HOME ASSIGNMENT-4

1. Solve the recurrence relation $a_n = 3a_{n-1} + 10a_{n-2} + 5n$

sol Solving the homogenous part :-

$$(a_n^h) = 3a_{n-1} + 10a_{n-2}$$

$$a_n = r^k \quad \Rightarrow a_n = r^2 \quad (k=2)$$

$$3a_{n-1} = 3.r^{k-1} = 3.r^{2-1} = 3.r$$

$$10a_{n-2} = 10.r^{k-2} = 10.r^{2-2} = 10$$

$$r^2 = 3r + 10$$

$$r^2 - 3r - 10 = 0$$

$$(r-5)(r+2) = 0 \quad \Rightarrow r_1 = 5, \quad r_2 = -2$$

$$\boxed{(a_n)^h = A \cdot 5^n + B \cdot (-2)^n} \quad [F(n) = 5^n]$$

$$a_n^p = An(5^n)$$

Subs. a_n^p to non-homogenous eqn :-

$$An(5^n) = 3A(n-1)(5^{n-1}) + 10A(n-2)(5^{n-2}) + 5^n$$

Divide by 5^{n-2}

$$25A \cdot n = 15A(n-1) + 10A(n-2) + 25$$

$$25A \cdot n = 15An - 15A + 10An - 20A + 25$$

$$\cancel{25An} = \cancel{25An} - 35A + 25$$

$$0 = -35A + 25$$

$$35A = 25 \quad \Rightarrow A = \frac{25}{35} = \frac{5}{7}$$

$$(a_n)^p = \frac{5}{7} n(5^n)$$

G.S $\Rightarrow$ $\boxed{a_n = c_1 \cdot (5)^n + c_2(-2)^n + \frac{5}{7} n(5^n)}$

2. Solve the recurrence relation using generating function technique. $a_n = a_{n-1} + a_{n-2} + 3$ for $n \geq 2$ with conditions $a_0 = 0$ & $a_1 = 1$.

Sol ~~Solve the~~ Let generating function be

$$G(x) = \sum_{n=0}^{\infty} a_n \cdot x^n$$

Multiply with x^n & sum over all valid n

$$\sum_{n=2}^{\infty} a_n \cdot x^n = \sum_{n=2}^{\infty} a_{n-1} \cdot x^n + \sum_{n=2}^{\infty} a_{n-2} \cdot x^n + \sum_{n=2}^{\infty} 3x^n$$

interms of $G(x)$

$$\sum_{n=2}^{\infty} a_n \cdot x^n = G(x) - a_0 - a_1 x$$

$$= G(x) - x$$

$$\sum_{n=2}^{\infty} a_{n-1} \cdot x^n = x \sum_{n=2}^{\infty} a_{n-1} \cdot x^{n-1}$$

$$= x \sum_{k=1}^{\infty} a_k \cdot x^k = x\left(G(x) - a_0\right) = x(G(x))$$

$$\sum_{n=2}^{\infty} a_{n-2} \cdot x^n = x^2 \sum_{n=2}^{\infty} a_{n-2} \cdot x^{n-2}$$

$$= x^2 \sum_{k=0}^{\infty} a_k \cdot x^k = x^2 \left(G(x)\right)$$

$$\sum_{n=2}^{\infty} 3x^n = 3 \sum_{n=2}^{\infty} x^n = 3\left(\frac{1}{1-x} - 1 - x\right) = 3\left(\frac{x^2}{1-x}\right)$$

to find $G(x)$:-

$$G(x) - x = x(G(x)) + x^2 \cdot G(x) + \frac{3x^2}{1-x}$$

$$G(x) - x \cdot G(x) - x^2 \cdot G(x) = x + \frac{3x^2}{1-x}$$

$$G(x)\cdot(1-x-x^2) = x + \frac{3x^2}{1-x} = \frac{x(1-x)+3x^2}{1-x} = \frac{x-x^2+3x^2}{1-x}$$

$$= \frac{x+2x^2}{1-x}$$

$$G(x) = \frac{x+2x^2}{(1-x)(1-x-x^2)}$$

writing into partial fractions.

roots of $1-x-x^2=0$ are

$$x = \frac{-1\pm\sqrt{5}}{2} = \frac{1\pm\sqrt{5}}{2}$$

let $\phi = \frac{1+\sqrt{5}}{2}$ & $\varphi = \frac{1-\sqrt{5}}{2}$ then

$$1-x-x^2 = (1-\phi x)(1-\varphi x)$$

$$G(x) = \frac{A}{1-x} + \frac{B}{1-\phi x} + \frac{C}{1-\varphi x}$$

$$A = -3B \quad , \quad B = \frac{3\phi+2}{\sqrt{5}} \quad , \quad C = \frac{3\varphi+2}{-\sqrt{5}}$$

$$B = \frac{3\left(\frac{1+\sqrt{5}}{2}\right)+2}{\sqrt{5}} = \frac{\frac{3+3\sqrt{5}+4}{2}}{\sqrt{5}} = \frac{7+3\sqrt{5}}{2\sqrt{5}} = \frac{7\sqrt{5}+15}{10}$$

$$C = \frac{3\left(\frac{1-\sqrt{5}}{2}\right)+2}{-\sqrt{5}} = \frac{\frac{3-3\sqrt{5}+4}{2}}{-\sqrt{5}} = \frac{7-3\sqrt{5}}{-2\sqrt{5}} = \frac{-7\sqrt{5}+15}{10}$$

$$\frac{1}{1-kx} = \sum_{n=0}^{\infty} k^n \cdot x^n$$

$$a_n = -3(1)^n + B(\phi)^n + C(\varphi)^n$$

$$a_n = -3 + \frac{7\sqrt{5}+15}{10}\phi^n + \frac{-7\sqrt{5}+15}{10}\varphi^n$$

$$a_n = -3 + \frac{7\sqrt{5}+15}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{-7\sqrt{5}+15}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

3. If A bacteria population doubles every hour. If the intial count is 150. find a recurrence relation & express the number of bactelia after n hours?

Sol: let B_n = bactelia after n hours

$B_0 = 150$

no. of bacteria at hour "n" = twice no. at $n \geq 1$

$B_n = 2B_{n-1}$ for $n \geq 1$

$\boxed{B_n = 150 \cdot 2^n}$ $[B_n = B_0 \cdot 2^n]$

∴ The no. of bactelia after n hours ⇒ $\boxed{B_n = 150 \cdot 2^n}$

4. If a graph has 10 vertices, each of degree 3, find number of edges?

Sol:

$\sum deg(V) = 2e$

$V = 10$, each vertex of deg = 3

$\sum deg(V) = 10 \times 3$

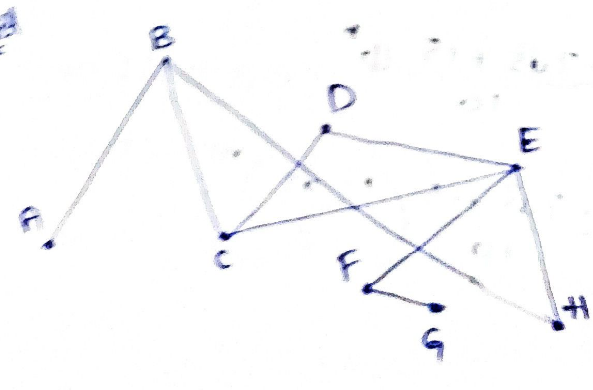
$deg(v) = 30$

$30 = 2e$

$e = 15$

5. Determine degrees of all vertices from graph?

Sol:



$deg(A) = 3$ $deg(B) = 2$

$deg(C) = 2$ $deg(D) = 3$

$deg(E) = 3$ $deg(F) = 2$

$deg(G) = 2$ $deg(H) = 3$

every hour. If
rrence relation
n hours?

no. at n = 1

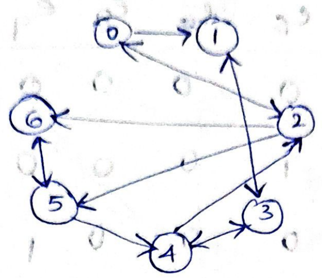
$B_n = 150 \cdot 2^n$

ee 3, find

=3

ph?
g(B) = 2
(D) = 3
g(F) = 2
g(H) = 3.

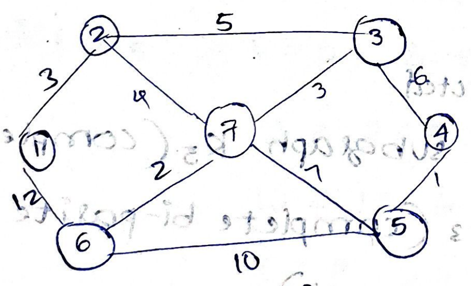
6. Tabulate in-degrees & out-degrees for following graph:-



Sol:

Vertex	In-degree	Out-degree
0	1	0
1	1	0
2	0	4
3	1	1
4	1	1
5	1	1
6	1	1

7. Write the adjacency & incidency matrix for graph:-



11 edges total;

$(e_1, e_2, \ldots e_{11}) = \{(2,1)(1,6)(6,5)(3,5)(3,2)(2,7)(7,6)$
$(7,5)(7,3)(3,4)(4,5)\}$

Adjacency matrix:- $A_{ij} = 1$ if vertices i & j are connected.

	1	2	3	4	5	6	7
1	0	1	0	0	0	1	0
2	1	0	1	0	0	0	1
3	0	1	0	1	1	0	1
4	0	0	1	0	1	0	0
5	0	0	1	1	0	1	1
6	1	0	0	0	1	0	1
7	0	1	1	0	1	1	0

Incidency matrix:

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
1	1	1	0	0	0	0	0	0	0	0	0
2	1	0	0	0	1	1	0	0	0	0	0
3	0	0	0	1	1	0	0	0	1	1	0
4	0	0	0	0	0	0	0	0	0	1	1
5	0	0	1	1	0	0	0	1	0	0	1
6	0	1	1	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1	0	0

8. Determine whether the follow. graph is planar or not.

Acc^n to Kuratowsaki's theorm,

a graph has 6 vertices i.e

K_6 graph (every vertex connected to each other vertex) with subgraph K_5 (complete graph on 5 vertices) or $K_{3,3}$ (complete bi-partite graph with 3 vertices in each set.)

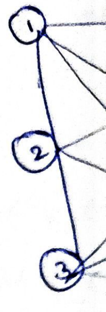
Acc^n to kuratowsaki's theorm:-

if a graph contains subgraph K_5 or $K_{3,3}$ then it is said to non-planar

As K_5 is non-planar

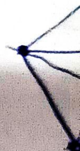
9. Draw K

Sol:



10. Give an planar

Sol:



from

for

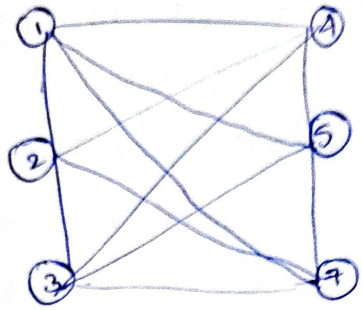
11. Dete Han

Sol

(B)

9. Draw $K_{3,4}$ verify it's planar or not?

Sol:

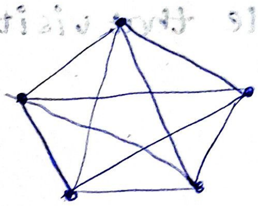


Since the graph $K_{3,4}$, we have $m=3$ & $n=4$. Because both have $m>2$ & $n>2$ the condn for planarity not satisfied.

$K(3,4)$ (although $K_{3,3}$ is a sub-graph) by Kuratowski's theorm it's not planar.

10. Give an example of K_5 graph & determine it is planar or not?

Sol:



K_5 graph is non-planar because it is impossible to draw it on-plane without edges crossing.

from Euler's formula;- $(e \leq 3v-6)$

for K_5 :- $v=5$ & $e=10$,

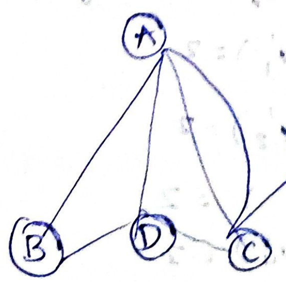
$10 \leq 3(5)-6$

$10 \leq 15-6$

$10 \leq 9$ which is false.

11. Determine whether following graph is euler graph & Hamilton graph. If so give path?

Sol



Euler graph:-

vertex A: deg 4 (even)

vertex B: deg 2 (even)

vertex C: deg 4 (even)

vertex D: deg 2 (even)

vertex E: deg 2 (even)

Since all vertices have even degree the graph is an Euler graph

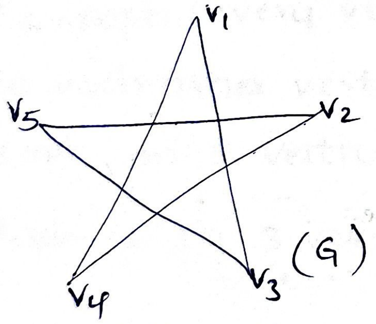
Hamilton graph:-

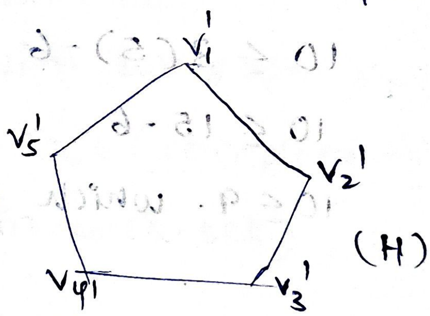
a hamilton graph contains a cycle that visits every vertex once

Hamilton cycle: $A \to B \to D \to C \to E \to A$

∴ has both euler & hamilton graph.

12. Identify given graph's are isomorphic or not?





i) no. of vertices of $G = H = 5$

ii) no. of edges of $G = H = 5$

iii) degree sequence of G:-

deg of $(V_1) = 2$

deg of $(V_2) = 2$

" $(V_3) = 2$

" $(V_4) = 2$

" $(V_5) = 2$

degree sequence of H:-

deg. of $(V_1') = 2$

deg of $(V_2') = 2$

" $(V_3') = 2$

" $(V_4') = 2$

" $(V_5') = 2$

degree sequence of G & H = 2, 2, 2, 2, 2

iv) One-one mapping:-

deg. of $V_1 = V_1'$ vertices with deg "2" are V_1, V_2, V_3, V_4, V_5

all are satisfying so choose order correctly

deg of $V_2 = V_4'$

deg of $V_3 = V_2'$

deg of $V_4 = V_5'$

deg of $V_5 = V_3'$

(or)

deg of $(V_1) = V_1'$

deg of $(V_2) = V_3'$

deg of $(V_3) = V_5'$

deg of $(V_4) = V_2'$

deg of $(V_5) = V_4'$

iv) adjacency matrix of G:-

	V_1	V_2	V_3	V_4	V_5
V_1	0	0	1	1	0
V_2	0	0	0	1	1
V_3	1	0	0	0	1
V_4	1	1	0	0	0
V_5	0	1	1	0	0

*Matrix for H:-

	V_1'	V_4'	V_2'	V_5'	V_3'
V_1'	0	0	1	1	0
V_4'	0	0	0	1	
V_2'	1	0	0	0	1
V_5'	1	1	0	0	0
V_3'	0	1	1	0	0

∴ the adjacency & the incidence matrix's are same the given graphs are isomorphic. //