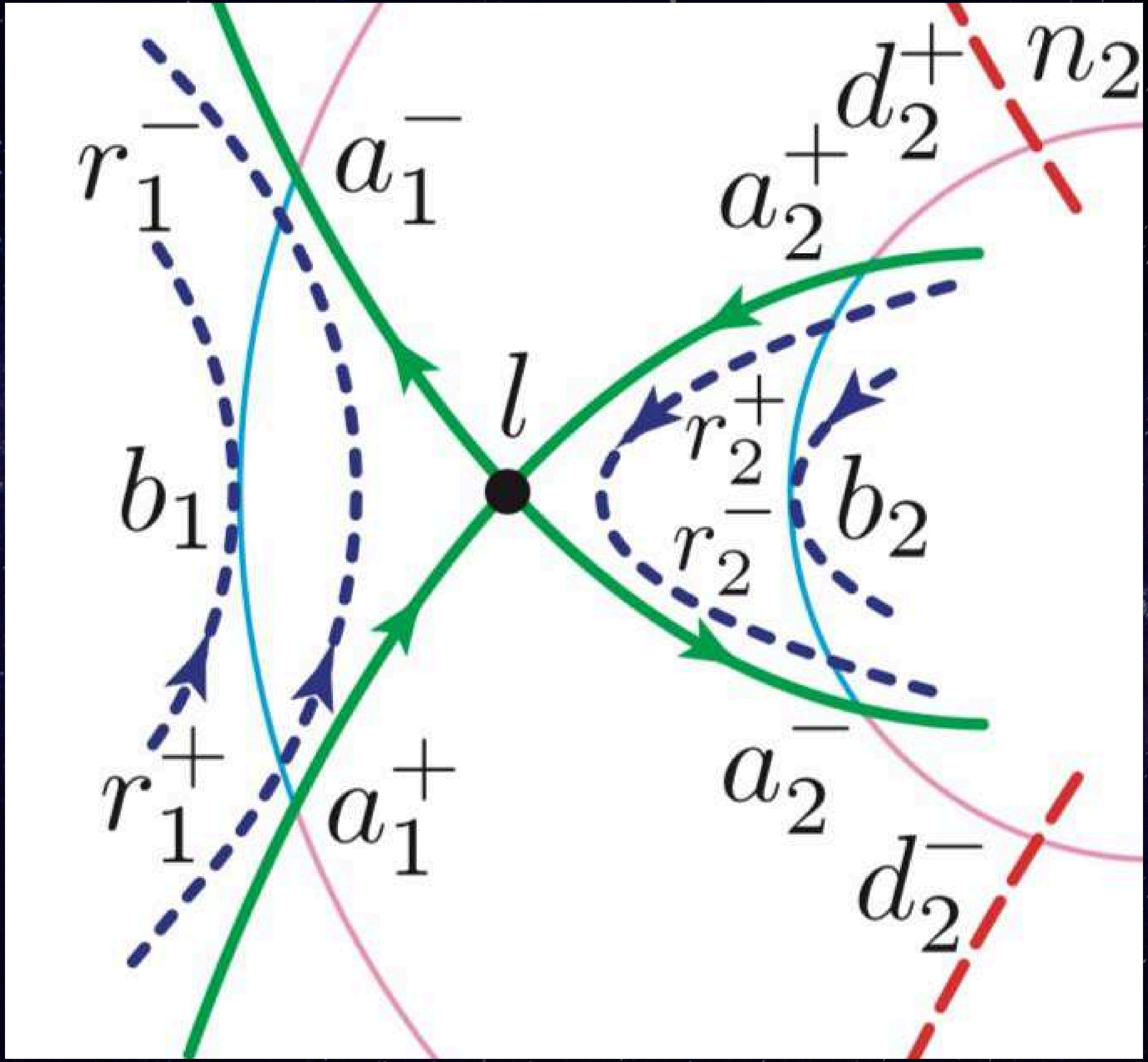

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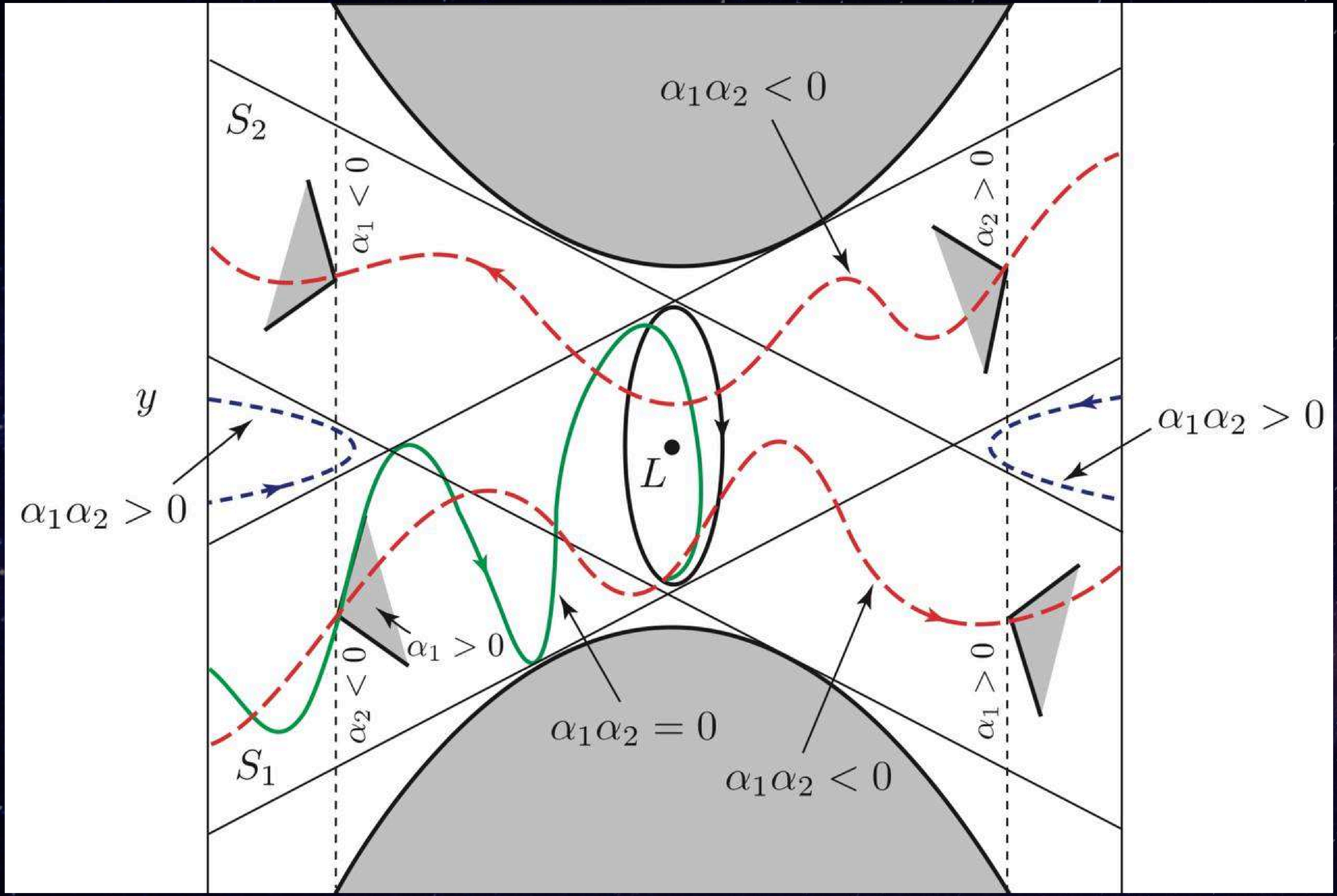
COMING BACK TO THE EUCLIDEAN SPACE

TYPES OF ORBIT



$$x(t) = \alpha_1 e^{\lambda t} + \alpha_2 e^{-\lambda t} + 2(\beta_1 \cos \nu t - \beta_2 \sin \nu t)$$

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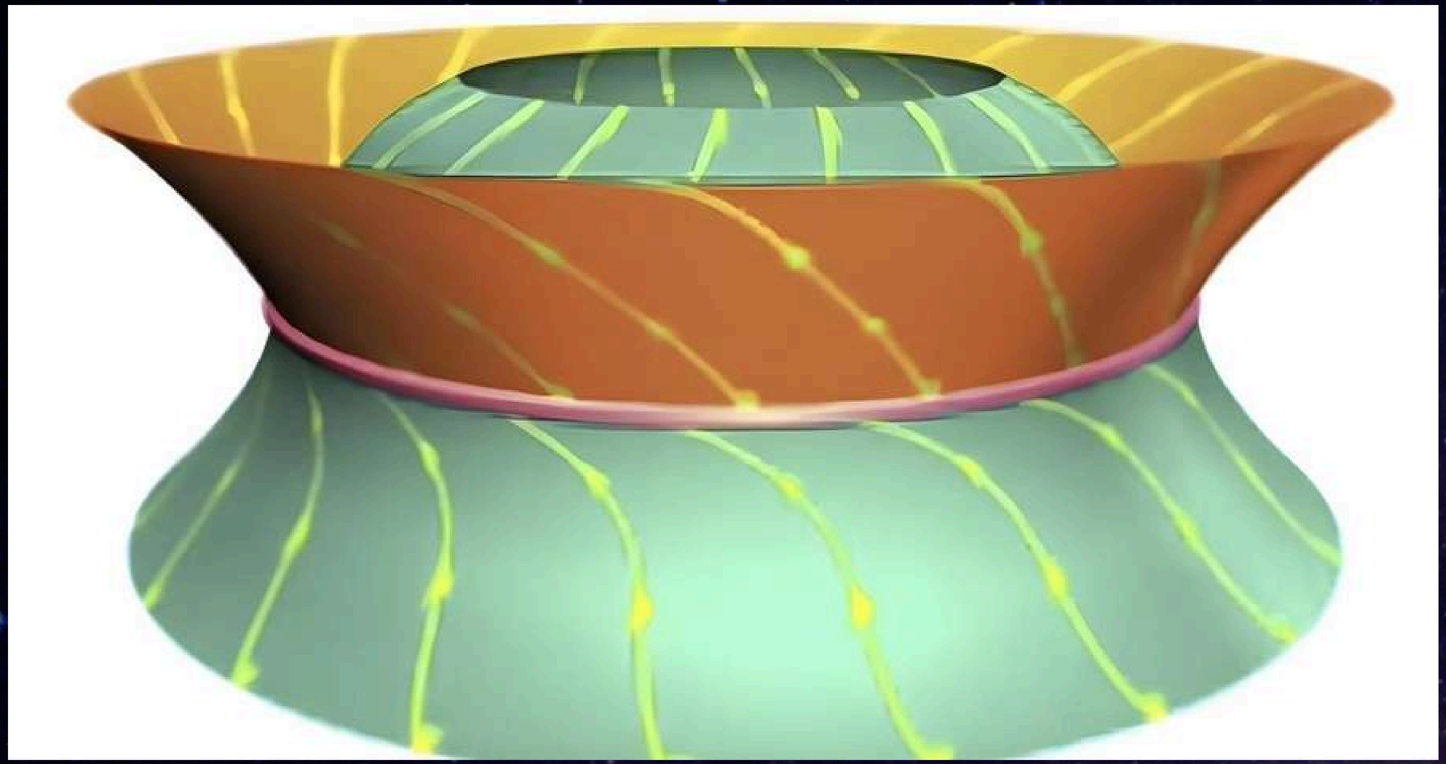
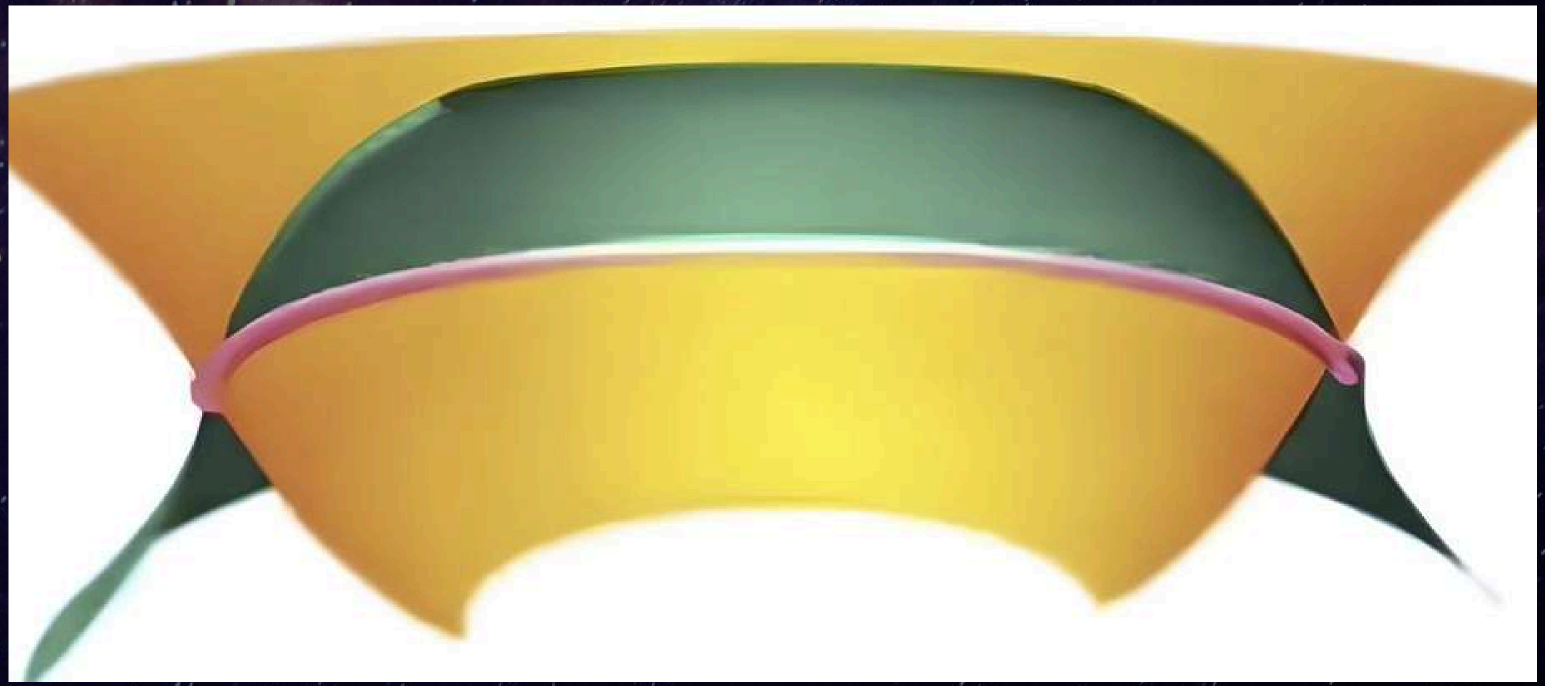
LYAPUNOV ORBIT





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MOVING TO THREE DIMENSIONAL ORBITS

$$\ddot{x} - 2\dot{y} - (1 + 2c_2)x = 0$$
$$\ddot{y} + 2\dot{x} + (c_2 - 1)y = 0$$
$$\ddot{z} + c_2 z = 0$$

$$x = -A_x \cos(\omega_p t + \phi)$$
$$y = \kappa A_x \sin(\omega_p t + \phi)$$
$$z = A_z \sin(\omega_v t + \psi)$$

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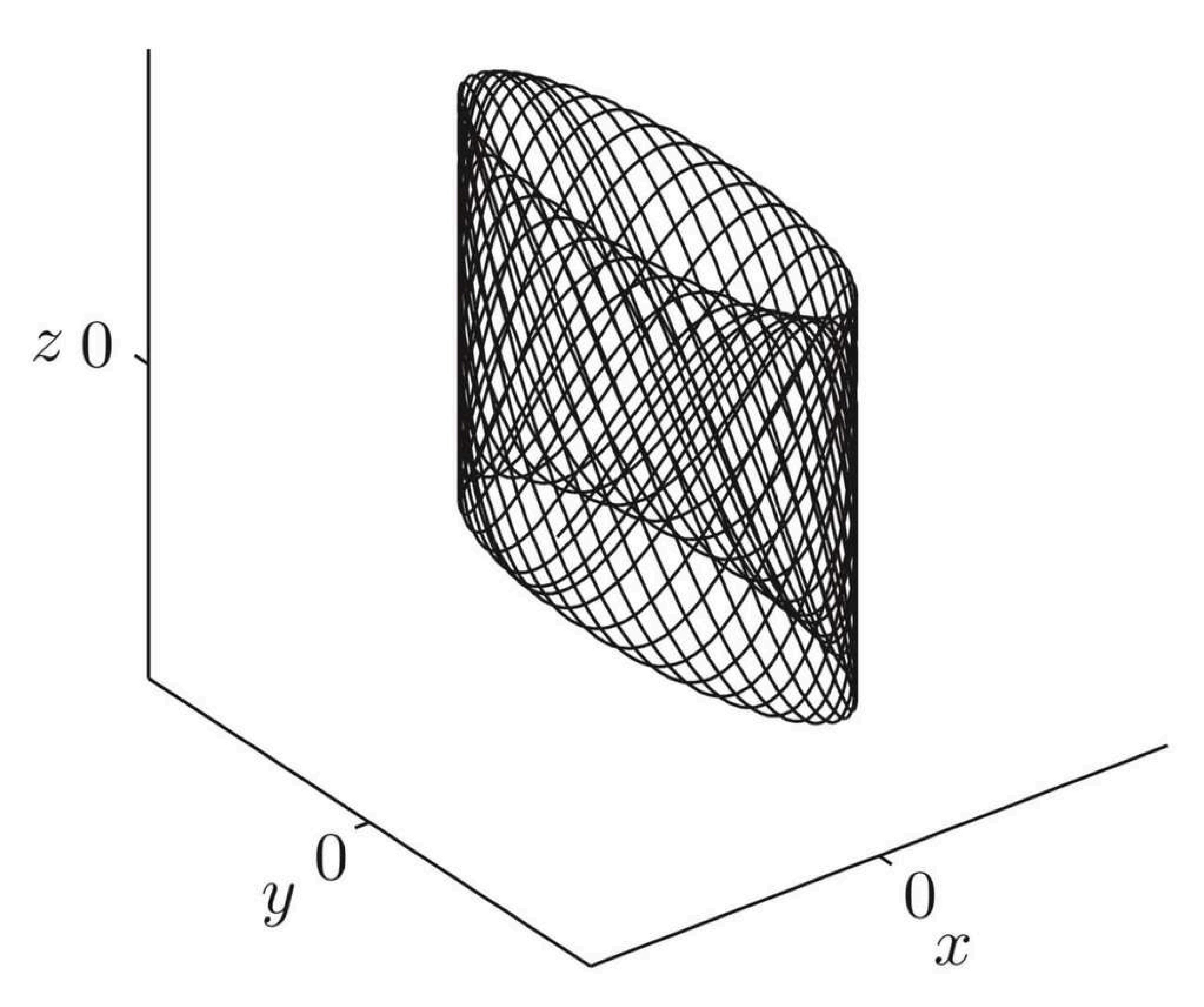
LISSAJOU ORBIT

THESE ARE QUASI-PERIODIC ORBIT



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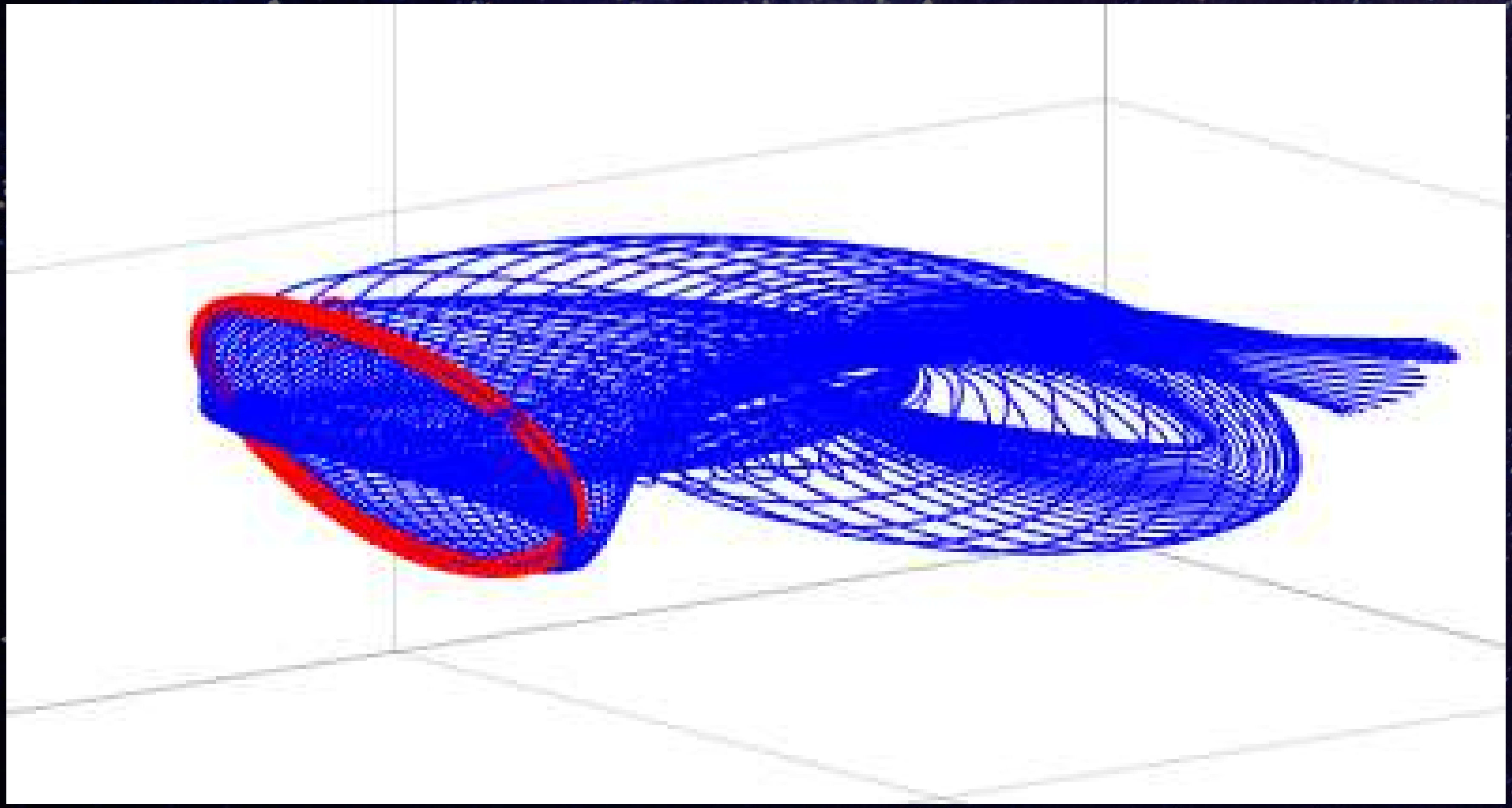
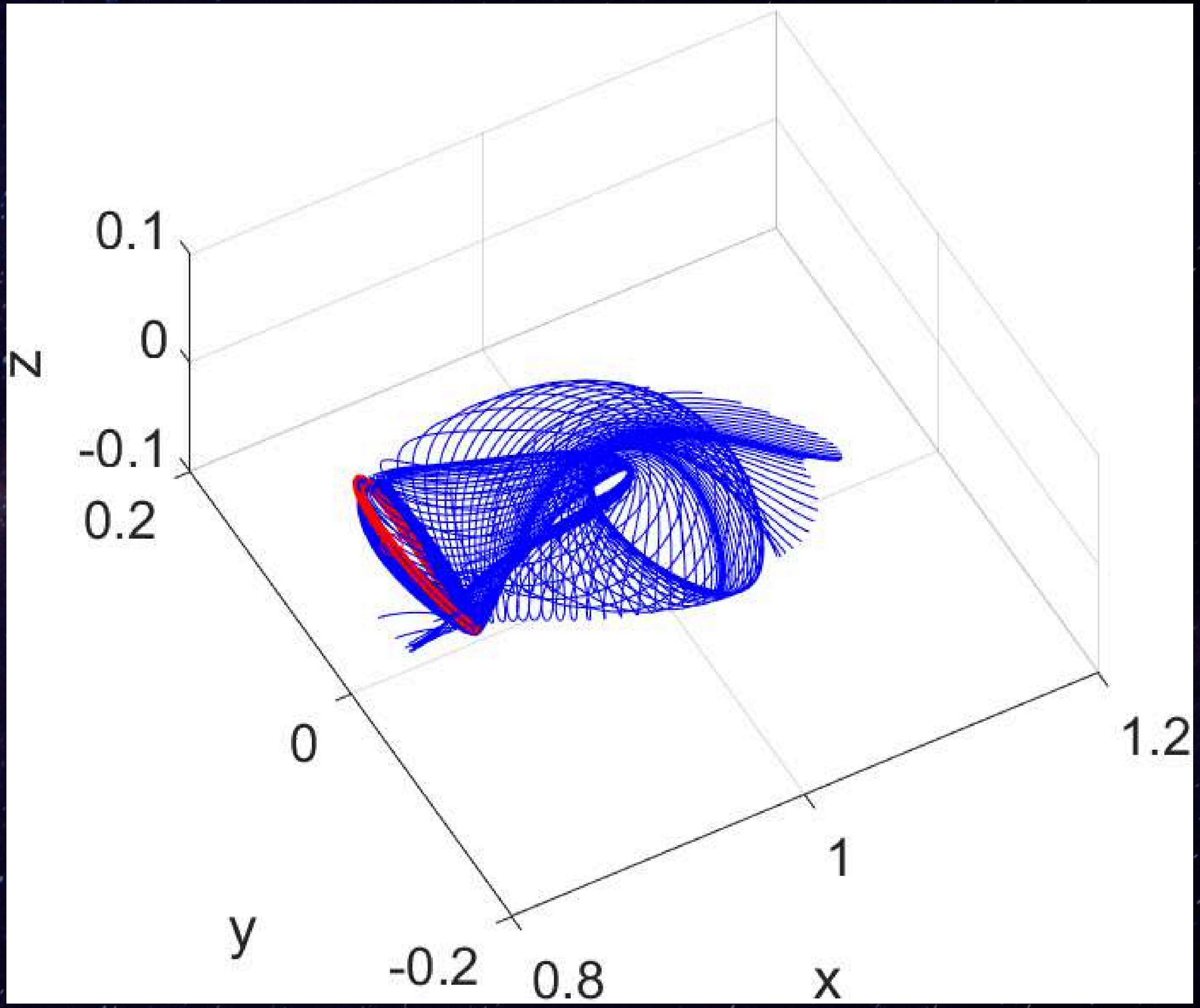
HALO ORIBITS

NOW YOU SHOULD BE ABLE TO VISUALISE THE TUBES





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LETS DESIGN A SPACE MISSION

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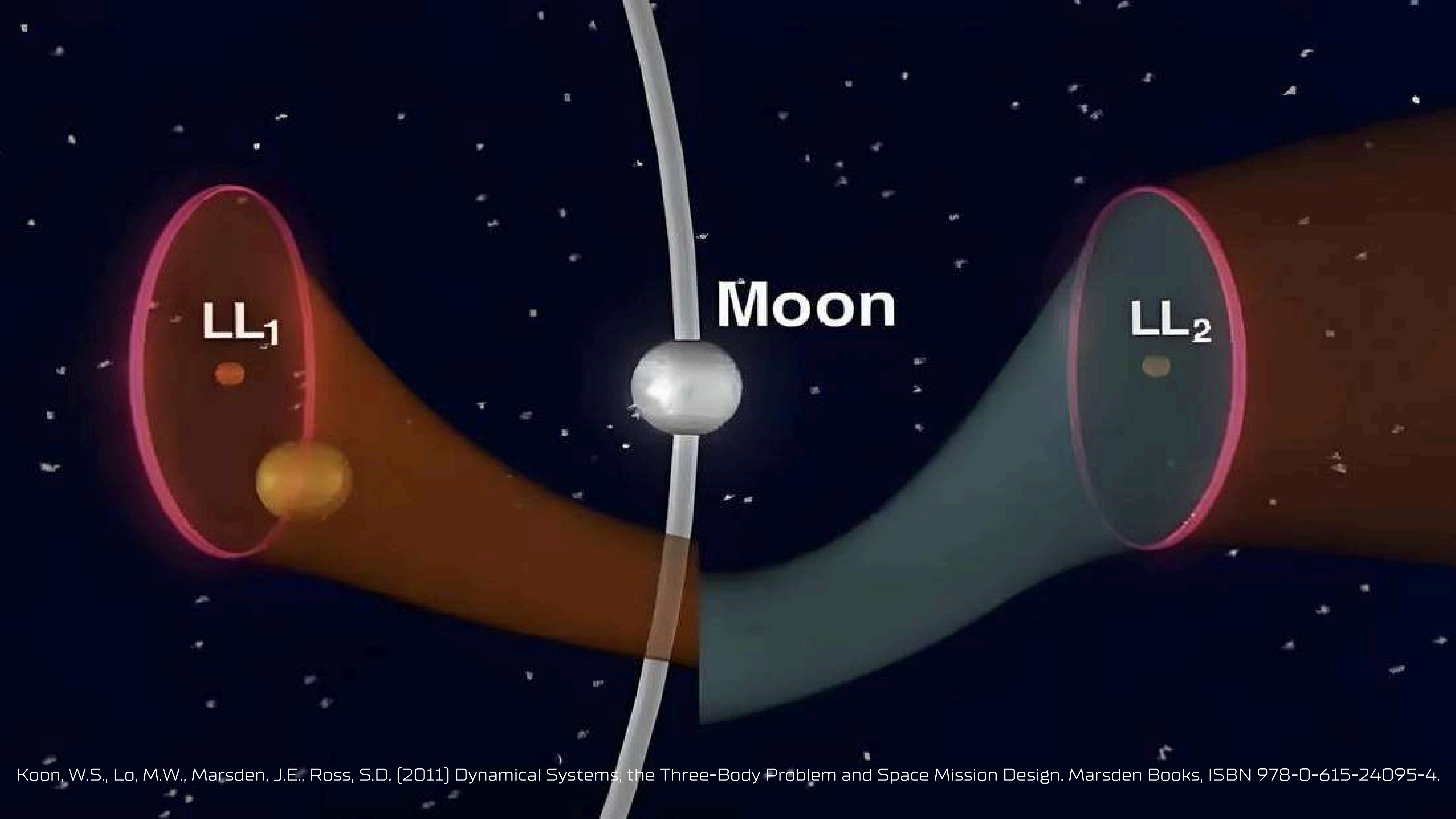

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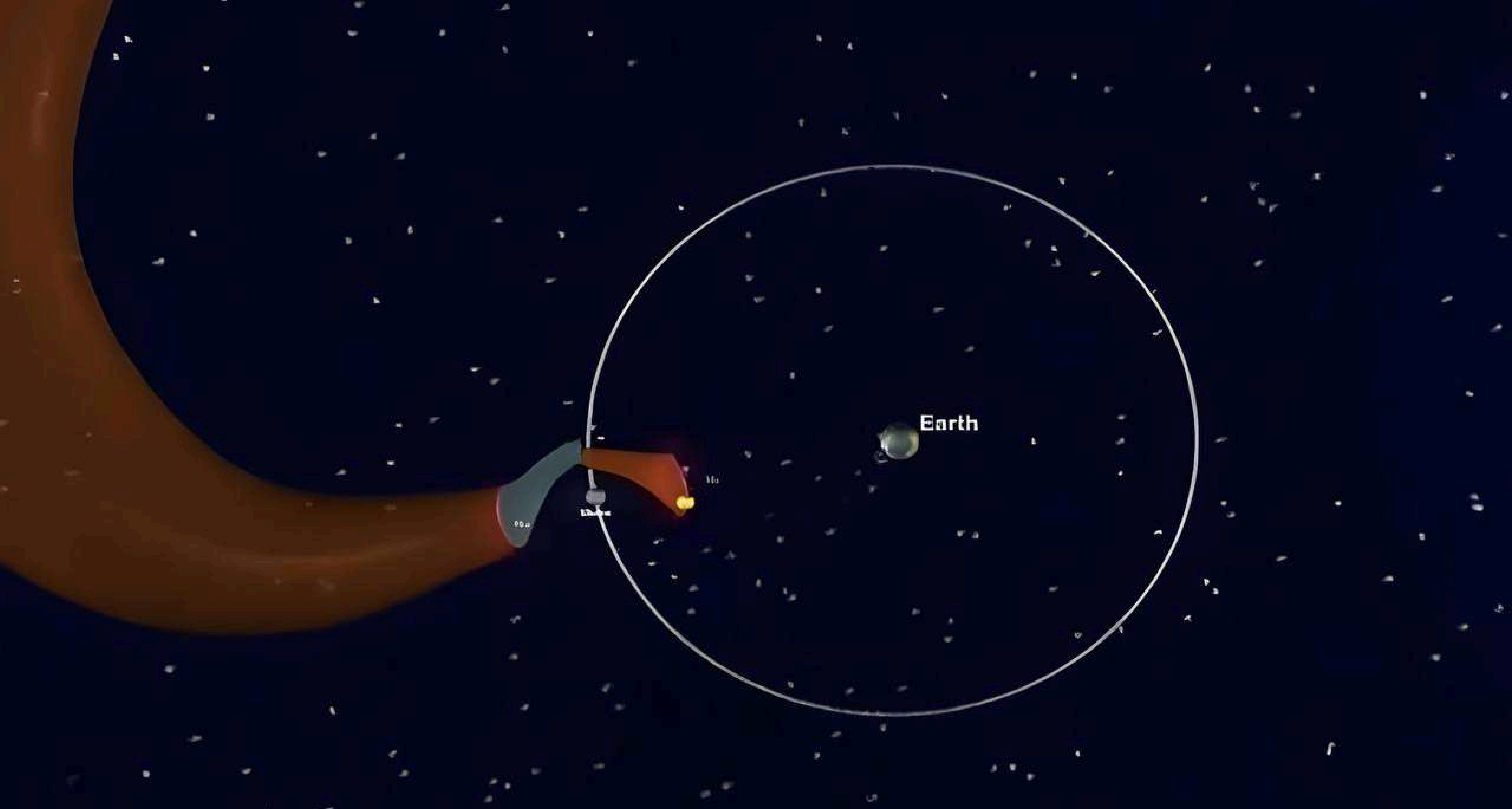

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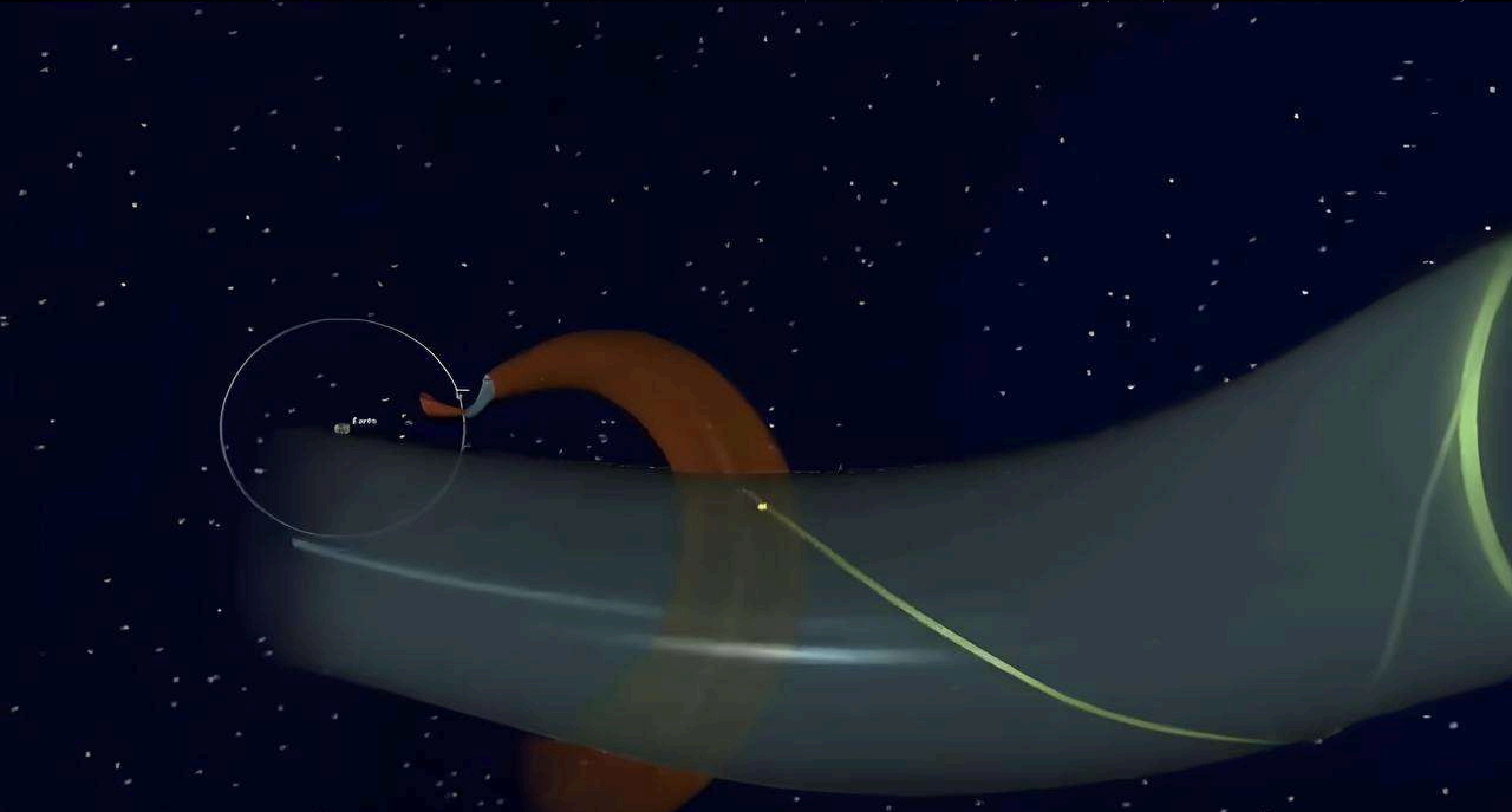

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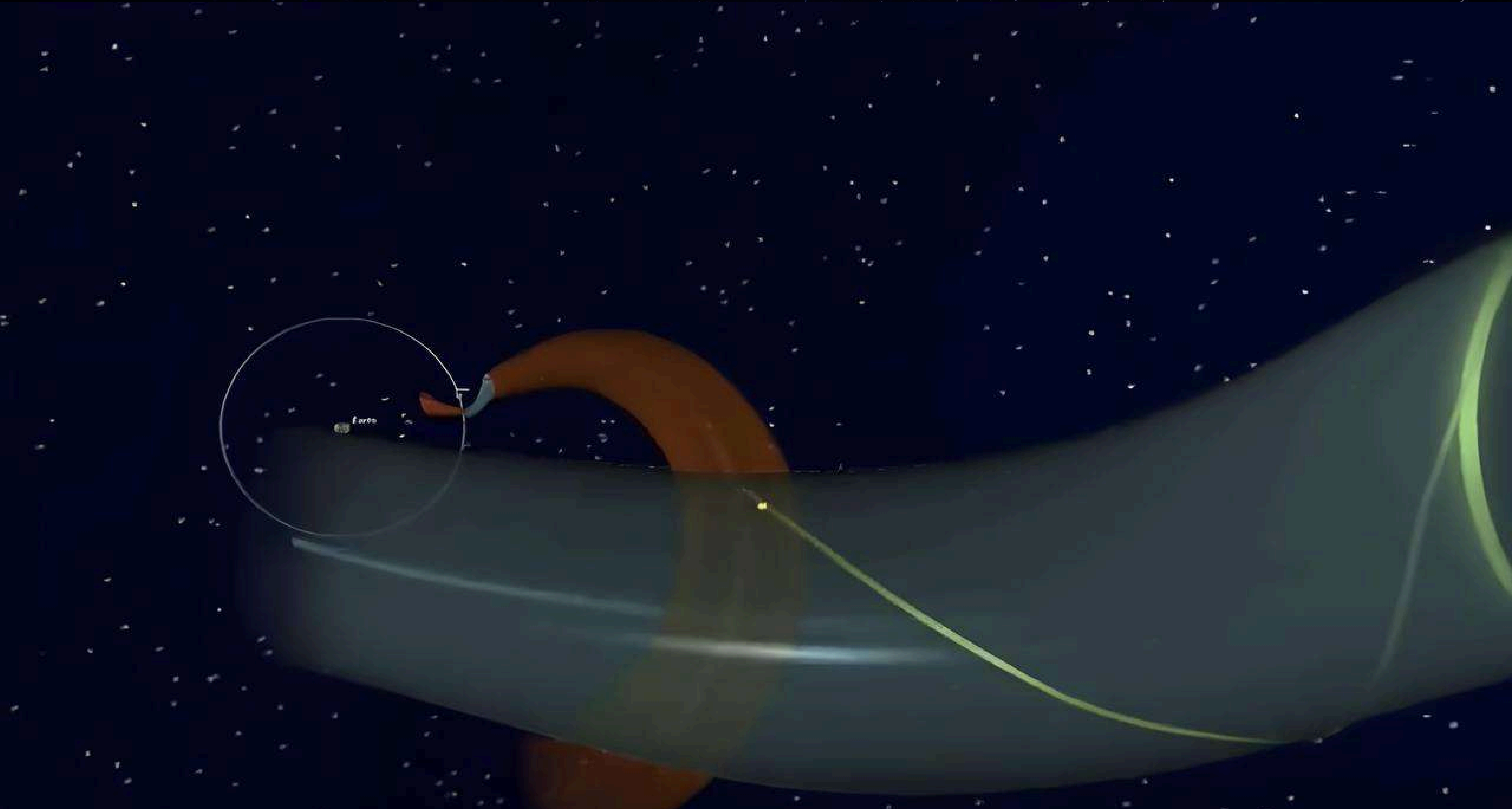

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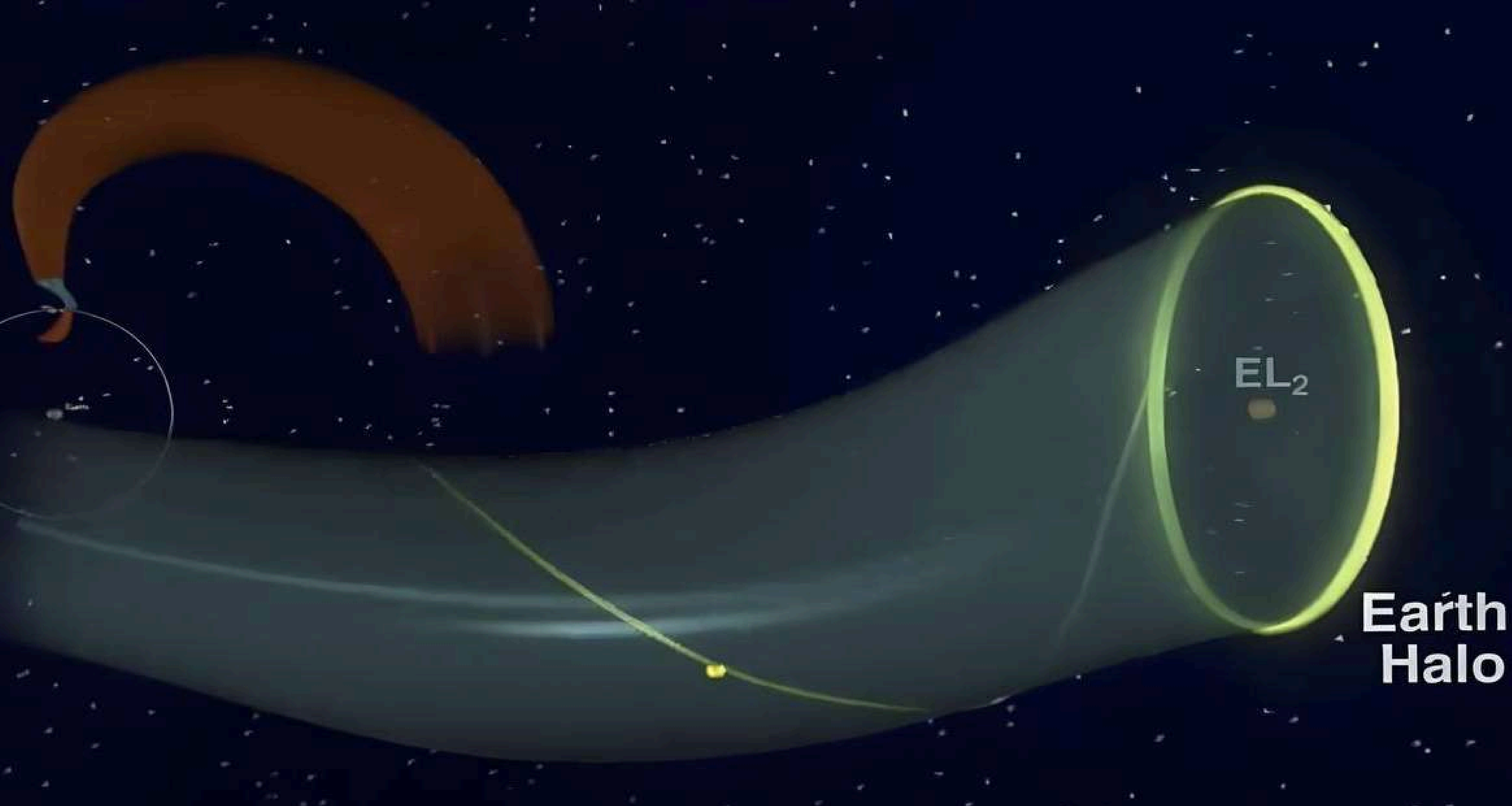

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NOW,
YOU ALL ARE JUNIOR SPACE MISSION DESIGNER

THANK YOU